Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communications
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Lecture – 30
Introduction to Orthogonal Frequency Division Multiplexing (OFDM) – Cyclic Prefix (CP) and Circular Convolution

Hello welcome to another module in this massive open online course. In the previous modules, we have looked at equalization and equalization to overcome the Inter Symbol Interference of a wireless channel and today we are going to look at a very different technology, a very modern technology. In fact, to overcome the same problem that is Inter Symbol Interference in a wireless channel very efficiently and this technology is OFDM or Orthogonal Frequency Division Multiplexing. So, in today's module we will start looking at OFDM.

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So, we are going to start looking at estimations specifically for OFDM that is where, that is our aim starting this module, that is estimation for OFDM where OFDM is an abbreviation it stands for Orthogonal Frequency Orthogonal where OFDM stands for Orthogonal Frequency Division Multiplexing and in fact OFDM is a very modern technology and OFDM is a very important technology. As you might all be familiar

OFDM is used in the 4G wireless cellular standards such as LTE Wimax and it is also used in the modern wireless Wi-Fi standards such as 802.11 n 802.11 ac and so on.

So, OFDM this is a modern, very modern wireless technology or very 1 can say it is the latest wireless technology and this is used in 4G standards such as the 4th-generation cellular standard such as for example, it is used in your LTE which stands for Long Term Evolution (Refer Time: 2.37) it is used in the Wimax standard and is also used in some other it is also used in other standards such as Wi-Fi.

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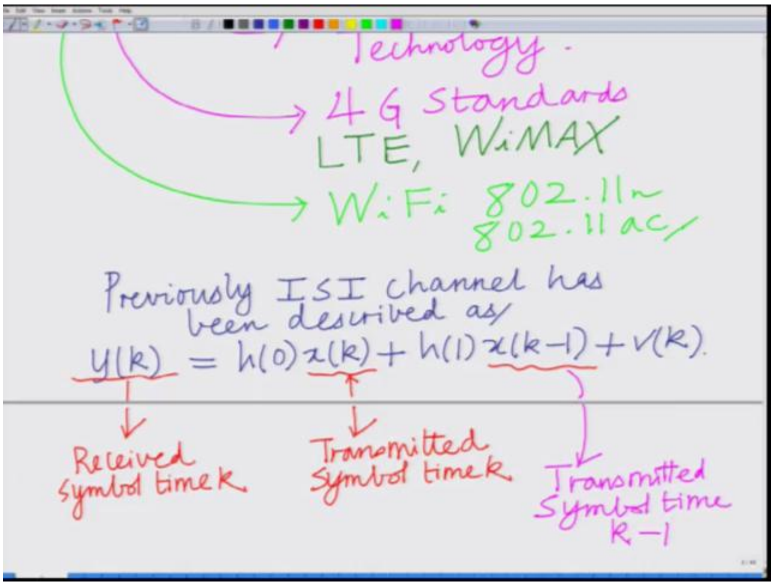

For instance, 802.11 n 802.11 ac and. So, so OFDM is a very powerful and a very important technology because its wide applicability in the modern 4G wireless standards as well as the Wi-Fi standards.

So, (Refer Time: 3.05) to understand estimation for OFDM we have to first understand what OFDM is about or what Orthogonal Frequency Division Multiplexing is about. So, to understand that let us again start with our ISI or Inter Symbol Interference Limited channel and as we have already seen many times before we have described it previously, the ISI channel has been described, it has been described as your y k that is received symbol at time k equals h 0 X k plus h 1 X k minus 1 plus v k that is y k.

This is the received symbol at time k, X k is transmitted symbol at k, this is the transmitted X k is the transmitted symbol at time k and X k minus 1 is the previous symbol, this is the transmitted symbol, this is the transmitted symbol at time k minus 1 therefore what we have is that the transmitted symbol at time k minus 1 which is X k minus 1 is interfering with X k which is the transmitted symbol at time k and this is what we term as Inter Symbol Interference alright. So, what we said is basically Inter Symbol Interference is caused by the fact that X k minus 1 which is the transmitted symbol at time k minus 1 is interfering with X k which is the present symbol. So, the past symbol is interfering with the present symbol this is Inter Symbol Interference.

