Baysesian/MMSE Estimation for MIMO/OFDM Wireless Communications
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 22
System Model for Multiple-Input Multiple-Output (MIMO) Downlink Wireless Channel Estimation

Hello, welcome to another module in this massive open online course on Baysesian MMSE estimation for wireless communications. So, previously in the previous modules we have completed the analysis of LMMSE or MMSE estimation for multi antenna wireless channels. Let us now proceed to look at the corresponding estimation; that is LMMSE and MMSE estimation for Mimo channels

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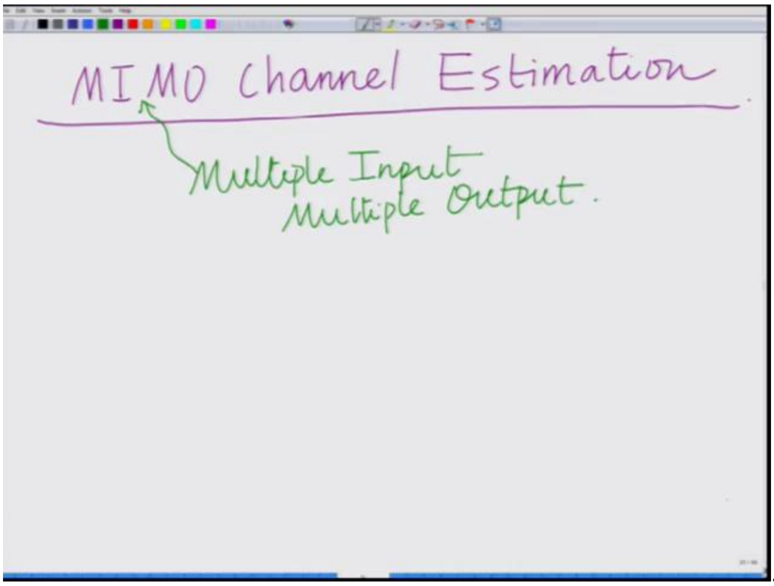

So, what we are going to do, starting today is look at mimo channel estimation, where mimo stands for multiple inputs multiple output. So, what you want to do, is we want to start looking at, the concept or the framework of mimo channel estimation, where mimo stands for multiple input multiple output wireless communication system. And what it means, basically is that you have multiple antenna. Multiple antenna is a transmitter and all as well as multiple antennas in the receiver. Previously, we have considered multiple antennas only at the transmitter, right in the downlink wireless channel now; we are saying we have multiple antennas both at the transmitter and also the receiver. For instance if we look at the downlink channel, we have multiple antennas both the transmitter that is the base station, and also the mobile device which is the receiver. So, multiple input multiple output.

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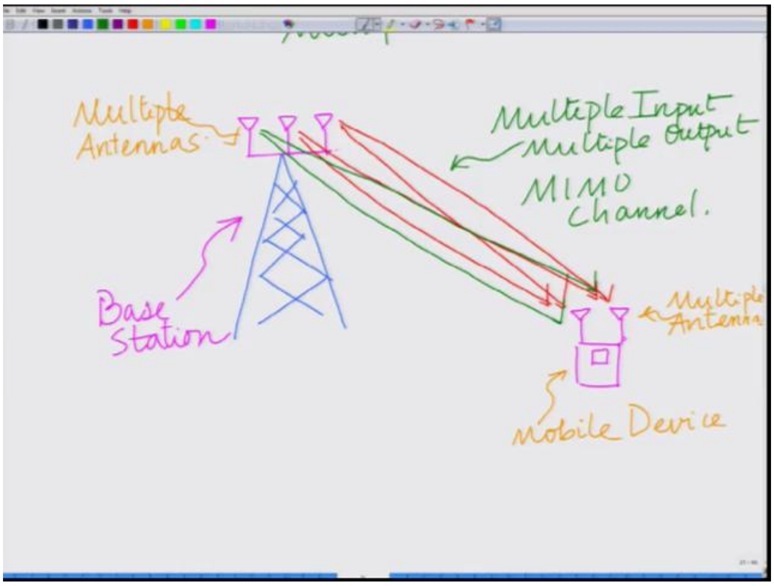

