Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communications
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Lecture - 17
Vector Parameter Estimation – System Model for Multi-Antenna Downlink Channel Estimation

Hello. Welcome to another module in this massive open online course. So far, we have talked about parameter estimation, the estimation of a scalar parameter h. So, now, we are going to start talking about a much more general scenario, that is, a vector parameter estimation; that is, where the parameter to be estimated is actually a parameter vector, which we are going to denote by h bar. So, from today's module, we are going to start talking about vector parameter estimation.

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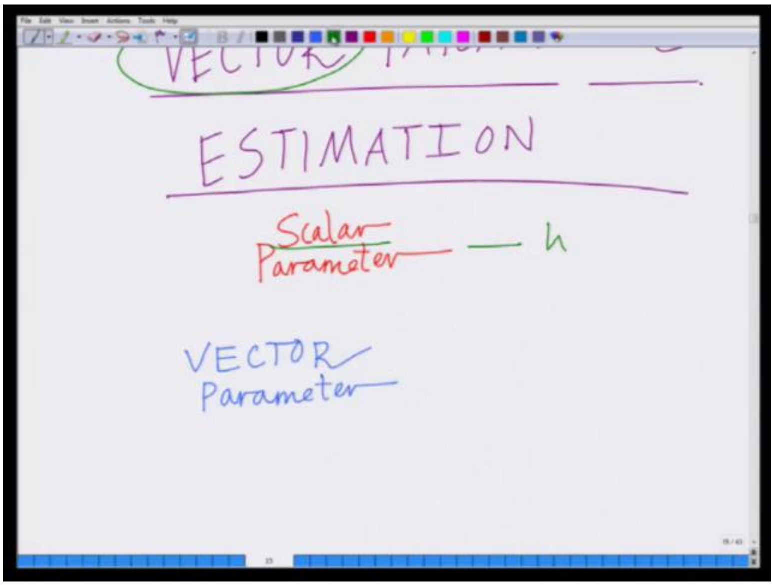

So, what we are going to start talking about from today is, basically, to develop a framework for vector, vector parameter, vector parameter estimation. And, in fact, the key word here is this word, which is basically a vector. So, far, we have talked about a parameter, or rather a scalar to be more precise. We have talked about a scalar parameter h, that is, estimation of a scalar parameter h. Now, and from this module, we are going to start talking about a vector parameter, estimation of a vector parameter; that is, let me write it; emphasis this. This is a vector parameter, and similar to the notation that we use for vectors, we are going to denote this by vector h bar. This is the parameter vector h bar; and therefore, h bar is, let us say, it is an m dimensional vector, h 1, h 2, up to h m, which is basically a stacking, or a collection of m scalar parameter.

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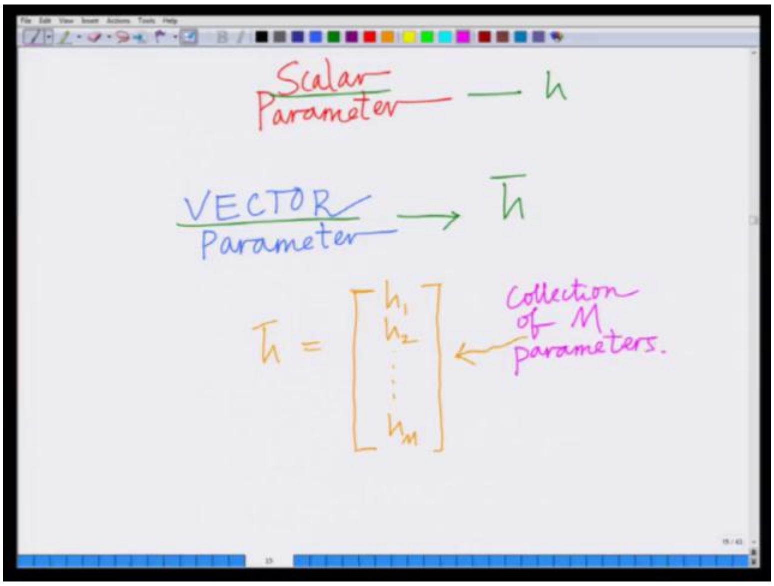

So, basically, this is a collection of m parameters h 1, h 2, up to h m. So, what we are going to start talking about from today is, basically, the estimation of a vector parameter, which we are going to denote by h bar. And, this is an m dimensional vector, which means, it contains m parameters, which we are denoting by h 1, h 2, up to h m. So, to understand this better, I mean, to understand, to motivate this concept, or how to motivate, or to motivate why we need to estimate a vector parameter, rather than a scalar parameter, and let us try to look at it from the perspective of an example. I think, an example will clarify this concept, and the framework related to it, much better. So, let us start with a simple example, in the context of wireless communication system. So, let us look at an example.