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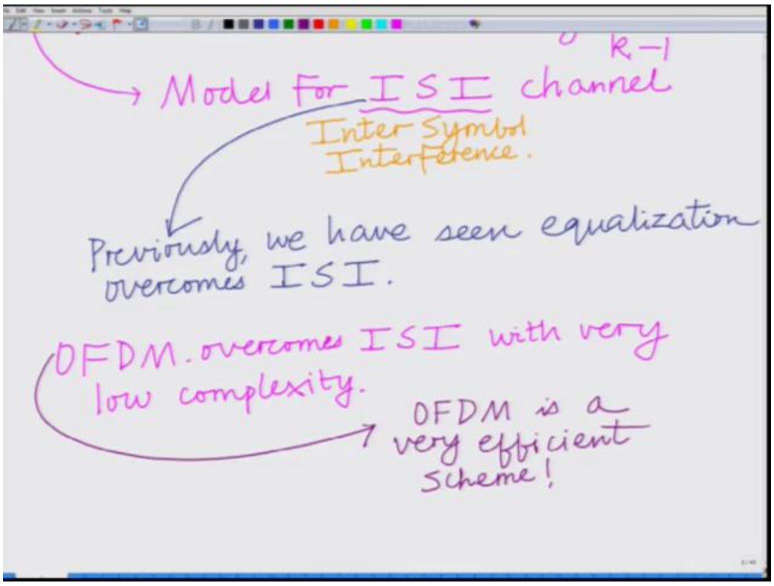

So, this is the model for your inter symbol, this is the model for you ISI channel where ISI stands for and this is also something that we have seen before where ISI stands for inter symbol where ISI stands for Inter Symbol Interference. And previously we have seen that OFDM is a techno previously we have seen that equalization is a technology to overcome Inter Symbol Interference. And now what we are saying is that OFDM is another technology which can overcome this Inter Symbol Interference and very efficiently right. So, we are going to see that.

So previously, previously we have seen equalization overcomes ISI and OFDM is another technology to overcome ISI even more efficiently, more efficiently meaning lower computational complexities so that is easier to implement. OFDM overcomes ISI OFDM has many advantages, one is that it overcomes ISI with very low, with very low complexity that is the advantage of OFDM or Orthogonal Frequency Division Multiplex in that it is a very low complexity or a very efficient scheme to overcome the Inter Symbol Interference in the wireless channel.

So, this is a very low complexity OFDM is a very efficient scheme. OFDM is a very efficient scheme and that is what has in made OFDM such a cutting edge wireless technology for employment in the various 4G wireless standards as LTE Wimax and also the Wi-Fi standards that we have talked about. And now what we are going to do we are slow we are going to first I am going to describe the model, the system model that is how described how this OFDM based wireless communication works and then we are going to look at how to perform estimation in the context of an OFDM system.

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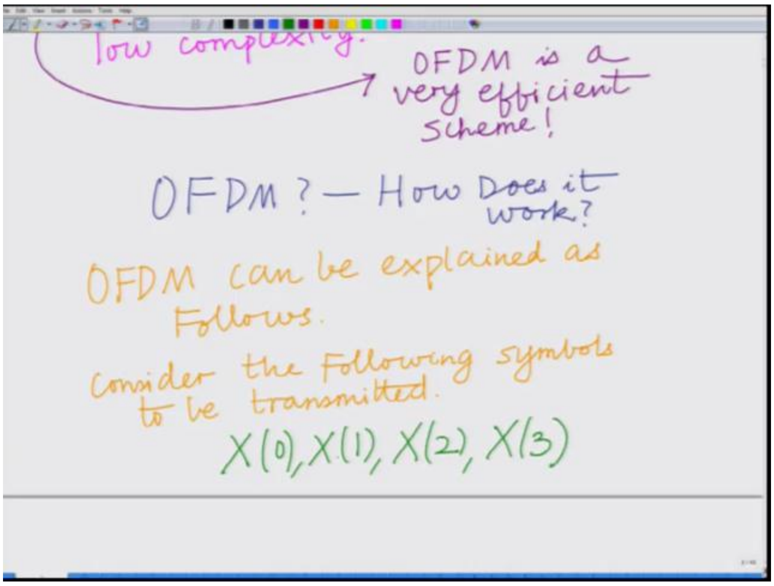

So, first we are going to look at what is this OFDM or what is an OFDM that is how does how does it work, that is what is the transmission scheme for OFDM and that can be explained as follows. So, OFDM can be explained as follows. So, consider the following symbols. So, consider the following symbols to be transmitted in the OFDM symbol, let us say we have 4 symbols which we are going to denote by X 0 X 1 X 2 X 3. So, these are the 4 symbols which are to be transmitted.

