Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communications
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Lecture – 20
Example of Linear Minimum Mean Squared Error (LMMSE)
Estimate for Multi-Antenna Downlink Wireless Channel

Hello. Welcome to another module in this massive open online course on Bayesian MMSE Estimation for Wireless Communication Systems. So far we have looked at the MMSE estimate or even the LMMSE estimate for a non-Gaussian vector parameter h bar, where h bar where h bar and we particularly illustrated this for the case of estimation of a multi-antenna downlink wireless channel. So, now to understand it better let us do a simple example to see how this vector channel estimated is computed.

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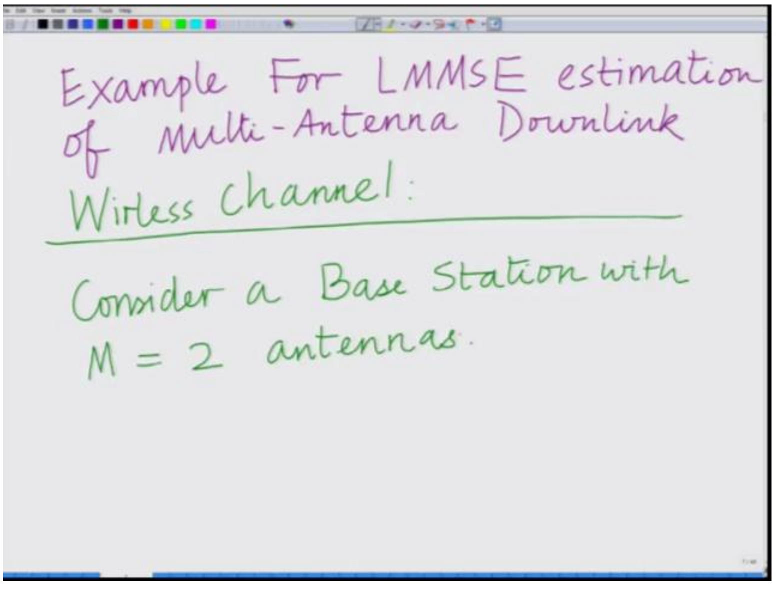

So, in today's module we will be doing example for the multi-antenna downlink vector parameter channel estimation. Of course the movement I say LMMSE you have to understand that if the parameter is Gaussian then it becomes MMSE, example for LMMSE estimation of multi-antenna downlink wireless channel, we would like to an example for this.

Consider towards this example consider a base station with M equal to 2 antennas, so we want to consider a base station with M is equal to 2 antennas. That means, the base station has 2 transmitted antennas. And of course as per the scenario we considering the user only have a single receive. Then later we will extend to this a scenario where the user also as the multiple antennas then it will become a MIMO channel; that is the multiple inputs multiple output wireless channel. So, right now we are considering only multiple and transmit antenna, so it is a MISO channel; multiple input single output channel.

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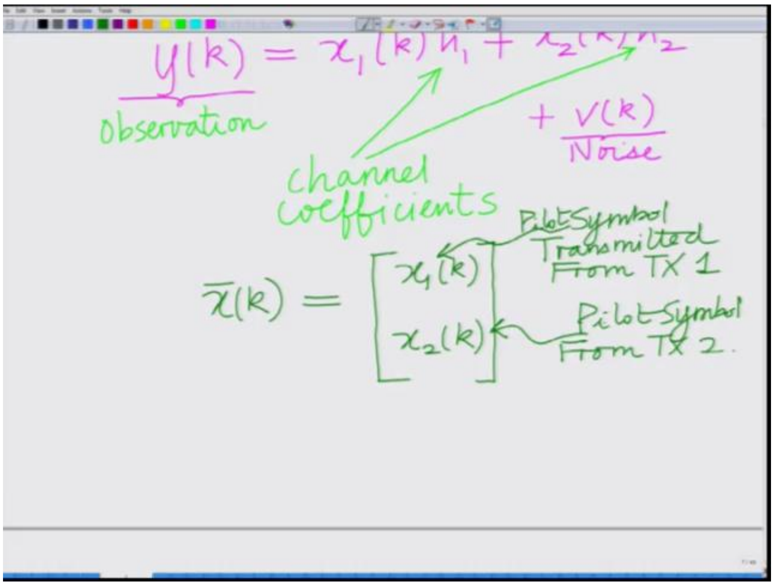

So, the system module for this can be given as y k which is the received symbol at time instant k equals x 1 k times h 1, h 1 is the channel coefficient corresponding to the first transmit antenna plus x 2 k times h 2 plus of course v k; v k is the noise, y k is the observation. And we have h 1 and h 2 these are the two channel coefficients of the multi-antenna channel. Further, what we have is that our pilot vector x of k or x bar of k this is equal to the pilot vector x bar of k is equal to x 1 k x 2 k, where time instant k x 1 k is symbol transmitted from transmit antenna 1 from transmit antenna 1 and this is the pilot symbol in fact both are pilot symbols from transmit antenna is a pilot symbol from transmit antenna 2.