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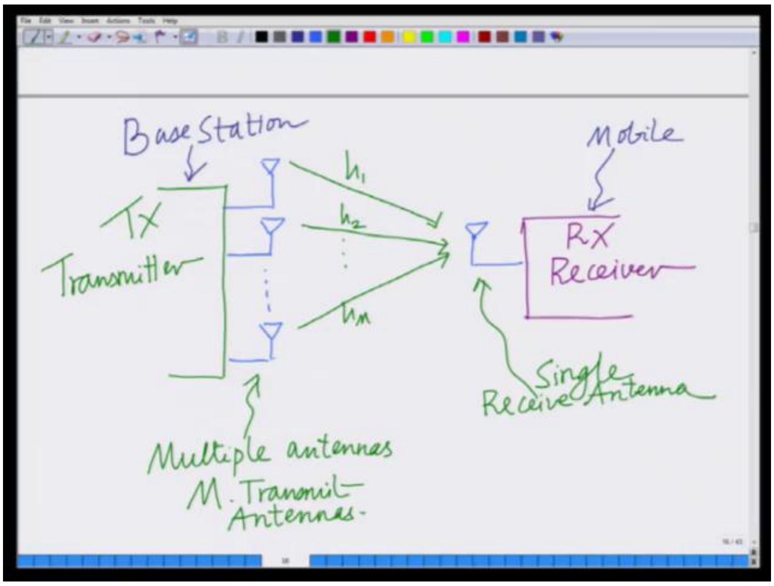

And, in this example, again, similar to our example previously, let us look at the channel estimation. Let us look at, remember the estimation of the fading channel coefficient in the wireless system is known as channel estimation. However, let us look at channel estimation in a multi-antenna system.

So far, we have considered only a single antenna communication system. So, let us look at channel estimation in a multi -antenna system. So, what we are saying is, let us try to understand this framework of vector parameter estimation. And, to explore and understand that, let us begin with a simple example in the context of channel estimation in a wireless communication system, but channel estimation in a multiple antenna system; that is, a wireless communication system with multiple antennas; which means, more than one antenna, alright. And, a simple such scenario is for instance, let us say, I have a transmitter; I have my transmitter, my wireless transmitter; and, I also have my receiver, my wireless, my wireless receiver. So, let us say, this is my, this is my wireless receiver, which I am going to denote by R x. And, I have, let us say, multiple antennas at the base station; that is, not just a single antenna, but more than one antenna at the base station; and, I have a single antenna.

So, each of these triangles is basically an antenna, and, what I am saying is, for instance, this is my transmitter, and there are multiple antennas, let us say, m transmit antennas. And, there is a single antenna at the receiver, a single receive antenna. For instance, we can consider a simple scenario, where this is basically your, this system is something like a base station; for instance, your transmitter is a base station, and the receiver is, let us say, this is simply your mobile, or mobile station. So, the transmitter is basically larger in size. So, it can have multiple antennas. And, in fact, current generation, for instance, 3G wireless systems have the provision to go up to 2 to 4 antennas; L T E is exploring about 4 and antennas beyond 4.

So, people are already talking about systems with antenna, with about 4 antennas and also going up to 8 antennas and more, right. So, we can have a transmitter, such as a base station, a wireless transmitter, which has more than one antenna, alright. So, we are considering multiple antennas at the transmitter. The number of antennas at the transmitter is being denoted by m; and, we have a single antenna at the receiver, which is a mobile, alright. And, the transmission, let us say that, transmission is from the base station to the mobiles; basically, we are considering a downlink scenario; that is what we said, even in the previous channel estimation scenario, when the transmission is from the base station to the mobile, it is the downlink; when the transmission is from the mobile to the base station, it is the uplink.

So, we are considering a multi antenna downlink wireless communication scenario, where the base station has the multiple antennas, and the mobile has a single antenna. Now, naturally, since there are multiple antennas, there are going to be multiple channel coefficients corresponding to these multiple antennas. And, since there are m antennas, there are going to be m channel coefficients, h 1, h 2, up to h m.

