Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communications
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Lecture - 26
Channel Equalization and Inter Symbol Interference (ISI) Model

Hello. Welcome to another module in this massive open online course. So far in the previous modules we have looked at channel estimation in particular MIMO channel estimation. In this module we will start looking at other aspects of wireless communications that is channel equalization; that is equalization of a wireless channel.

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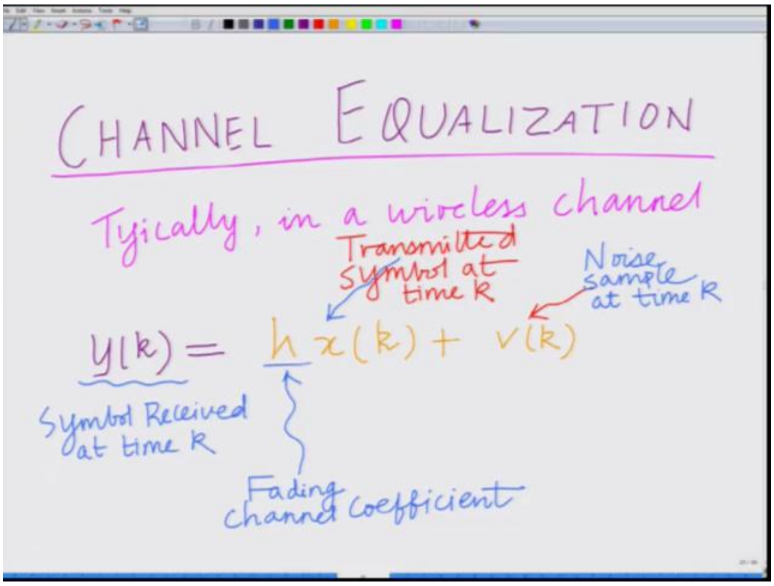

So, let us start looking at channel what we term as channel equalization. And this can be motivated as follows channel equalization. Typically, in a wireless channel as we have already seen before the received output symbol at time y k equals h this is the model for typically wireless channel h times x k plus v k which is the noise. And what is y k? Y k is symbol received at time k at just refresh your memory symbol received at time k, h is your fading channel coefficient; h is the fading channel coefficient like the flat fading channel coefficient. X k is the transmitted symbol at time instant k what is this, this is transmitted symbol at time k and v k is basically the noise sample the Gaussian noise sample at time k x.

So, I have this system value which can be described has y k equals h times x k plus v k. Y k is the received symbol, h is the fading coefficient, x k is the transmitted symbol v k is the noise sample. And observe that is in this module the received symbol y k depends only on the current transmitted symbol x k. So, this is the very subtle, but an important aspects that the received symbol depends only on the current input x k.

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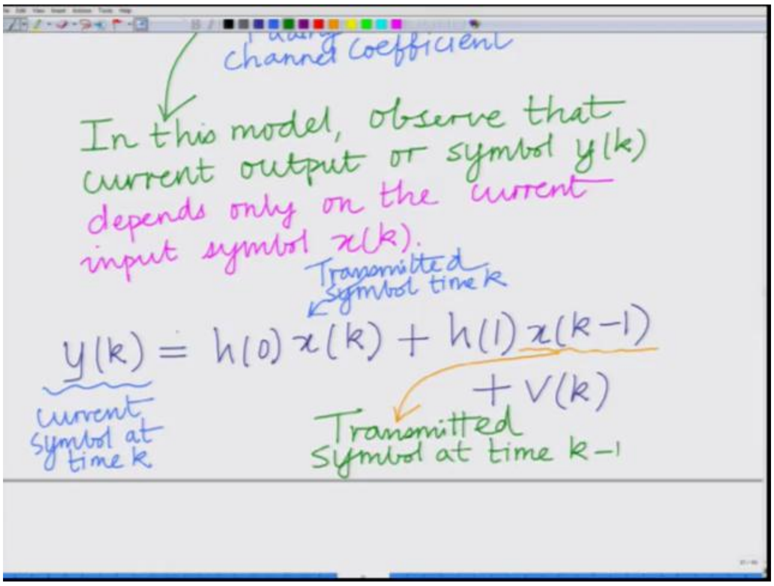

So, let us observe this let us note this about this module, so that in this module in this observe that the current output y k or the current symbol output or symbol y k depends only on the current input symbol x k, so the current output symbol y k it depends only on the current input symbol x k.