So, we have pilot symbol from transmit antenna 1 and pilot symbol. This is the pilot vector x bar of k which consist of two pilot symbols; x 1 k from transmit antenna 1 x 2 k from transmit antenna 2.

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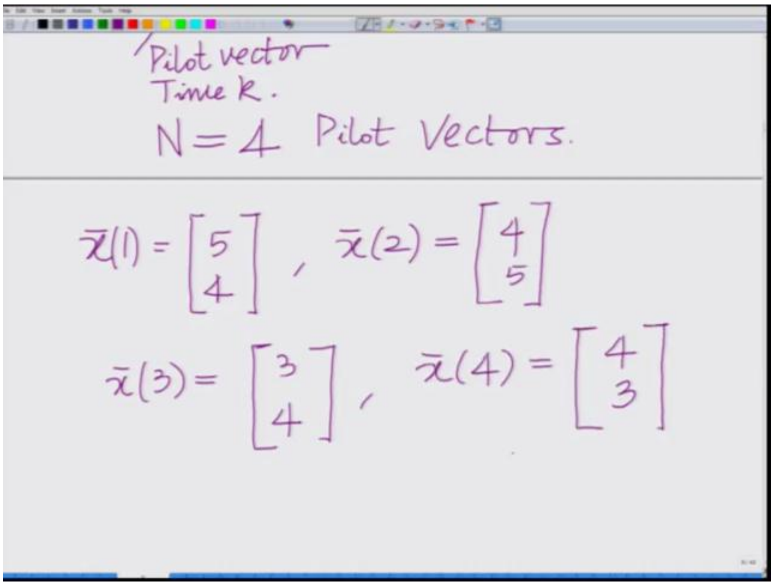

Now in this particular example, so each of this is a pilot vector at time k. Now let us consider four such pilot vectors. Considers in our example N equal to 4 pilot vectors, let us say the pilot vectors are x 1 bar equals 5 coma 4, 5 4 this is a first two pilot vector x 2 bar equals 4 5 x bar 3. Let us say x bar 3 equals 3 4 and x bar 4 equals 4 3. These are the 4 pilot vectors. And consider the observations then then corresponding to this we will have see where N equal to 4 pilot vectors corresponding to this we will have N equal to 4 observations.

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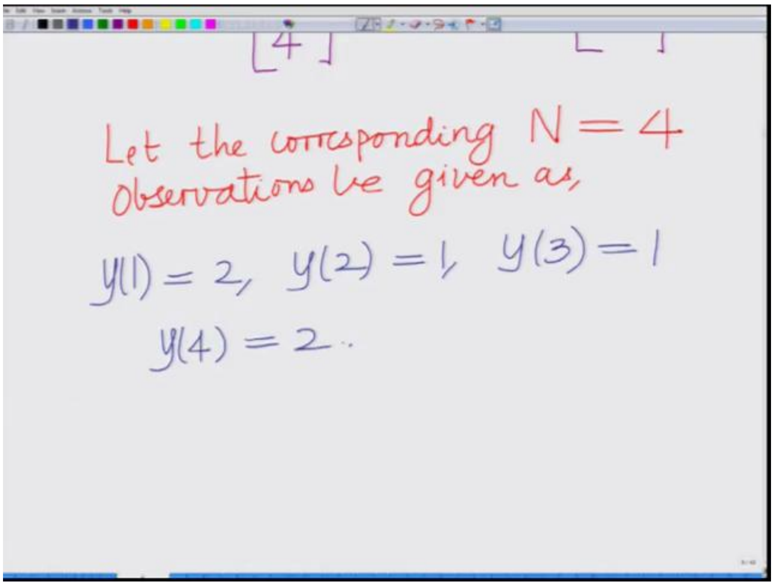

So, let the corresponding N equal to 4 observations these be given as y 1 equals 2, y 2 equals 1, y 3 equals 1, y 4 equals 2.