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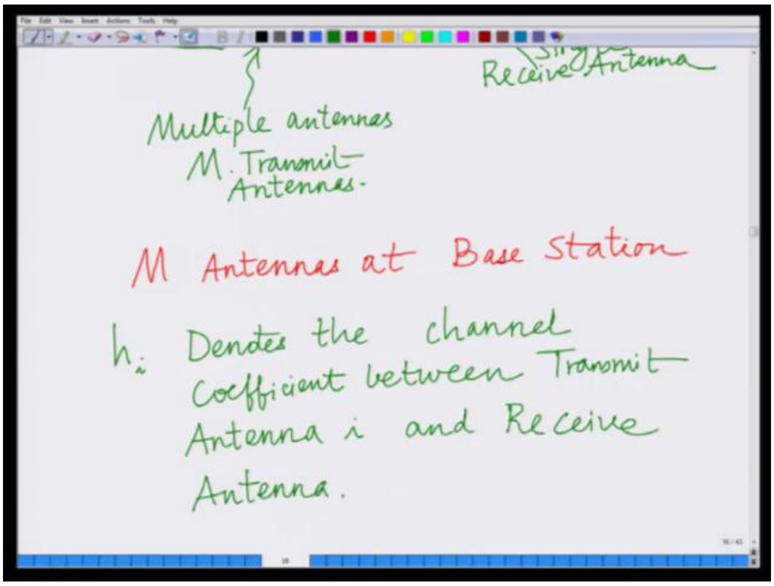

So, basically, what we are saying is, we have a total of m antennas; that is, m antennas; we have m antennas at the base station. And, h i denotes, h i denotes the channel coefficient between transmit antenna i and the receive antenna; h i denotes the channel coefficient between the transmit antenna i and the receive antenna; that is, the channel coefficient. For instance, let us say, let us consider h 2; h 2 is the channel coefficient between the transmit antenna 2 at the base station, and the single receive antenna.

Similarly, h 3 is the channel coefficient between the transmit antenna 3 and the single receive antenna, and, so on. And, since we have m antennas, m transmit antennas, therefore, we naturally have m channel coefficients corresponding to the m transmit antennas. So, what we have is, we have m channel coefficients h 1, h 2, h m.

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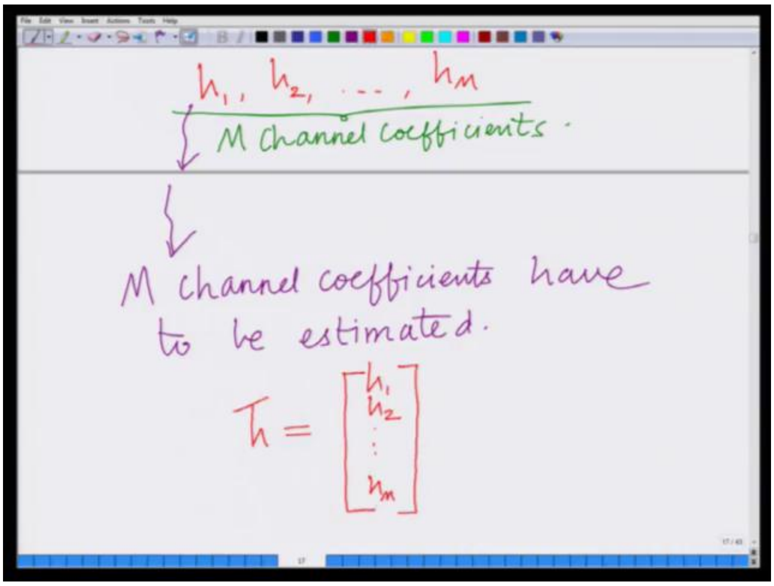

These are the, these are your m channel coefficients, correct. In this wireless, in this multi antenna wireless communication system, we have m channel coefficients, and to begin with, these m channel coefficients are unknown; that is the paradigm of channel estimation. Therefore, this m channel coefficients can now be stacked as a vector. This can be considered as a vector h bar, and that channel vector h bar has to now be estimated. Therefore, this leads to vector parameter estimation.

So, what do we have is, to put it more explicitly, m channel coefficients have to be estimated. So, now, these m channel coefficients can be denoted by the vector which is h bar equals h 1, h 2 up to h m. So, this is the channel vector which has to be estimated.

