Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communications
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Lecture – 29
Linear Minimum Mean Square Error (LMMSE)
Channel Equalization – Example Part – II

Hello, welcome to another module in this massive open online course on Bayesian MMSE estimation for wireless communications. So, we are doing an example to illustrate the derivation of the l MMSE equalizer, alright.

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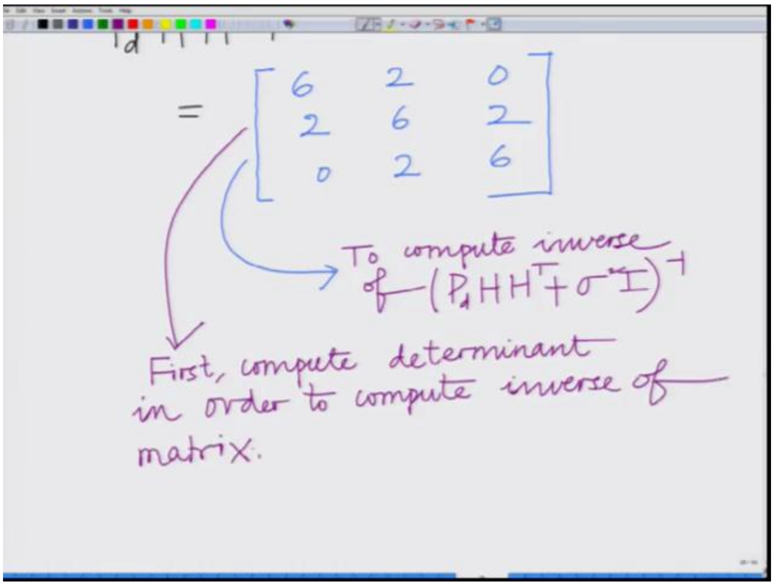

We have considered or we have demonstrated that this matrix Pd HH transpose plus sigma square identity that this matrix is well this matrix is given as let me just write this down clearly this matrix is is a 3 cross 3 matrix, with as a 6 2 0, 2 6 2, 0 2 6, and now we have to do we have to compute the inverse of this matrix we have to compute inverse of to derive the l MMSE equalizer we have to compute the inverse of Pd HH transpose plus sigma square identity this matrix inverse. Now what we have to do to compute this inverse first we have to compute the determinant first we have to compute determinant in order to compute the inverse of this matrix alright. So, previously whenever we computed a matrix inverse we considered 2 cross 2 matrices, but this is a 3 cross 3 matrix. So, computation of the inverse is still possible, but slightly more involved alright. So, the first step is to compute the determinant.

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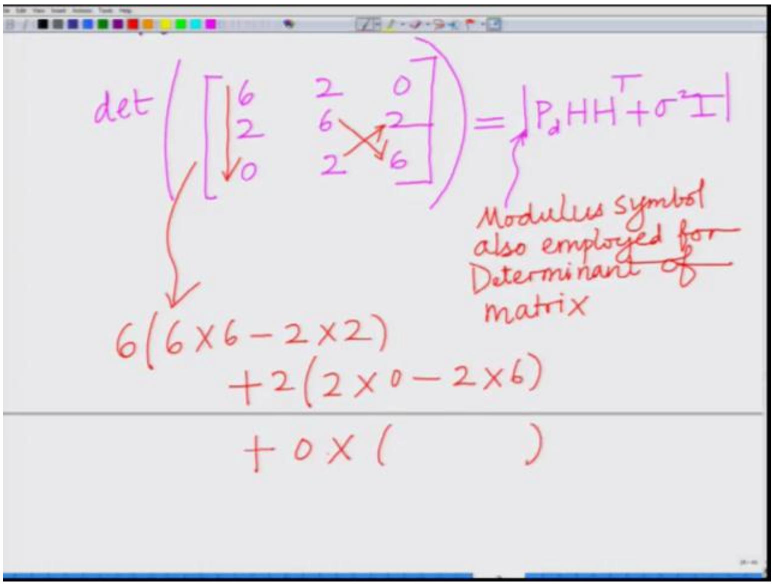

So, let us start by computing the determinant that is the let us denote the determinant by det dt of this matrix 6 2 0, 2 6 2, 0 2 6 and this determinant is also denoted by this symbol sometimes the determined is also denoted by kind of modulus sort of symbol of course, modulus is for the real numbers. So, this is your modulus symbol which is also used to denote the determinant of a matrix this is also and we will also employ this is also, we can explicitly write the determinant or this modulus symbol also employed for determinant of the matrix.