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So, I can draw simple scenario depicting multiple input multiple output wireless communication as follows. So, let say this is my base station. Previously we have considered the base station has multiple antennas, and that remains same as before. So, I still have multiple antennas at the base station. But now in addition my device also has multiple antennas. So, if I have a mobile device I also considered multiple antennas at the mobile device. So, this is my base station, which has your multiple antennas, or also this is becomes your multiple input, because these are the input throw the channel and this is mobile device, and this also has multiple antennas.

So, you have multiple antennas in the transmitter and multiple antennas at the receiver, and therefore, what you have now is basically a full-fledged mimo channels. So, what you have now, is basically a multiple [nose] a multiple input. This is a multiple input, multiple output; that is basically a mimo channels. So, mimo stands for multiple input multiple output where the multiple input mean your multiple transmit antennas multiple output you mean your multiple receive antennas, right and we have illustrated this for the downlink wireless communication scenario, where we have base station which have multiple transmit antennas, and the mobile device which has multiple received antennas.

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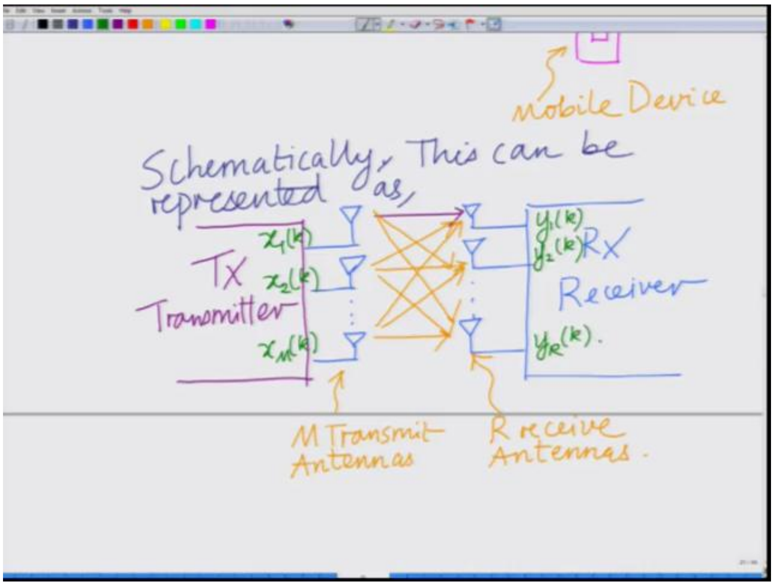

And schematically this can be represented as, let me just write draw a schematically diagram, schematically this can be represented as, let say you have a transmitter. This is your t x; that is your transmitter, and you have your r x; that is the receiver. So, I have transmitter, I have the receiver, I have multiple antennas at the transmitter, I have multiple antennas and the receiver, and between these naturally what I have is the mimo channel.

Look at this I have a path between every transmit received antenna, I have a path between every transmit received antenna pair, between every transmit antenna and every received antenna I have a channel correct. So, I have a path. So, if I have m transmit antennas and r received antennas, I am going to have total of a collection of m cross m times r channels. So, let say I have m transmit antennas, similar to before and I have r receive antennas. I have total of product of m cross r m times r channel is possible paths or possible links where the signal can propagate from the transmitter to the receiver. So, let us consider the symbols transmitted on this m transmit antennas to be x 1 k x 2 k. So, on up to x m k and the received symbols is y 1 k y 2 k y r k.