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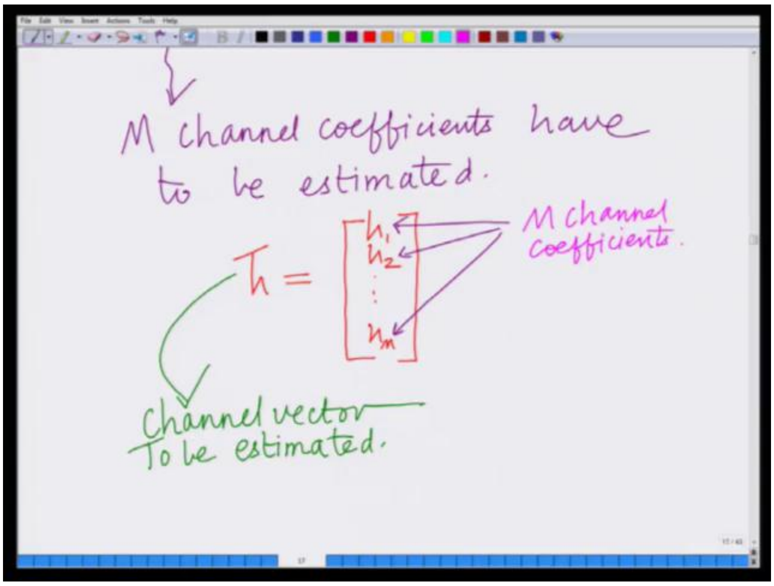

So, this is your channel vector which has to be estimated, and these, and these are basically your m, alright. So, naturally, a multiple antenna system in which there are multiple antennas, and therefore, multiple channel coefficients, this naturally leads to a necessity, and it naturally leads to a paradigm of vector parameter estimation, right. So, we can no longer be satisfied by simply considering the scalar parameter, that is, individual parameters, but we have to look at a group of parameters, that is, parameters which are grouped as a vector, that naturally leads to the requirement and the necessity of vector parameter estimation.

Now, how do we estimate this channel coefficient vector? Let us formulate this problem for estimation of this channel vector h bar, and, that can be seen as follows. Similar to the single input that, similar to the single transmit antenna and single receive antenna, we are going to transmit pilot symbols from the transmitter, which is, in this case is that, it is the downlink scenario; the transmitter is at the base station.

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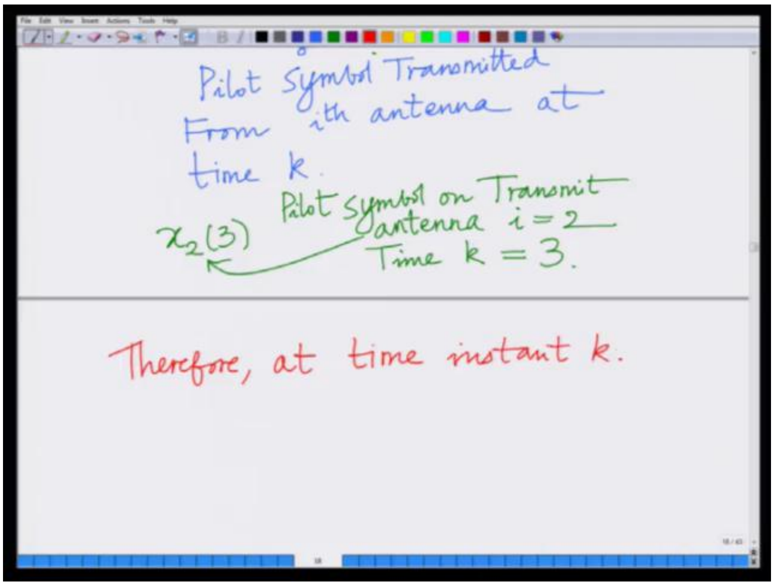

So, let us consider the transmission of pilot symbols. However, I have been slightly inaccurate, because we have multiple antennas. So, we need the transmission of one pilot symbol from each transmit antenna. So, rather than the pilot symbols, we will actually be transmitting pilot vectors, where the vector of transmitted symbols corresponds to one transmit symbol from each transmit antenna.

