

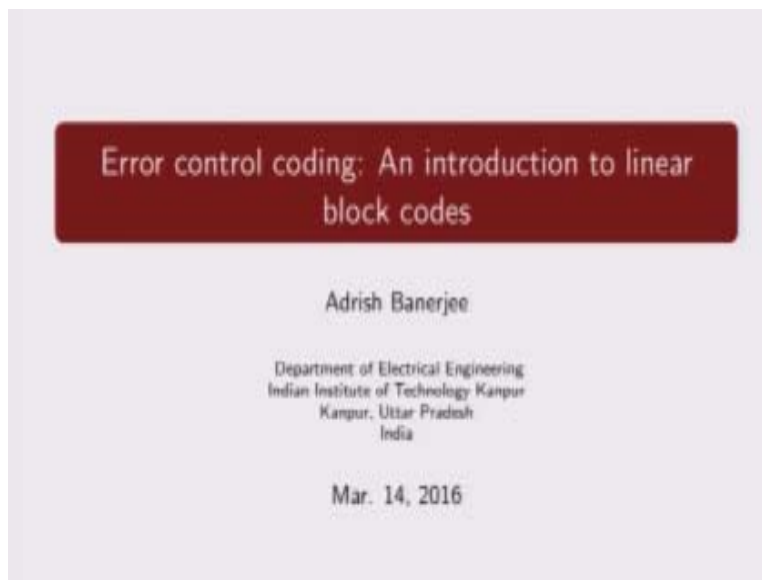
Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Error Control Coding: An Introduction to Linear Block Codes

Lecture – 2
Generator Matrix and Parity Check Matrix

by
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Welcome to the course on error control coding, an introduction to linear block codes.

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Today in this lecture we are going to describe what we mean by generator matrix and parity check matrix, so we will continue our discussion with introduction to linear block codes.

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Lecture #2: Introduction to linear block codes, generator matrix
and parity check matrix

We will first describe, what is a generator matrix and what is a parity check matrix and how are they related, so as we described in the last class an encoder for a linear block codes what it does it takes a block of k bits and maps it to the two end bit.

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Linear block codes

- An (n,k) linear block code can be defined by a $k \times n$ generator matrix.

$$\mathbf{G} = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{k-1} \end{bmatrix} = \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & \cdots & g_{0,n-1} \\ g_{1,0} & g_{1,1} & g_{1,2} & \cdots & g_{1,n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{k-1,0} & g_{k-1,1} & g_{k-1,2} & \cdots & g_{k-1,n-1} \end{bmatrix}$$

Now the matrix we can use a matrix $k \times n$ matrix to define this mapping from k information bits to n coded bits and this matrix is basically our generator matrix.

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Linear block codes

- An (n,k) linear block code can be defined by a $k \times n$ generator matrix,

$$\mathbf{G} = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{k-1} \end{bmatrix} = \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & \cdots & g_{0,n-1} \\ g_{1,0} & g_{1,1} & g_{1,2} & \cdots & g_{1,n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{k-1,0} & g_{k-1,1} & g_{k-1,2} & \cdots & g_{k-1,n-1} \end{bmatrix}$$

For a so for an (n, k) linear block code the mapping of k information bits to encoded bits is defined by this generator matrix G which is of rank k so if we denote the information bits by u .

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Linear block codes

- An (n,k) linear block code can be defined by a $k \times n$ generator matrix.

$$V = UG$$

$$\begin{matrix} 1 \times n & 1 \times k & k \times n \end{matrix}$$

$$G = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{k-1} \end{bmatrix} = \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & \dots & g_{0,n-1} \\ g_{1,0} & g_{1,1} & g_{1,2} & \dots & g_{1,n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{k-1,0} & g_{k-1,1} & g_{k-1,2} & \dots & g_{k-1,n-1} \end{bmatrix}$$

And we denote our coded bits by v then we can write v is u times G , where our u is $1 \times k$ vector and this is, generator matrix is $k \times n$ matrix and our output coded bit is $1 \times n$ vector. So as the name suggest basically generator matrix is used to generate our code word so we generate our code words using this generator map matrix and this generator matrix gives the mapping between the information bits u to the coded bits v , so how do we find code words?

We find code words by taking linear combinations of rows of these generator matrix, in case of binary codes so then these entries in the generator matrix are either zero or one depending upon which bits are used to generate a particular coded sequence. So we form a set of 2^k code words by taking linear combinations of rows of these generator matrix.

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Linear block codes

- An (n,k) linear block code can be defined by a $k \times n$ generator matrix.

$$\mathbf{G} = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{k-1} \end{bmatrix} = \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & \cdots & g_{0,n-1} \\ g_{1,0} & g_{1,1} & g_{1,2} & \cdots & g_{1,n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{k-1,0} & g_{k-1,1} & g_{k-1,2} & \cdots & g_{k-1,n-1} \end{bmatrix}$$

- The set of 2^k binary codewords is formed by taking the *linear combinations* of the rows of $\mathbf{G}$.

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Linear block codes

- An (n,k) linear block code can be defined by a $k \times n$ generator matrix,

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_0 \\ \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_{k-1} \end{bmatrix} = \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & \cdots & g_{0,n-1} \\ g_{1,0} & g_{1,1} & g_{1,2} & \cdots & g_{1,n-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ g_{k-1,0} & g_{k-1,1} & g_{k-1,2} & \cdots & g_{k-1,n-1} \end{bmatrix}$$

- The set of 2^k binary codewords is formed by taking the *linear combinations* of the rows of $\mathbf{G}$.
- For the binary information sequence $\mathbf{u} = (u_0, u_1, \dots, u_{k-1})$, the corresponding binary codeword sequence is given by

$$\mathbf{v} = \mathbf{uG} = u_0\mathbf{g}_0 + u_1\mathbf{g}_1 + \cdots + u_{k-1}\mathbf{g}_{k-1} \quad (\text{modulo-2})$$

So we can, as I said we can write our coded sequence as $\mathbf{u}$ times $\mathbf{G}$ which is basically linear combination of rows of the generator matrix. So these are basically linearly independent k rows and the rank of this generator matrix is k , since we are without loss of generality since we are talking about binary linear block codes.

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Linear block codes

- An (n,k) linear block code can be defined by a $k \times n$ generator matrix,

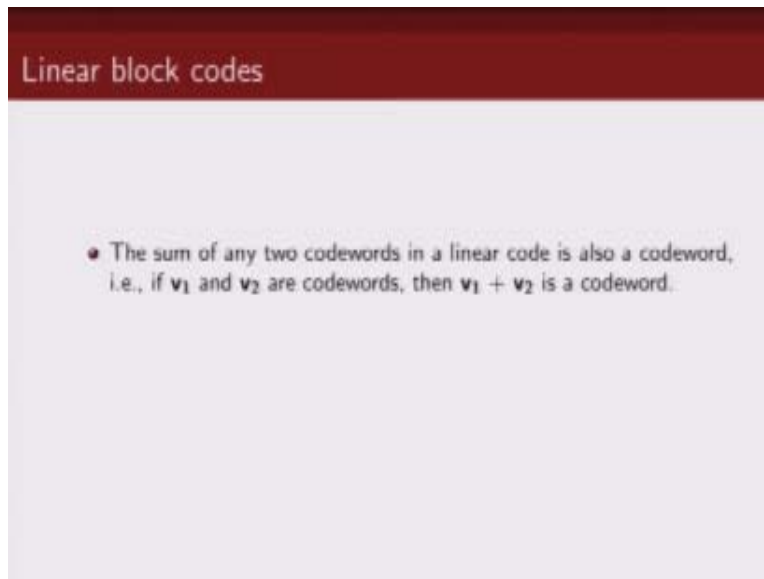
$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_0 \\ \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_{k-1} \end{bmatrix} = \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & \cdots & g_{0,n-1} \\ g_{1,0} & g_{1,1} & g_{1,2} & \cdots & g_{1,n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{k-1,0} & g_{k-1,1} & g_{k-1,2} & \cdots & g_{k-1,n-1} \end{bmatrix}$$

- The set of 2^k binary codewords is formed by taking the *linear combinations* of the rows of $\mathbf{G}$.
- For the binary information sequence $\mathbf{u} = (u_0, u_1, \dots, u_{k-1})$, the corresponding binary codeword sequence is given by

$$\mathbf{v} = \mathbf{uG} = u_0\mathbf{g}_0 + u_1\mathbf{g}_1 + \cdots + u_{k-1}\mathbf{g}_{k-1} \quad (\text{modulo-2})$$

So we will be doing this addition modulus – 2.

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Linear block codes

- The sum of any two codewords in a linear code is also a codeword, i.e., if $\mathbf{v}_1$ and $\mathbf{v}_2$ are codewords, then $\mathbf{v}_1 + \mathbf{v}_2$ is a codeword.

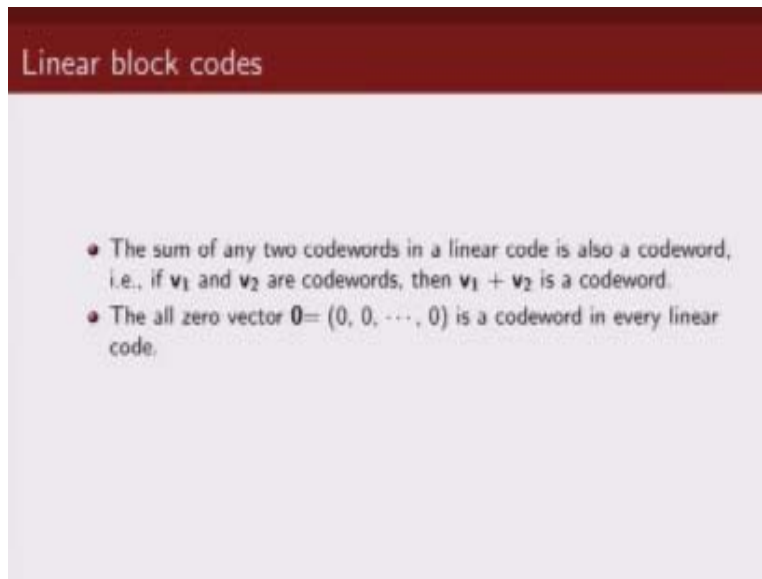
So what are the properties of a linear block code? Sum of any two code words in a linear code is also a valid code word, so if $\mathbf{v}_1$ and $\mathbf{v}_2$ are valid code words then $\mathbf{v}_1 + \mathbf{v}_2$ will also be a valid code word.