Now if you look at the determinant the determinant of this is nothing, but 6 times 6 into 636 minus. So, this minus 6 into 6 you can see you refresh your knowledge about the computation of the determinant of a 3 cross 3, 6 into 63, 6 minus 2 into 2 plus considering expanding along this column plus 2 times well 2 into 0 minus that is 2 into 0 minus 2 into 6 that is 12 plus 0 times. Of course, whatever this is it does not make any difference because it is multiplied by 0.

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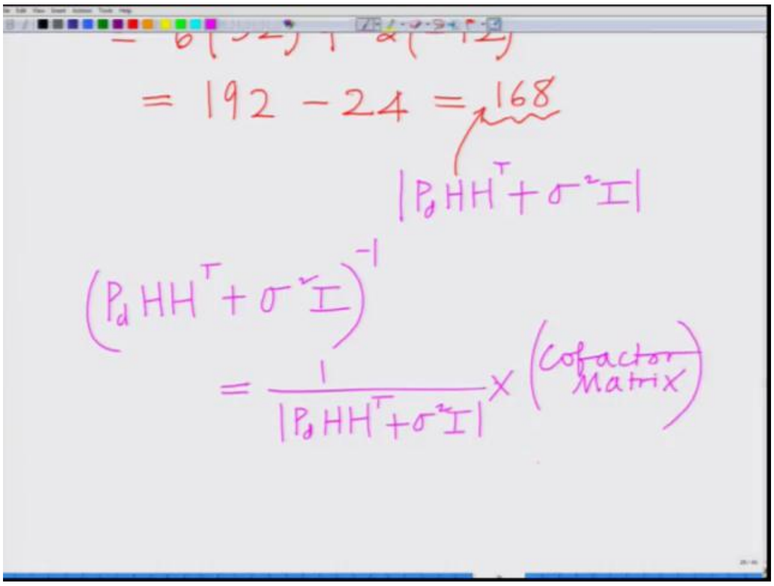

So, net this will be 6 into 36 minus 4 that is 32 plus 2 into 0 minus 12, 2 into minus 12 that is basically your 192 minus 24 and this will be equal to 168. So, this is the. So, this is this is the determinant of. So, this is your determinant of this matrix Pd h s transpose plus sigma square times identity.

Now, we have to compute the inverse of this matrix. So, we have to divide the inverse of this matrix is given by 1 over the determinant of the matrix times the co-factor matrix. So, that inverse is basically Pd HH transpose plus sigma square identity inverse is equal to 1 over the determinant that is 1 over 168, Pd HH transit determinant of this matrix times the co-factor times the co-factor matrix again you can refresh your knowledge about the co-factor matrix.

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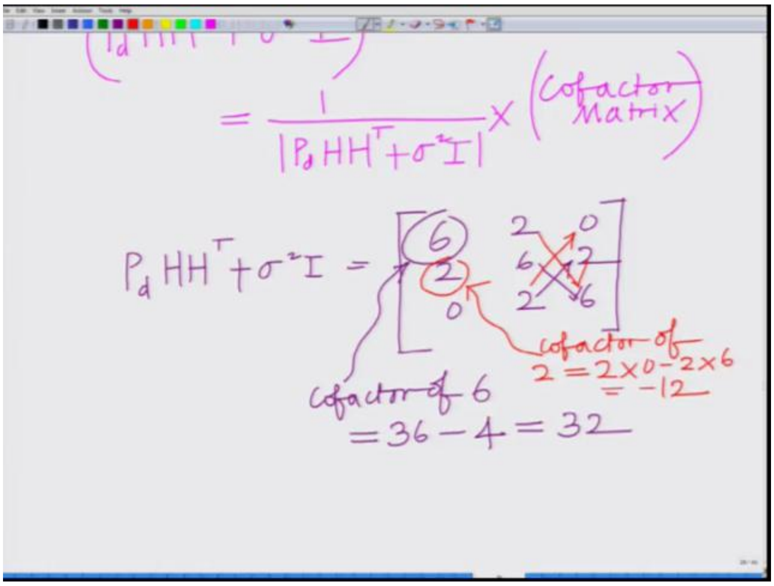

