Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communication
Prof. Adithya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 23
Channel/ Noise Statistics for Multiple-Input Multiple-Output (MIMO) Downlink Wireless Channel Estimation

Hello. Welcome to another module in this massive open online course on Bayesian MMSE Estimation for MIMO DM Wireless Communications. So, what we are being doing or what we are done in the previous module is to describe the MIMO wireless communication system module. We have laid down the system module for the MIMO system. What we are going to do todays module is to explore it a bit more that is statistically speaking derived the statistical properties of the more specifically the statistical properties of the channel matrix and the statistical properties of the noise matrix, because these are going to be important for us to derive the MMSE or the LMMSE.

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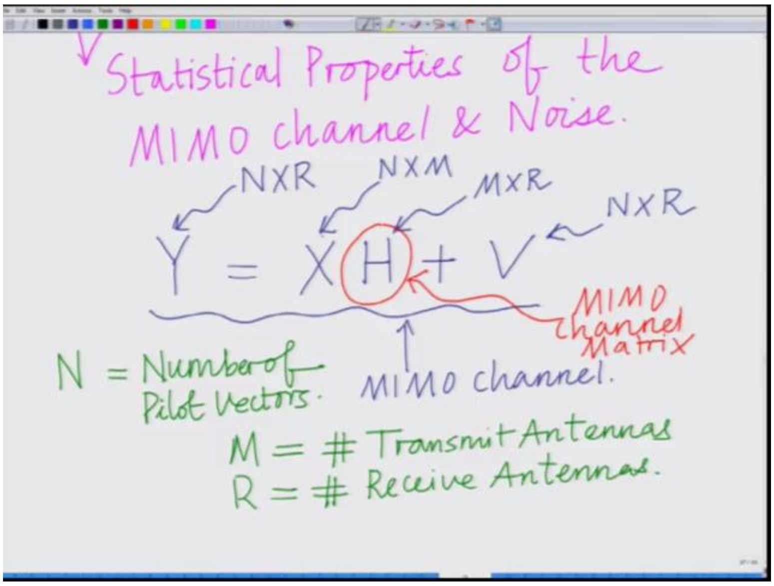

So, we are looking at MIMO channel estimation, and what we would like to do is you would like to examine the statistical properties of the MIMO channel and the noise. We have the MIMO channel which is the model which is Y equals X H plus V, this is our MIMO channel model remember. This is the model of the MIMO channel which we have derived in the previous module this is Y which is N cross R observation matrix, X which is N cross M, H which is M cross R MIMO channel matrix, V which is N cross R noise matrix. And what we have the various quantities we have N equals number of pilot vectors, M equals number of transmit antennas, and R equals number of received antennas and H is the MIMO channel matrix, this is the M cross R MIMO channel matrix.

And we would like to estimate this MIMO channel matrix that is termed as MIMO channel estimation. But before we do that let us examine the statistical properties of the MIMO channel matrix and also the noise matrix that is this is V which is the noise matrix.

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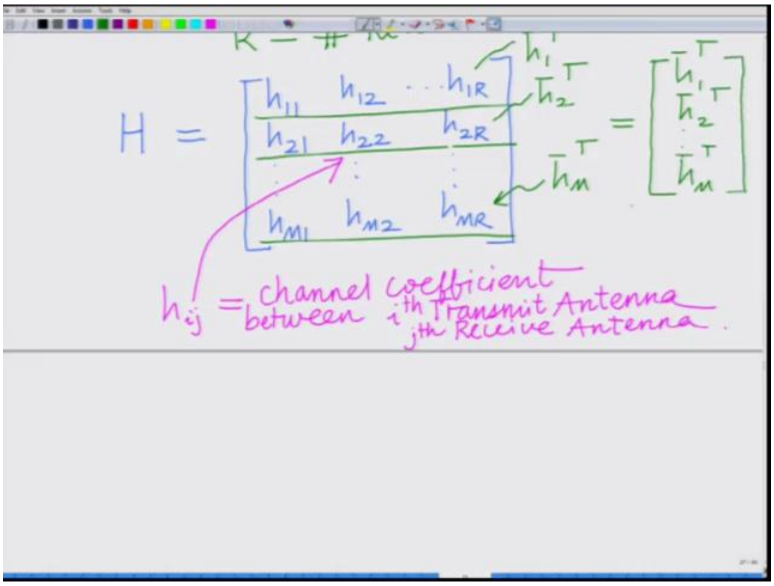