However, we might frequently have another scenario which is a decidable scenario in which the following occurs. So frequently also it might happen that we have the received symbol y k is given as h 0 times x k plus h 1 times x k minus 1 plus v k which is not. And if you look at this model you will observed that this is y k this is again the current symbol at time k. X k is the current input symbol that is transmitted symbol at time L that aspect is fine.

However, we also something very interesting we have also x k minus 1 that is the transmitted symbol or the input symbol at time k minus 1 we have; write it down this is the transmitted symbol at time instant k minus 1. So, what we observing is that unlike previously where the received output symbol y k depends only on the transmitted symbol x k times instant k. What we have in this scenario is the output symbol y k depends not only on x k, but it also depends on the previous symbol that is x k minus 1.

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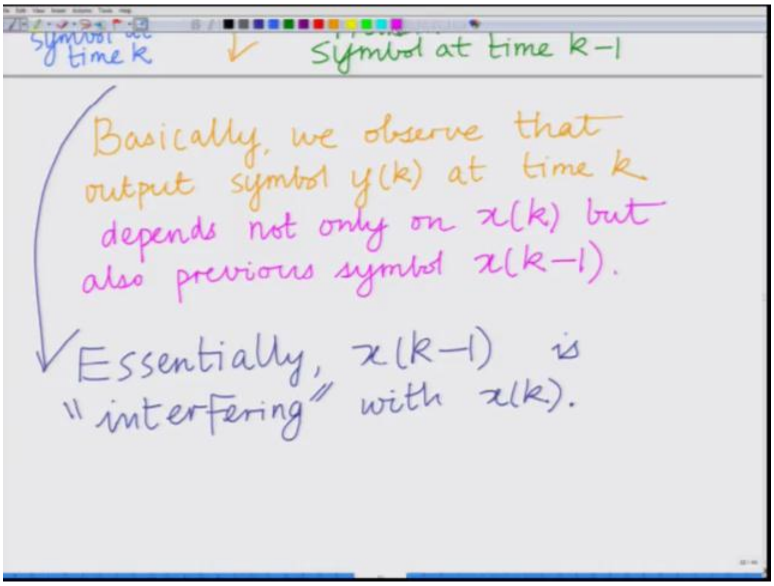

So, let us write this, now this is a important aspects that is what we are observing basically we observe that the output symbol y k at time k also depends not only on x k, but also in the previous symbol, but also depends not only on x k, but also the previous symbol x k minus one 1 is y k depends on not only x k, but also the previous symbol x k minus 1. Basically, which means that the previous symbol x k minus 1 is interfering with the current symbol x k that is it is causing interference in the detection in the current symbol x k this is known as inter symbol interference.

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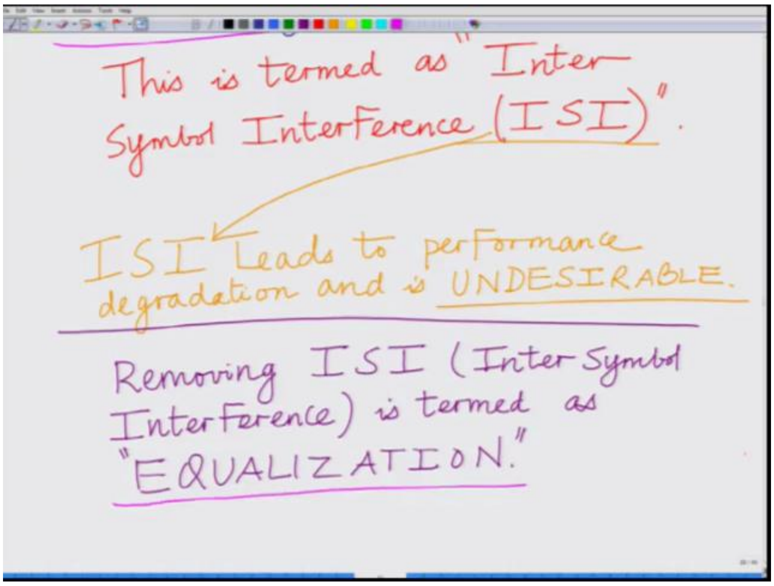

So, essentially x k minus 1 is to put it this is interfering with x k, x k minus 1 is interfering with x k this is termed as inter symbol interference or ISI. So, this phenomenon is termed as inter symbol, this phenomenon is termed as inter symbol interference or ISI when the previous symbol x k minus 1 is also is interfering with current symbol x k that is y k as the effect of x k and x k minus 1 this is termed as inter symbol interference. And naturally this ISI inter symbol interference leads to performance degradation because there is interference because x k minus 1 is interfering with x k this leads to performance degradation at the receiver. So, this it is an undesirable effect.