For example, if you look at this matrix Pd HH transpose plus sigma square identity Pd HH transpose plus sigma square identity which is equal to your 2 or 6 2 0, 0 2 6, 2 0 2. 6 for instance co-factor of 6 equals that is equals your 6 into 6 minus 2 into that is 36 minus 4 that is equal to 30, that is equal to 32 similarly the co-factors can before instance co-factor of 2 equals this co-factor of 2 equals 2 into 0 minus 2 into 6 that is 2 into 0 minus 2 into 6 equals minus 12. In fact, these co-factors are nothing, but what you multiply the corresponding element by to get the determinant alright. So, you can look into this thing if you are not familiar with this it is very simple. So, basically to compute the inverse we need to compute 1 the determinant and 2 what is known as the co-factor matrix.

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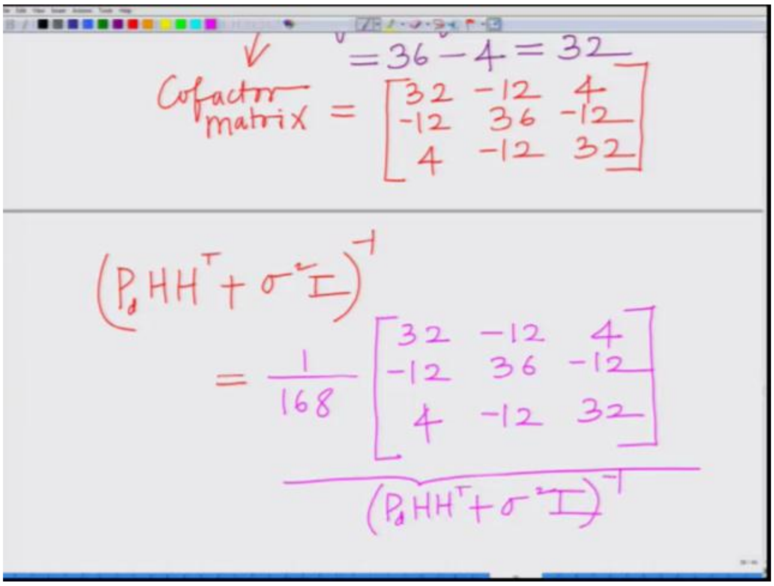

And the co-factor matrix for this is given by the matrix of co-factors the co-factor matrix is 32 minus 12, 4 minus 12, 36 minus 12, 4 minus 12, 32 this is the matrix of co-factors and therefore, the inverse HH transpose plus sigma square identity inverse is basically 1 over the determinant which is 1 over 168 times the co-factor matrix that is 32 minus 12, 4 minus 12, 36 minus 12, 4 minus 12, 32 this is the required inverse. So, this is basically your Pd HH transpose plus sigma square identity inverse of course, you can take 4 common out of this and you can simplify it, but we can do it later.

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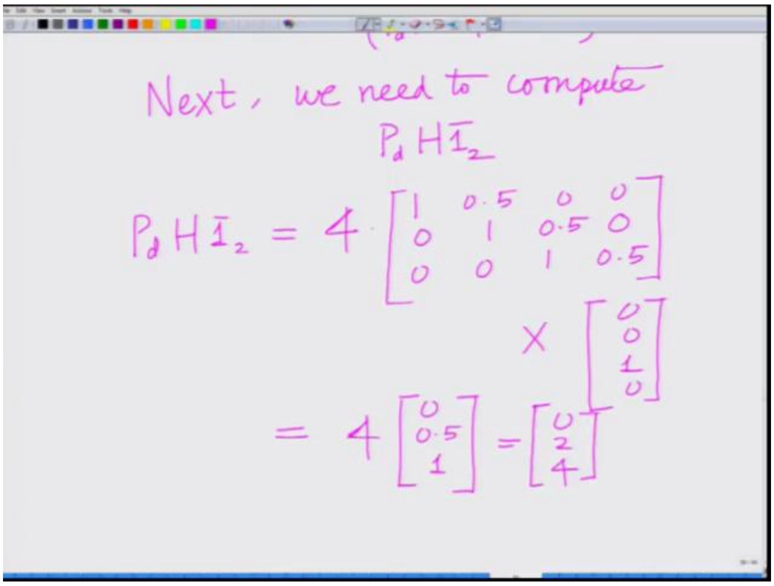

Now, we need to compute further next, we need to compute for computing the equalizer this quantity Pd h into this 1 bar 2 to compute this equalizer remember that is given as product of 2 quantities 1 is this well the equalizer vector remember is equalizer is basically this Pd HH transpose sigma square identity inverse into Pd 1 bar 2 transpose times h transpose side think this is, there is an error over here this is simply h times 1 bar 2 this is simply h times 1 bar 2 that is the transpose of this vector this vector transpose of this vector.