So, what we have is let us write this channel matrix down explicitly we have H which is your M cross R matrix we have already seen this h 11, h 21 so on up to h M; h 12, h 22, h M2; we have h 1R, h 2R, h MR. And we have also said that h ij is the channel coefficient between the ith transmit antenna and the jth received antenna. And therefore, naturally we have M transmit antennas R received antennas, so this as M rows and R columns specifically; so it is M cross R channel matrix. What we are going to do now is organize this as a grouping of rows.

So, let us call the first row h 1 transpose, second row h 2 transpose last row h M transpose; so I call this h 1 bar transpose that is the first row, h 2 bar transpose that is second row last row, I have M rows h M bar transpose. Because each row is a row vector I am denoting it whether transpose of a column vector; h 1 bar transpose, h 2 bar transpose. So, I can write this h as equal to a collection of M rows that is h 1 bar transpose, h 2 bar transpose, h M bar transpose.

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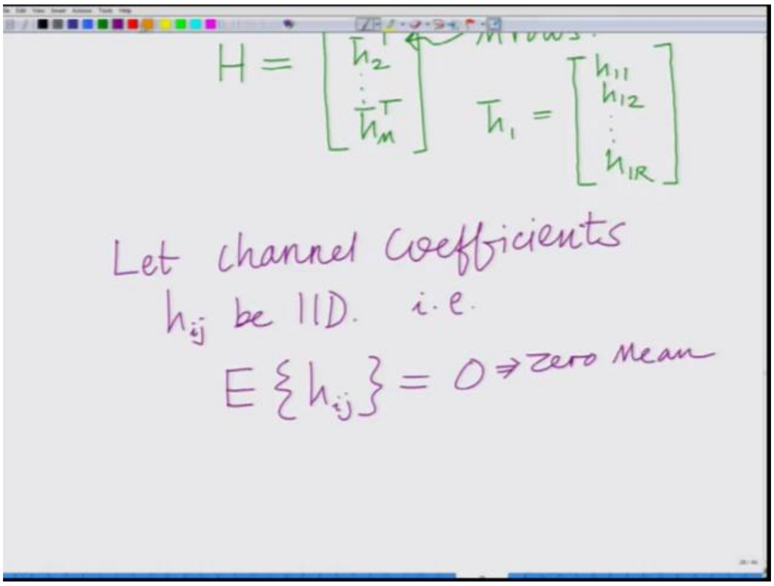

For example, that is H is equal to let me just write this thing over here explicitly H is equal to h 1 bar transpose, h 2 bar transpose, h M bar transpose. This is collection of M rows for example, you have h 1 bar equals from our definition h 1 bar is a first h 11, h 12, naturally it is of size R; h 11, h 12 h 1R and so on this is h 1 bar. Similarly, one can define h 2 bar h 3 bar up to h M bar. So, h bar is collection them matrix h this is M cross are collection of M rows is h 1 bar transpose h 2 bar transpose so on h M bar transpose.

Now, let us assume the channel coefficients, let the channel coefficients h ij be IID not necessarily Gaussian they can be independent identically distributed with variance of each term that is expected value of. So, let us first start with mean let us assume zero mean expected value of h ij equals 0 which basically implies zero mean.