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Linear block codes

- The sum of any two codewords in a linear code is also a codeword, i.e., if $\mathbf{v}_1$ and $\mathbf{v}_2$ are codewords, then $\mathbf{v}_1 + \mathbf{v}_2$ is a codeword.

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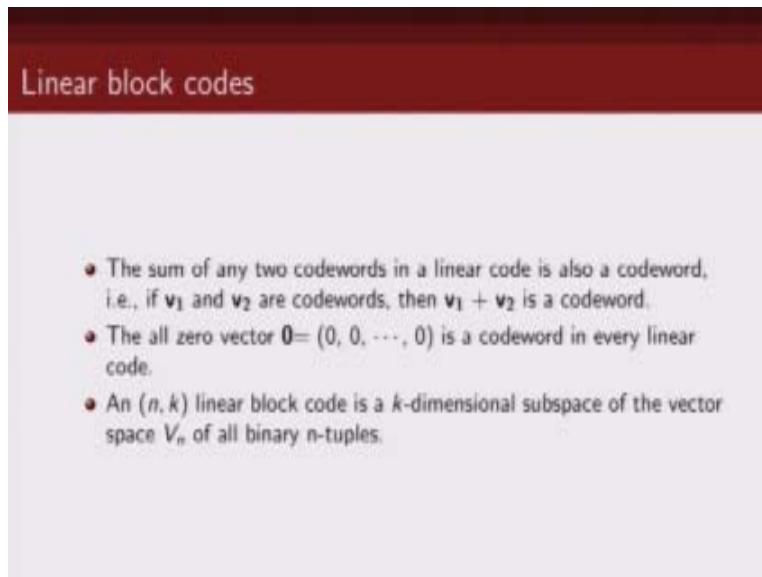


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- The sum of any two codewords in a linear code is also a codeword, i.e., if $\mathbf{v}_1$ and $\mathbf{v}_2$ are codewords, then $\mathbf{v}_1 + \mathbf{v}_2$ is a codeword.
- The all zero vector $\mathbf{0} = (0, 0, \dots, 0)$ is a codeword in every linear code.

Also an all zero code word is a valid code word in any linear block codes.

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- The sum of any two codewords in a linear code is also a codeword, i.e., if $\mathbf{v}_1$ and $\mathbf{v}_2$ are codewords, then $\mathbf{v}_1 + \mathbf{v}_2$ is a codeword.
- The all zero vector $\mathbf{0} = (0, 0, \dots, 0)$ is a codeword in every linear code.
- An (n, k) linear block code is a k -dimensional subspace of the vector space V_n of all binary n -tuples.

So we can define a linear block code (n, k) linear block code as a k dimensional subspace of vector space V_n of all binary n – tuples so we can define a linear binary block codes as a k dimensional subspace of vector space V_n of all binary n – tuples.

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Linear block codes	
Example 2.1: Let $k = 3$ and $n = 6$. The table gives a $(6, 3)$ linear block code.	
Message (u_0, u_1, u_2)	Codewords $(v_0, v_1, v_2, v_3, v_4, v_5)$
(0 0 0)	(0 0 0 0 0 0)
(1 0 0)	(0 1 1 1 0 0)
(0 1 0)	(1 0 1 0 1 0)
(1 1 0)	(1 1 0 1 1 0)
(0 0 1)	(1 1 0 0 0 1)
(1 0 1)	(1 0 1 1 0 1)
(0 1 1)	(0 1 1 0 1 1)
(1 1 1)	(0 0 0 1 1 1)

Now let us take an example to illustrate what is a generator matrix, so in this example we have considered.

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Linear block codes	
Example 2.1: Let $k = 3$ and $n = 6$. The table gives a $(6, 3)$ linear block code.	
Message (u_0, u_1, u_2)	Codewords $(v_0, v_1, v_2, v_3, v_4, v_5)$
(0 0 0)	(0 0 0 0 0 0)
(1 0 0)	(0 1 1 1 0 0)
(0 1 0)	(1 0 1 0 1 0)
(1 1 0)	(1 1 0 1 1 0)
(0 0 1)	(1 1 0 0 0 1)
(1 0 1)	(1 0 1 1 0 1)
(0 1 1)	(0 1 1 0 1 1)
(1 1 1)	(0 0 0 1 1 1)

Three information bits and six coded bits and in this table I have given you the set of eight information sequences and their corresponding code words, so how do we find the generator matrix in this case? So we will have to look at each of these code bits and see how are we generating these code bits in terms of message bits u_0 , u_1 and u_2 .

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Linear block codes

Example 2.1: Let $k = 3$ and $n = 6$. The table gives a $(6, 3)$ linear block code.

Message (u_0, u_1, u_2)	Codewords ($v_0, v_1, v_2, v_3, v_4, v_5$)
(0 0 0)	(0 0 0 0 0 0)
(1 0 0)	(0 1 1 1 0 0)
(0 1 0)	(1 0 1 0 1 0)
(1 1 0)	(1 1 0 1 1 0)
(0 0 1)	(1 1 0 0 0 1)
(1 0 1)	(1 0 1 1 0 1)
(0 1 1)	(0 1 1 0 1 1)
(1 1 1)	(0 0 0 1 1 1)

So first thing we are going to do is, look at each of these code bits $v_0, v_1, v_2, v_3, v_4, v_5$ and write them in terms of u_0, u_1, u_2 , okay so let us look at each of these.

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Linear block codes

Example 2.1 (contd.): We can write the coded bits in terms of information bits as follows

$$v_0 = u_1 + u_2$$

$$v_1 = u_0 + u_2$$

$$v_2 = u_0 + u_1$$

$$v_3 = u_0$$

$$v_4 = u_1$$

$$v_5 = u_2$$

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5] = [u_0 \ u_1 \ u_2] \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & g_{0,3} & g_{0,4} & g_{0,5} \\ g_{1,0} & g_{1,1} & g_{1,2} & g_{1,3} & g_{1,4} & g_{1,5} \\ g_{2,0} & g_{2,1} & g_{2,2} & g_{2,3} & g_{2,4} & g_{2,5} \end{bmatrix}$$

So v_0 is u_1+u_2 .

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Linear block codes

Example 2.1: Let $k = 3$ and $n = 6$. The table gives a $(6, 3)$ linear block code.

Message (u_0, u_1, u_2)	Codewords $(v_0, v_1, v_2, v_3, v_4, v_5)$
(0 0 0)	(0 0 0 0 0 0)
(1 0 0)	(0 1 1 1 0 0)
(0 1 0)	(1 0 1 0 1 0)
(1 1 0)	(1 1 0 1 1 0)
(0 0 1)	(1 1 0 0 0 1)
(1 0 1)	(1 0 1 1 0 1)
(0 1 1)	(0 1 1 0 1 1)
(1 1 1)	(0 0 0 1 1 1)

We can see easily v_0 is this column and we can see this is same as u_1+u_2 , so u_1+u_2 in this case is zero, u_1+u_2 is zero, $1+0$ is 1, $1+0+1$ is 1, $1+1$ is 0 modulo 2 and $1+1$ is zero modulo 2, so this v_0 is basically given by u_1+u_2 .

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Linear block codes

Example 2.1 (contd.): We can write the coded bits in terms of information bits as follows

$$\begin{aligned}v_0 &= u_1 + u_2 \\v_1 &= u_0 + u_2 \\v_2 &= u_0 + u_1 \\v_3 &= u_0 \\v_4 &= u_1 \\v_5 &= u_2\end{aligned}$$
$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5] = [u_0 \ u_1 \ u_2] \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & g_{0,3} & g_{0,4} & g_{0,5} \\ g_{1,0} & g_{1,1} & g_{1,2} & g_{1,3} & g_{1,4} & g_{1,5} \\ g_{2,0} & g_{2,1} & g_{2,2} & g_{2,3} & g_{2,4} & g_{2,5} \end{bmatrix}$$

Similarly we can see v_1 is given by u_0+u_2 and v_2 is given by u_0+u_1 .

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Linear block codes

Example 2.1: Let $k = 3$ and $n = 6$. The table gives a $(6, 3)$ linear block code.

Message	Codewords
(u_0, u_1, u_2)	$(v_0, v_1, v_2, v_3, v_4, v_5)$
(0 0 0)	(0 0 0 0 0 0)
(1 0 0)	(0 1 1 1 0 0)
(0 1 0)	(1 0 1 0 1 0)
(1 1 0)	(1 1 0 1 1 0)
(0 0 1)	(1 1 0 0 0 1)
(1 0 1)	(1 0 1 1 0 1)
(0 1 1)	(0 1 1 0 1 1)
(1 1 1)	(0 0 0 1 1 1)

So let us just check and say v_2 , v_2 we can see is given by, v_2 is given by $u_0 + u_1$.

