Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Error Control Coding: An Introduction to Convolutional Codes

Lecture – 2B
Introduction to Convolutional Codes-II
State Diagram, Trellis Diagram

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Welcome, to the course on error control coding an introduction to convolutional code. Today we are going to talk about how to draw the state diagram and trellis diagram of a convolutional code.

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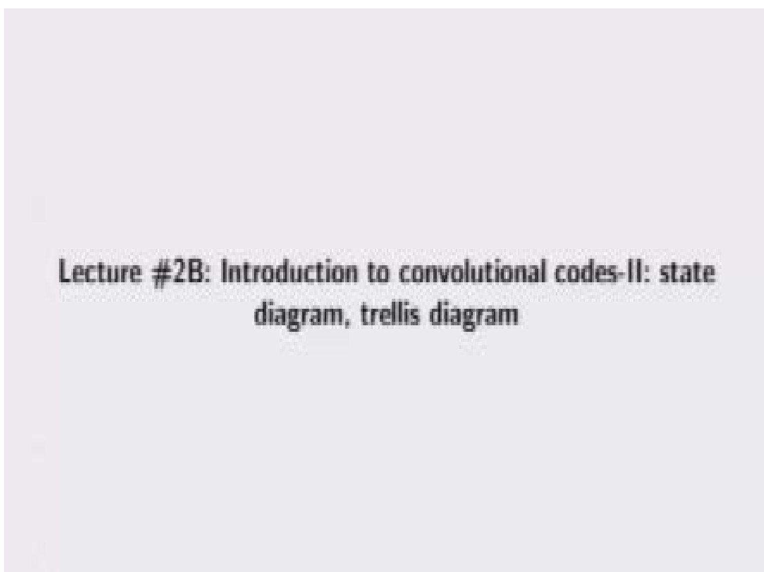

So today's lecture deals with state diagram and trellis diagram of a convolutional code.

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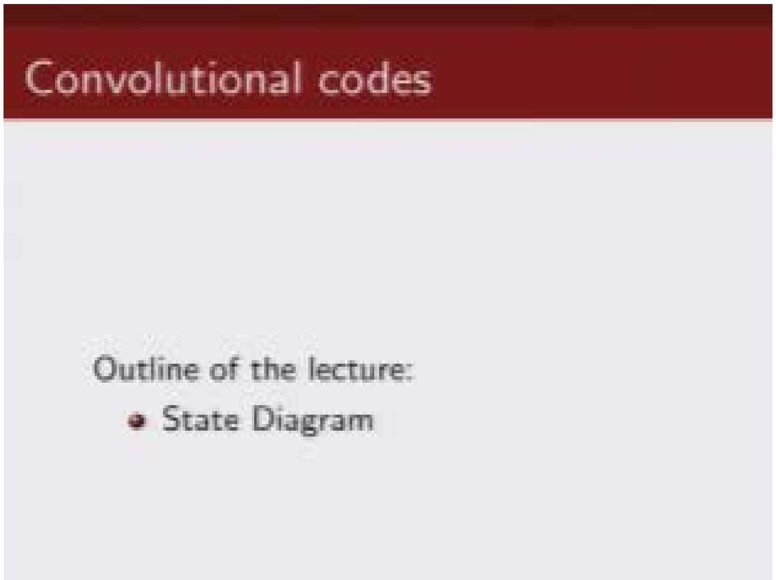

So we will start of this lecture with description of how to construct the state diagram for convolutional code.

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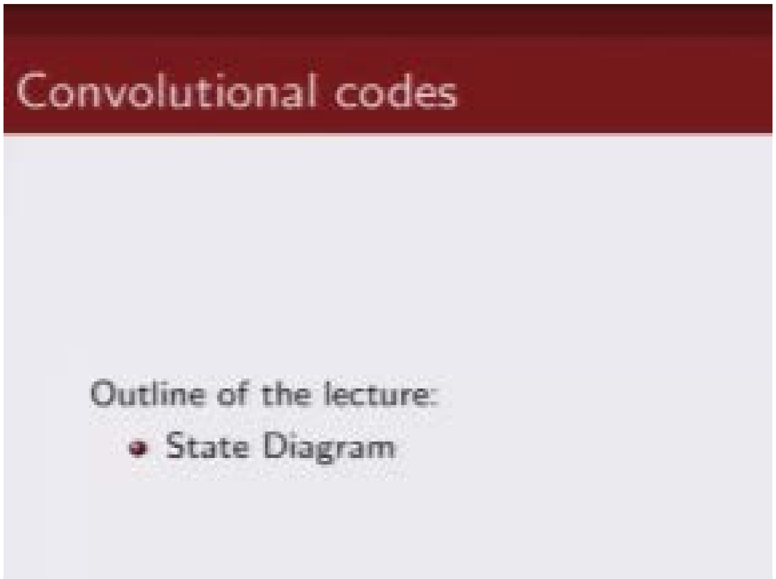

And then we will talk about the Trellis diagram.