So, we will talk about pilot vectors. So, let us consider the transmission of pilot vectors, and, in fact, there will be multiple such vectors. So, what will we have, let us say, now, we denote by x i of k; that is, x i of k is the pilot symbol transmitted from the, x i of k, this is the pilot symbol transmitted from your ith antenna, at time k. So, now, you have multiple antennas; we cannot consider the transmission of a single pilot symbol. Previously, we had a single transmit antenna. So, we said, s k is the pilot symbol transmitted at time instant k. However, now, we have a multiple antennas. So, I am using the subscript i to denote by x i k, the pilot symbol transmitted at time instant k on antenna i, alright. So, x i k denotes the pilot symbol transmitted on transmit antenna i, at time instant k, by the transmitter, which is the base station in a downlink.

For instance, again, let us take a simple example, x 2 of 3. If I look at x 2 of 3, this is symbol, or rather, pilot symbol on transmit antenna 2, on transmit antenna 2, that is, i equal to transmit antenna; your i equals 2, at time, or discrete time k equals 3; that is, x 2 of 3 is a symbol transmitted, pilot symbol transmitted on transmit antenna 2 at time instant 3. Therefore, at time instant k, we have, we are going to have, therefore, at time, therefore, at time instant k, if we have the observed symbol y k, let me write this clearly, y k which is the received symbol, this is equal to h 1 x 1 k, plus h 2 x 2 k, plus so on, plus h m x m k, plus v k.

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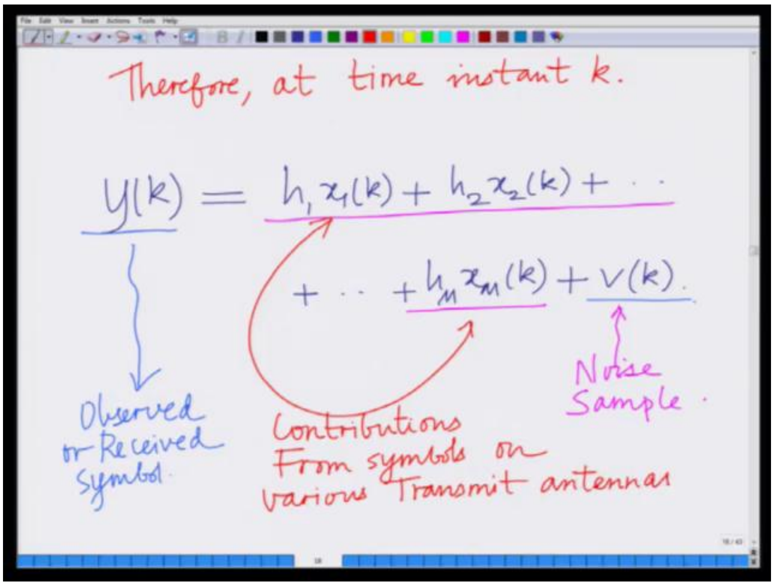

Let me explain this - y k is the observed symbol at the time instant k; v k is the corresponding noise sample at time instant k. These two things are simple. Now, in between, the rest of these things are contributions from the various channel coefficients. For instance, h 1 x 1 k corresponds to x 1 k, that is the symbol transmitted at time instant k on transmit antenna one. And, that goes to the channel coefficient h 1. And that, in addition to that, we have h 2, channel coefficient h 2 times x 2 k, which is the symbol, pilot symbol transmitted on transmit antenna two at time instant k. All these contributions from the various transmit antennas add up at the receiver; therefore, we are going to see the composite, the sum signal.

So, let me describe that, h 1 k is the observed symbol at the receiver. This is the observed, or one can say, your received symbol v k is, basically, your noise sample. And, this quantity here is, basically, these are the contributions from the various receive antennas; or, it is transmit antennas; contributions from symbols on, transmitted on various, transmitted on the various transmit antennas.

For instance, x 1 k is the symbol transmitted on transmit antenna one at time instant k, that goes through channel h 1 k; for instance, h 2 x 2 k, basically refers to symbol x 2 k transmitted on transmit antenna two, at time instant k, going through channel h 2, and so on, until h m x m k; that is what we have. And now, therefore, this is the system at time instant k. This is the model corresponding to the received symbol y k, at time instant k. I can write it succinctly again using vector notation y k, as follows - y k, we are already familiar with vector notation; y k equals h 1 h 2, or, let us write it this way; x 1 k, x 2 k, x m k times, the row vector times h 1 h 2 up to h m.