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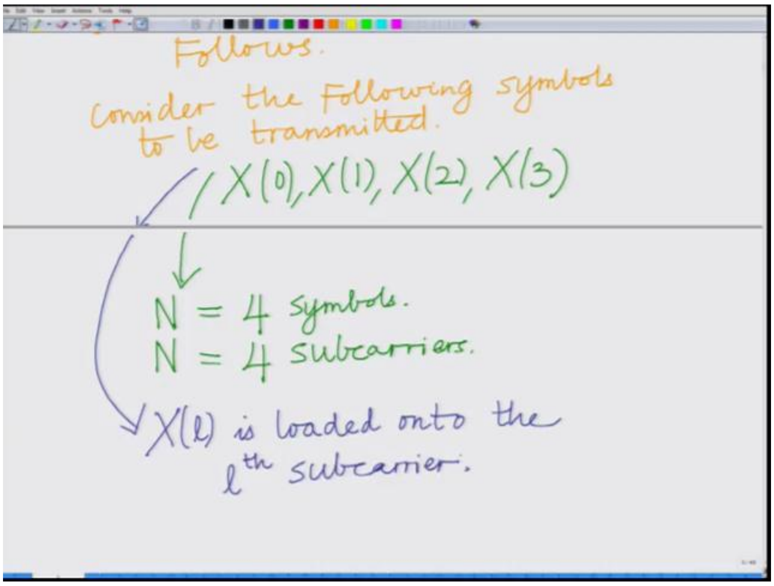

So, we have capital N equal to 4 symbols and this is also termed as the number of sub carriers. Capital N is equal to 4, capital N equal to 4 subcarriers. So, we are looking at. So, this capital N is an important parameter in this OFDM system we are saying we are going to use capital N equal to 4 symbols, these are denoted by X 0 capital X 0 capital X 1 capital X 2 capital X 3. It is important to note that we are representing them by capital X because we will use small X to represent something else. So, we have n equal to 4 capitals N equal to 4 symbols correct and also this is known as capital N equal to 4 sub carriers, we say that these symbols are loaded on to the capital n subcarriers. That is the capital N symbols that is the n symbols are loaded into the onto the n subcarriers that is capital X 0 is loaded on to subcarrier 0 capital X 1 is loaded on to sub carrier 1 and so on capital X 4 is loaded on to sub carrier 4.

So, this let us clarify this also capital X l is loaded, the terminology used is loaded onto the lth this is loaded onto the lth subcarrier 4 symbols they are loaded onto the 4 subcarriers alright. So, in a general scenario you have capital N symbols and they are loaded onto capital N subcarriers. So, now, what does it mean to say they are loaded onto the subcarriers and this is what it means to say they are loaded onto the subcarriers?

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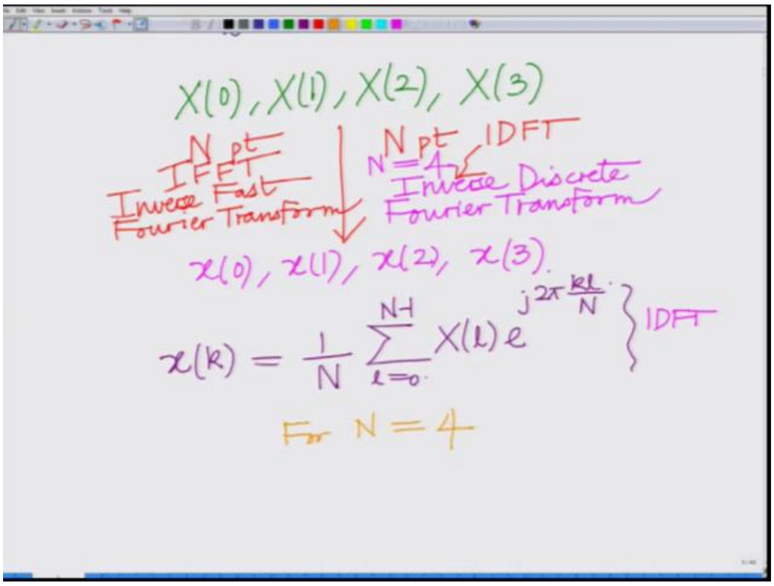

