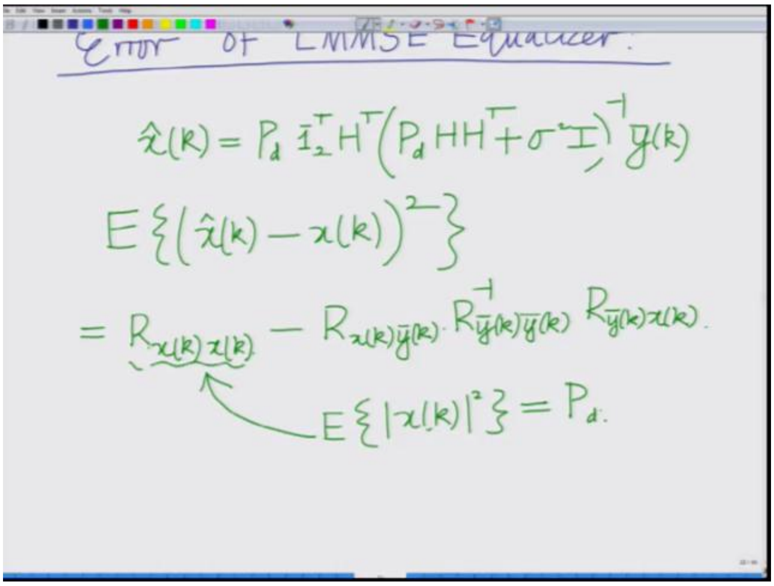
Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communications
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 28
Linear Minimum Mean Square Error (LMMSE) Channel Equalizer and Example – Part – I

Welcome to another module in this massive open online course in Bayesian MMSE Estimation for Wireless Communications. So, we are looking at the LMMSE equalizer for a wireless communications system with inter symbol interference all right we have derived the LMMSE equalizer to remove the inter symbol interference you know wireless channel. Now, let us look at the error for this LMMSE equalizer.

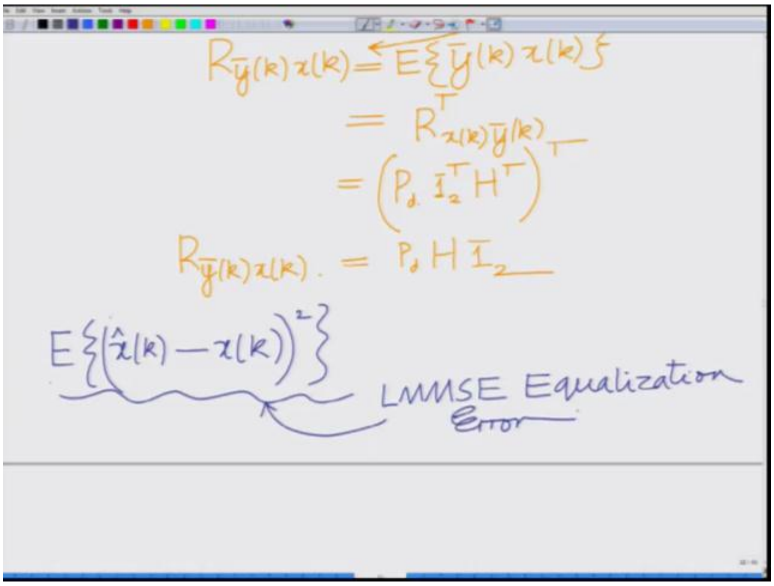
(Refer Slide Time: 00:40)



So, today first thing that we are going to do in this module is the error of LMMSE. If we define the error of the LMMSE equalizer as right expected we have derived the LMMSE equalizer as x hat of k equals lets write about P d 1 bar 2 transpose H transpose P d H h transpose sigma square I inverse y bar of k and now we want to derive the error for the LMMSE equalizer that is expected value of x hat of k minus x of k whole square which is equal to R remember x k x k you know the explanation for the error of the LMMSE estimate R x k x k minus R x k y bar k times R y bar k y bar k inverse into R y bar k x k where R x k x k, this is never expected value of, this is nothing but expected value of magnitude x k square which is equal to P d that is the power of the data symbols all right R x k x k is expected value x k into x k or x k square which is equal to the power that is P d.