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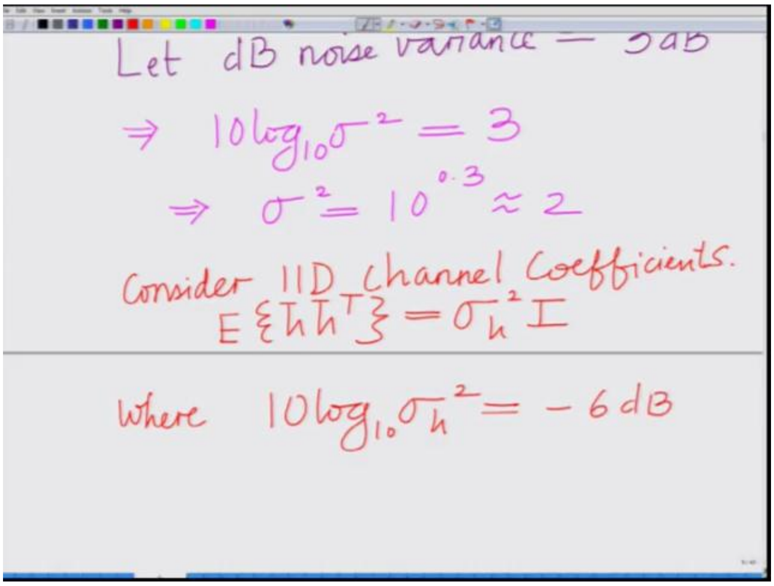

Let the dB noise variance be equal to 3 dB, this implies that dB noise variance will basically log 10 of sigma square equals thr3ee which implies sigma square equals 10 to the power of point 3 which is approximately equal to 2. We already done this several times that is 3 dB is approximately equal to 2. So, 10 to the power of 0.3 is approximately equal to 2, so 3 dB noise variance means noise variance is equal to 2.

And also we are considering IID channel coefficients, not necessarily Gaussian that is expected value of h bar h bar transpose is if you consider IID channel coefficients and the covariance matrix will be proportional to identity. So, expected value of x bar h bar h bar transpose is equal to sigma h square I, where sigma h square in dB that is the average channel variance of the average channel gain in dB equals minus 6 dB.

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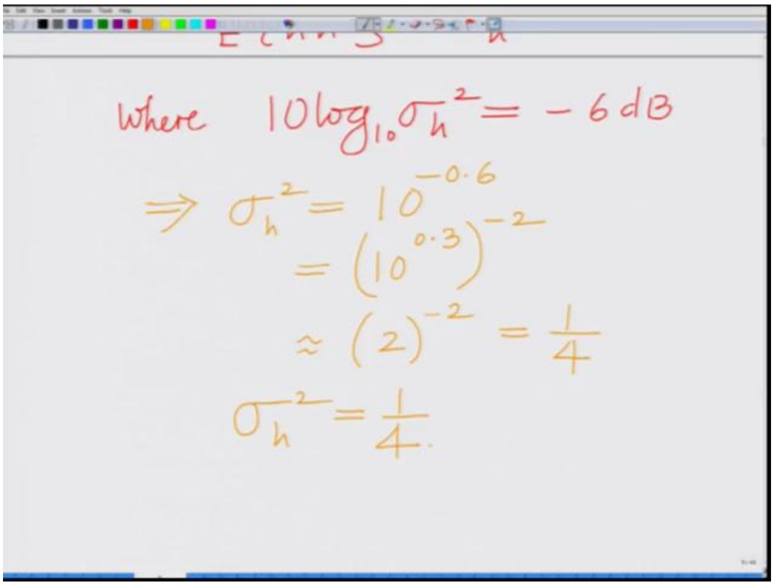

Now, what does this imply? This implies that sigma h square equals 10 to the power of minus 0.6 this is equal to 10 to the power of 0.3 whole raise to the I am writing this in terms of 10 to the power of minus 0.3, because we know 10 to the power of 0.3. This is 10 to the power of 0.3 is approximately to 2, so 10 to the power of 2 raise to the power of minus 2 this is equal to 1 over 4. So, sigma h square equals 6 dB in minus 6 dB implies sigma h square equals 1 over 4. So, that is what we have.

Now what we need to now, we need to compute the LMMSE estimate of the channel vector h bar. Of course, I have said this many times if the channel vector h bar is a

Gaussian channel vector then LMMSE estimate that is a linear minimum mean squared error estimate itself is the MMSE estimate is the minimum mean squared error estimate. Of course, if it is not Gaussian then it is simply the LMMSE estimate. This is the best among the class of linear estimates. This is the something to keep in mind I will keep repeating this frequently because it is a certain, but very important point.