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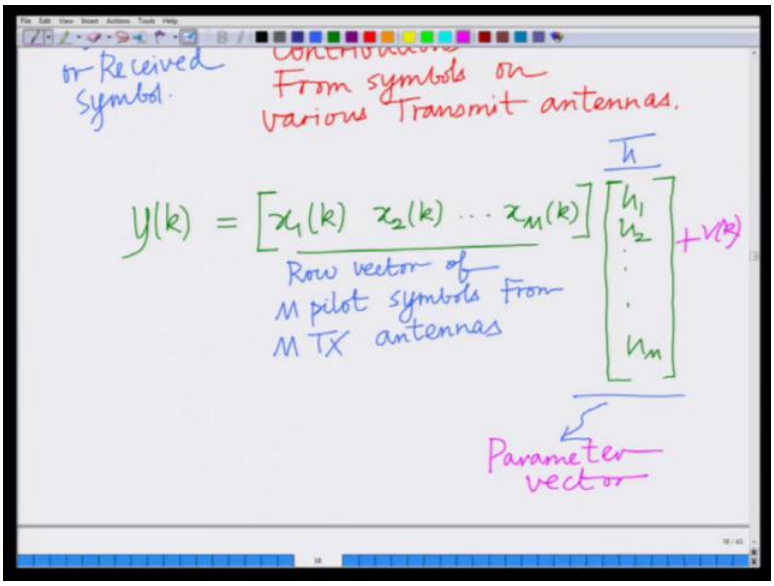

So, this is the row vector of m pilot symbols transmitted from the m transmit antennas. So, row vector of m pilot symbols, from m transmit antennas. This, of course, is h bar; this is basically your unknown parameter vector. This is your, plus, of course, there is going to be your noise as usual, that is v k. So, we have, y k is basically the received symbol, which is equal to x 1, x 2, up to x k, which is the row vector of pilot symbols transmitted at time instant k, from the m transmit antennas, times the column vector of the channel coefficients h 1, h 2, up to h m, which we are denoting by the vector h bar, plus v k, which is the noise sample.

Now, let us say, we have n such pilot vectors that are transmitted; that is, pilot transmission corresponding to n time instants; that is, multiple pilot symbols, or multiple transmit times 1, 2, 3, up to N, and multiple transmit antennas 1, 2, 3, up to m. Now, we can concatenate all these, and write the composite system model as follows. So, therefore, consider now N pilot symbols.

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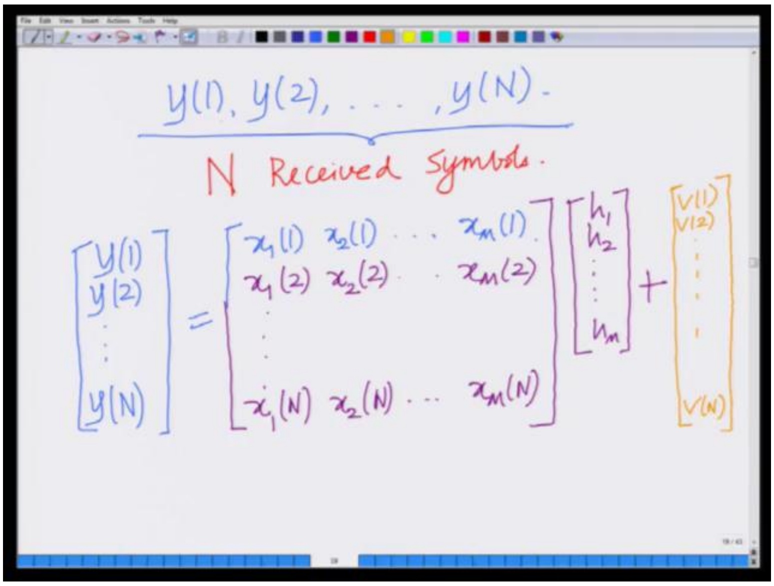

Consider now, N pilot symbols. Therefore, I can now, I have N observed symbols, y 1, y 2, y N, these are the, what are these? These are your N received symbols. In fact, I should say, consider N pilot vectors, because at each time instant, we are transmitting a vector from the m antennas.