So, this ISI leads to performance degradation naturally and is undesirable. And is basically ISI is inter symbol interference is basically undesirable. Hence therefore, we would like to remove inter symbol interference that is what we would like to do at the receiver for the deduction of x k that is you want to infer what x k you would like to interlay remove the interference x k minus 1, and this process of inter symbol interference that is this removing inter symbol interference is termed as equalization of channel equalization.

So, channel equalization is nothing but removing the effect of inter symbol interference to improve the performance at the receivers. So removing this inter symbol interference essentially removing this ISI or inter symbol interference is basically what is termed as equalization. So let me write it down removing inter removing inter symbol interference is termed as; this process of removing intra symbol interference is basically what is termed as equalization. Process of removing the effect of x k minus 1 on x k termed as or basically removing this interference from x k minus 1 on x k or the intra symbol interference as termed as equalization.

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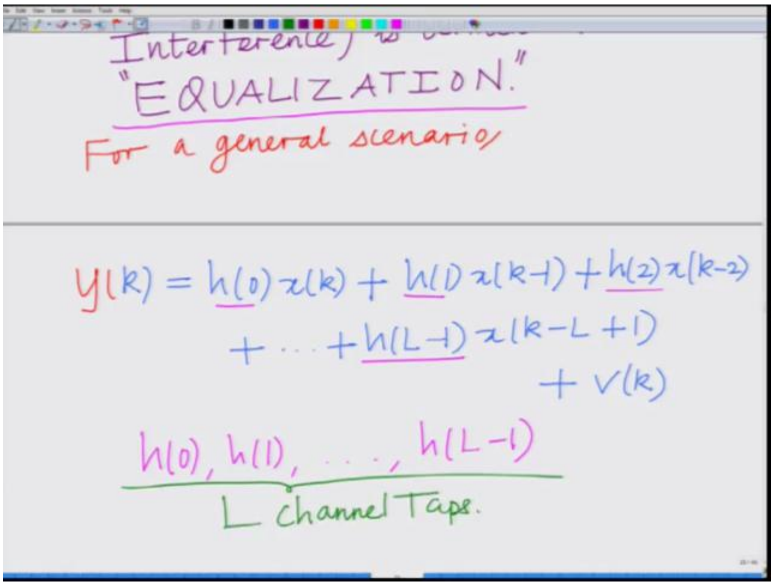

Further, I would like to also point out that the inter symbol interference will not be restricted to x k minus 1 it can be on a larger number of previous symbols.

So, here in this system model if you look at, I can in general have that is y k that is for a general scenario I can have y is y k equals h 0 x k plus h 1 x k minus 1 plus h 2 x k minus 2 plus so on; h L minus 1 x k minus L plus 1 plus v k. Therefore, you have L channel taps so you have h 0, h 1, h 2 h L minus 1 these are the coefficients these are known as channel taps. So, h 0, h, 1 h L minus 1 these are known as the channel taps these are basically your L channel taps.

So, we are saying for a general system with into symbol interference y k equals as 0 x k plus h 1 x k minus 1 so on so forth until h L minus 1 times x k minus L plus 1 plus the noise sample v k and then for you have L channel taps h 0 h 1 up to h L minus 1. And therefore, as L increases the severity of ISI or inter symbol interference increases which means more and more previous symbols x k minus 1 x k minus 2 so on interfere with the current symbol x k.