So, now let us compute this quantity over here Pd times h into 1 bar 2 that is equal to. So, Pd times h into 1 bar 2 that will be equal to your well Pd is 4 times h which is 1, 0.5000 1.5000, 1.5 times 0 0 1 0 which is equal to 4 into 0.5 into 1 which is equal to 0 2 4.

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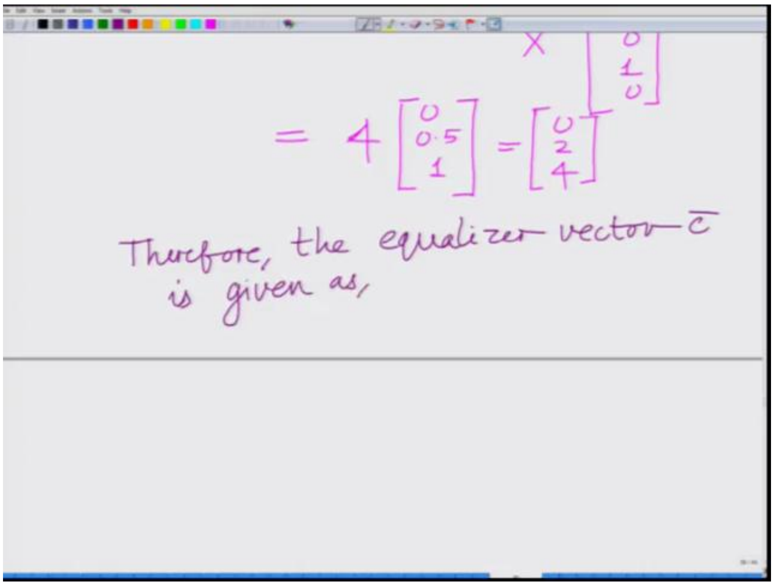

Now, therefore, the equalizer now we need to compute the equalizer vector c bar such that c bar transpose v bar k remembers, that is your net equalizer that is the net implementation of the equalization process.

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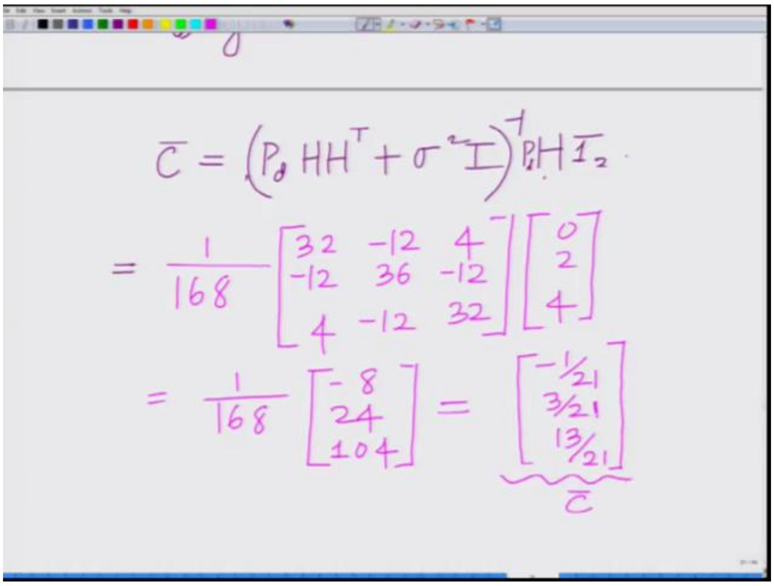

So, the equalizer vector c bar therefore, c bar is given as equalizer vector is given as c bar equals Pd HH transpose plus sigma square identity inverse into h 1 bar 2 into of course, this is a scalar constant Pd. So, we can this is the power of the symbols we can bring this outside by the way this is your vector 1 bar 2, 0 0 1 0 this is the vector 1 bar 2 which basically has 1 in the second position all 0 basically all 0 except 1 in second position counting from 0 except 1 in second position counting from 0.