So, we have the symbols X 0 X 1 X 2 X 3. Now to these symbols what we do is basically what we are going to do with this symbol these symbols are we are going to perform the N point IDFT that is n point IDFT which is IDFT stands for as you all have should be familiar with, this stands for the Inverse Discrete Fourier. IDFT stands for the Inverse Discrete Fourier Transform. To get the IDFT samples X 0 X 1 X 2 these are the small X's small X 0s small X 1 small X 2 small X 3. So, N equal to N point IDFT or in this case basically you have N equal to 4 point IDFT. So, you take the symbols which are the capital X 0 capital X 1 capital X 2 capital X 3 yeah alright, and you load them onto the subcarrier which means basically you formed we take the N point IDFT that is the Inverse Discrete Fourier Transform. This can also be performed very efficiently, very fast in a very fast manner using the Inverse Fast Fourier Transform.

So, typically you will also see basically it is the N point IDFT which is the same as the N point IFFT. IFFT is simply a fast algorithm to perform the discrete Fourier transform inverse fast this performs the Inverse Fast Fourier Transform. Therefore, now what you have is you have capital X that is the X of k is the sample is basically corresponds the Inverse Fast Fourier Transform, as you know this is given as 1 over N summation l equal to 0 to N minus 1 capital X l e to the power of j, 2 phi kl divided by N; this is the expression for the IDFT all of you should be very familiar with, which is basically now setting N equal to 4, for N equal to 4.

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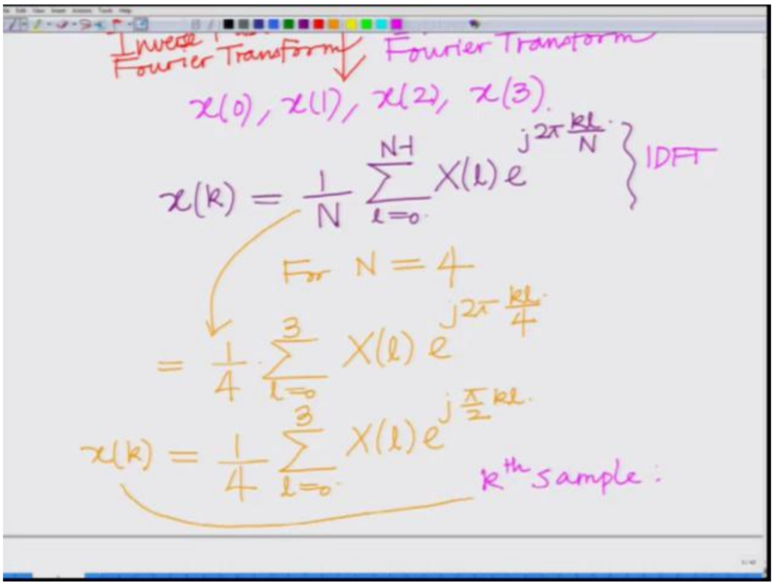

This is basically given as X k equals 1 over 4 summation l equal to 0 to N minus 1 which is equal to 3 X to the ray X l e raised to j 2 phi kl divided by N which is 4 which is equal to 1 over 4 for this specific example. Please note that this is only for this specific example corresponding to n equal to 4 sub carrier's l equal to 0 to 3 X l e raised to j phi by 2 kl alright. So, this is the expression for the remember X k is the kth sample. So, small xk denotes the sample capital X l denotes the symbol loaded on the lth subcarrier small X k is the kth sample which is generated by the kth IIFT point of the capital X capital X 0 capital X 1 up to capital X N minus 1.

And now these samples are transmitted over the channels subsequently, but before transmission over the channel there is another important operation to be performed and that is as follows.