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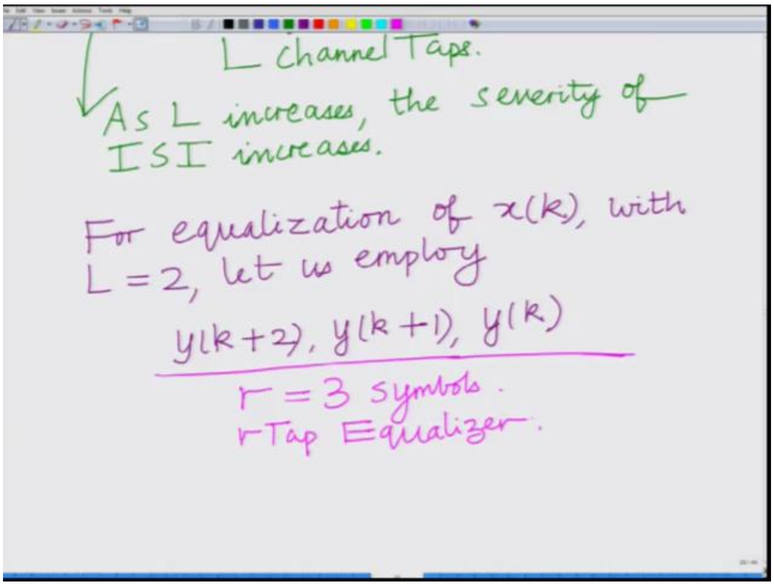

So, as L increases the severity of the ISI increases, that is basically the idea. And then that what you said is basically we want to remove that the effect of the intra symbol and at the receiver that is termed as equalization, and that can be done as follows. What we will do is to basically deduct x k we will also employ x k y k plus 1 and y k plus 2. So, previously for flat fading scenario to detects x k we simply employ the receive symbol y k. However, now because there is a inter symbol interference we will not only employ y k, but we will also employ y k plus 1 and y k plus 2 which is what I am going to illustrate.

Now for equalization k, of course let us go back to a simple scenario with L equal two let us employ y k plus 2 y k plus 1 y k that is we are employing 3 symbols or basically r equal to 3 symbols. So, this is known as 3 tap equalization. So, we are employ r equal to 3 symbols that is y k plus 2 y k plus 1 y k this is known as a r tap or basically 3 tap this is known as an r tap or 3 tap equalizer.

And what we are going to do is basically we are going to linearly combine these symbols, so we are going to linearly combine these symbols to eliminate the effect of inter symbol interference. So, first let us write the system model for these 3 symbols y k plus 2 y k plus 1 y k so you can see that.

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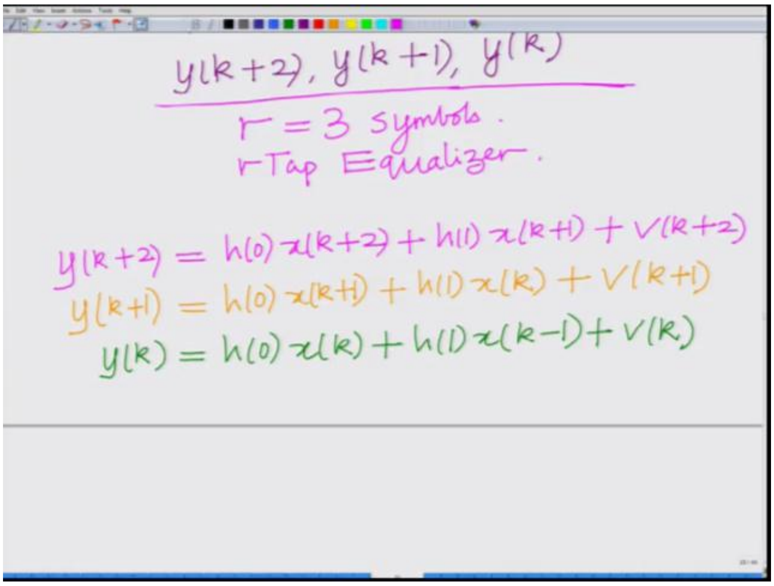

Well, y k equals L h 0 x k plus h 1 x k minus 1 plus v k now y k plus 1 will be similarly h 0 x k plus 1 plus h 1 times a previous symbol that is x k plus v k plus 1. And similarly y k plus 2 equals h 0 times x k plus 2 plus h 1 times x k plus 1 plus v k plus 2. Now what I am going to do is I am going to stack these symbol y k plus 2 y k plus 1 y k as a vector and you can see now I will get the system models.