Now, therefore, this will become Pd. So, now, this will become Pd HH transpose plus sigma square I inverse into Pd h into 1 bar 2. So, this is 1 over 168 times 32 minus 12, 4 minus 12, 36 comma minus 12, 36 minus 12 into 4 minus 12 into 32 into 0 2 4. If you look at this, this will be 1 by 168 times minus 820, 4 1 0 4 taking I think 8 common this will be well minus 1 divided by 21 3 divided by 21 13 divided by 21 this is your c bar which is the equalizer vector.

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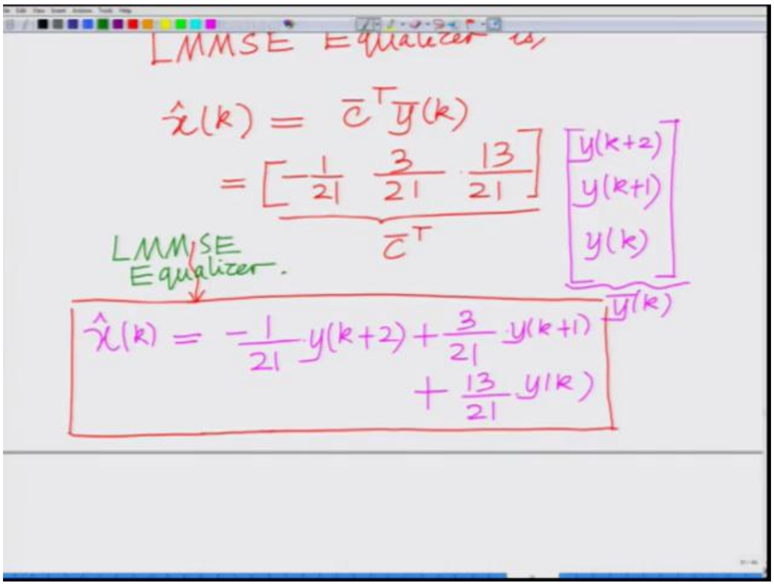

So, this is your c bar which is this is the equalizer vector c bar. So, your equalizer vector c bar has been derived as minus 1 over 21, 3 over 21, and 13 over 21 this is the equalizer vector alright. We have basically substituted all the quantities used it in the expression that we have derived previously for the equalizer vector.

Now, the equalizer has been has to be implemented using this equalizer remember the equalizer is nothing, but equalizer performs nothing, but the estimate of the symbol x k yields the estimate x hat k x hat k means c bar transpose y bar k.

So, equalizer is. So, net your l MMSE equalizer is x hat k equals c bar transpose y bar k that is which is equal to now c bar we have derived above. So, c bar transpose is nothing, but column vector gets converted to row vector that is minus 1 over 31, 13 over 31 sorry 3 over 31, 13 over 31, 13 over 31 this is your c bar transpose and your y bar k is nothing, but y k plus 2 y k plus 1 and y k and therefore, x hat k equals minus 1 over 31 times. So, the net equalizer is c bar transpose into this is your y bar k which is minus 1 over 31 into y k plus 2 plus 3 over 31 into y k plus 1 plus thirteen over 31 into y k and this is basically h hat k and this if finally, what is your l MMSE equalizer this is your. So, this is basically your l MMSE equalizer.

So, once we have derived this vector c bar the l MMSE equalizer is simply c bar transpose into y bar k and that yields the equalized estimate that x hat k in order to remove the inter symbol interference or basically I s I removal in the wireless channel.

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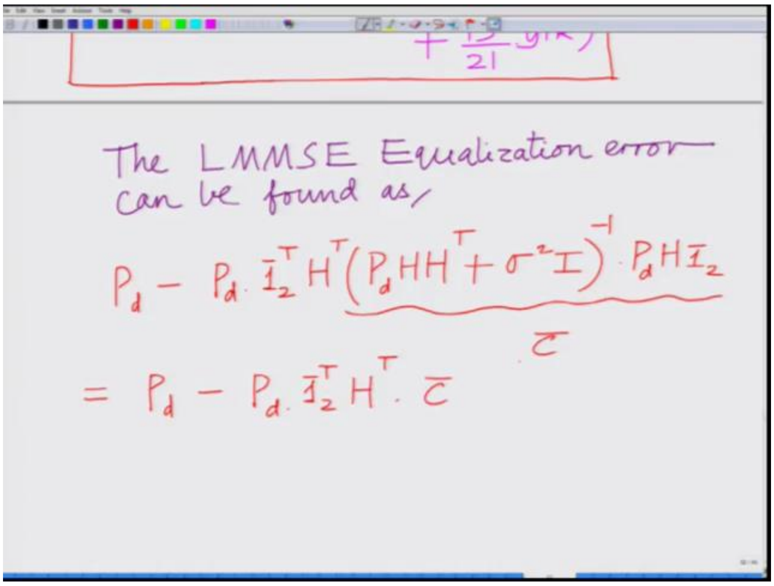