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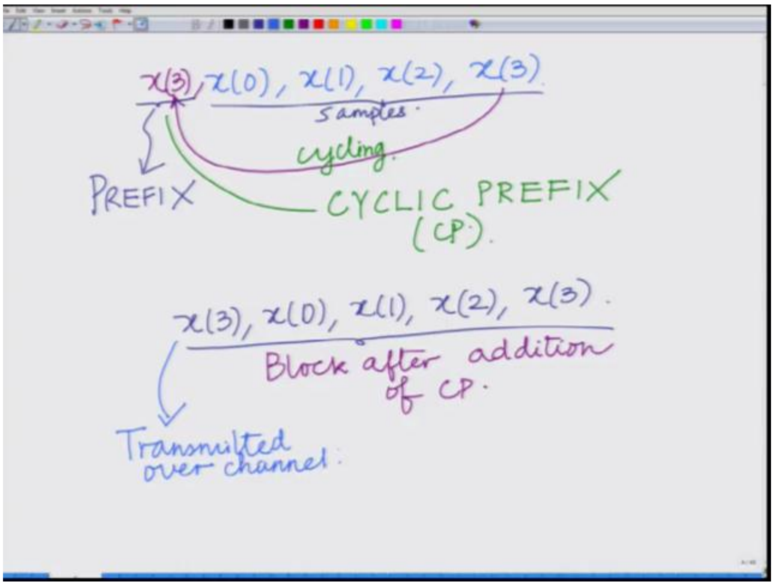

So, we have the samples x 0 x 1 x 2 x 3 and what we are going to do now is basically we are going to take the last sample x 3 and again place it before x 0. So, we are doing a prefix. So, what are we essentially doing is we are prefixing the samples. So, these are your samples and what is this this is your, this is a very important thing we are doing a prefix and we are cycling the samples from the end towards the beginning. So, this is termed as a cyclic and this is a very important concept in OFDM this is termed as the cyclic prefix simply denoted by the term CP.

So, what we are doing is basically we have the samples small x 0, small x 1, small X 2, small X 3. Now we are prefixing this by taking X 3 from the back that is we are copying X 3 from the tail of the block and repeating it again at the head that is we are prefixing the block with X 3 alright. So, it is a prefix and since we are cycling the sample from the end of the block towards the beginning this is known as the cyclic prefix. So, we are simply copying some samples from the tail of the block and prefixing them at the head of the block, this mechanism or this step in OFDM transmission is known as a cyclic prefix, the addition of the cyclic prefix.

So, first you take the symbols which are loaded onto the subcarriers, perform the IFFT after the IFFT you add the cyclic prefix, following the cyclic prefix the samples are transmitted over the channel, now, after addition of the cyclic prefix. So, we have the samples x 0 x 1 x 2. So, this is your block after addition, this is your block after addition of the cyclic prefix and this is then transmitted over the, transmitted up to the addition of the cyclic prefix. These is transmitted over the channel and remember our channel is basically the Inter Symbol Interference channel where y k equals h 0 times x k plus h 1 times X k minus 1 plus v k, this is our channel.

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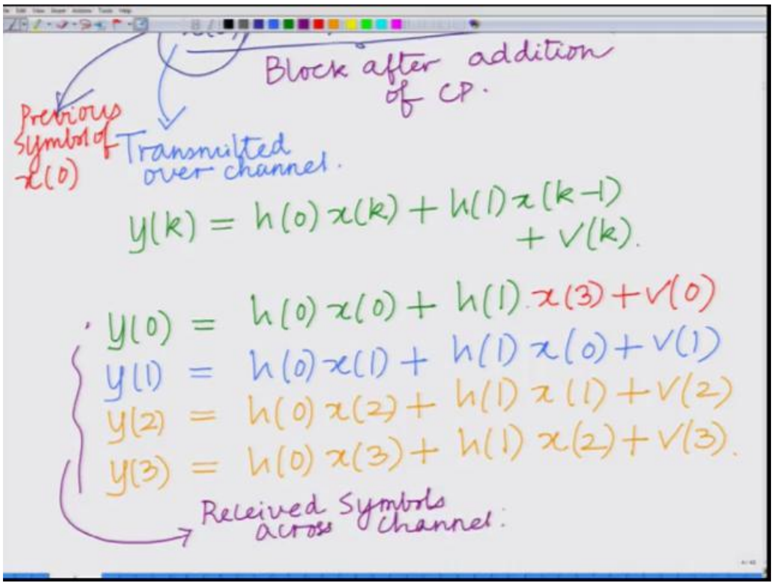