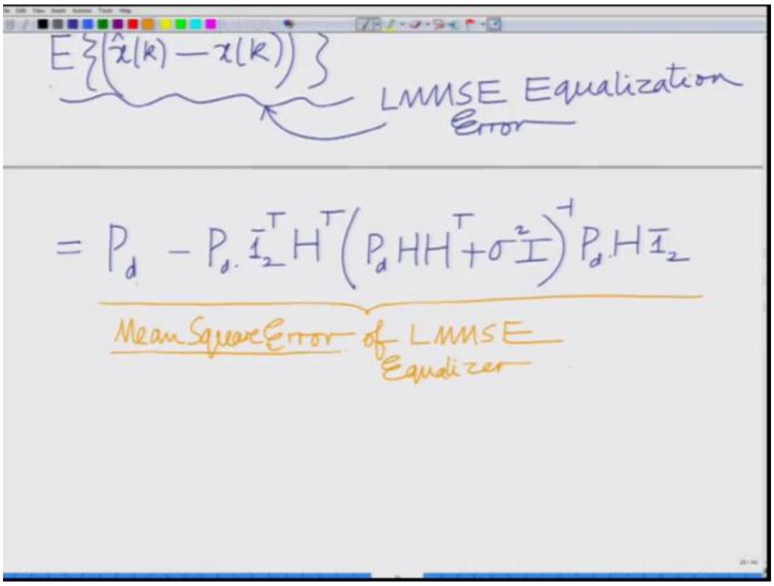
Now, these co-ordinates we have also derived, now R y bar k x k, remember we have derived previously without R x k y bar k R y bar k x k, this quantity R y bar k x k.

(Refer Slide Time: 03:11)



This is nothing but expected value of y bar k into x k which is nothing but expected R x k y bar k transpose and we know R x k y bar k we have already derived that R x k y bar k is P d 1 bar 2 transpose into H transpose is R x k y bar k transpose of this which is basically of course, P d is a scalar times the matrix h, P d is a scalar times the matrix H times the vector 1 bar 2 that is R y k x bar k. So, therefore, we have expected value of x hat k minus x k whole square, this is the LMMSE equalization error, this is the error of LMMSE equalization which is equal to R x k x k P d minus this expression.

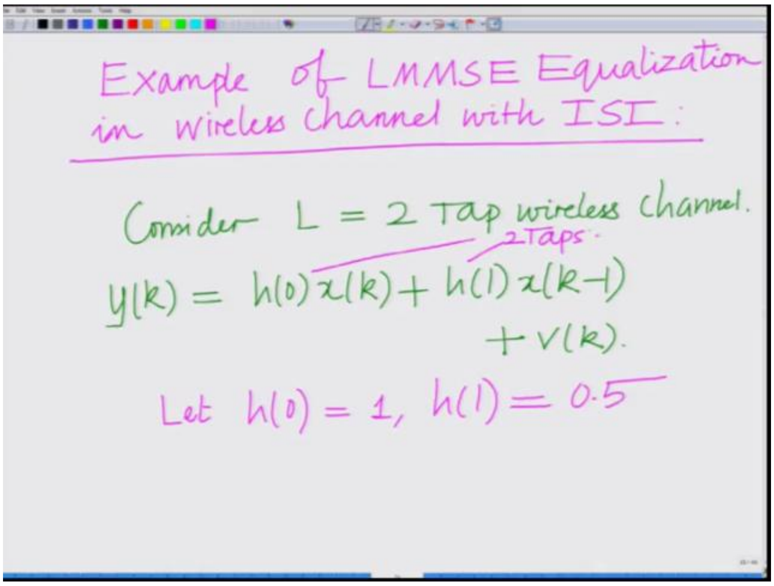
(Refer Slide Time: 04:52)



Look at this R x k y bar k R y bar k y bar k inverse, this is nothing but the LMMSE equalizer that is this quantity P d one bar two transpose H transpose into P d H transpose plus sigma square I inverse. So, this will become minus P d 1 bar 2 transpose H transpose times P d H h transpose plus sigma square I inverse into P d into H into 1 bar 2. This is the expression error of LMMSE equalizer or rather mean square error, this is the mean square error of the LMMSE equalizer that is P d minus P d 1 bar 2 transpose H transpose into P d H H transpose plus sigma square I inverse into P d H into 1 bar 2. So, that is the error of the LMMSE equalizer.