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Linear block codes

Example 2.1 (contd.): We can write the coded bits in terms of information bits as follows

$$v_0 = u_1 + u_2$$

$$v_1 = u_0 + u_2$$

$$v_2 = u_0 + u_1$$

$$v_3 = u_0$$

$$v_4 = u_1$$

$$v_5 = u_2$$

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5] = [u_0 \ u_1 \ u_2] \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & g_{0,3} & g_{0,4} & g_{0,5} \\ g_{1,0} & g_{1,1} & g_{1,2} & g_{1,3} & g_{1,4} & g_{1,5} \\ g_{2,0} & g_{2,1} & g_{2,2} & g_{2,3} & g_{2,4} & g_{2,5} \end{bmatrix}$$

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Linear block codes	
Example 2.1: Let $k = 3$ and $n = 6$. The table gives a $(6, 3)$ linear block code.	
Message (u_0, u_1, u_2)	Codewords $(v_0, v_1, v_2, v_3, v_4, v_5)$
(0 0 0)	(0 0 0 0 0 0)
(1 0 0)	(0 1 1 1 0 0)
(0 1 0)	(1 0 1 0 1 0)
(1 1 0)	(1 1 0 1 1 0)
(0 0 1)	(1 1 0 0 0 1)
(1 0 1)	(1 0 1 1 0 1)
(0 1 1)	(0 1 1 0 1 1)
(1 1 1)	(0 0 0 1 1 1)

You can check v_2 is given by so u_0+u_1 , $0+0$ is 0 , $1+0$ is 1, $0+1$ is 1, $1+1$ is 0, $0+0$ is 0, $1+0$ is 1, $0+1$ is 1 and $1+1$ is 0.

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Linear block codes

Example 2.1 (contd.): We can write the coded bits in terms of information bits as follows

$$v_0 = u_1 + u_2$$

$$v_1 = u_0 + u_2$$

$$v_2 = u_0 + u_1$$

$$v_3 = u_0$$

$$v_4 = u_1$$

$$v_5 = u_2$$

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5] = [u_0 \ u_1 \ u_2] \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & g_{0,3} & g_{0,4} & g_{0,5} \\ g_{1,0} & g_{1,1} & g_{1,2} & g_{1,3} & g_{1,4} & g_{1,5} \\ g_{2,0} & g_{2,1} & g_{2,2} & g_{2,3} & g_{2,4} & g_{2,5} \end{bmatrix}$$

Similarly we noticed that v_3 , v_4 , v_5 are nothing but information bits u_0 , u_1 , and u_2 respectively.

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Linear block codes	
Example 2.1: Let $k = 3$ and $n = 6$. The table gives a $(6, 3)$ linear block code.	
Message (u_0, u_1, u_2)	Codewords $(v_0, v_1, v_2, v_3, v_4, v_5)$
(0 0 0)	(0 0 0 0 0 0)
(1 0 0)	(0 1 1 1 0 0)
(0 1 0)	(1 0 1 0 1 0)
(1 1 0)	(1 1 0 1 1 0)
(0 0 1)	(1 1 0 0 0 1)
(1 0 1)	(1 0 1 1 0 1)
(0 1 1)	(0 1 1 0 1 1)
(1 1 1)	(0 0 0 1 1 1)

So let us go back, v_3 is this column and we can see this is same as u_0 , 01, 01, 01, 01, similarly v_4 is equal to u_1 and v_5 is same as u_2 .

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Linear block codes

Example 2.1 (contd.): We can write the coded bits in terms of information bits as follows

$$v_0 = u_1 + u_2$$

$$v_1 = u_0 + u_2$$

$$v_2 = u_0 + u_1$$

$$v_3 = u_0$$

$$v_4 = u_1$$

$$v_5 = u_2$$

$$\begin{bmatrix} v_0 & v_1 & v_2 & v_3 & v_4 & v_5 \end{bmatrix} = \begin{bmatrix} u_0 & u_1 & u_2 \end{bmatrix} \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & g_{0,3} & g_{0,4} & g_{0,5} \\ g_{1,0} & g_{1,1} & g_{1,2} & g_{1,3} & g_{1,4} & g_{1,5} \\ g_{2,0} & g_{2,1} & g_{2,2} & g_{2,3} & g_{2,4} & g_{2,5} \end{bmatrix}$$

So now we have written our coded bits in terms of our information bits, this set of six equations I can write it in a matrix form so I can write my coded bits in terms of information bits and this matrix G which is our generator matrix will tell us how are we generating each of these coded bits as a linear combination of these information bits, so if we compare each equation let us look at v_0 .

So what is v_0 ? V_0 is $u_0 g_{00} + u_1 g_{10} + u_2 g_{20}$ and what do we see here? v_0 is $u_1 + u_2$ so that means g_{00} is 0 because there is no u_0 term here, g_{10} is 1 because there is a u_1 term here and g_{20} is 1 because there is a u_2 term here, so this will be 0, 1, 1.

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Linear block codes

Example 2.1 (contd.): We can write the coded bits in terms of information bits as follows

$$\begin{aligned}
 v_0 &= u_1 + u_2 \\
 v_1 &= u_0 + u_2 \\
 v_2 &= u_0 + u_1 \\
 v_3 &= u_0 \\
 v_4 &= u_1 \\
 v_5 &= u_2
 \end{aligned}$$

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5] = [u_0 \ u_1 \ u_2] \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & g_{0,3} & g_{0,4} & g_{0,5} \\ g_{1,0} & g_{1,1} & g_{1,2} & g_{1,3} & g_{1,4} & g_{1,5} \\ g_{2,0} & g_{2,1} & g_{2,2} & g_{2,3} & g_{2,4} & g_{2,5} \end{bmatrix}$$

Similarly look at v_1 , v_1 is $u_0 g_{01} + u_1 g_{11} + u_2 g_{21}$ and if we compare it with v_1 here we see v_1 is $u_0 + u_2$ that means this g_{01} should be 1, g_{11} should be 0 and this should be 1, likewise we build up the other columns of this matrix.

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Linear block codes

A generator matrix for this code is

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_0 \\ \mathbf{g}_1 \\ \mathbf{g}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The codeword for the message $\mathbf{u} = (1 \ 0 \ 1)$ is

$$\begin{aligned} \mathbf{v} &= \mathbf{u} \cdot \mathbf{G} \\ &= 1 \cdot (0 \ 1 \ 1 \ 1 \ 0 \ 0) + 0 \cdot (1 \ 0 \ 1 \ 0 \ 1 \ 0) + 1 \cdot (1 \ 1 \ 0 \ 0 \ 0 \ 1) \\ &= (0 \ 1 \ 1 \ 1 \ 0 \ 0) + (0 \ 0 \ 0 \ 0 \ 0 \ 0) + (1 \ 1 \ 0 \ 0 \ 0 \ 1) \\ &= (1 \ 0 \ 1 \ 1 \ 0 \ 1) \end{aligned}$$

So if we do that what we get is something like this, we can verify basically let us take second last column.

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Linear block codes

Example 2.1 (contd.): We can write the coded bits in terms of information bits as follows

$$\begin{aligned}
 v_0 &= u_1 + u_2 \\
 v_1 &= u_0 + u_2 \\
 v_2 &= u_0 + u_1 \\
 v_3 &= u_0 \\
 v_4 &= u_1 \\
 v_5 &= u_2
 \end{aligned}$$

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5] = [u_0 \ u_1 \ u_2] \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & g_{0,3} & g_{0,4} & g_{0,5} \\ g_{1,0} & g_{1,1} & g_{1,2} & g_{1,3} & g_{1,4} & g_{1,5} \\ g_{2,0} & g_{2,1} & g_{2,2} & g_{2,3} & g_{2,4} & g_{2,5} \end{bmatrix}$$

So what is v_4 ? v_4 is u_0 times g_{04} + u_1 times g_{14} + u_2 times g_{24} and what is v_4 ? v_4 is u_1 so then this should be 1 and this should be 0 and this should be 0 and this is what we have.

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Linear block codes

A generator matrix for this code is

$$\mathbf{G} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The codeword for the message $\mathbf{u} = (1 \ 0 \ 1)$ is

$$\begin{aligned}
 \mathbf{v} &= \mathbf{u} \cdot \mathbf{G} \\
 &= 1 \cdot (0 \ 1 \ 1 \ 1 \ 0 \ 0) + 0 \cdot (1 \ 0 \ 1 \ 0 \ 1 \ 0) + 1 \cdot (1 \ 1 \ 0 \ 0 \ 0 \ 1) \\
 &= (0 \ 1 \ 1 \ 1 \ 0 \ 0) + (0 \ 0 \ 0 \ 0 \ 0 \ 0) + (1 \ 1 \ 0 \ 0 \ 0 \ 1) \\
 &= (1 \ 0 \ 1 \ 1 \ 0 \ 1)
 \end{aligned}$$

Zero one zero, so now we can basically find out the generator matrix. So a linear block code is completely describe by a, it is generator matrix and as we said we can use the generator matrix to generate our code words for example if my

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Linear block codes

A generator matrix for this code is

$$G = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The codeword for the message $u = (1 \ 0 \ 1)$ is ✓

$$\begin{aligned} \underline{v} &= \underline{u} \cdot G \\ &= 1 \cdot (0 \ 1 \ 1 \ 1 \ 0 \ 0) + 0 \cdot (1 \ 0 \ 1 \ 0 \ 1 \ 0) + 1 \cdot (1 \ 1 \ 0 \ 0 \ 0 \ 1) \\ &= (0 \ 1 \ 1 \ 1 \ 0 \ 0) + (0 \ 0 \ 0 \ 0 \ 0 \ 0) + (1 \ 1 \ 0 \ 0 \ 0 \ 1) \\ &= \underline{(1 \ 0 \ 1 \ 1 \ 0 \ 1)} \end{aligned}$$

Information sequence is one zero one, what should be my corresponding coded bits for the information sequence one zero one, how do I find that so as I know my output code word is basically u times G so I will take linear combinations of rows of my generator matrix. What are the rows of my generator matrix, these are the three rows of my generator matrix, so my coded bit corresponding to this information sequence would be one times G zero plus zero times G one plus one times G 2.