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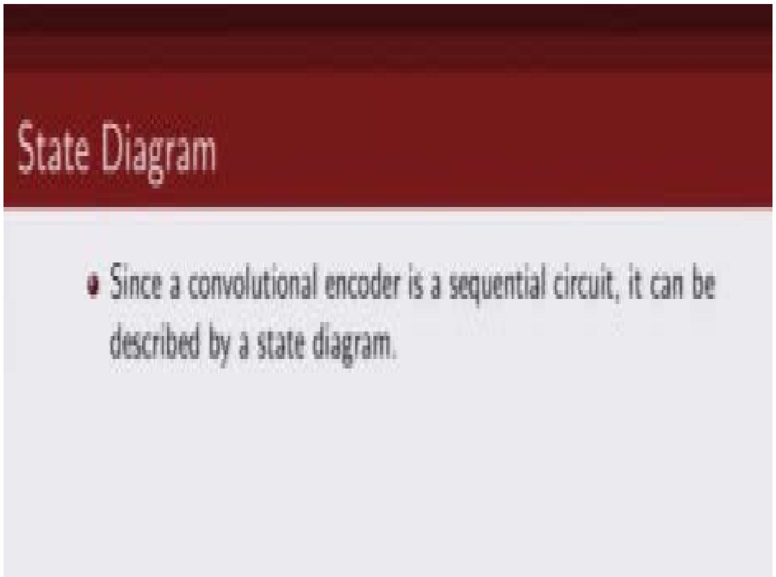

So before we talk about state diagram and trellis diagram, just few words about convolutional code and its equivalent state diagram. Since it is a sequential circuit the convolutional encoder we can represent it using a state diagram and we are going to make use of this state diagram and trellis diagram when we will be decoding the convolutional code.

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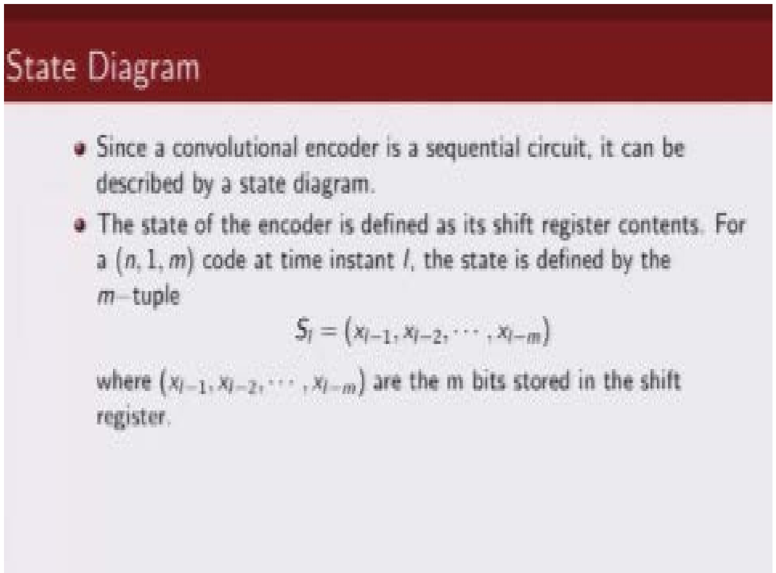

So state diagram is described as follows, a state of an encoder is basically defined by the contents of its shift register. So if you have a rate 1/n code whose memory order is m.

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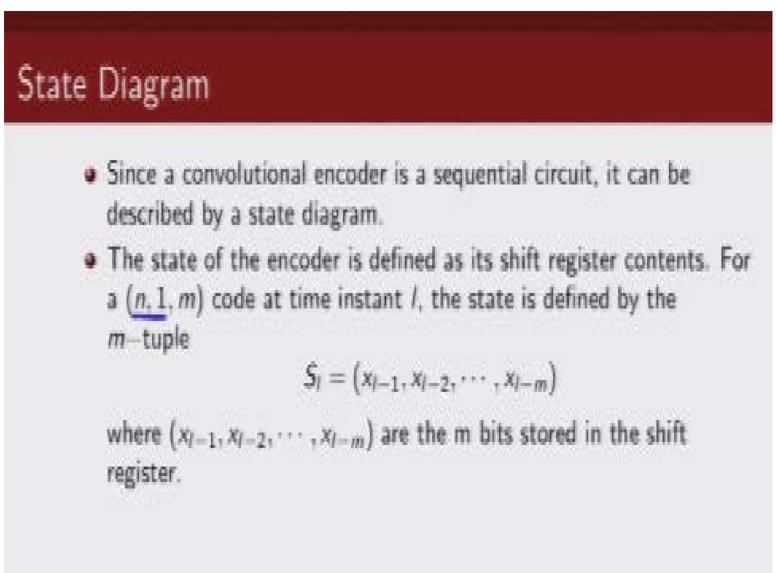

Then we define the state at time l by S sub l where xl-1, x1-2 up to xl-m these are essentially the contents of the shift registers.