So, what we have done is basically we have done two things one is we have derived the LMMSE equalizer and also the corresponding error for the LMMSE equalizer.

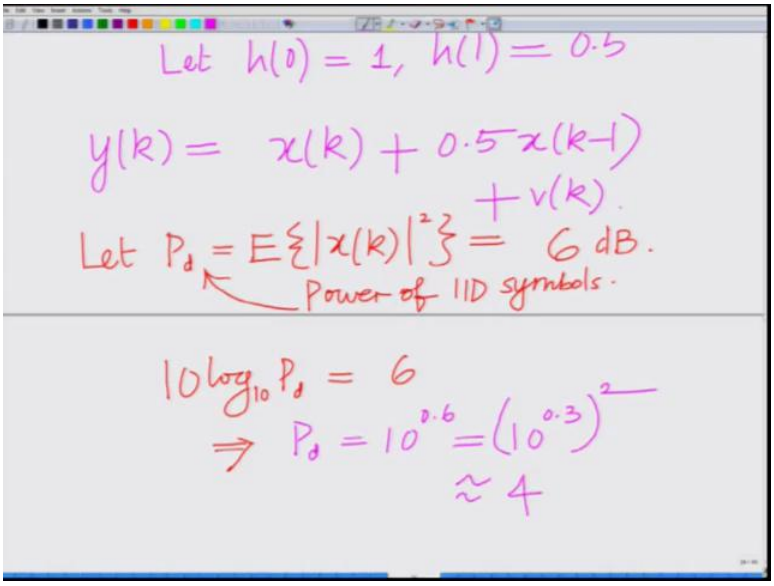
(Refer Slide Time: 06:45)



Now, let us do a simple example to understand the application of this concept of LMMSE equalization in a wireless channel with inter symbol interference. We are going to introduce a simple example of LMMSE equalization in wireless channel with ISI. So, let us consider a wireless channel of L equal to 2 tap, consider an L equal to 2 tap wireless channel with y k equals h 0 x k plus h 1 x k minus one plus v k.

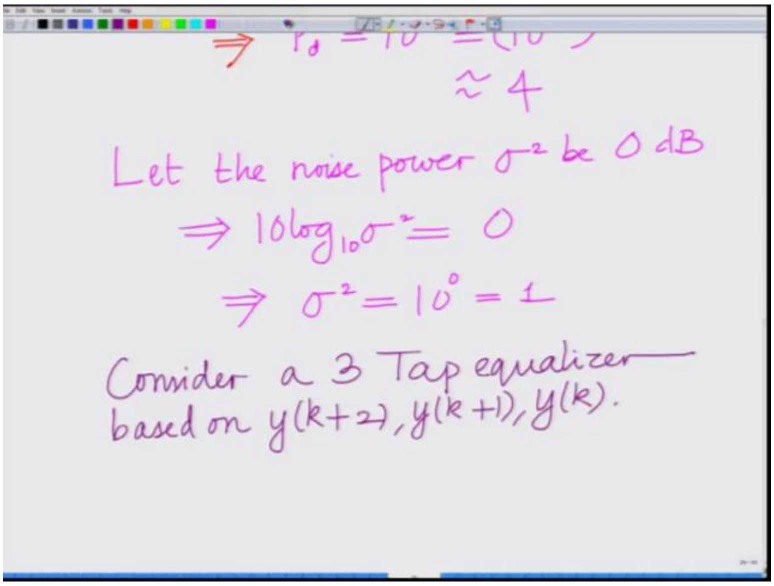
Let h 0 is equal to 1 h 1 equal to 0.5. So, we are considering a two tap wireless channel similar to what we have considered before that is h 0 and h 1, these are the two taps with h zero equals to 1 h 1 equals 0.5.