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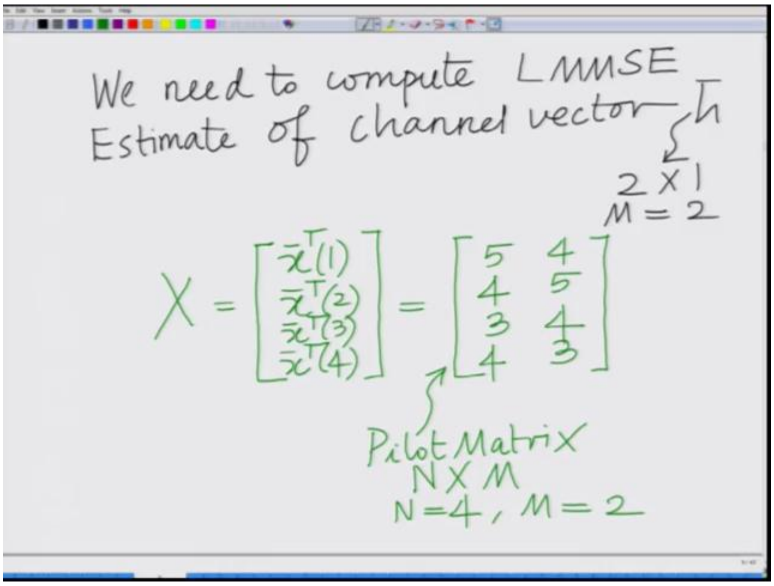

So, we need to compute the LMMSE estimate of channel vector h bar by the way this is M cross 1 channel vector which means it is 2 cross 1, since M is equal to 2; we have 2 antennas at the base station. Now for this first let us form the pilot matrix x which is equal to x bar transpose of 1 x bar transpose of 2 x bar transpose of 3 x bar transpose of 4, where x bar 1, x bar 2, x bar 3, x bar 4 are the 4 pilot vectors. So, this will simply be 5 4, 4 5, 3 4, 4 3 and this is a pilot matrix.

This is a pilot matrix which is remembered N cross M that is N is equal to 4 and M is equal to this is a N cross M pilot matrix. So, now we have the pilot matrix. Remember this is your x which is the pilot matrix. This is the first important aspect of multi-antenna channel distribution we have to form the pilot matrix.

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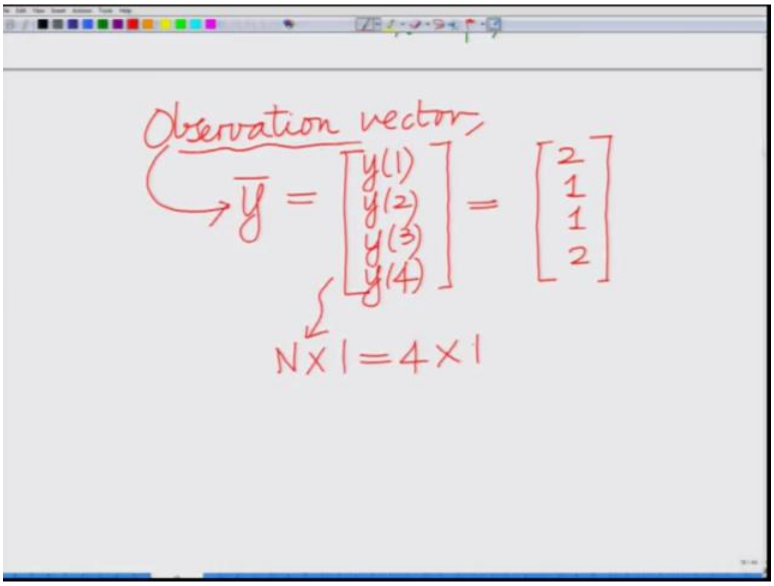

Next we form the observation vector that is y bar. Remember y bar is N cross one where N is the number of observation. Next we form the observation vector y bar equals y 1 y 2 y 3 coma y 4 which is basically your 2 1 1. And now this is your observation vector. So, observation vector is y bar and y bar is N cross 1 this is equal to 4 cross 1 observation vector.

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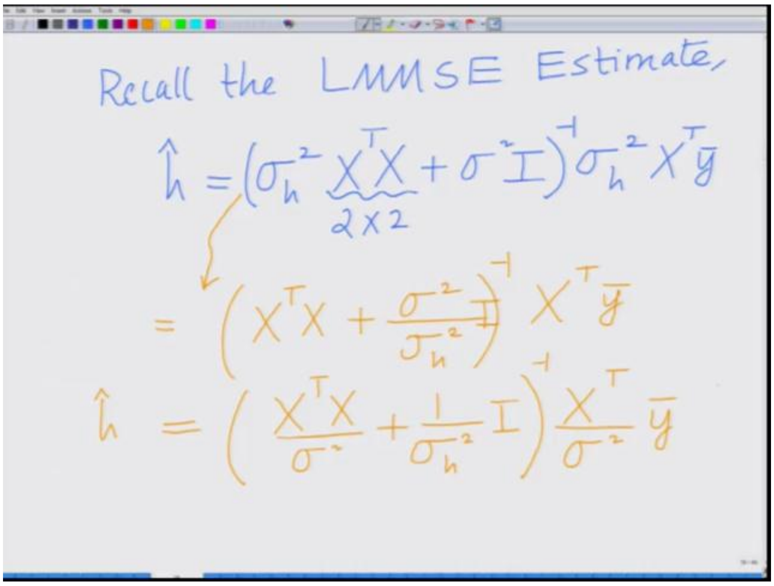