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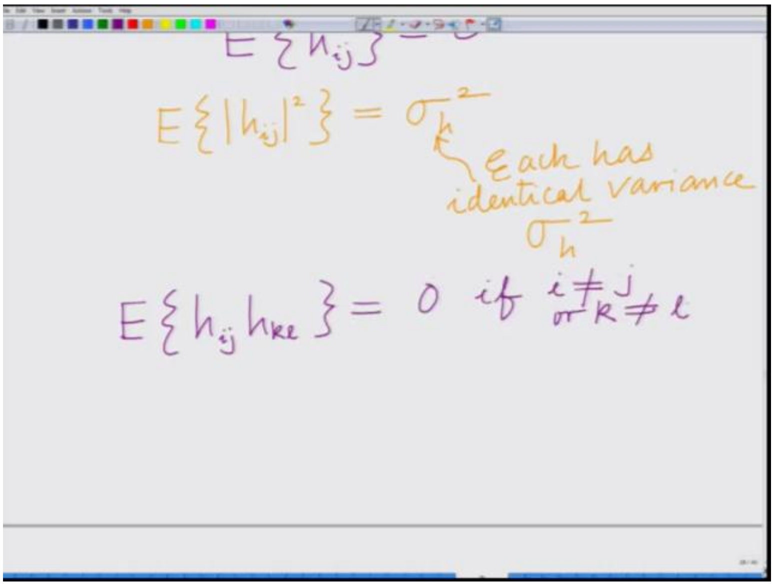

Let us also assume they are in identical so they have identical variance expected value of magnitude h ij square equals sigma X square that is each has identical variance prior variance sigma h square. Further, since they are independent naturally they are uncorrelated let us assume that h that is any two channel coefficients are independent h ij the product expected value of h ij h kl equal to 0, if i not equal to j or k not equal to l. That is if you look at any two distinct channel coefficients h ij h kl the expected value the correlation between h ij h kl the expected value of h ij h kl the product is 0. Of course, we are assuming that independent so naturally it also follows that they are uncorrelated.

So, if you take any two distinct channel coefficients the correlation between them is 0 rather the cross correlation between them is the correlation is 0.

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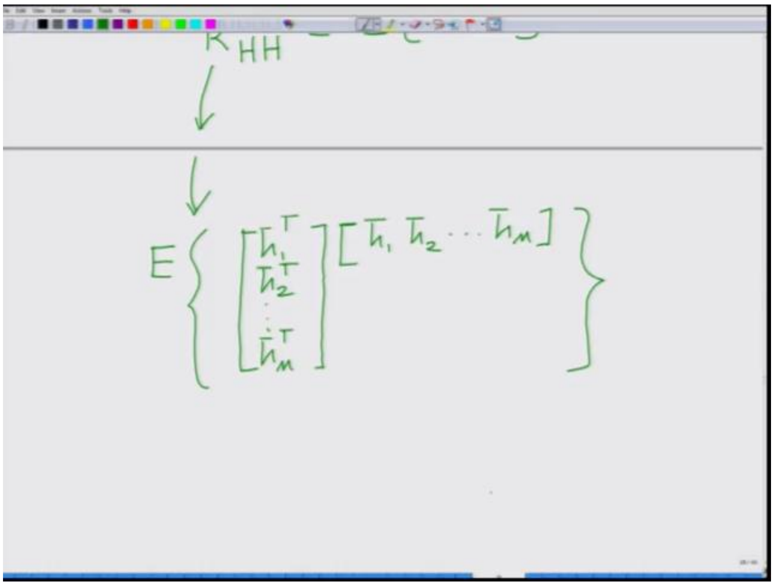

And let us say now we want to compute this quantity R HH equals expected value of HH transpose which I can write as R HH which we can write as now remember H is basically your this is your matrix h 1 bar transpose h 2 bar transpose so on up to h M bar transpose times h 1 bar h 2 bar so on up to h M bar.