(Refer Slide Time: 08:45)



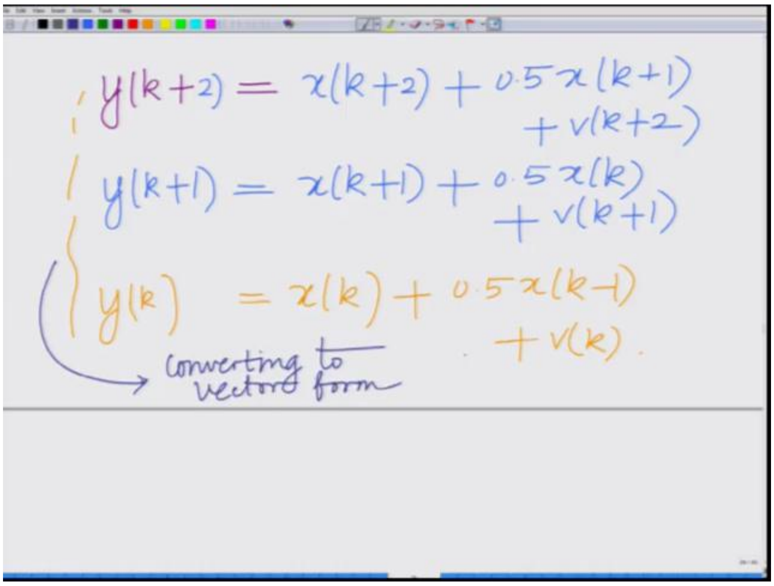
So, h 0 and h 1, these are the two taps, we have y k substituting h 0 equals 1, we have y k equals one times that is x 0 times x k that is one times x k plus h 1 times x k minus 1.5 times x k minus 1 plus 2 because 3 d B corresponds to two, this is approximately equal to 4, 2 square this is what we know. Let the noise power sigma square be 0 d B, additionally we are also when the noise form, remember we need the power of the symbols in the noise power to compute the LMMSE equalizer.

(Refer Slide Time: 09:26)



So, we are assuming the noise power sigma square to be 0 degree which implies that 10 log to the base 10 sigma square equal to 0 which implies that sigma square equals 10 to the power of 0 which is equal to 1. Now, further consider a 3 tap equalizer based on y k plus 2 y k plus 1 comma y k which is equal to.

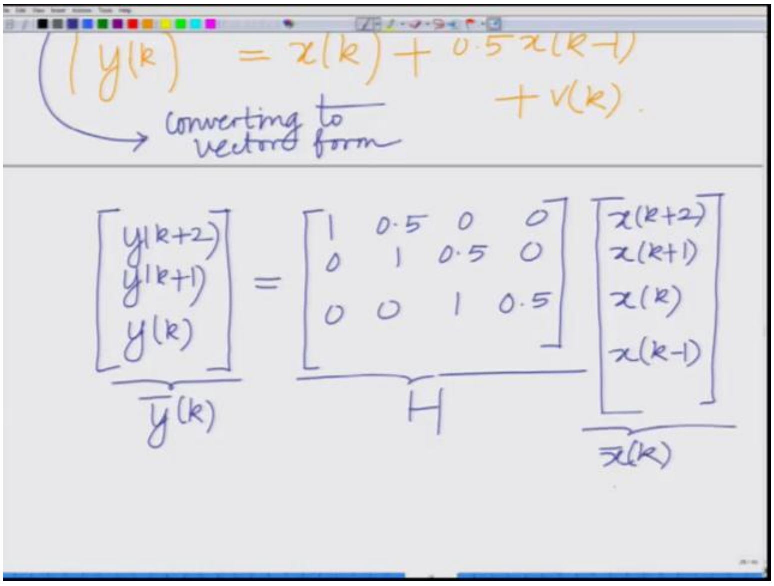
(Refer Slide Time: 10:30)



Now writing the equations we have y k plus 1 equals we have h 0 into or y k plus 2 rather equals h 0 into x k plus 2 that is x k plus 2 plus 0.5 into x k plus 1 plus v k plus 2 y k plus 1 equals x k plus 1 plus 0.5 x k plus v k plus 1 and y k plus 3 x or y k rather equals x k plus 0.5 x k minus 1 plus v k.