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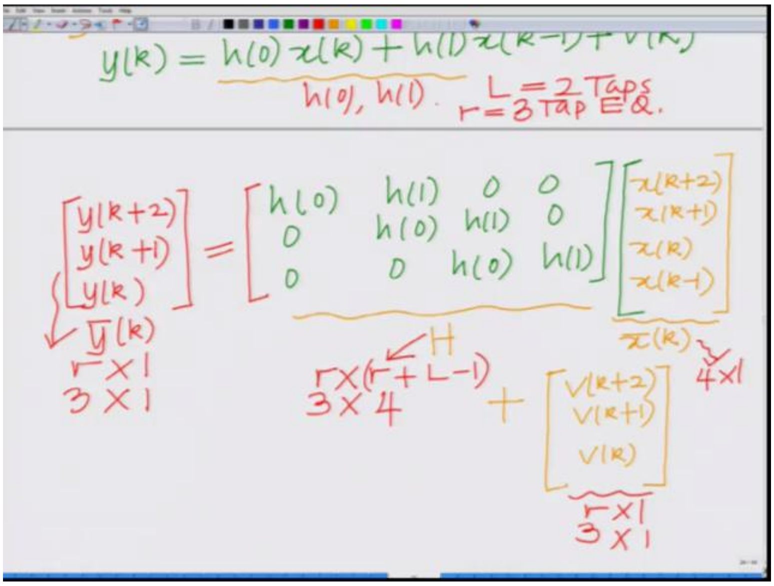

So, when I stack these a vector what I will get is y k plus 2 y k plus 1 this is the vector remember we can call this is the vector y bar k which is an r cross 1 or basically 3 cross

1. Remember we are looking at a 2 taps equalizer so I have r cross 1 vector this is equal to the following matrix h 0 h 1, 0 h 0 h 1 0, 0 0 h 0 h 1 times.

Now, my symbols x k plus 2 x k plus 1 x k and x k minus 1 let us called this your matrix H, let us call this your matrix your vector x bar k plus your noise vector is basically v k plus 2 v k plus 1 v k. So, basically what you can observe is here we are considering and L equal to 2 tap channel because we have h 0 coma h 1 as the channel taps, so, we have L equal to 2 taps r equal to 3 tap equalizer, so we have your r equal to 3 tap equalizer. So, this y bar k is basically 3 cross 1. Now this matrix h you can see is r cross r plus L minus 1 that is 3 cross 3 plus L equal to 2 minus 1 so 3 cross 4 x bar is r plus L plus minus 1 cross 1. So, this is 4 cross one and v bar k is simply r cross 1 that is 3 cross 1.

So, now, we have written the vector model for this basically inter symbol interference channel which is basically y bar k which contains the symbol y k plus 2 y k plus 1 y k equals which are the matrix h is found out the channel coefficient which is r cross r plus L minus 1 matrix. Where, r is the number of taps of channel coefficient times x bar k which is r plus L minus 1 cross 1 plus v bar k which is noise factor which is r cross 1. And now we would like to design or equalize.

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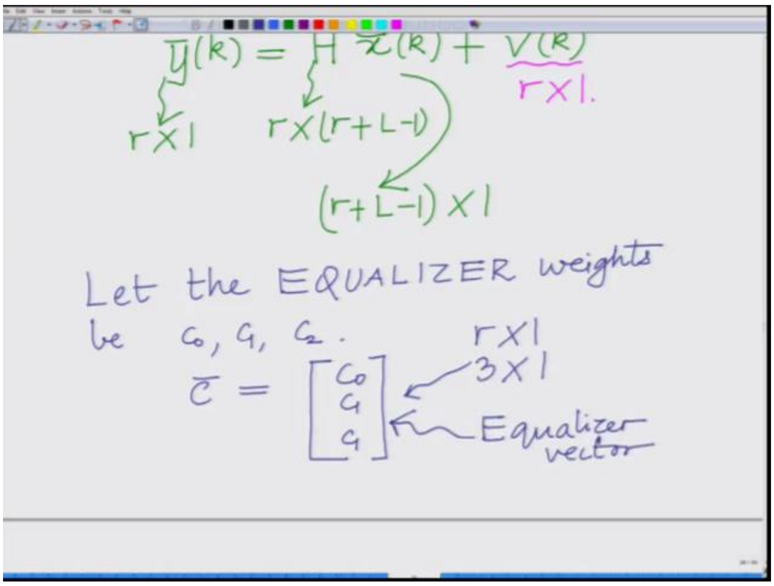