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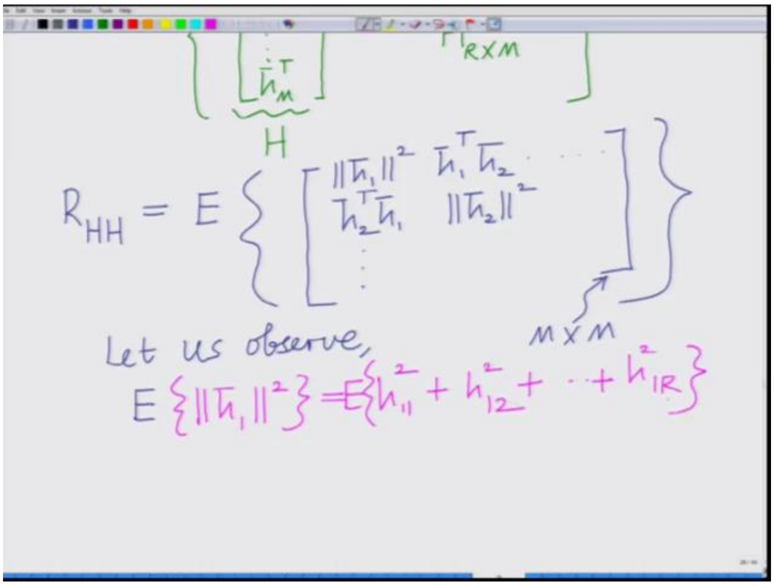

Now look at this is H which is M cross R and this is h transpose naturally H transpose this is an R cross M matrix. And now I am going to simplify this further. If you simplify this this is going to be expected value of well h 1 bar transpose h 1 bar that is norm h 1 bar square h 1 bar transpose h 2 bar that is simply h 1 bar transpose h 2 bar h 2 bar transpose h 1 bar h 1 bar and norm h 2 bar square so on and you can write the elements of this matrix so on.

This will be by the way product of M cross R R cross M this will be an M cross M matrix, this is your R HH covariance matrix that is covariance matrix of the channel matrix h bar. Now let us look at this, now observe let now you want compute this covariance matrix of the expected value of this matrix correct now let us observe this quantity. expected value for instance let us observe and simplify this quantity, let us observe expected value of norm h 1 bar square this is equal expected value of h 11 square plus h 12 square plus h 1R square. Remember that is your h 1 bar we have already specified here this matrix this vector h 1 bar.

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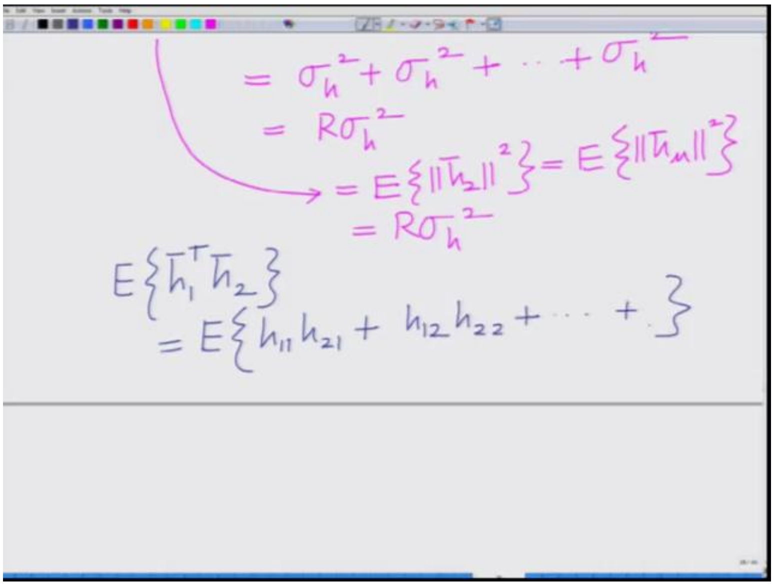

Similarly, you can compute for other vectors also and expected values of each of these remember these are IID random variables each of these quantities as a variance sigma square. So, this is well basically sigma h square each of these quantities is equal to sigma h square so this is sigma h square R times so this is R times sigma h square expected value. And similarly this is also equal to expected value of because all of them are identically expected value of norm h 2 bar square so on up to norm expected value of norm h M bar square each of these is basically equal to your R sigma h square.

Which means basically if you look at these diagonal entries expected value of all these diagonal entries all these diagonal entries will be R sigma h square that much is clear; all the diagonal entries of the covariance matrix will be R sigma h square.