Now, if I look at, and I am going to explain this again; let me write the system model. I will have y 1, y 2, up to y N; that is, the received symbol at time instant N, which is equal to this matrix, that is, x 1 1, x 1 2, up to x 1 m; or, let me write it this way. I think, we have to write it as x 1 1, x 2 1, so on, up to x m 1, that is, the pilot symbols transmitted at time instant one; and, x 2 or, and x 1 2 transmitted at time instant two, x 2 2, at time instant two, so on, x m 2 time instant two; and, x N 1, x N 2; I am sorry, x 1 N, x 2 N, so on, x m N, times your parameter vector.

Let me just draw it a little bit shorter; I will explain the reason - h 1, h 2, up to h m; plus your noise vector, noise vector is naturally going to be of dimension N, that is, v 1, or noise sample at time instant one, v 1, v 2, so on, up to v N. And, this is basically your input output pilot symbol model. So, now, let me denote this vector, vector of received symbol as your vector y bar. This is your vector of received symbols. Let us write this down. What is this? This is an N cross m matrix of pilot symbols. This is N cross m matrix; this is N cross m matrix of pilot symbols. Let us denote this by x bar. This is now your pilot matrix. This is a pilot matrix, and, and look at this. In this pilot matrix, each row corresponds to a particular time instant; each row corresponds to particular time. And, each column corresponds to a, let me write it here; each column corresponds to an antenna; each column corresponds to an antenna.

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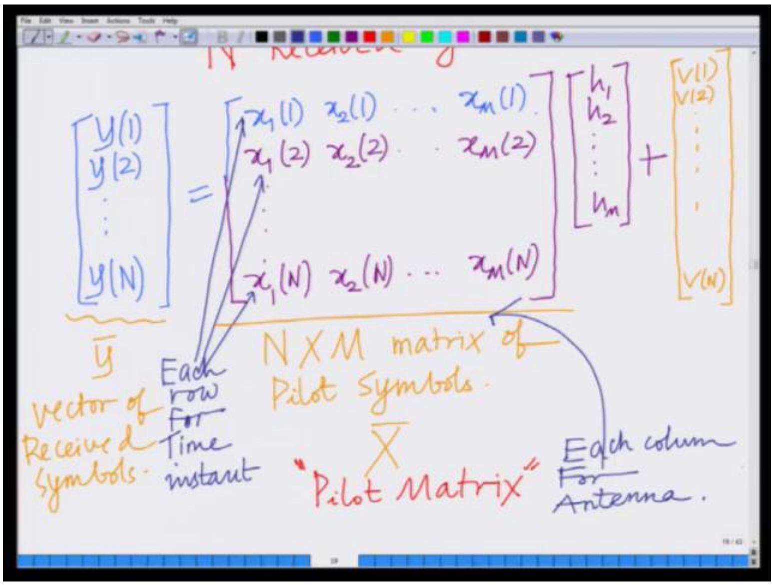

So, for instance, at time instant, time instant two, the row is basically x 1 2, x 2 2, and so on, x m 2, basically, which means that, x 1 1 is the symbol transmitted, and transmit antenna one, at time instant one; x 2 2 is basically the symbol transmitted at, from transmit antenna two, at time instant two and so on, until x m 2, which is basically symbol transmitted from transmit antenna m, at time instant two. So, each row, basically, if you observe this matrix, each row of this pilot matrix corresponds to the pilot symbols transmitted from the m transmit antennas at time instant k; that is, the k th row corresponds to the m transmit, m pilot symbols transmitted from the m transmit antennas at time instant k. Therefore, we have m entries in each row, which means, we have basically m columns, alright. And similarly, we have N such rows; each row corresponds to one particular time instant, and therefore, we are saying, we have N, capital N such time instants; that is, basically, that corresponding to the transmission of capital N pilot vectors.

So, this matrix, pilot matrix is N cross m. And also, now, we are going to, I said this, I am going to draw this column vector, this h bar slightly smaller, because, typically we assume N is larger than, or equal to m; the reason being very simple, because, we have m unknown channel coefficients, and N, capital N is a number of observations. To estimate m unknown channel coefficients, which we need at least N observations or more; this is, this is the simple property from equations, that is, basically, to solve, to have a unique solution for m unknown quantities, we need at least m equations or more, right. So, basically, we have N pilot vectors, the N pilot observations. These observations, that is, number of transmitted pilot vectors has to be at least equal to, or basically, equal to, or greater than m, where m is the number of unknown channel coefficients.