Now, recall the LMMSE estimate or MMSE estimate recall that to be more precise the LMMSE estimate; linear minimum mean squared error. Recall that this is given as h hat which is the LMMSE estimate equals, we derived this in the previous module sigma h square x transpose x plus sigma square identity inverse times sigma x square times x transpose y bar. Now remember again once again we point out the previous module that this is much more simpler version that is computing x transpose that sigma h square x transpose x plus the sigma square identity inverse because look at this x is 4 cross 2 therefore x transpose x will be 2 cross 2 matrix; this is a 2 cross 2 matrix.

So, inversion of these 2 cross 2 matrix is much simpler, but if you are considering x x transpose that is other version other formulation based on x x transpose then that will be a 4 cross 4 matrix. So, we have to compute the inverse of 4 cross 4 matrix which is much more complicated compared to this which is much more simpler.

Now I will proceed, basically now of course one can also now notice that I can also simplify this slightly by bringing out this sigma x square sigma square. I can write this also as first let us bring out sigma x square, so x transpose x plus sigma square by sigma h square inverse, of course when sigma x square comes out it is comes in the denominator that cancels with sigma x square so this becomes x transpose y bar. And now bring out sigma square so this becomes x transpose x by sigma square plus 1 over sigma h square times identity, of course here there is an identity means sigma x square times identity into inverse into x transpose by sigma y square times or slightly more convenient to use this formulation.

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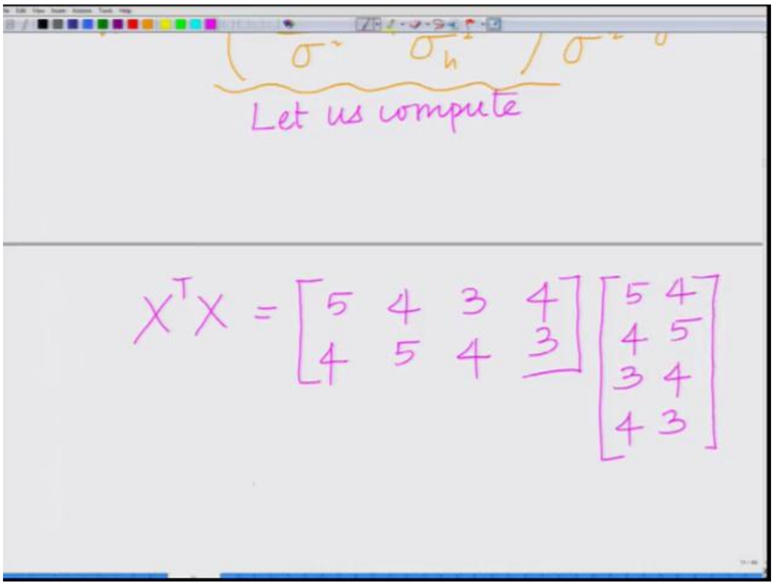

Now, let us compute this matrix. So, let us compute this matrix x transpose x divided by sigma square plus sigma 1 over sigma x square I inverse. So, now towards this first let us start by computing x transpose x; we now x that is the pilot matrix so x transpose x simply take the transpose of x. So, this is 5 4, 4 5, 3 4, 4 3; 5 4, 4 5, 3 4, 4 3 is transpose of the matrix into itself you can compute this. This is your x transpose this is your x.

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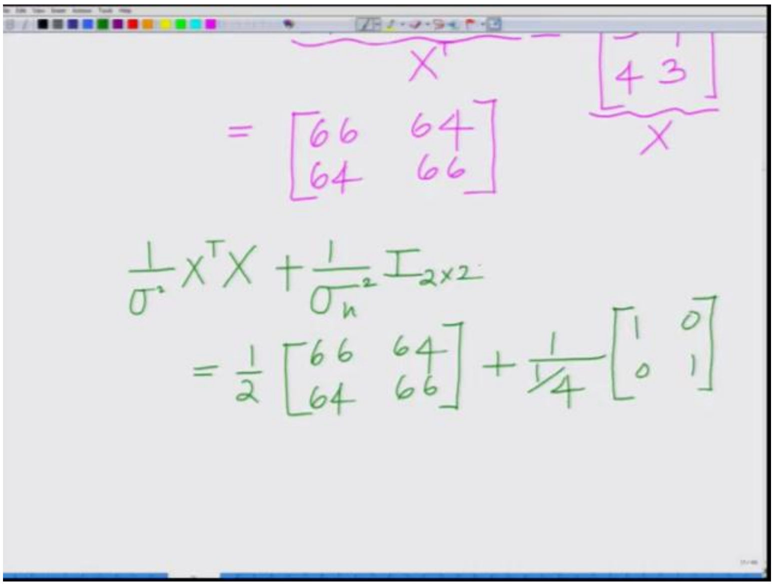