Now, let us look at the off diagonal entries. For instance, expected value of let us look at the off diagonal entries, off diagonal means off the principal diagonal for instance I think again that is also clear for instance if you look at this matrix this is the diagonal or the principle diagonal when we talk about diagonal we talk about the principle rest of the entries are the off diagonal entries. Typically the diagonal entries X demonstrate slightly different properties compare to the off diagonal entry entries as we are going to say expected value of h 1 bar transpose h 2 bar this is equal to expected value of well I can write this h 11 h 21 plus h 12 h 22 plus so on.

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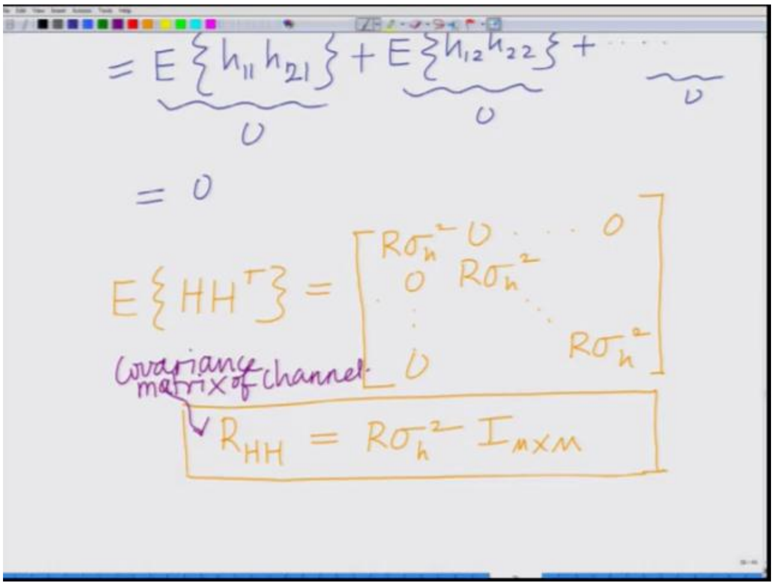

Now, you can say I can split in to the some of the expectations that is the expected value of h 11 h 21 plus expected value of h 12 h 22 so on. And now we can see each of this quantities this is equal to 0 when we are using this independence or basically it raise to un correlated that is expected value of h ij into h kl equal to 0 if i not equal to j or k not equals. Therefore, all this terms expected value of h 11 h 21 - expected value of h 12 h all this terms of 0. So similarly you have 0, so basically this thing is 0. So, what you have net is all the diagonal entries are equal to R sigma h square all the off diagonal entries are 0 in this covariance matrix; off diagonal is equal to 0.

And therefore, that simplifies a covariance matrix therefore now we have nice expression for the covariance matrix that is we have expected value of HH transpose equals expected value of this thing the matrix that we described above that is R sigma h square, R sigma h square, R sigma h square and all the off diagonal entries are basically 0 that is this has the structure R sigma h square times identity M cross M. So, this is your R HH. This is the covariance matrix of your channel R HH equals R sigma h square this is the covariance matrix of the channel or this is basically your channel covariance.

Now let us look at the covariance matrix of the noise. That is expected value of VV transpose because remember to compute the MMSE or LMMSE estimate we need two quantities one is R YY this covariance of the output we need R hy cross covariance between channel h and the output y. In order to compute that we also need some other properties that is basically the channel covariance which we have computed R HH and also now the noise covariance which is equally important.

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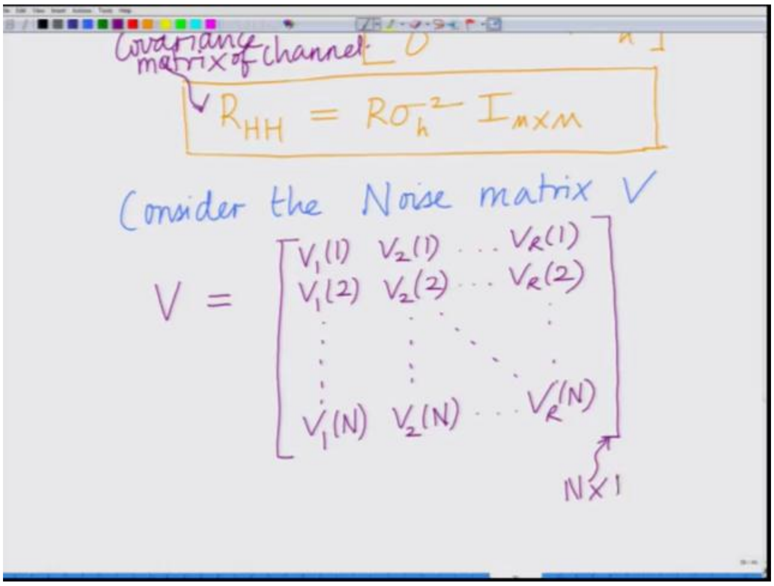