So that is what I have written here this is one times G zero, zero times G one plus one time G 2 so this is basically zero, so what I have is then this plus this right, so let us look at 0 plus 1 would be 1, 1+1 would be 0, 1+0 is 1, 1+ 0 is 1, 0 + 0 is 0 and 0 plus 1 is 1, so my code word corresponding to this information message bits information bit is given by 1 0 1, 1 0 1 okay.

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Linear block codes

- An (n,k) linear block code is in *systematic form*, if its generator matrix is in the following form:

$$\mathbf{G} = [\mathbf{P} : \mathbf{I}_k]$$

$$= \begin{bmatrix} p_{0,0} & p_{0,1} & \dots & p_{0,n-k-1} & 1 & 0 & 0 & \dots & 0 \\ p_{1,0} & p_{1,1} & \dots & p_{1,n-k-1} & 0 & 1 & 0 & \dots & 0 \\ p_{2,0} & p_{2,1} & \dots & p_{2,n-k-1} & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{k-1,0} & p_{k-1,1} & \dots & p_{k-1,n-k-1} & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

Now what do we mean by a linear code in systematic form? Now if we are able to, among the coded bits if we are able to separate them out into, if the message bits appear directly in the coded bits sequence then we can separate out the message bits from the parity bits, for example.

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Linear block codes

A generator matrix for this code is

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_0 \\ \mathbf{g}_1 \\ \mathbf{g}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The codeword for the message $\mathbf{u} = (1 \ 0 \ 1)$ is

✓

$$\begin{aligned} \underline{\mathbf{v}} &= \underline{\mathbf{u}} \cdot \mathbf{G} \\ &= 1 \cdot (0 \ 1 \ 1 \ 1 \ 0 \ 0) + 0 \cdot (1 \ 0 \ 1 \ 0 \ 1 \ 0) + 1 \cdot (1 \ 1 \ 0 \ 0 \ 0 \ 1) \\ &= (0 \ 1 \ 1 \ 1 \ 0 \ 0) + (0 \ 0 \ 0 \ 0 \ 0 \ 0) + (1 \ 1 \ 0 \ 0 \ 0 \ 1) \\ &= \underline{(1 \ 0 \ 1 \ 1 \ 0 \ 1)} \end{aligned}$$

Go back to this example.

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Linear block codes

Example 2.1 (contd.): We can write the coded bits in terms of information bits as follows

$$\begin{cases} v_0 = u_1 + u_2 \\ v_1 = u_0 + u_2 \\ v_2 = u_0 + u_1 \\ v_3 = u_0 \\ v_4 = u_1 \\ v_5 = u_2 \end{cases}$$

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5] = [u_0 \ u_1 \ u_2] \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & g_{0,3} & g_{0,4} & g_{0,5} \\ g_{1,0} & g_{1,1} & g_{1,2} & g_{1,3} & g_{1,4} & g_{1,5} \\ g_{2,0} & g_{2,1} & g_{2,2} & g_{2,3} & g_{2,4} & g_{2,5} \end{bmatrix}$$

What do we have here, we have 3 of these coded bits exactly same as information bits and the other 3 bits parity bits are linear combination of this message bits, so from the output code word we, we can clearly separate out the information sequence which is in this case we given by V3, V4 and V5 so in this case V1, V2, V3 are these N – K parity bits.

(Refer Slide Time: 13:05)

Linear block codes

Example 2.1 (contd.): We can write the coded bits in terms of information bits as follows

$$\begin{cases} v_0 = u_1 + u_2 \\ v_1 = u_0 + u_2 \\ v_2 = u_0 + u_1 \\ v_3 = u_0 \\ v_4 = u_1 \\ v_5 = u_2 \end{cases}$$

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5] = [u_0 \ u_1 \ u_2] \begin{bmatrix} g_{0,0} & g_{0,1} & g_{0,2} & g_{0,3} & g_{0,4} & g_{0,5} \\ g_{1,0} & g_{1,1} & g_{1,2} & g_{1,3} & g_{1,4} & g_{1,5} \\ g_{2,0} & g_{2,1} & g_{2,2} & g_{2,3} & g_{2,4} & g_{2,5} \end{bmatrix}$$

And v_3, v_4, v_5 are my information bit, so in this particular example we can see that we are able to separate out information bits directly from the coded bits, so in a systematic, a block

(Refer Slide Time: 13:20)

Linear block codes

A generator matrix for this code is

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_0 \\ \mathbf{g}_1 \\ \mathbf{g}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The codeword for the message $\mathbf{u} = (1 \ 0 \ 1)$ is

✓

$$\begin{aligned} \mathbf{v} &= \mathbf{u} \cdot \mathbf{G} \\ &= 1 \cdot (0 \ 1 \ 1 \ 1 \ 0 \ 0) + 0 \cdot (1 \ 0 \ 1 \ 0 \ 1 \ 0) + 1 \cdot (1 \ 1 \ 0 \ 0 \ 0 \ 1) \\ &= (0 \ 1 \ 1 \ 1 \ 0 \ 0) + (0 \ 0 \ 0 \ 0 \ 0 \ 0) + (1 \ 1 \ 0 \ 0 \ 0 \ 1) \\ &= (1 \ 0 \ 1 \ 1 \ 0 \ 1) \end{aligned}$$

(Refer Slide Time: 13:21)

Linear block codes

- An (n,k) linear block code is in *systematic form*, if its generator matrix is in the following form:

$$G = [P : I_k]$$

$$= \begin{bmatrix} p_{0,0} & p_{0,1} & \dots & p_{0,n-k-1} & 1 & 0 & 0 & \dots & 0 \\ p_{1,0} & p_{1,1} & \dots & p_{1,n-k-1} & 0 & 1 & 0 & \dots & 0 \\ p_{2,0} & p_{2,1} & \dots & p_{2,n-k-1} & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{k-1,0} & p_{k-1,1} & \dots & p_{k-1,n-k-1} & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

Code in a systematic form we are able to separate out the information bit part from the coded bits, so a generator

(Refer Slide Time: 13:33)

Linear block codes

- An (n,k) linear block code is in *systematic form*, if its generator matrix is in the following form:

$$G = [P \mid I_k] \quad [I_k \mid P']$$

$$= \begin{bmatrix} p_{0,0} & p_{0,1} & \dots & p_{0,n-k-1} & 1 & 0 & 0 & \dots & 0 \\ p_{1,0} & p_{1,1} & \dots & p_{1,n-k-1} & 0 & 1 & 0 & \dots & 0 \\ p_{2,0} & p_{2,1} & \dots & p_{2,n-k-1} & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{k-1,0} & p_{k-1,1} & \dots & p_{k-1,n-k-1} & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

Matrix for a linear block code in systematic form will be of the form like this or it would be basically I times I K times something of this, either of this one, now why do we say that? So only when we have our a part of our generator matrix of the form of identity then what is going to happen, when we multiply our information sequence with this sort of generator matrix you will see

(Refer Slide Time: 14:13)

Linear block codes

- An (n,k) linear block code is in *systematic form*, if its generator matrix is in the following form:

$$G = [P : I_k]$$

$$= \begin{bmatrix} p_{0,0} & p_{0,1} & \dots & p_{0,n-k-1} & 1 & 0 & 0 & \dots & 0 \\ p_{1,0} & p_{1,1} & \dots & p_{1,n-k-1} & 0 & 1 & 0 & \dots & 0 \\ p_{2,0} & p_{2,1} & \dots & p_{2,n-k-1} & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{k-1,0} & p_{k-1,1} & \dots & p_{k-1,n-k-1} & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

- Every codeword consists of two parts: a message part and a parity check part.

Part of my coded bits will just depend on one particular information bit sequence, so if I write down the corresponding equations for coded sequence what you will see that some coded bits directly depend on the message bits and then rest R which are parity bits are linear combination of these message bits.

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Linear block codes

- An (n,k) linear block code is in *systematic form*, if its generator matrix is in the following form:

$$\mathbf{G} = [\mathbf{P} : \mathbf{I}_k]$$
$$= \begin{bmatrix} p_{0,0} & p_{0,1} & \dots & p_{0,n-k-1} & 1 & 0 & 0 & \dots & 0 \\ p_{1,0} & p_{1,1} & \dots & p_{1,n-k-1} & 0 & 1 & 0 & \dots & 0 \\ p_{2,0} & p_{2,1} & \dots & p_{2,n-k-1} & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{k-1,0} & p_{k-1,1} & \dots & p_{k-1,n-k-1} & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

- Every codeword consists of two parts: a message part and a parity check part.

So in a systematic form basically we can separate out the message part from the parity bit part, so as I said

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For a systematic linear block codes the message part will consist of the K information bits and the remaining $N - K$ bits which are

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Linear block codes

- For systematic linear block code, the message part consists of the k unaltered message bits, and the parity check part consists of $n - k$ parity check bits.
- The encoding equations for a systematic code are given by (parity check equations):

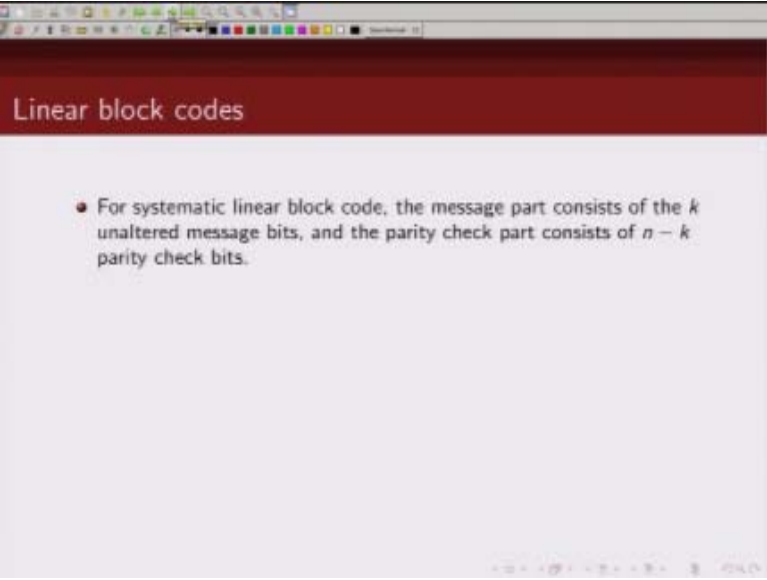
$$v_j = u_0 p_{0,j} + u_1 p_{1,j} + \dots + u_{k-1} p_{k-1,j}, \quad 0 \leq j \leq n - k - 1$$

(message bits.)

$$v_{n-k+i} = u_i, \quad 0 \leq i \leq k - 1$$

The parity bits basically they will be linear combination of these message bits, so we can write down the encoding equations for these matrix's for these, for systematic code. So if you look at what is our encoding equation, our V is U times G where G is a form like this, okay.