And this will become basically your 66 you can compute this 66, 64, 64, 66 is a symmetric matrix. Of course x transpose x is symmetric matrix transpose of this matrix is equal to itself and naturally you can say that is a symmetric matrix, it is a symmetric 2 cross matrix.

Now, let us compute this quantity 1 over sigma square x transpose x plus 1 over sigma h square identity this becomes half 66, 64 into 64, 66 plus 1 over sigma h square; sigma h square is 1 1 by 4, so 1 over 1 by 4 times identity 1 0 0 1. Of course, this is the 2 cross 2 identity matrix that is the N cross M identity matrix which will be equal too.

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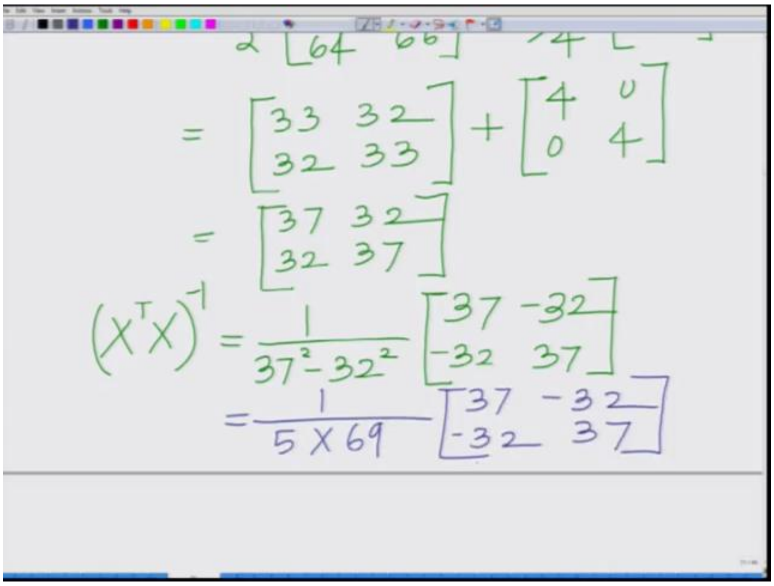

Now you can compute this thing, this will be 33 32, 32 33 half of x transpose x plus 1 over 1 over 1 by 4 times identity matrix is 4 0 0 4, this will be equal to 37 32, 32 37. And now we can say we can complete the inverse of this matrix easily. The inverse of this matrix is 1 over the determents since this is the 2 cross 2 matrix that is 1 over 37 square minus 32 square that is 37 square minus 32 square times switch the diagonal elements 37 37, because both are same negative of the off diagonal elements minus 32 coma minus 32. At this will becomes 1 over 37 minus 32 square is 37 minus 32 that is 5 times 37 plus 32 that is 69 times of course 37 minus 32 minus 32 37.

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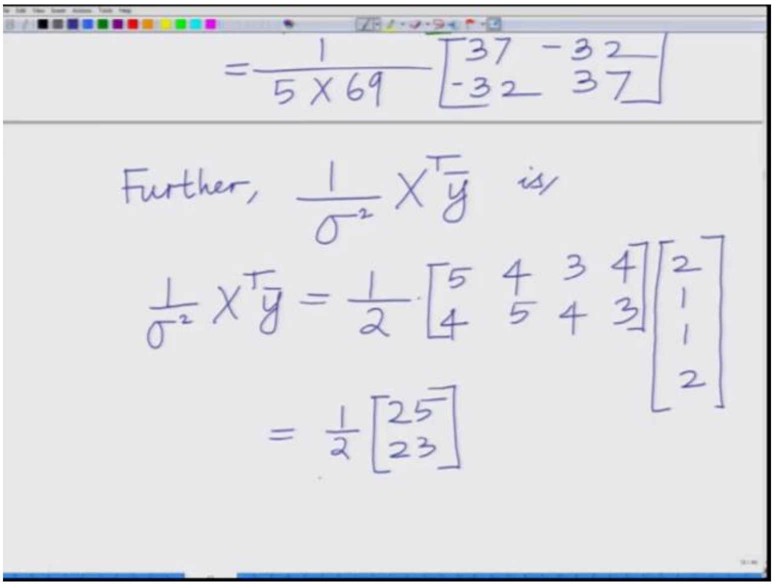