Now, if you look the symbol y 0, we have y 0 equals x h 0 x 0 plus h 1 times the previous symbol, but the previous symbol you can see is x 3 because of the addition of the cyclic prefix x 3 becomes previous symbol of x 0. So, this is the previous symbol of x 0. So, this is h 1 into x 0 plus v 0 that is, I am sorry h 1 into x3 plus v 0 because x 3 because of the addition of the cyclic prefix where x 3 has been moved that is copied from the tail of the block and prefixed in the head of the block because of the cyclic nature of the prefix x 3 also is the previous symbol to x 0. Therefore, you have y 0 equals h 0 x 0 plus h 1 times the previous symbol which is x 3 plus v 0.

And of course, now for the rest of the symbols we can write it as usual, that is y 1 equals h 0 x 1 plus h 1 x 0 which is the previous symbol of x 1 plus v 1, y 2 equals h 0 x 2 plus h 1 x 1 plus v 2 y 3 equals h 0 x 3 plus h 1 x 2 which is the previous symbol to x 3 plus v 3 and this now gives us, this now basically gives us the received symbols across the. So, y 0 these are the received symbols these are the received symbols across the, these are the received symbol symbols across the channel alright. So, we have the smaller y 0 small y 1 small y 2 small y 3 which are the received symbols.

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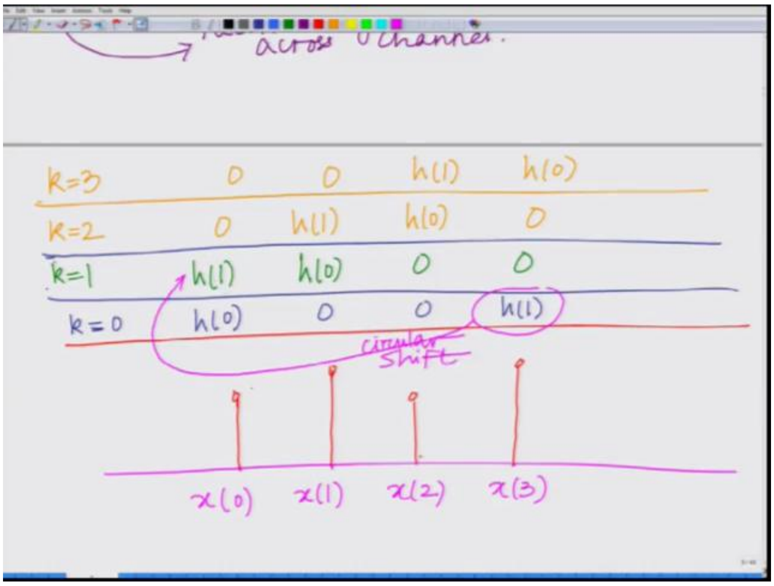

Now, let us observe an important property of this small y 0 small y 1 small y 2 small y 3 which are the n received symbols. Now what I am going to do is I am going to illustrate with a figure because with a figure it is best illustrated pictorially with the aid of a figure. So, let us use, I think let us have here what I am going to draw is the time axis with x 0, let me draw it clearly. I have x 0 x 1 x 2 x 3 and let us say this is your x 0, let us say this is your x 1 this is your x 2 and this is your x 3. Now look at this at time k equal to 0, that is for y k at time k equal to 0 what do we have at time k equal to 0 if we can look at the expression above you have h 0 times x 0 plus h 1 times x 3. So, I can write it as h 0 times x 0 plus h 1 times x 3 plus 0 times x 1 plus 0 times x. So, what I get is h 0 times x 0 plus h 1 times x 3 that is the effect of the channel on the symbols.