So, let we just write down this term again. To summarize this thing if we consider an r tap equalizers so I have y bar k equals H times x bar k plus v bar k where this is r cross 1 this is r cross r plus L minus 1, this x is basically r plus L minus 1 cross 1. And this v bar k this is simply r cross 1 vector. And we would now like to design an equalizer, remember we said we would like to do design the equalizer based on the 3 symbols y k plus 2 y k plus 1 y k and we would like to linearly combine these symbols y k plus 2 y k plus 1 y k so let the combining weight to C 0, C 1, C 2.

So, let the equalizer weights or this is the combing weights; let the equalizer weights be C 0, C 1, C 2, therefore now you can denote this equalizer I can denote this equalizer by C bar which is C 0, C 1, C 2 this is remember observe this is an r cross 1 vector this is r cross 1 or basically 3 cross 1 vector. This is the equalizer also we can call this as the equalizer vector.

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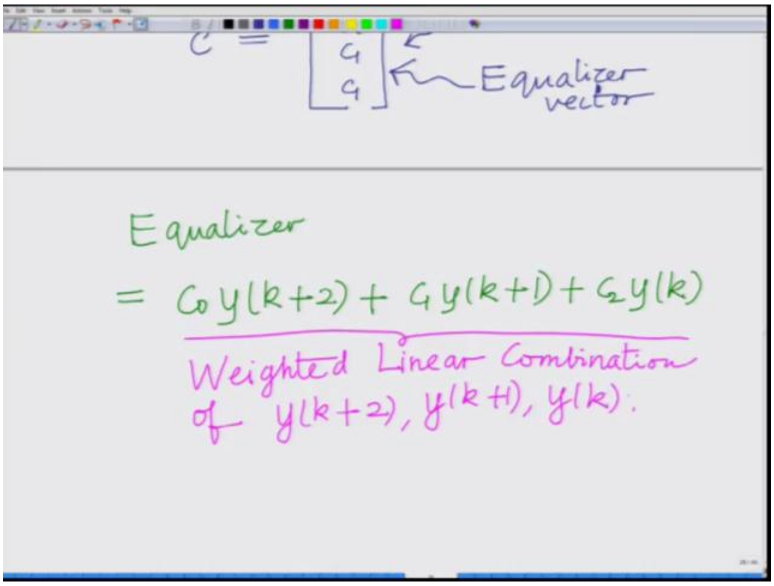

And our equalizer therefore is now given has C 0 times y k plus 2 plus C 1 times y k plus 1 plus C 2 times y k which is this basically the linear combination of y k plus 2 y k plus 1 and y k. This is basically your or weighted linear combination let we let me put it that way, this is your weighted linear combination of the outputs y k plus 2 y k plus 1 and y k. So, what we are saying is basically we have a 3 tap equalizer which comprises of the weights C 0, C 1, C 2; these are the three taps of equalizer vector C bar. And now what we are doing is we are taking the outputs y k plus 2 y k plus 1 y k we are linearly combining them using this weights as C 0 times y k plus 2 plus C 1 times y k plus 1 plus C 2 times y k this is the operation of the equalizer. So, we would like to design the weights basically C 0, C 1, C 2 so as to eliminate the inter symbol interference for the detection of x k and that is the problem of equalizer (Refer Time: 25:09).

So, what we have what we have done in this module so far is to basically motivate this problem of intra symbol interference where we have the current symbol y k it depending not only on the current input symbol x k, but also the past symbol x k minus 1 and so on. This is known as a inter symbol interference and inter symbol interference is to performance degradation which as to be removed at the receiver this is known as equalization. And for equalization we have to design this equalizer, equalizer is basically nothing but these are the co efficient or the weights C 0, C 1, C 2 which linearly combine the output symbols; that is we consider a three tap equalizer linearly combine the output symbols y k plus 2 y k plus 1 y k basically equalize or remove the intra symbol and interference towards the detection of x k.

So, how exactly do a design this weights C 0, C 1, C 2 or how exactly we design this equalizer that we are going to look at the next module. So, let us stop this module here.

Thank you.