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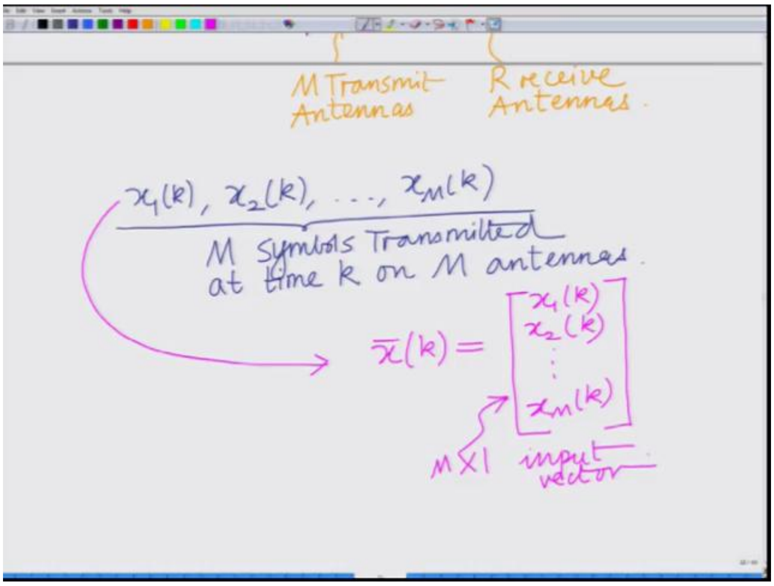

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So, what we are saying is, x 1 k x 1 k x 2 k up to x m k, these are the symbols transmitted. These are the m symbols transmitted on the m transmitted antennas at time k on the m antennas or rather the transmit antennas, and these. Since you have m transmit antennas, you can transmit m symbols and these I can write as the transmit vector x bar k equals x 1 k x 2 k up to x m k. This is your m cross 1 vector or this is the same as your m cross 1 input vector that we have seen previously. So, x 1 k x 2 k x m k are the m symbols transmitted from the m transmitter. Naturally y 1 k y 2 k y r k are the r received symbols on the r receive antennas. Now, since we have multiple received antennas we will have multiple received symbols.

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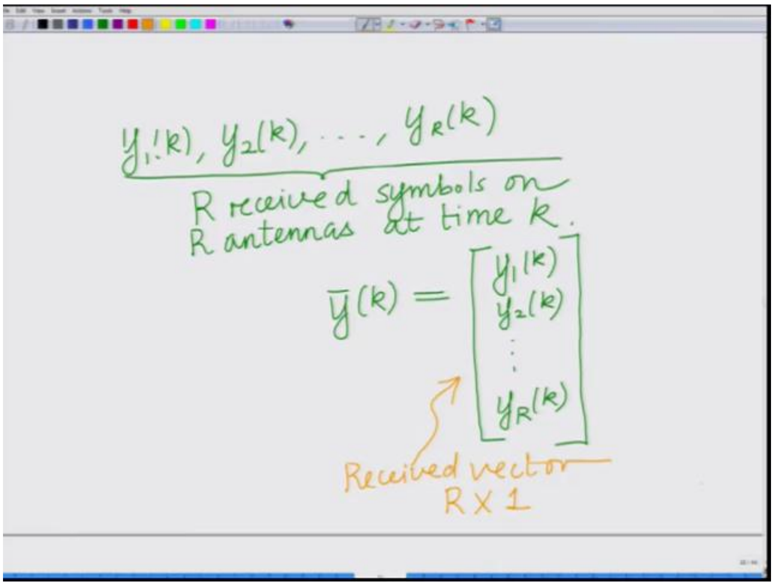

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So, what we have r y 1 k y 2 k y r k, these are r received symbols on r antennas at time k. And the received vector y bar k, the received vector y bar k equals, this equals y 1 k. this is y 1 k y 2 k y r k this is the received vector, and this is r cross 1 dimensional. So, we have an x bar k which is the transmitter vector which m dimensional, we have an r cross 1 vector y bar k which is r dimensional which is the received vector correct.