So, we have N cross m pilot matrix, where N is greater than or equal to m, that is, at least going to be the assumption going forward, alright. So, let me also write this down. It is, even though it is a subtle point, it is important N is greater than or equal to m, which basically means that, number of pilot symbols is greater than or equal to your number of channel coefficients. It is greater than or equal to number of channel coefficients.

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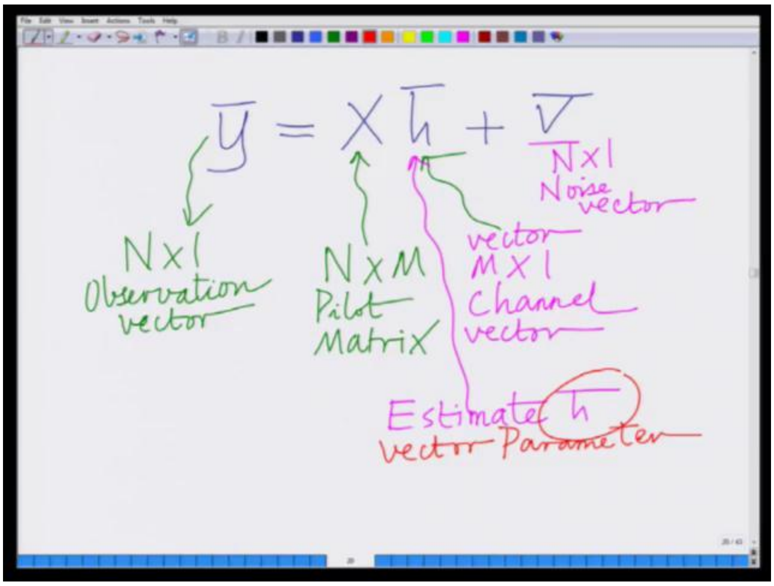

And, of course, this we have already seen before. This is h 1, h 2, h m. This is, which is the pilot, which is the channel coefficient, the channel vector h bar. This is the noise vector v bar, which contains the N noise coefficients v 1, v 2, v N, corresponding to the N received samples y 1, y 2, y N. And now, therefore, if I write this entire thing together succinctly in the vector notation and let me write that clearly. I have, y is equal to, let me just write this clearly, because this is an important to understand.

Understanding this is very important, because we are going to use this notation going forward in vector parameter estimation. X h bar plus v bar, where, let me again, for the sake of clarity, define this things. This is the N cross 1 observation vector, or received symbol vector. This is x, which is the N cross m pilot matrix corresponding to m pilot vectors; vectors of m pilot symbol are transmitted over N time instants. This is the vector of m channel coefficients, m cross 1, this is the channel coefficient vector, or simply called the channel vector; and this v bar is your N cross 1 noise vector.

So, this is your N cross 1 noise vector, and this is your input output, this is the matrix input output vector, input output system model; and estimation, we need to estimate, estimate h bar, which is basically your vector, channel vector, or which is a vector parameter. So, this, basically, h bar is the vector parameter which we are interested in estimating. Basically now, I formulated this, very succinctly and very, in a very compact, and in a very tractable fashion, where y bar is the observation vector, which is basically y 1, y 2, up to y N corresponding to the N transmitted pilot vectors, which is equal to x, which is the N cross m pilot matrix, right; N rows corresponding to the N time instants and m columns corresponding to the m transmit antennas, times h bar, which is the channel vector, which is contains the channel coefficients, the unknown channel coefficients, h 1, h 2, up to h m.

And, that denotes the parameter vector, the unknown parameter vector h bar, plus v bar, where v bar is the noise vector. This is a compact representation of this vector parameter estimation model, and therefore, what we have done in this module is simply to motivate the necessity, the need for the estimation of a vector parameter because we have previously, we only considered the estimation of the scalar parameter. And also through an example, basically, considering the example of channel vector estimation for a multiple antenna downlink wireless communication scenario.

We have built up the framework, and motivated the necessity, or illustrated the need to develop a framework for vector parameter estimation. And, we have also developed this succinct, this elegant compact vector model, vector system model, we might also call it, which is y bar equals x times h bar plus v bar, which we are going to subsequently explore towards the estimation of this vector parameter h bar, alright. So, we will stop with this problem formulation here, and how is the vector parameter estimation done, that is, how do we come up with a likelihood function, and how do we do the exact vector parameter estimation; that, we are going to explore in the subsequent modules.

Thank you very much.