Now, let us look at time let us again now look at time k equal to 1 at time k equal to 1. I have h 0 times x 1 plus h 1 times x 0. So, this is h 0 times x 1 plus h 1 time x 0 plus 0 times x 2 plus 0 times x 3. I can also include x 2 x 3 by simply putting zeroes correct, and at time k equal to 2 now look at something interesting at time k equal to 2 I have h 0 times x 2 plus h 1 times x 1. So, h 0 times x 2 plus h 1 times x 1 and now if you can see at time k equal to 3 I have h 0 times x 3 plus h 1 times x 2 plus 0 times x 1 plus 0 times X 0. And now as you can see as a result of this and if you observe closely this figure what you can see is basically I have the symbols x 0 x 1 x 2 x 3 and I have the channels taps h 0 or the channel filter h 0 h 1 and you can see is that the channel filter is rotating over the transmitted symbol.

So, you can see in the first step we have h 0 h 1 in the next step h 1 has circularly shifted to the left. So, h 1 has circularly shifted. So, if you can observe this, this is basically a circular shift, h 1 has circularly shifted to the left h 0 has moved to the right, in the next step both h 1 and h 0 have moved to the right and then in the next step h 1 and h 0 have moved to the. So, this is basically a circular shift of the filter, channel filter over the symbols x 0 and x 1.

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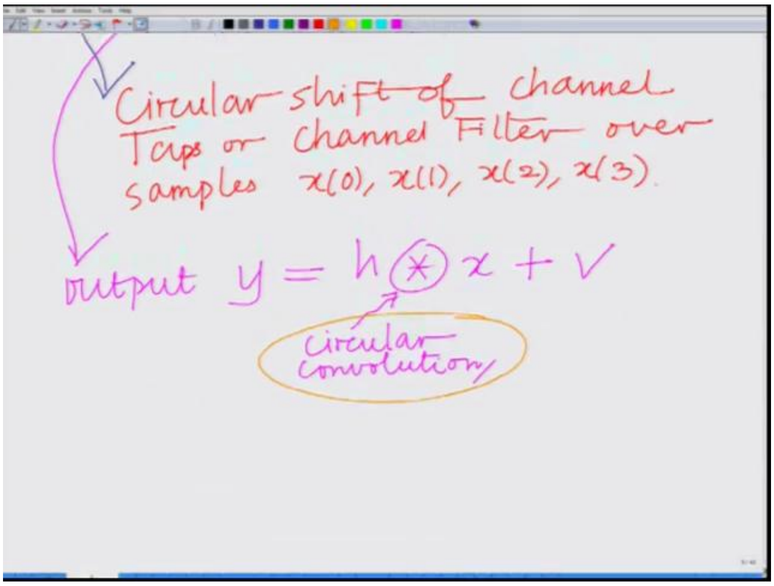

So, if you look at this what this represents is something very important; that is a circular shift of channel filter or basically your channel taps or basically channel filter over the samples, over the samples x 0 x 1 x 2 x 3 and therefore the important point that it is a circular convolution, it represents a circular convolution.

So, the output y equals h circularly convolved with x plus of course there is the noise with v. So, this represents your circular, this represents your circular convolution. So, what we have is if you can look at this is basically the action, because of the addition of the cyclic prefix the action of the channel filter on the input symbol is such that basically it every point in time you are rotating the that is you are circularly shifting basically the channel filter right. So, h 0 is first to the left, h 1 is to the right and the next step h 1 has circularly shifted to the left h 0 has shifted to the right and then they are shifting then at with every time instant they are shifting 1 step to the right and of course towards when there is the end circularly shift shifts back to the beginning and that is basically the very definition of a circular convolution before the action.

Because of the addition of the cyclic prefix the action of the channel on the input symbol is such that it represents a circular convolution and therefore, the output y is basically the circular convolution of the channel filter h with the samples x plus the noise and this is the important point. What is this? this is the circular convolution by addition of cyclic prefix we have converted the output of a channel into a circular convolution. So, addition of and this is also important to realize why is this cyclic pre, why is the circular convolution arising? addition of cyclic prefix has converted the linear channel or linear con convolution, convolution has converted a linear convolution and otherwise a liner convolution to circular convolution and this is the important point.