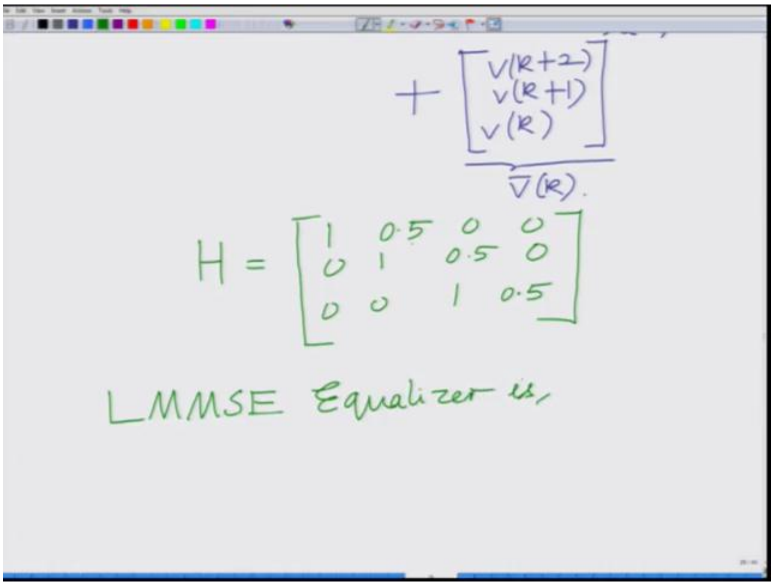
Now, putting these in a matrix form, these three things stacking this, thus converting this to a matrix form or converting to vector form, we have the output vector y bar of k which is y k plus 2.

(Refer Slide Time: 12:11)



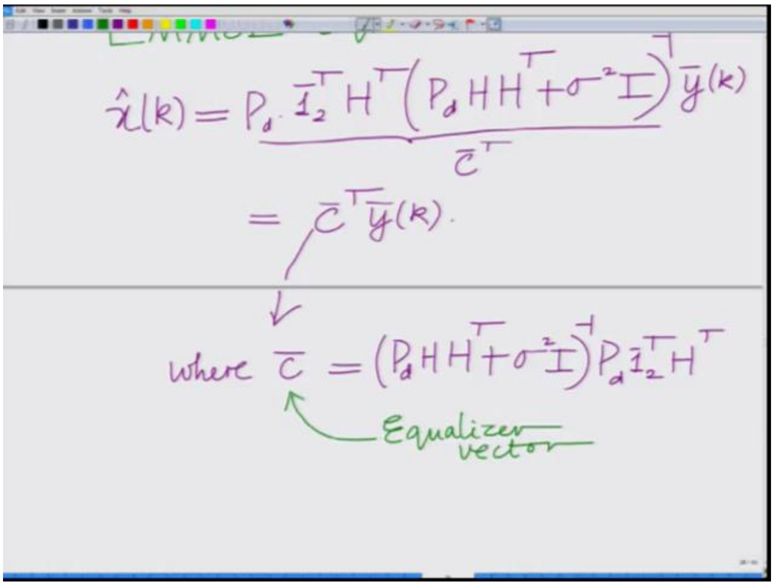
Remember that is k y plus 2 y k plus 1 y k which is equal to 1 0.5 0 0 0 1 0.5 0 0 0 1 0.5, this is our matrix H, this is our matrix y bar k times your matrix x k plus 2 x k plus 1 x k minus 1, this is your vector x bar of k plus the vector v bar of k, remember v bar of k is v k plus 2 v k plus 1 v k v bar of k.

(Refer Slide Time: 13:07)



So, the three thing here to notice that this is your matrix H, let us rewrite the matrix H, again H is basically equal to the matrix H is equal to 1 0.5 0 0 0 1 0.5 0 0 0 1 0.5, notice that the LMMSE equalizer is given as x hat of k equals P d into the vector 1 bar 2 transpose H transpose P d H H transpose plus sigma square identity inverse into 1 bar k.