So, the noise covariance again that can be computed in similar fashion; fashion similar to the channel covariance. Consider the noise matrix V we have V equals again V is N cross R matrix I hope you remember from the previous module that is V 1 1, V 2 1 so on up to V R 1; V 1 2 V 2 2 up to V R 2 so on up to V 1 N that is noise sample on antenna one at time N, V 2 N so on up to V R N; this is your noise matrix which is N cross R noise matrix.

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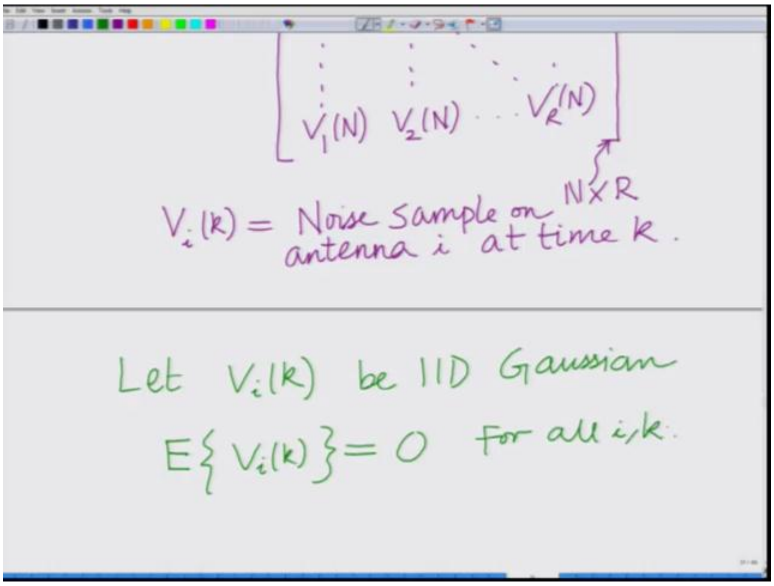

And in this V i coma k equals noise samples on antenna I at time k. And let us now assume the noise is Gaussian, let similar to previous let V i k the different noise samples V i k be IID Gaussian, noise samples at all antennas at all time instance are Gaussian and also independent of each other which means and let us assume that their zero mean again similar follows largely the development of channel coefficient expected V i k equal to 0 for all i coma k.

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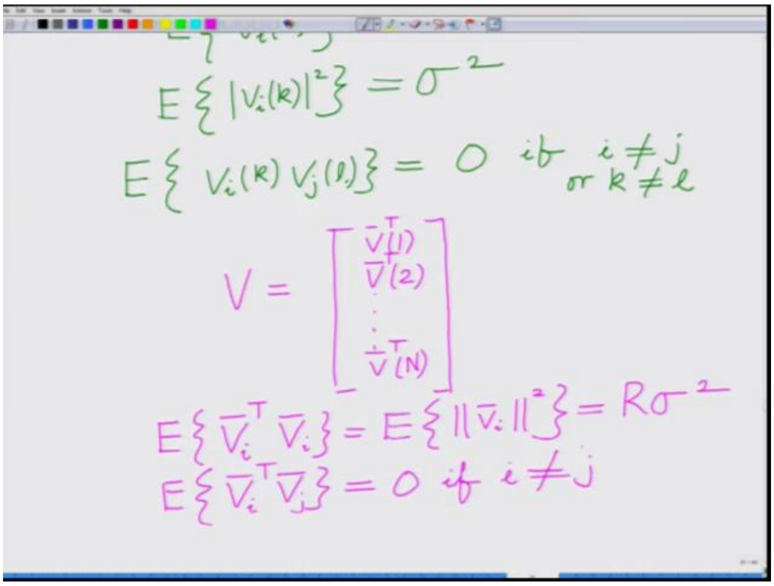