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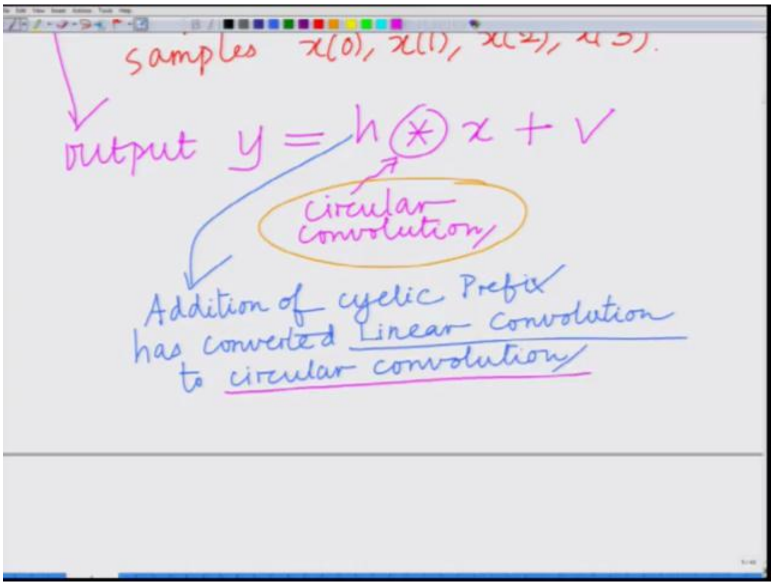

And therefore, now we know from the properties of the circular convolution that is if you take the FFT of h and x that is circular convert then the FFT in the FFT domain it is simply the product of the FFT's of h and x. So, what we have is because we have circular convolution, if we take the FFT of y that is equal to FFT of h circularly convolved with x plus v which is the noise.

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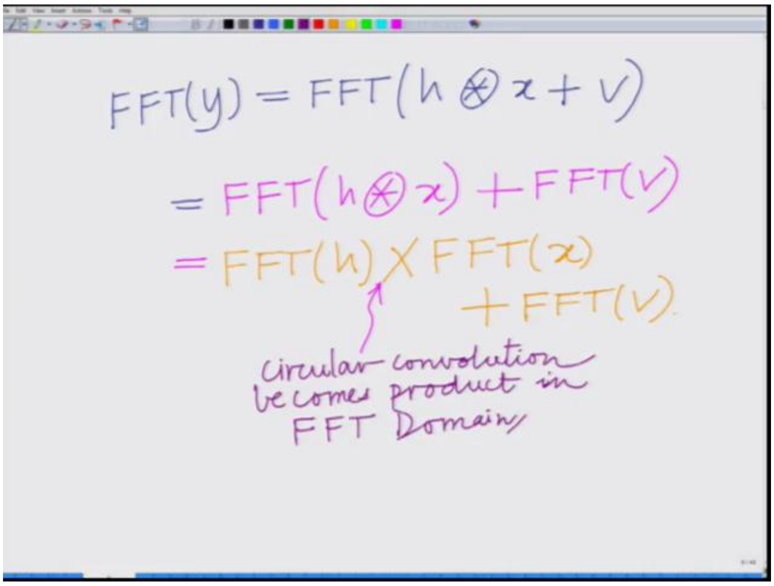

Now, the FFT operator is linear. So, that is simply the FFT of h circularly convolved with X plus FFT of v which is now if you look at this the FFT of a circular convolution is the product of the FFT. So, this is basically your FFT for h times the product, remember the circular convolution in the FFT domain becomes the product plus the FFT of v and this is a very important. So, this circular convolution has now been converted into circular convolution becomes product in the FFT, FFT domain or basically your frequency domain alright.

So, basically that is the addition that is the interesting aspect of OFDM where we have taken a block of symbols the capital X's capital X 0 capital X 1 capital X 2 capital X 3 perform the IFFT perform the IFFT of the ID of T to generate the samples to the samples we have added the cyclic prefix and now once you look at the output of the Inter Symbol Interference channel you see that the that basically what the channel is performed is (Refer Time: 31.11) is performing is equivalent to a circular convolution. Therefore, if you take the FFT at the receiver in the FFT domain, the channel is basically the product. So, FFT of y is basically equal to the FFT of the channel filter h times the FFT of the samples that is the smallest plus of course the FFT of the noise that is v.

And this is the important this is the key equation in the OFDM which is described OFDM system model. So, with this model we will stop here and we will continue to explore this aspect and implications of this aspect in the next module.

Thank you, thanks very much.