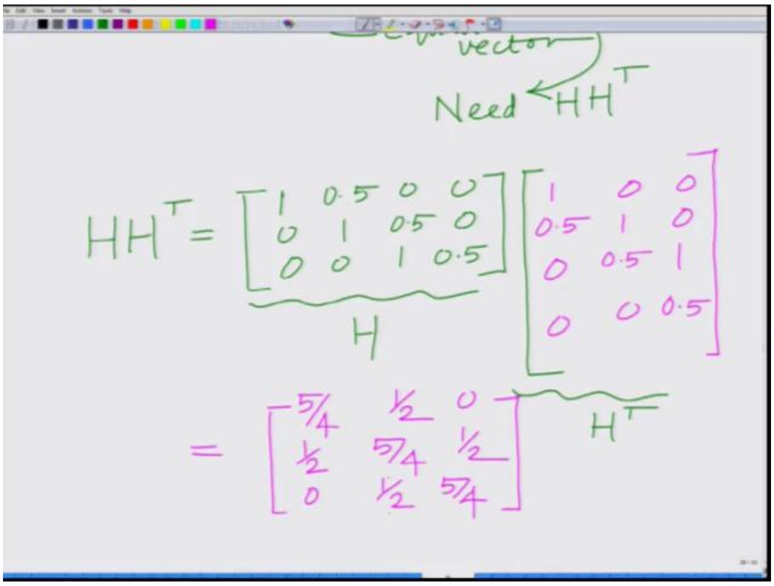
(Refer Slide Time: 14:15)



Let us call this as C bar transpose. So, this can be written as C bar transpose into 1 bar k where this vector C bar equals, it is clear what is C bar, C bar is P d H H transpose plus sigma square identity inverse into P d 1 bar 2 transpose into H transpose, we can call this vector P d and we can call this vector C bar as the equalizer vector.

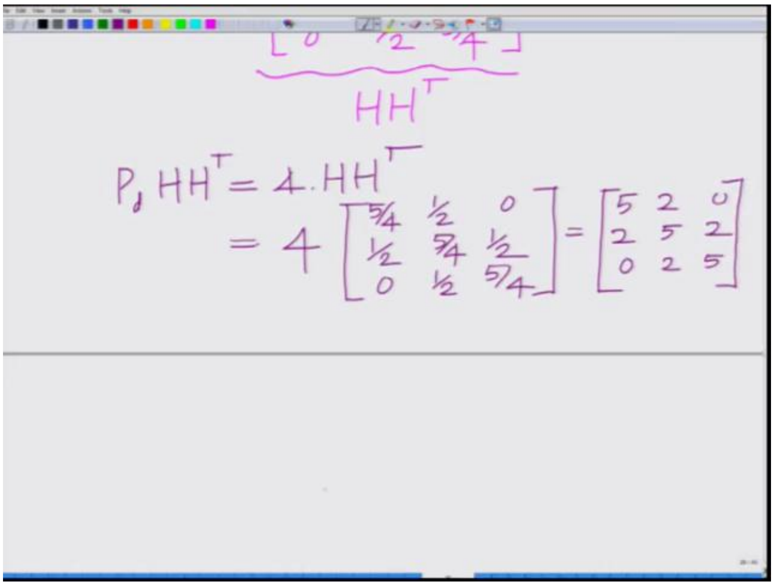
So, we are defining this vector C bar which is the equalizer vector, we can find the equalizer vector, from that we can implement the equalizer or that is the process of equalization across the wireless channel.

(Refer Slide Time: 15:45)