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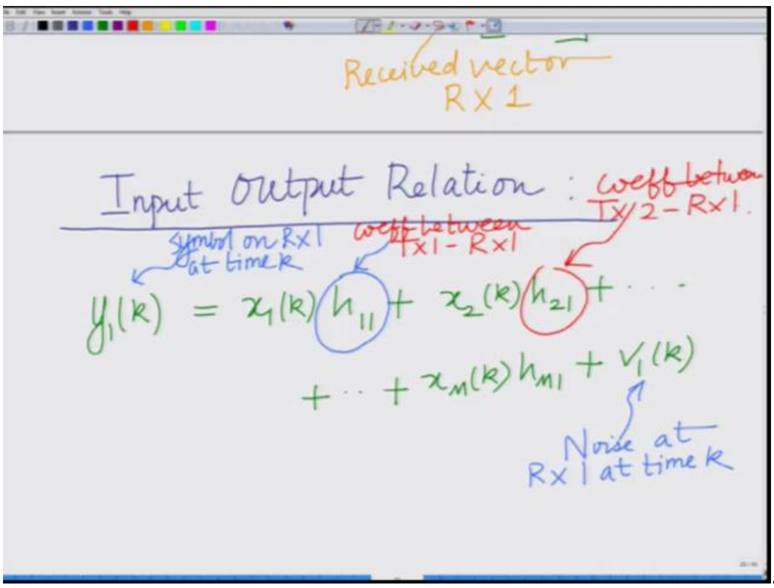

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Now, let us see how we can model the input output relation in the mimo system, input. Let us see how we can model the input output relation, and the input output relation can be modeled as follows. Let us look at y 1 of k symbol on receive antenna one at time instant k, this is given as x 1 k h 1 1 plus x 2 k h 2 1 plus so on plus so on x m k h m 1 plus v 1 k. And what is y 1 k y 1 k is symbol on received antenna 1 at time k. symbol on received antenna 1 at time k. what is v 1 k. v 1 k is noise at received antenna 1 at time k. Now look at k this quantity h 1 1, this is the coefficient between transmit antenna 1 and received antenna 1, h 2 1 coefficient between transmit antenna two and receive antenna 1 and so on and so forth h 1 1 coefficient between t x 1 r x 1. h 2 1, this is the coefficient between t x 2 and r x received antenna one.

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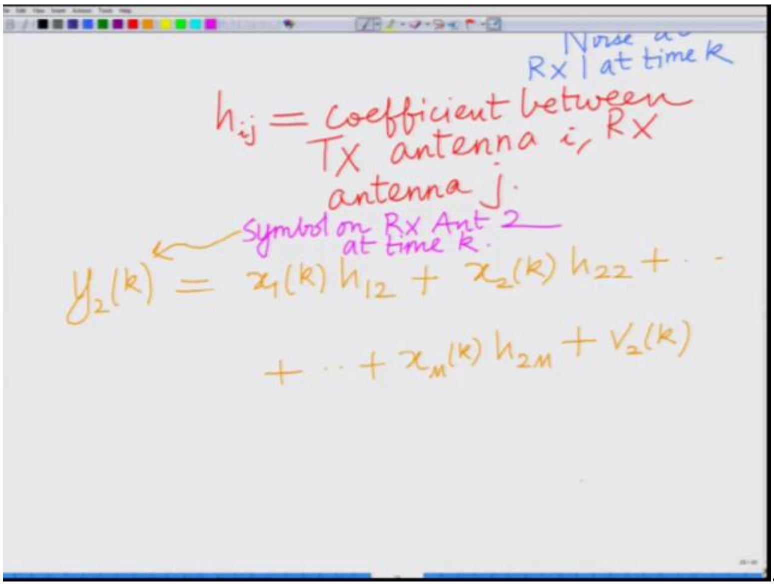

So, h j I or h I J equals coefficient between t x antenna I comma r x t x antenna I and r x antenna j. Similarly I can write y 2 k symbol received on receive antenna two. y 2 k is the symbol received on receive antenna two at time instant. Remember we have r received antennas, so at any point of time k we have r received symbols y 1 k y r k up to y 1 k. So, y 2 k, I can express it as x 1 k h 1 2 plus x 2 k h 2 2 plus so on plus x up m k h 2 m plus v 2 k. Remember this y 2 k symbol on received antenna 2, the symbol on receive antenna 2 at time symbol on received antenna 2 at time instant k so on and so forth.