Now, the error also the l MMSE equalization error, we have derived the expression for the l MMSE equalizer equalization error and this l MMSE equalization error that is given as Pd minus Pd into 1 bar 2 transpose h transpose Pd HH, transpose plus sigma square identity into inverse into Pd h bar into 1 bar 2, but now see this part is simply your equalization vector c bar. So, I can write this also as Pd minus Pd 1 bar 2 transpose h transpose into c bar.

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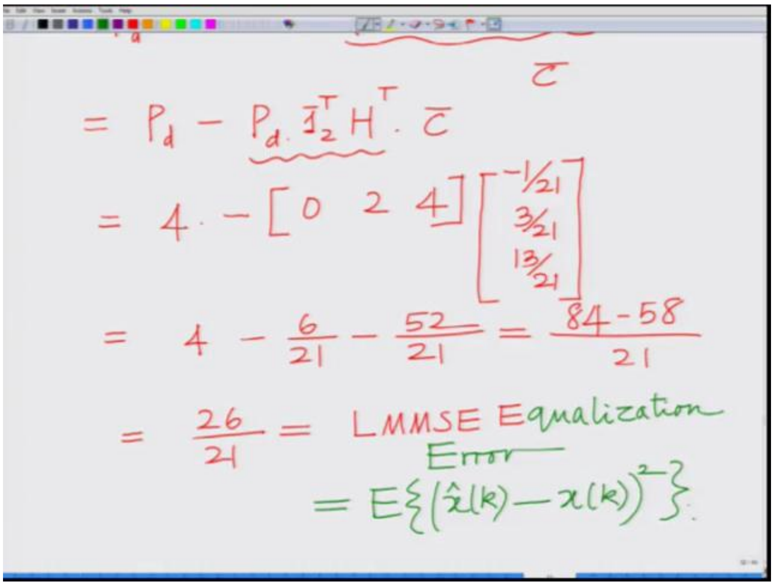

Both these quantities we have computed previously we have computed Pd 1 bar 2 transpose h transpose. In fact, we have computed the transpose the column vector of this we currently computed Pd into h into 1 bar. So, Pd into 1 bar to transpose h transpose is nothing, but the transpose of that quantity alright. If you can if you just to remind you this is basically your Pd into h into 1 bar 2, Pd into 1 bar 2 transpose h transpose simply the transpose of that quantity.

So, therefore, we know all this quantity. So, this is basically simply your Pd which is 4 minus Pd into 1 bar 2 transpose h transpose that is 0 rho vector 0 2 4 times c bar which is your equalizer vector and the equalizer vector is basically nothing, but that is something that we have just derived minus 1 by 31, 3 by 31, 13 by 21. So, this is 4 minus 6 by 31 minus 52 by 31 and that is equal to 84 minus 58 by 31 equals 36 by 31 and this is nothing, but your l MMSE equalization error which is basically again to define it more precisely. This is your x hat k expected value x hat k minus x k whole square. So, this is basically your l MMSE equalization error.

So, what we have done in this example is basically, we have completed this example of deriving the l MMSE equalizer alright. First derived the l MMSE equalization vector c bar then derived the l MMSE equalizer and also finally, derived the error of l MMSE equalization remember this is for the specific case of your inter symbol interference channel that is y k equals h 0 times x k that is h 0 is 1. So, y k is x k plus 0.5 times x k minus 1 h 1 is 0.5 plus v k, where v k is the noise, we have derived at 3 tab equalizer considering the various quantities for this inter symbol interference channel and as I have said previously this can also be extended for other scenarios general more general scenarios with a larger number of channel tabs and with larger number of equalizer tabs.

So, we will stop this module here and continue with other aspects in the subsequent lectures.

Thank you.