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Linear block codes

- For systematic linear block code, the message part consists of the k unaltered message bits, and the parity check part consists of $n - k$ parity check bits.

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Linear block codes

- An (n,k) linear block code is in *systematic form*, if its generator matrix is in the following form:

$$G = [P \mid I_k]$$

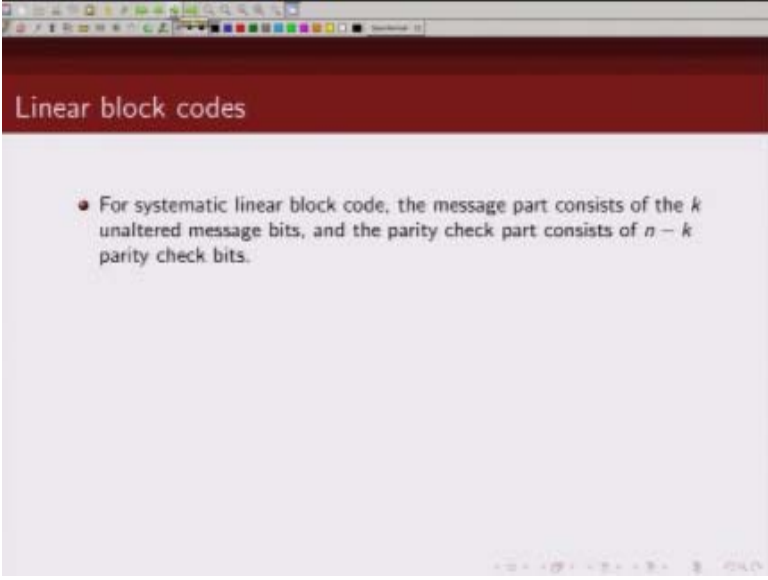
$$= \begin{bmatrix} p_{0,0} & p_{0,1} & \dots & p_{0,n-k-1} & 1 & 0 & 0 & \dots & 0 \\ p_{1,0} & p_{1,1} & \dots & p_{1,n-k-1} & 0 & 1 & 0 & \dots & 0 \\ p_{2,0} & p_{2,1} & \dots & p_{2,n-k-1} & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{k-1,0} & p_{k-1,1} & \dots & p_{k-1,n-k-1} & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

$$v = UG = [u_0 \ u_1 \ \dots \ u_{k-1}]$$

- Every codeword consists of two parts: a message part and a parity check part.

So if we write U which is basically $U(0) \ U(1), \dots, U_{k-1}$ times this G matrix what we will get.

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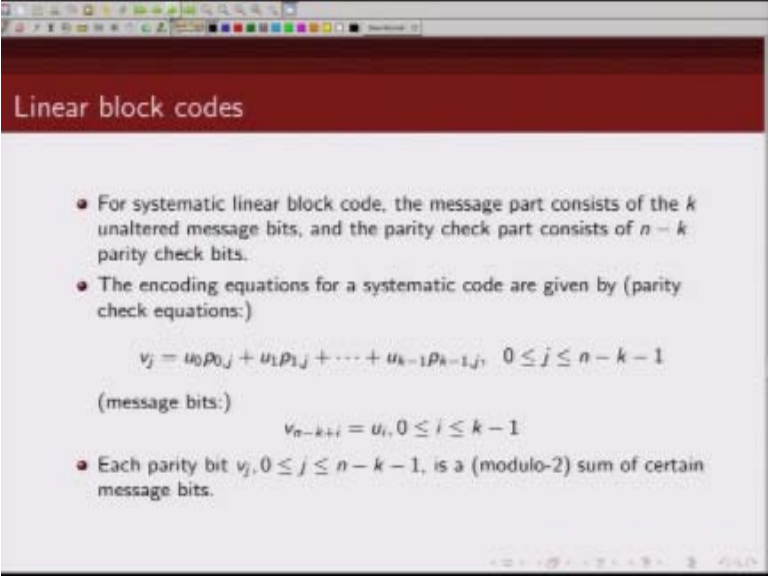


Linear block codes

- For systematic linear block code, the message part consists of the k unaltered message bits, and the parity check part consists of $n - k$ parity check bits.

Is a form.

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Linear block codes

- For systematic linear block code, the message part consists of the k unaltered message bits, and the parity check part consists of $n - k$ parity check bits.
- The encoding equations for a systematic code are given by (parity check equations):
$$v_j = u_0 p_{0,j} + u_1 p_{1,j} + \dots + u_{k-1} p_{k-1,j}, \quad 0 \leq j \leq n - k - 1$$
(message bits:)
$$v_{n-k+i} = u_i, \quad 0 \leq i \leq k - 1$$
- Each parity bit $v_j, 0 \leq j \leq n - k - 1$, is a (modulo-2) sum of certain message bits.

Like this so you will have $N - K$ parity equations, parity check equations which are given by this expression and then you will have remaining K unaltered message bit, so for a linear block code in a systematic form the encoding equations will be of this form, and as I said since we are restricting ourselves without any loss of generality to binary code words this addition is basically.

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Linear block codes

- For systematic linear block code, the message part consists of the k unaltered message bits, and the parity check part consists of $n - k$ parity check bits.
- The encoding equations for a systematic code are given by (parity check equations:)

$$v_j = u_0 p_{0,j} + u_1 p_{1,j} + \dots + u_{k-1} p_{k-1,j}, \quad 0 \leq j \leq n - k - 1$$

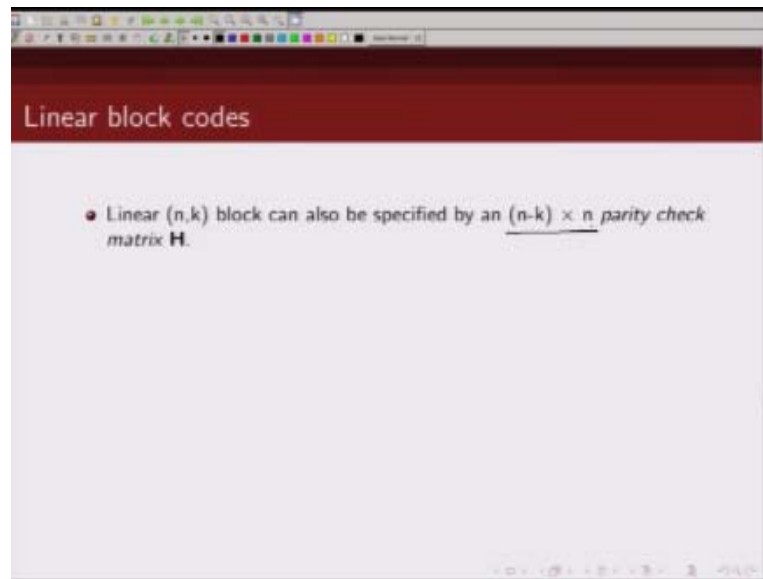
(message bits:)

$$v_{n-k+i} = u_i, \quad 0 \leq i \leq k - 1$$

- Each parity bit $v_j, 0 \leq j \leq n - k - 1$, is a (modulo-2) sum of certain message bits.

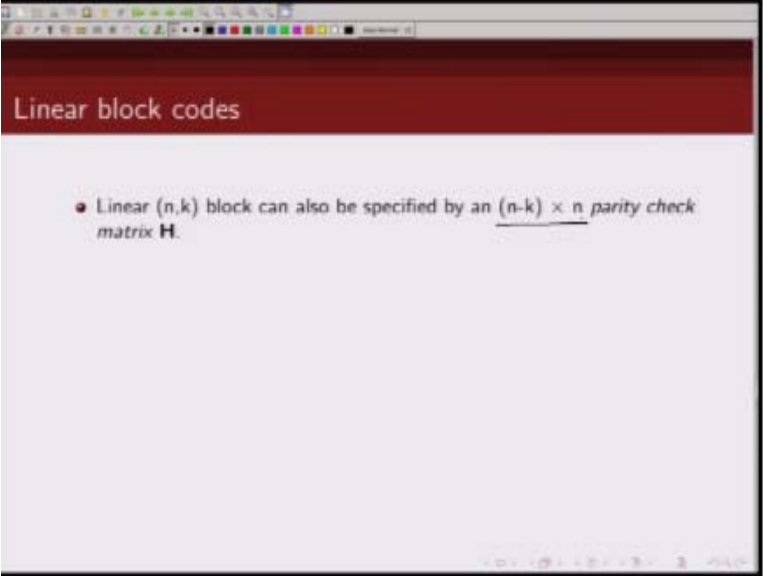
Done modulo-2, so what we have seen so far is we can describe a linear block code by its generator matrix which is a K cross n matrix, and we can use this generator matrix to generate our set of code words. Now there is another matrix which we call parity check matrix which is related to our generator matrix we will show which can also be used to completely describe a linear block code, so for a NK linear block.

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Code can be specified by a $N-K \times n$ parity check matrix which we denote by H , the generator matrix we denote by G and the parity check matrix we denote by H .