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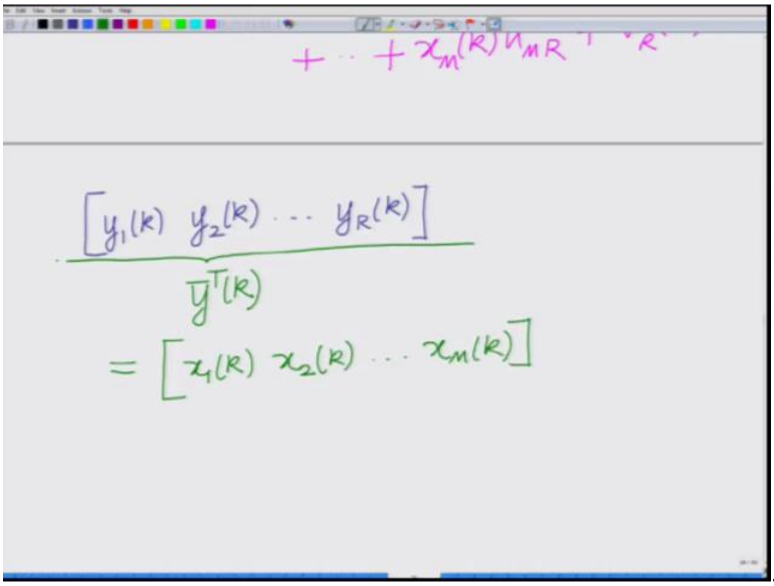

I can extend this model to y r k y r k equals x 1 k h 1 r plus x 2 k h 2 r plus x m k h m r plus v r k, y r k is the symbol received on received antenna r at times instant k . So, now, have the relations for y 1 k y n 2 k up to y r k. I can now put this together as a vector. So, now, I can form the vector form the model as follows. So, I can write and you can verify this, I can write y 1 k y 2 k y r k. I am writing this as a row vector you can check this. This is basically y bar transpose of k, where y bar of k we have defined previously that contains the r dimensional vector or received symbols at time instant k this is y bar transpose k. This is equal to x 1 k x 2 k so on up to x m k. this is the transmit vector which is x bar of k time instant k, x bar transpose k.

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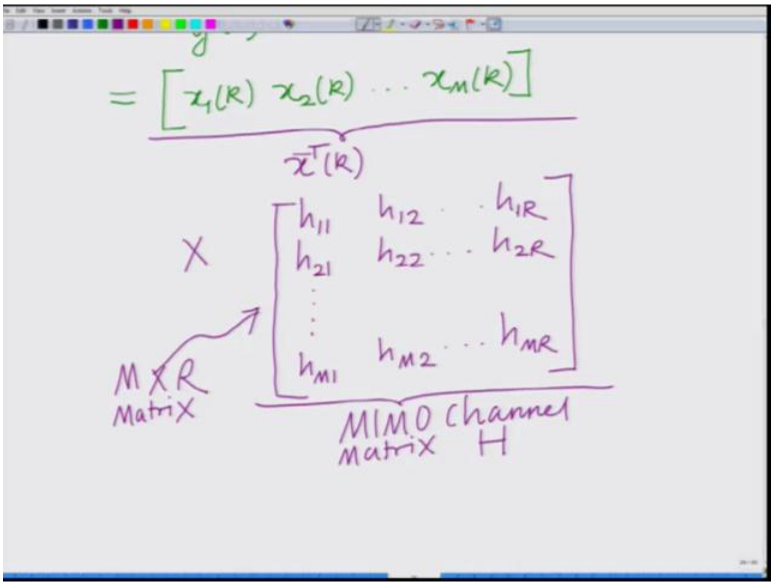