Expected the noise variance expected value of magnitude V i k square this is equal to identical noise statistically identical this independent identical, so all of them has zero mean all of them have power or variance sigma square. And again they are independent therefore expected value of V i k times V j l equal to 0 if i not equal or j or k not equal to l. So the nice sample again very simple model the noise samples V i k are IID Gaussian their zero mean variance sigma square each and the correlation between any two noise samples corresponding to either distinct antennas received antennas or distinct time instance or both is zero.

And now again we can split this into rows that are V 1 bar transpose V 2 bar transpose or are the each these row compare corresponds to different time instant therefore, we can denote this for as V 1 bar transpose, V 2 bar transpose, and V bar transpose of N. So, I can write V the noise matrix as V 1 bar transpose, V 2 bar transpose, V bar transpose of N is collection of N row vectors naturally I say N cross R matrix so it as N rows.

Similar to now I am not going to the entire derivation again because you have considered IID noise elements you can sure that expected value of V i bar transpose V i bar that is expected value of norm V i bar square equals R sigma square where sigma square is the noise variance. Similarly, expected value of V i bar transpose V j bar equal to 0 if i not equal to j.

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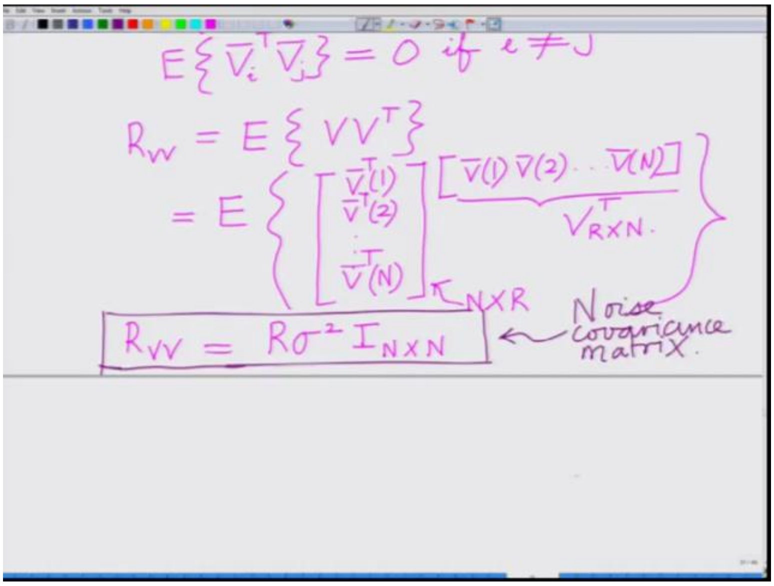

Therefore, the noise covariance R VV this similar to; so therefore, the noise covariance is expected VV transpose which is equal to expected value of V bar transpose 1, V bar transpose 2, V bar transpose N times V bar 1, V bar 2, V bar N. Again this will be where the off diagonal elements all are sigma square the off diagonal elements are 0, so this will be R sigma square I but this will be N cross N identity matrix because look at this, this is a N cross R matrix this is V which is R V transpose which is R cross N. Therefore, VV transpose is N cross N so this R sigma square identity N cross N and this is R VV which is your noise covariance matrix. So, this is your noise covariance matrix.

So, what you have done in this module is basically we have explored the statistical properties of the MIMO input output system module. Further, assuming IID channel coefficients we have derive the channel covariance assuming IID noise covariance, IID Gaussian coefficients we have derive the noise covariance. And now we are going to use this in the computation of MMSE or LMMSE estimate of MIMO channel matrix subsequent. So, we will stop this module here.

Thank you very much.