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Linear block codes

- Linear (n,k) block can also be specified by an $(n-k) \times n$ *parity check matrix* H .

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Linear block codes

- Linear (n,k) block can also be specified by an $(n-k) \times n$ *parity check matrix* $\mathbf{H}$.
- If $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ is a binary n -tuple, then $\mathbf{v}$ is a codeword if and only if
$$\mathbf{v}\mathbf{H}^T = (0, 0, \dots, 0).$$

Now this parity check matrix has this property that if $\mathbf{V}$ is your valid code word, if and only if $\mathbf{V}\mathbf{H}^T$ is going to be zero, so if $\mathbf{V}$ is a valid code word $\mathbf{V}\mathbf{H}^T$ will be zero so let us see how we can derive our parity check matrix.

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Linear block codes

- Linear (n,k) block can also be specified by an $(n-k) \times n$ *parity check matrix* $\mathbf{H}$.
- If $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ is a binary n -tuple, then $\mathbf{v}$ is a codeword if and only if
$$\mathbf{v}\mathbf{H}^T = (0, 0, \dots, 0).$$
- Example 2.3: Consider a $(7,4)$ linear systematic code with generator matrix
$$\mathbf{G} = \left[\begin{array}{cccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$
$$[P \mid I_k]$$

From a generator matrix and what is the relation of the generator matrix with parity check matrix. So we will take an example of a 7, 4 systematic linear block code whose generator matrix is given by this so since this is a systematic code we can write it of the form E times this I_k this generator matrix can be written of this form okay. Now from this generator equation we can write our coded bits in terms of our message bits so let us do that.

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Linear block codes

- Linear (n, k) block can also be specified by an $(n-k) \times n$ *parity check matrix* $\mathbf{H}$.
- If $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ is a binary n -tuple, then $\mathbf{v}$ is a codeword if and only if
$$\mathbf{v}\mathbf{H}^T = (0, 0, \dots, 0).$$
- Example 2.3: Consider a $(7, 4)$ linear systematic code with generator matrix
$$\mathbf{G} = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$[P \mid I_k]$

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Linear block codes

- The encoding equations can be written as

$$\underset{V}{[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6]} = \underset{U}{[u_0 \ u_1 \ u_2 \ u_3]} \underset{G}{\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}}$$

So the encoding equation says this one was our generator matrix G , this is our information bits message bits U and this is our coded bits V , so we can write V as U times G , so it is a 7, 4 code so there are 4 information bits, I denote them by U_0, U_1, U_2, U_3 and there are 7 coded bits I denote them by V_0, V_1, V_2 to V_6 and this generator matrix I have already given you this so I can write down the encoding equations.

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Linear block codes

- The encoding equations can be written as
$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6] = [u_0 \ u_1 \ u_2 \ u_3] \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$
- We can write this as
$$\begin{aligned} v_0 &= u_0 + u_2 + u_3 \\ v_1 &= u_0 + u_1 + u_2 \\ v_2 &= u_1 + u_2 + u_3 \\ v_3 &= u_0 \\ v_4 &= u_1 \\ v_5 &= u_2 \\ v_6 &= u_3 \end{aligned}$$

We do that, you can see from this what is v_0 it is $u_0 + u_2 + u_3$, there is a typing mistake here this would have been u_2 , so v_0 is u_0 times 1 plus 0 times u_1 + 1 times u_2 + 1 times u_3 so this V_0 is given by $u_0 + u_2 + u_3$. Similarly what is v_1 , v_1 is given by $u_0 + u_1 + u_2$, v_2 is given by $u_1 + u_2 + u_3$, so that is what I have here. What is v_3 , v_3 is given by $u_0 \times 1$ and rest are all 0, so v_3 is nothing but u_0 . Similarly v_4 is u_1 , v_5 is u_2 and v_6 is u_3 . So I now have set of seven coded bits and this shows the relation between the coded bits and information bits. Now we are since we are restricting ourselves to binary code.

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Linear block codes

- The encoding equations can be written as

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6] = [u_0 \ u_1 \ u_2 \ u_3] \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

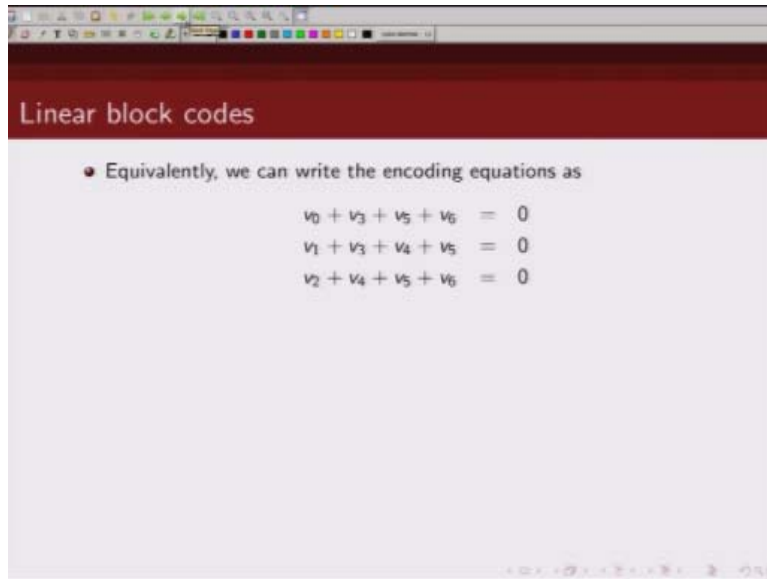
- We can write this as

$v_0 = u_0 + u_2 + u_3$	$v_0 + u_0 + u_2 + u_3 = 0$
$v_1 = u_0 + u_1 + u_2$	$v_1 + u_0 + u_1 + u_2 = 0$
$v_2 = u_1 + u_2 + u_3$	$v_2 + u_1 + u_2 + u_3 = 0$
$v_3 = u_0$	
$v_4 = u_1$	
$v_5 = u_2$	
$v_6 = u_3$	

We can even write this equation like this, $v_0 + u_0 + u_2 + u_3 = 0$, correct? Because this v_0 is nothing but parity bit which is basically like 1 or 0 so if we add this to this modulo 2 sum will be 0. So this similarly we can write as $v_1 + u_0 + u_1 + u_2 = 0$ and this can be written as $v_2 + u_1 + u_2 + u_3 = 0$. The next what we would try to do is we would try to write these parity check equations in terms of other coded bits. So we can see here, u_0 is nothing but v_3 so wherever u_0 appears we can replace it by v_3 .

Similarly, u_1 is equal to v_4 so wherever u_1 appears we can replace it by v_4 , u_2 is equal to v_5 so we can replace u_2 by v_5 and u_3 is equal to v_6 we can replace u_3 in terms of v_6 . By doing this what we will get is set of equations which basically are dependent on these coded bits. So if we do that.

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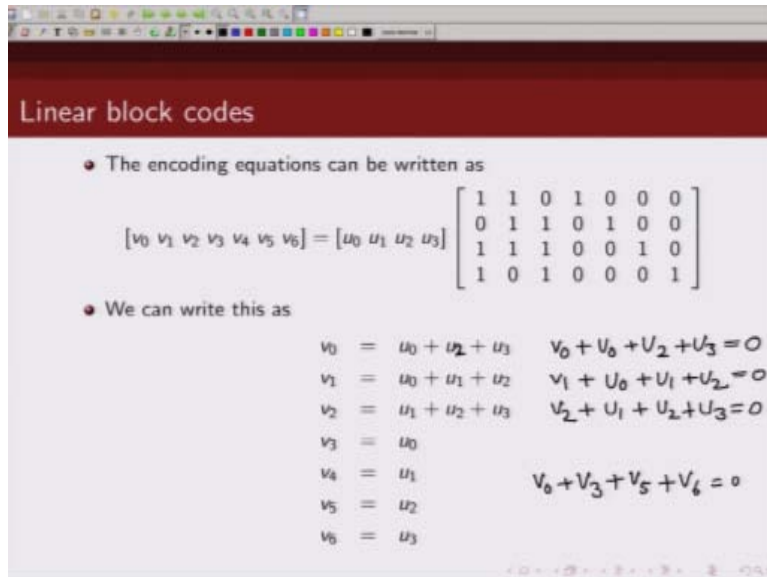
Linear block codes

- Equivalently, we can write the encoding equations as

$$\begin{aligned}v_0 + v_3 + v_5 + v_6 &= 0 \\v_1 + v_3 + v_4 + v_5 &= 0 \\v_2 + v_4 + v_5 + v_6 &= 0\end{aligned}$$

What we get is something like this. The first equation basically which was.

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Linear block codes

- The encoding equations can be written as

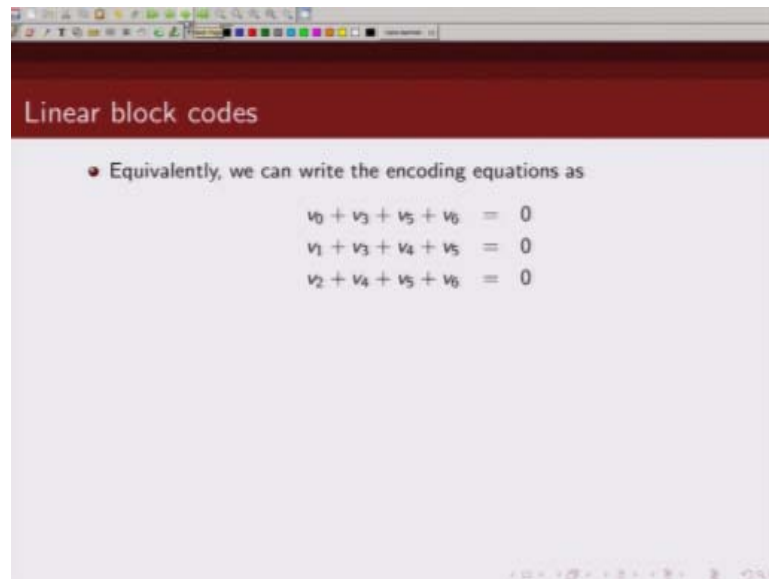
$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6] = [u_0 \ u_1 \ u_2 \ u_3] \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- We can write this as

$$\begin{aligned}v_0 &= u_0 + u_2 + u_3 & v_0 + u_0 + u_2 + u_3 &= 0 \\v_1 &= u_0 + u_1 + u_2 & v_1 + u_0 + u_1 + u_2 &= 0 \\v_2 &= u_1 + u_2 + u_3 & v_2 + u_1 + u_2 + u_3 &= 0 \\v_3 &= u_0 \\v_4 &= u_1 & v_0 + v_3 + v_5 + v_6 &= 0 \\v_5 &= u_2 \\v_6 &= u_3\end{aligned}$$

$v_0 + u_0 + u_2 + u_3$, now this can be re-written as $v_0 +$ what is u_0 , u_0 is v_3 , $v_3 +$ what is u_2 , u_2 is v_5 , $v_5 +$ what is u_3 it is v_6 so $v_0 + v_3 + v_5 + v_6 = 0$ and that is what we have here.