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Where x bar k is a transmit vector at time instant k, times the product h 1 1 h 1 2 so on to h 1 r h 2 1 h 2 2 so on up to h 2 r h m 1 h m 2 so on up to h m r. This is your mimo channel matrix h and you can clearly see this is an m cross r. So, we have now the m cross r mimo channel matrix h correct, where m is the number of transmit antennas r is the number of receive antennas correct

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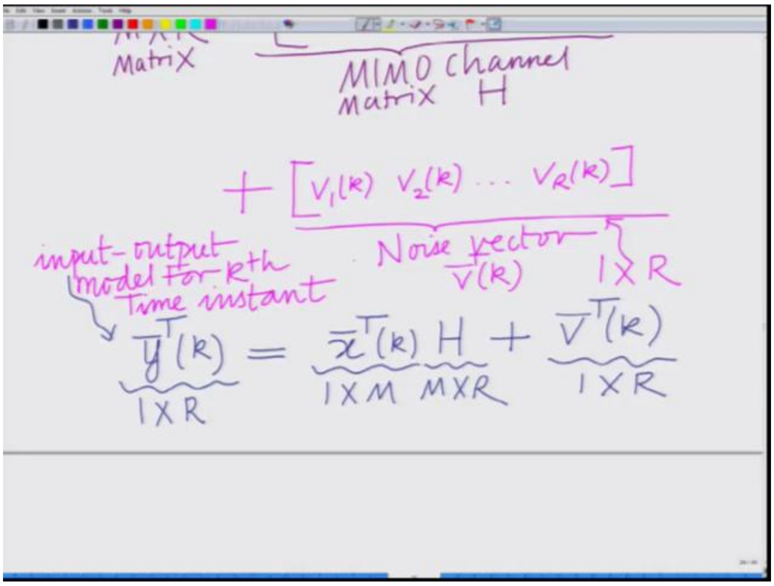

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And what is left is the noise vector, and the noise vector can be written as. So, this is basically your matrix h plus the noise vector, noise vector is v 1 k v 2 k so on up to v r k. This is the noise vector v bar transpose of k, and therefore, a time instant and this is also you can clearly this is; obviously, you have r receive antenna. So, you have r row noise sample. So, this is a 1 cross r vector, and therefore, what you have is y bar transpose k equals x bar transpose k times the mimo channel matrix h plus v bar transpose k. Just to confirm this again this is your one cross r received vector this is one crossed m transmit vector, this is m cross r mimo channel matrix this is one cross r vector noise vector v bar transpose k. . So, y bar transpose k equals x bar transpose k times mimo channel matrix h plus v bar transpose k. This is the model input output model for kth time instant; this is the input output model for the k th time instant

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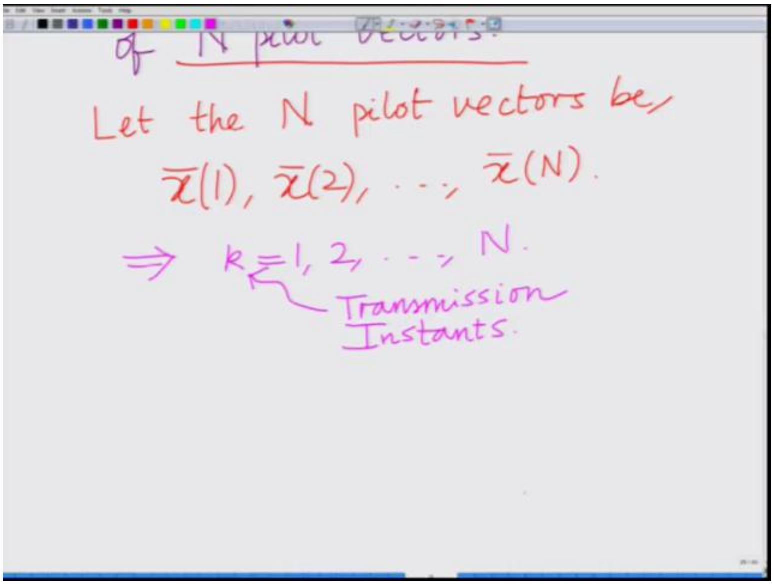