Now, let us compute this other quantity 1 over sigma square x transpose y. Further, 1 over sigma square x transpose y bar is what this is, this is 1 over sigma square times x transpose y bar; so sigma square is 2 so 1 over 2 x transpose is 5 4, 4 5, 3 4, 4 3 times 2 1 1 2 which is equal to half 25 23. And therefore we have computed these both this components 1 over sigma x transpose y and the other. So, now we put both these things together to compute the final LMMSE estimate.

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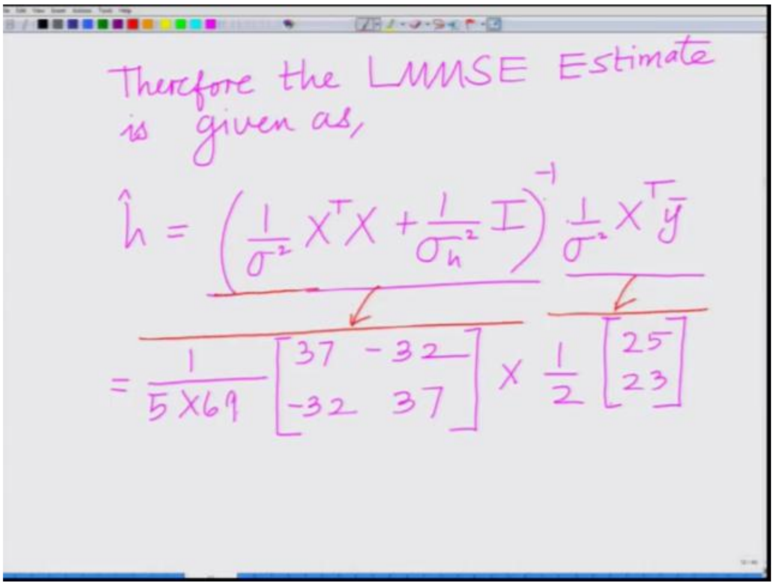

Therefore, the LMMSE estimate is given as h hat is equal to the first part is 1 over sigma square x transpose x plus 1 over sigma x square identity inverse into 1 over sigma square x transpose y bar. We have computed both these quantities above this will be basically your 1 over 5 times 69 into 37 minus 32 minus 32 coma 37 times this quantity; that is 1 over sigma square x transpose y that is half we have over here 25 23 that is a column vector 25. It should be clear to you that this is basically 1 over sigma square x transpose y, and this quantity is basically this 1 over sigma square x transpose x plus 1 over sigma h square I inverse.

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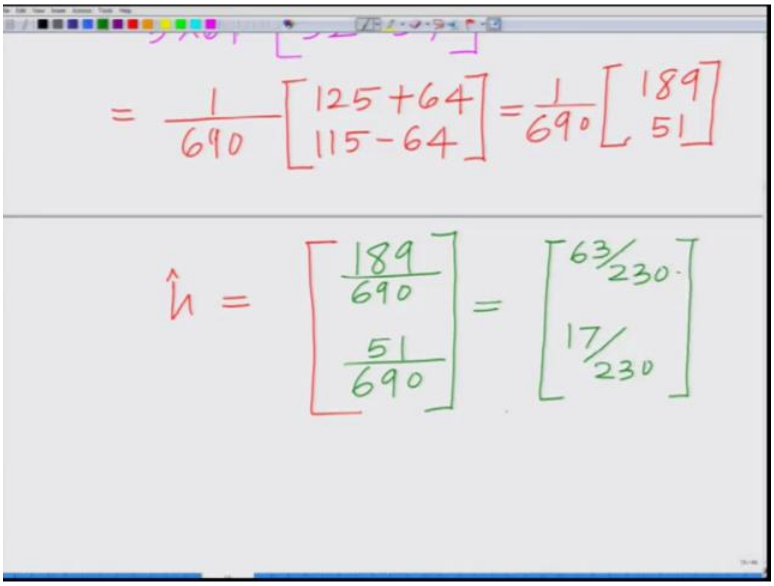

Now you multiply these two things together and what you are going to get is basically 1 over 690 times and you can check this 125 plus 64 115 minus 64 equals 1 over 690 times, well this becomes 189 coma 115 minus 64 that is 51.

So, your h hat is basically if you write it explicitly 99 divided by 690 and the second component is 51 divided by 690 and taking three common both the numerator and denominator of each component what we have is 63 divided by 230 and 17 over 230.