(Refer Slide Time: 23:22)

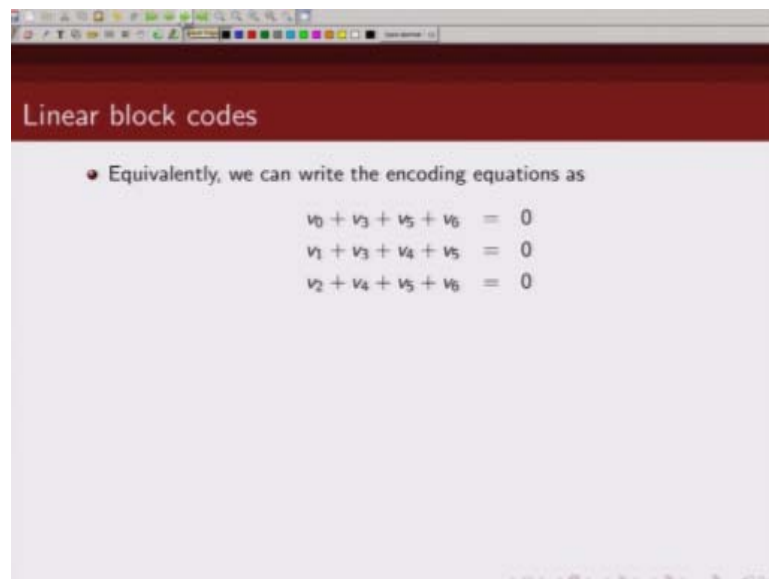


The screenshot shows a presentation slide with a dark red header containing the title "Linear block codes". Below the header, a bullet point states: "Equivalently, we can write the encoding equations as". Three equations are listed:

$$\begin{aligned}v_0 + v_3 + v_5 + v_6 &= 0 \\v_1 + v_3 + v_4 + v_5 &= 0 \\v_2 + v_4 + v_5 + v_6 &= 0\end{aligned}$$

$v_0 + v_3 + v_5 + v_6 = 0$.

(Refer Slide Time: 23:31)



This screenshot is identical to the one above, showing the same presentation slide with the title "Linear block codes" and the three encoding equations:

$$\begin{aligned}v_0 + v_3 + v_5 + v_6 &= 0 \\v_1 + v_3 + v_4 + v_5 &= 0 \\v_2 + v_4 + v_5 + v_6 &= 0\end{aligned}$$

(Refer Slide Time: 23:32)

Linear block codes

- The encoding equations can be written as

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6] = [u_0 \ u_1 \ u_2 \ u_3] \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- We can write this as

$$\begin{aligned} v_0 &= u_0 + u_2 + u_3 & v_0 + v_1 + v_2 + v_3 &= 0 \\ v_1 &= u_0 + u_1 + u_2 & v_1 + v_0 + v_1 + u_2 &= 0 \\ v_2 &= u_1 + u_2 + u_3 & v_2 + v_1 + v_2 + u_3 &= 0 \\ v_3 &= u_0 \\ v_4 &= u_1 & v_0 + v_3 + v_5 + v_6 &= 0 \\ v_5 &= u_2 \\ v_6 &= u_3 \end{aligned}$$

Similarly we can write the other equations as well, here also we will replace u_0, u_1, u_2 by v_3, v_4, v_5 and what we will get is.

(Refer Slide Time: 23:43)

Linear block codes

- Equivalently, we can write the encoding equations as

$$\begin{aligned} v_0 + v_3 + v_5 + v_6 &= 0 \\ v_1 + v_3 + v_4 + v_5 &= 0 \\ v_2 + v_4 + v_5 + v_6 &= 0 \end{aligned}$$

$v_1 + v_3 + v_4 + v_5 = 0$ and similarly the last parity check equation can be written as.

(Refer Slide Time: 23:55)

Linear block codes

- The encoding equations can be written as

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6] = [u_0 \ u_1 \ u_2 \ u_3] \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- We can write this as

$$\begin{aligned} v_0 &= u_0 + u_2 + u_3 & v_0 + v_1 + v_2 + v_3 &= 0 \\ v_1 &= u_0 + u_1 + u_2 & v_1 + v_0 + v_1 + v_2 &= 0 \\ v_2 &= u_1 + u_2 + u_3 & v_2 + v_1 + v_2 + v_3 &= 0 \\ v_3 &= u_0 \\ v_4 &= u_1 & v_0 + v_3 + v_5 + v_6 &= 0 \\ v_5 &= u_2 \\ v_6 &= u_3 \end{aligned}$$

$v_2 + u_1$ is $v_4 + u_2$ is $v_5 + u_3$ is v_6 , so that is what we have here.

(Refer Slide Time: 24:04)

Linear block codes

- Equivalently, we can write the encoding equations as
$$\begin{aligned}v_0 + v_3 + v_5 + v_6 &= 0 \\v_1 + v_3 + v_4 + v_5 &= 0 \\v_2 + v_4 + v_5 + v_6 &= 0\end{aligned}$$
- In matrix form,
$$\begin{bmatrix} v_0 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_2 + v_4 + v_5 + v_6 = 0$. So now we have set of encoding equations in terms of coded bits. Next the same thing we can write it in a matrix form, so I have my coded bits v_0 to v_6 , I have three sets of parity check equations this, this and this. And the same thing I can write it in a matrix form like this. Now you can see these are equivalent so look. Let us look at first equation, this is $v_0 + v_3 + v_5 + v_6 = 0$. So you can see which are the elements which are, so v_0 times 1, this is v_3 times 1, this is v_5 times 1 + v_6 times 1. So that is what define this equation. Similarly you can see this equation, this $v_1 + v_3 + v_4 + v_5 = 0$ and this last equation this is $v_2 + v_4 + v_5 + v_6 = 0$. And what did we say

(Refer Slide Time: 25:22)

Linear block codes

- Equivalently, we can write the encoding equations as

$$\begin{aligned}v_0 + v_3 + v_5 + v_6 &= 0 \\v_1 + v_3 + v_4 + v_5 &= 0 \\v_2 + v_4 + v_5 + v_6 &= 0\end{aligned}$$

(Refer Slide Time: 25:23)

Linear block codes

- The encoding equations can be written as

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6] = [u_0 \ u_1 \ u_2 \ u_3] \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- We can write this as

$$\begin{aligned}v_0 &= u_0 + u_2 + u_3 & v_0 + v_3 + v_5 + v_6 &= 0 \\v_1 &= u_0 + u_1 + u_2 & v_1 + v_3 + v_4 + v_5 &= 0 \\v_2 &= u_1 + u_2 + u_3 & v_2 + v_4 + v_5 + v_6 &= 0 \\v_3 &= u_0 \\v_4 &= u_1 \\v_5 &= u_2 \\v_6 &= u_3\end{aligned}$$

(Refer Slide Time: 25:24)

Linear block codes

- The encoding equations can be written as

$$\underbrace{[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6]}_{\mathbf{v}} = \underbrace{[u_0 \ u_1 \ u_2 \ u_3]}_{\mathbf{u}} \underbrace{\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{G}}$$

About parity check matrix.

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Linear block codes

- Linear (n,k) block can also be specified by an $(n-k) \times n$ parity check matrix $\mathbf{H}$.
- If $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ is a binary n -tuple, then $\mathbf{v}$ is a codeword if and only if

$$\mathbf{v}\mathbf{H}^T = (0, 0, \dots, 0).$$
- Example 2.3: Consider a $(7,4)$ linear systematic code with generator matrix

$$\mathbf{G} = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$[P \mid I_k]$

We said that if $\mathbf{H}$ is a parity check matrix it is an $(n-k) \times n$ matrix and it has this property that $\mathbf{v}\mathbf{H}^T$ is 0.

(Refer Slide Time: 25:41)

Linear block codes

- The encoding equations can be written as

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6] = [u_0 \ u_1 \ u_2 \ u_3] \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- We can write this as

$$\begin{aligned} v_0 &= u_0 + u_2 + u_3 & v_0 + u_0 + u_2 + u_3 &= 0 \\ v_1 &= u_0 + u_1 + u_2 & v_1 + u_0 + u_1 + u_2 &= 0 \\ v_2 &= u_1 + u_2 + u_3 & v_2 + u_1 + u_2 + u_3 &= 0 \\ v_3 &= u_0 \\ v_4 &= u_1 & v_0 + v_3 + v_5 + v_6 &= 0 \\ v_5 &= u_2 \\ v_6 &= u_3 \end{aligned}$$

So we have.