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Now let as consider the transmission or. So, this is for a particular time instant k. now let us consider the transmission of n pilot vectors; that is let us consider n transmission instance. So, let us consider the transmission of n pilot vectors. By the way we have not mentioned it explicitly, but it should be clear to you that this x bar of k this is nothing, but a pilot vector. Since your considering this for channel estimation, we are assuming this to be known or rather we are going to be assuming this to be known, similar to your multiple antenna downlink channel estimation scenario, where we have the transmission of pilot symbols. Now, we have the transmission of pilot vectors. Each x bar k is an m dimensional pilot. We already seen in the context of multi antenna downlink channel estimation where each vector is a pilot vector, and that continues to be the case x bar k is the pilot vector transmitted at time instant k, the input vector or for the purpose of channel estimation this is also the transmit of known symbols, and these are known as pilot symbol. So, naturally we have a pilot vector.

We considered the transmission of n pilot vectors. So, let these pilot vectors. Let these n pilot vectors be x bar 1 x bar 2 so on up to x bar of n. These are the n pilot vectors, implies these correspond to the transmission of well k equals s 1 2 up to n transmission instants. These are the n transmission instance, and let the n corresponding vectors the received vectors.

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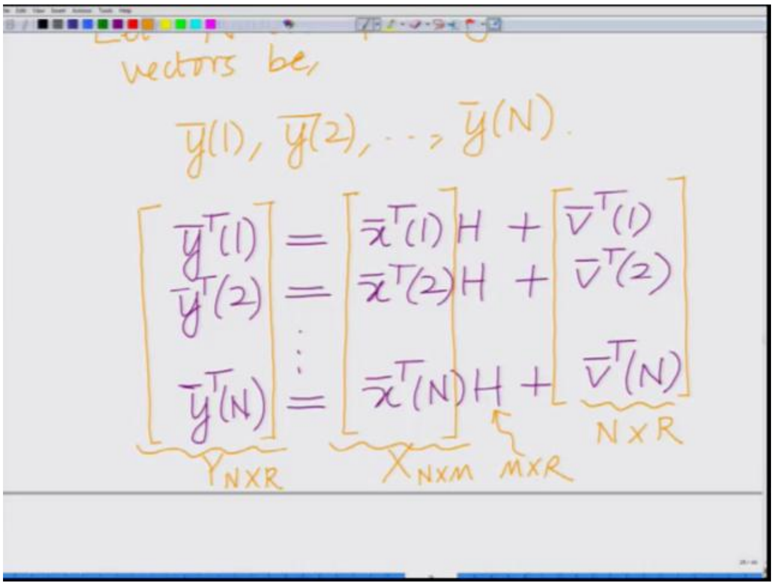

Let the n corresponding received vectors be y bar 1 y bar 2 up to y bar n. Then what we have, is basically the input output system model, can now be formulated as well for every transmission instant we have y bar transpose 1 equals x bar transpose 1 h v h v bar transpose 1 y bar transpose 2 x bar transpose 2 h plus v bar transpose 2. So, on y bar transpose n equals x bar transpose n plus h plus v bar transpose n. Now, I can stack all these as a matrix. This is your matrix y which as n rows each of size r. So, this is n cross r x bar transpose 1 x bar transpose 2 up to x bar transpose n. This is your pilot matrix, which is n cross m, mimo channel matrix which is m cross r, and we can stack this noise vectors as n rows, each row of dimension r, this is m cross r noise matrix; therefore, stacking these as matrices what we have