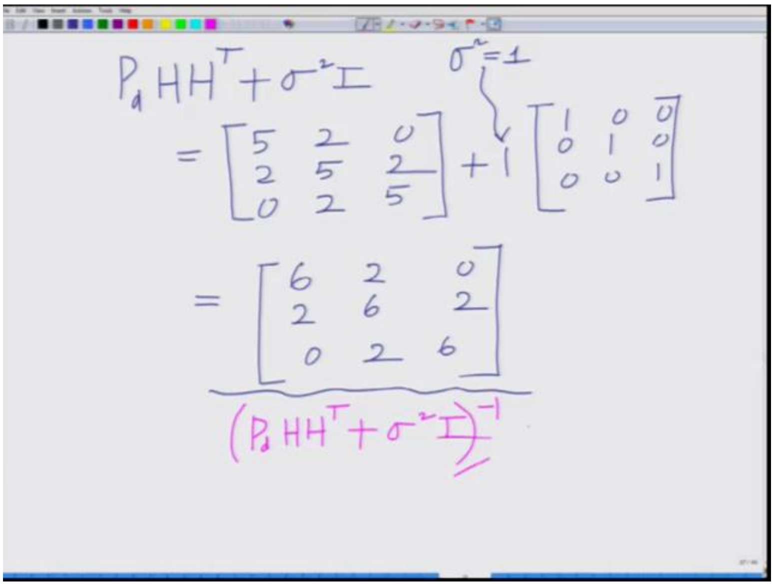
Now, the key is to compute this C bar and to compute this C bar you will see that we will need H H transpose and H h transpose can be obtained as now let us write this down H H transpose is 1 0.5 0 0 0 0 or 0 1 0.5 0 0 0 1 0.5 into 0 or 1 H transpose. So, this is H, we need to get H transpose. H transpose is the transpose of H that is 1 0.5 0 0 0 1 0.5 0 0 0 1 0.5 which is equal to let us write it as fractions 5 by 4 half 0 half 5 by 4 half 0 half 5 by 4, this is your matrix H H transpose.

(Refer Slide Time: 17:09)



Now, we need P d times H H transpose ,look at this, this is P d times H H transpose and P d times H H transpose is rather 4 into H H transpose which is equal to your 4 times H H transpose has been derived above. So, that is 4 times, let us just write this once more for the sake of clarity 5 by 4 half 0 half 5 by 4 half 0 half 5 by 4 equals 5 2 0 2 5 2 0 2 5, this is P d times H H transpose.

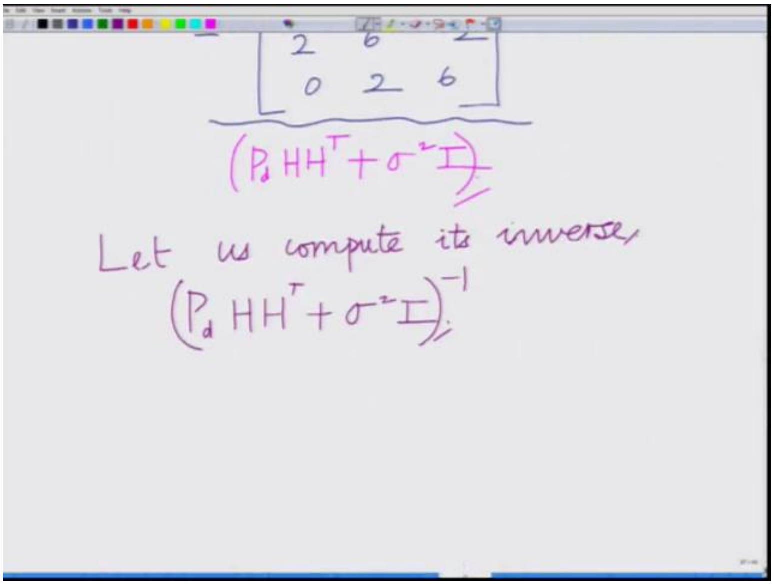
(Refer Slide Time: 18:19)



Now, we need P d times H H transpose plus sigma square I which is 5 2 0 2 5 2 0 2 5 plus sigma square is 1. Remember we have derived earlier sigma square is 0 d B which is equal to 1 times the identity matrix 0 0 0 0 1 0 0 0 1 which is equal to now, if you do this what we will get is this is equal to 6 2 0 2 6 2 0 2 6 and this is the matrix, this is your P d times H H transpose plus sigma square identity matrix inverse.

So, basically we have computed until this part that is P d times H H transposes. Now, we want to calculate the inverse. This is P d times H H transpose plus sigma square I.

(Refer Slide Time: 19:59)



Now, let us compute the inverse of this matrix. We need to compute the inverse and then we can use this in the computation of the equalizer vector. So, once we compute the inverse, we can use it in the computation of the equalizer vector. Let us stop this module here because this is a long example, probably it is s lightly easier to understand all aspects of it in one module.

So, let us stop this module here and we will continue with this example, we will complete the rest of the derivation that is derive the equalizer vector as well as the mean squared error of equalization, the mean squared error of equalization of the symbol x k in the subsequent module.

Thank you very much.