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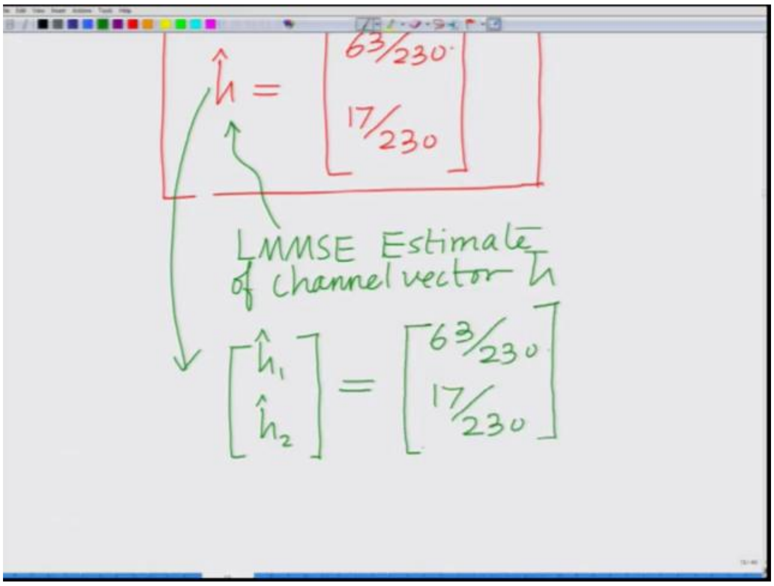

And therefore, what we have is net in LMMSE estimate h hat equals let us this write this down 63 divided by 230 17 divided by 230. And slightly more complicated to compute than the simple least squares estimate people will realize because of the complicated nature of the expression for the LMMSE estimates. So, this is basically your LMMSE estimate of your channel LMMSE estimate of the channel vector h bar. Therefore, now if you look at this h hat is basically estimate with channel vector h bar h bar itself is a channel vector corresponding to two transmit antennas therefore it has two components. So, h hat is basically of the form h 1 hat h 1 hat which is estimate of channel coefficient 1 and h 2 hat which is estimate of channel coefficient 2, so this is 63 by 230 and 17 by 230 which implies.

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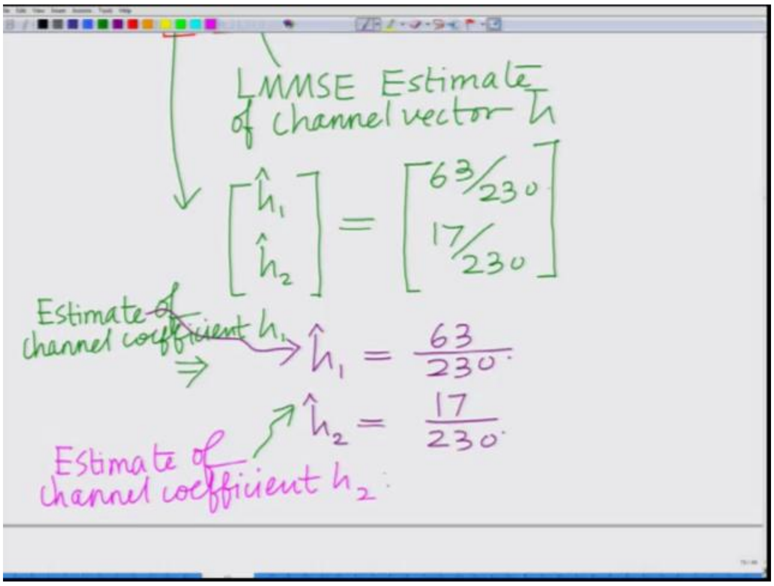

Basically now if you just write it explicitly again although all of these are very clear from the example just to write this a little more explicitly this 63 by 230 and h 2 hat equals 17 by 230. What is the h 1 hat estimate of h 1 that is channel coefficient 1 estimate of channel coefficient h 1, and what is h 2 hat this is your estimate of channel coefficient h 2; this is estimate of channel coefficient of h 2. So, this completes our example for the LMMSE estimate of the vector parameter which is a multi-antenna channel vector for the downlink.

So, what we have done in this we have consider simple example for multi-antenna downlink channel estimation with M equal to 2 antennas, N equal to 4 pilot vectors, consider the pilot matrix, the observation vector illustrated how to compute the LMMSE estimate. We have derived the LMMSE estimate from which the channel vector which one can extract naturally the LMMSE estimates of the individual channel coefficients.

So, we will stop with this and continues with the other aspects in the subsequent modules.

Thank you.