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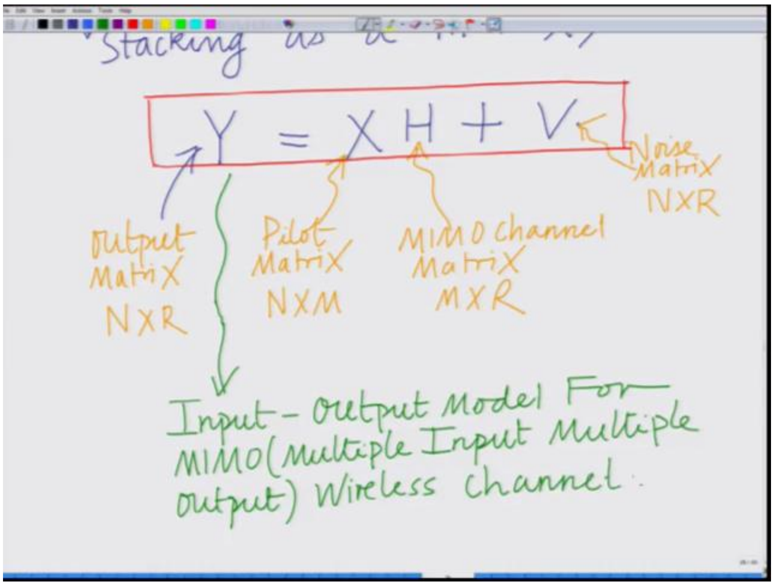

So, stacking as matrices, rather stacking as matrix, all the input output vectors corresponding to n time instance we have y equals x times h plus n. You can contrast this with the multi antenna downlink channel estimation problem, where we have y bar equals x times h bar plus v bar plus v bar. Now we have matrices. Let me just correct this noise matrix.

This noise matrix we are going to call v correct naturally. So, this is your y x, this is your channel estimate and this is your matrix, let us call this noise matrix as v. So, this is your output matrix or your observation matrix or received matrix, which is your n cross r, this is your pilot matrix which is con which continues to v similar to the multi antenna channel estimations in our pilot matrix which is n cross m; where n is the number of pilot vectors m is the number of transmit vector. This is now a mimo channel matrix we have already seen this. This is now a mimo channel matrix, which is m cross r m is the number of transmit antenna r is the number of receive antennas and this is the noise matrix, which is basically your n cross r noise matrix, and therefore, this is your final this mimo input output system model.

This is your input output system model for the mimo channel; that is the multiple input multiple output wireless channels. Let me just write that on also, this is the input output model for your mimo or basically multiple. This is the input output model for a mimo or multiple input multiple output wireless channel. So, this is the input output model for the multiple input multiple wireless channel. So, what we have done in this model is, basically we have developed this input output model for the mimo wireless channel, and there is a multiple input multiple output wireless channel, considering m transmit antennas, and considering n received antennas, because you have multiple transmit antennas and multiple received antennas m transmit antennas and r received antennas, because you have multiple transmit antennas and multiple received antennas this is becomes a multiple input multiple output wireless channel model. We have considered the transmission of n for pilot vectors.

We first develop the input output system model for a single transmitted pilot vector single received vector y bar; that is y bar transpose k equals x bar transpose k times the pilot mimo channel matrix h plus v bar transpose k, where v bar is the noise at time instant k. Stacking all these, considering the transmission of n pilot vectors stacking all these for m time instance we have developed the mimo input output system model, where y the received the pilot matrix, the receive the matrix is, the observation matrix y which is n cross r is equal to x the pilot matrix n cross m times h the mimo channel matrix m cross r plus the noise matrix which is n cross r . So, this is mimo model channel that we have set up, we are going to use the subsequently carry out mimo channel estimation that is present the LMMSE or MMSE estimation for the Mimo channel matrix h. So, we will stop this module here and continue in the subsequent lectures.

Thank you