(Refer Slide Time: 25:43)

Linear block codes

- Equivalently, we can write the encoding equations as

$$\begin{aligned} v_0 + v_3 + v_5 + v_6 &= 0 \\ v_1 + v_3 + v_4 + v_5 &= 0 \\ v_2 + v_4 + v_5 + v_6 &= 0 \end{aligned} \quad (7,4)$$

- In matrix form,

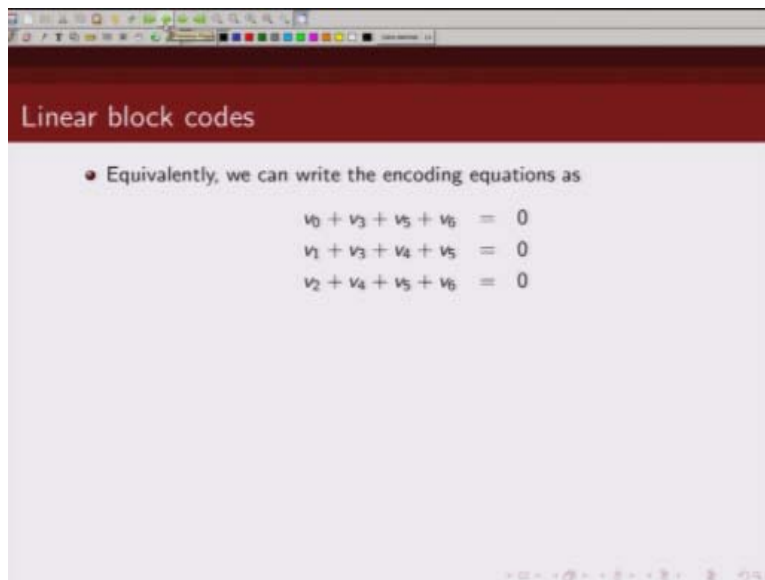
$$\underbrace{[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6]}_V \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}^H = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$
3x7

Can write this as, this is my v, this is my H^T vH^T is 0. So then what is my H matrix, H matrix is a transpose of this matrix so this will be 100 010 001 110 011 111 and 101 and this is my, so for

the (7, 4) code (7, 4) code this is basically 3x7, as I said (n-k) x n matrix this is my parity check matrix corresponding to this same code.

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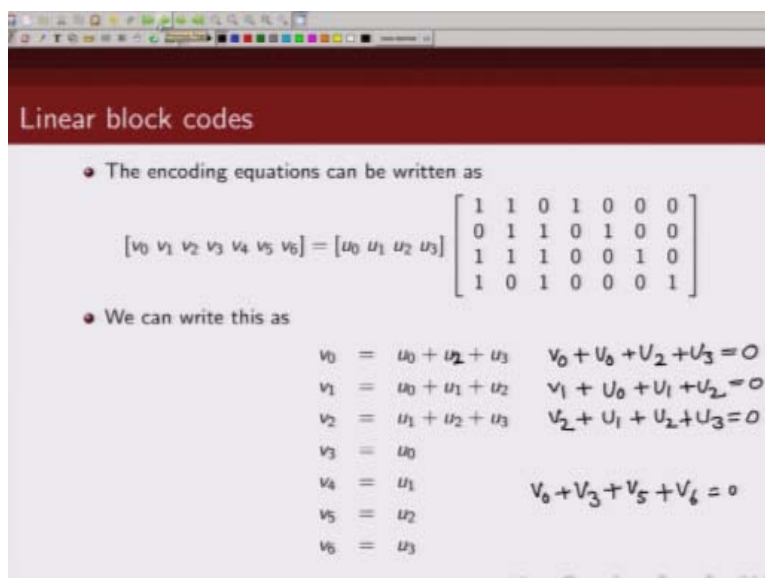
Linear block codes

- Equivalently, we can write the encoding equations as

$$\begin{aligned}v_0 + v_3 + v_5 + v_6 &= 0 \\v_1 + v_3 + v_4 + v_5 &= 0 \\v_2 + v_4 + v_5 + v_6 &= 0\end{aligned}$$

Which is generated by this

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Linear block codes

- The encoding equations can be written as

$$[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6] = [u_0 \ u_1 \ u_2 \ u_3] \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- We can write this as

$$\begin{aligned}v_0 &= u_0 + u_2 + u_3 & v_0 + v_3 + v_5 + v_6 &= 0 \\v_1 &= u_0 + u_1 + u_2 & v_1 + v_3 + v_4 + v_5 &= 0 \\v_2 &= u_1 + u_2 + u_3 & v_2 + v_4 + v_5 + v_6 &= 0 \\v_3 &= u_0 \\v_4 &= u_1 \\v_5 &= u_2 \\v_6 &= u_3\end{aligned}$$

(Refer Slide Time: 26:36)

Linear block codes

- The encoding equations can be written as

$$\underbrace{[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6]}_{\mathbf{v}} = \underbrace{[u_0 \ u_1 \ u_2 \ u_3]}_{\mathbf{u}} \underbrace{\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{G}}$$

(Refer Slide Time: 26:38)

Linear block codes

- Linear (n, k) block can also be specified by an $(n-k) \times n$ parity check matrix $\mathbf{H}$.
- If $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ is a binary n -tuple, then $\mathbf{v}$ is a codeword if and only if

$$\mathbf{v}\mathbf{H}^T = (0, 0, \dots, 0), \quad \mathbf{u}\mathbf{G}\mathbf{H}^T = 0$$
- Example 2.3: Consider a $(7, 4)$ linear systematic code with generator matrix

$$\mathbf{G} = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$[P \mid I_k]$

Another interesting property that you can basically see is so $\mathbf{v}\mathbf{H}^T$ is 0, I can write this is $\mathbf{u}\mathbf{G}\mathbf{H}^T = 0$. In other words $\mathbf{v}\mathbf{H}^T$ is 0 so what does that mean, the rows of $\mathbf{G}$ matrix and rows of $\mathbf{H}$ matrix are orthogonal to each other.

(Refer Slide Time: 27:03)

Linear block codes

- Linear (n, k) block can also be specified by an $(n-k) \times n$ parity check matrix H .
- If $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ is a binary n -tuple, then $\mathbf{v}$ is a codeword if and only if

$$\mathbf{vH}^T = (0, 0, \dots, 0), \quad \mathbf{uGH}^T = 0$$
- Example 2.3: Consider a $(7, 4)$ linear systematic code with generator matrix

$$\mathbf{G} = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$[P \mid I_k]$

(Refer Slide Time: 27:03)

Linear block codes

- The encoding equations can be written as

$$\underbrace{[v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6]}_{\mathbf{V}} = \underbrace{[u_0 \ u_1 \ u_2 \ u_3]}_{\mathbf{U}} \underbrace{\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{G}}$$

So the H lies in the null space of G .

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Linear block codes

- Equivalently, we can write the encoding equations as

$$\begin{aligned}v_0 + v_3 + v_5 + v_6 &= 0 \\v_1 + v_3 + v_4 + v_5 &= 0 \\v_2 + v_4 + v_5 + v_6 &= 0\end{aligned}\quad (7,4)$$

- In matrix form,

$$\begin{array}{c} \mathbf{V} \\ \hline [v_0 \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6] \end{array} \begin{array}{c} \mathbf{H}^T \\ \left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{array} \right] \end{array} = \begin{array}{c} \mathbf{H} \\ \left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{array} \right] \end{array} \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{c} 0 \\ 0 \\ 0 \end{array}$$

3×7

So as we can see from this that generator matrix and parity check matrix are related to each other and they have this property that rows of G matrix and H matrix are basically orthogonal to each other.

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Linear block codes

- For a systematic code with generator matrix $\mathbf{G} = [\mathbf{P} : \mathbf{I}_k]$, the parity check matrix can be written as,

$$\mathbf{H} = [\mathbf{I}_{n-k} : \mathbf{P}^T]$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \dots & 0 & p_{0,0} & p_{1,0} & \dots & p_{k-1,0} \\ 0 & 1 & 0 & \dots & 0 & p_{0,1} & p_{1,1} & \dots & p_{k-1,1} \\ 0 & 0 & 1 & \dots & 0 & p_{0,2} & p_{1,2} & \dots & p_{k-1,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & p_{0,n-k-1} & p_{1,n-k-1} & \dots & p_{k-1,n-k-1} \end{array} \right]$$

So if you have a systematic code whose generator matrix can be written in this form because the $\mathbf{H}$ lies in the null space of $\mathbf{G}$ we can write down its corresponding $\mathbf{H}$ matrix very easily and this is basically given by, so if a generator matrix can be written of the form $\mathbf{Pn}$ identity matrix we can write its parity check matrix as identity matrix and $\mathbf{P}^T$.

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Linear block codes

Example 2.3: Consider a (7, 4) linear systematic code with generator matrix

$$\mathbf{G} = \left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

Then the parity-check matrix in systematic form is

$$\mathbf{H} = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{array} \right] \checkmark$$

So let us take an example of a generator matrix for a systematic code, this is systematic code we can see, we can separate out this generator matrix as some matrix P and some identity matrix. So this we can write as the H matrix we can write as identity matrix and P^T . So then this can be written as 110 is 110, 011, 011, 111, 111, 010, so this is my H matrix corresponding to this. So whether you are given a generator matrix or a parity check matrix your linear block code is completely specified by either of them.

And as I said we use the generator matrix we generate our code set of code words whereas parity check matrix as the name suggest is used to check whether the parity check constraints are satisfied.

(Refer Slide Time: 28:58)

Linear block codes

Example 2.3: Consider a (7, 4) linear systematic code with generator matrix

$$G = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

Then the parity-check matrix in systematic form is

$$H = \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array} \right] \quad \checkmark \quad vH^T = 0$$

As we said basically parity check matrix has this property that is if v is a valid code word if and only if vH^T is 0. And we use this property in decoding and that is why you see the name parity check matrix because this matrix H is essentially used to in some sense check whether the parity check constraints of the code are satisfied or not. Thank you.

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