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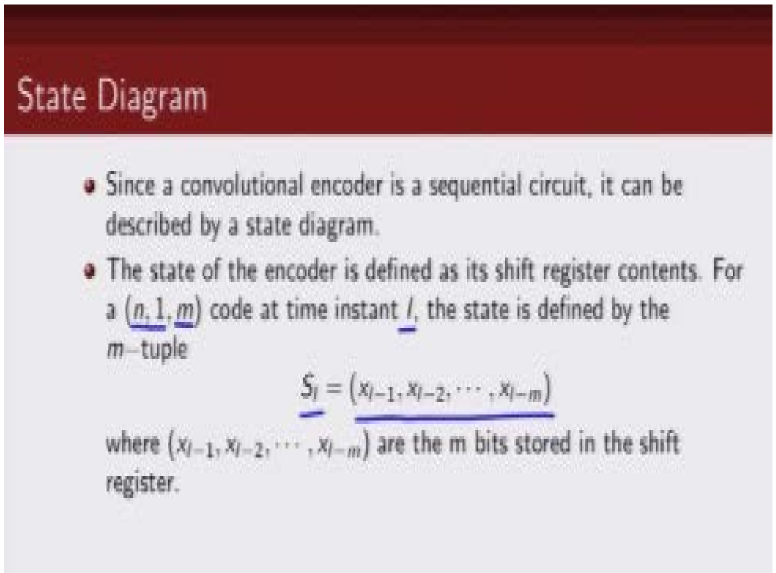

So the contents of the shift register will define what is the state of the convolutional encoder.

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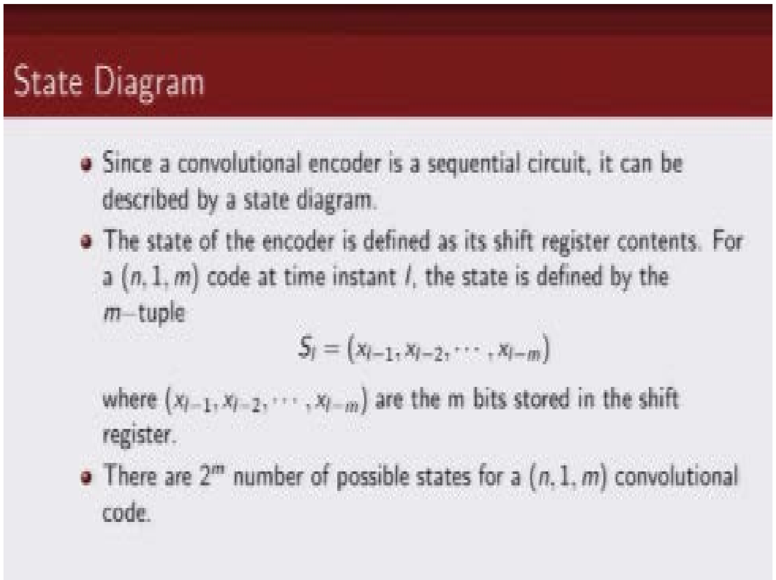

So if we have a convolutional code whose memory is m a rate 1/n code then essentially we will have total 2^m states, because we will have one set of shift register to represent this convolutional code because k=1 and since memory is m, so total there are 2^m different possibilities for the state depending on whether the bit is 0 or 1 which is stored in the shift register.

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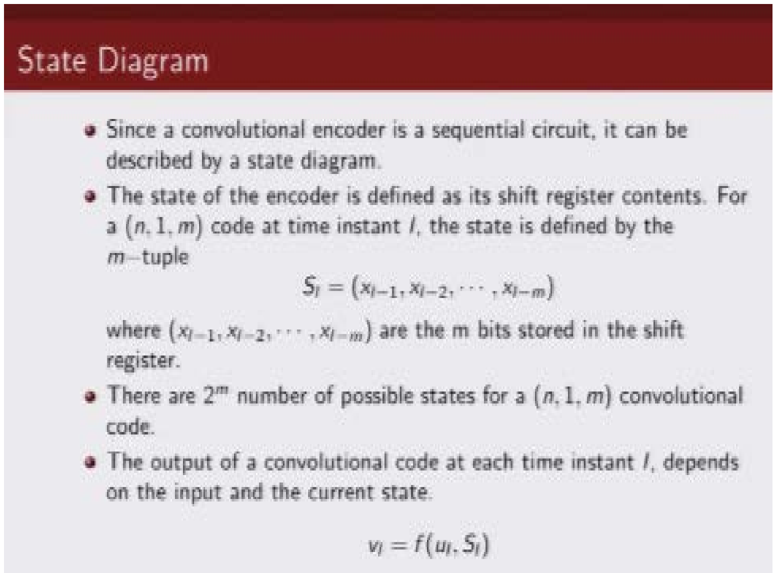

And then output of a convolutional code.

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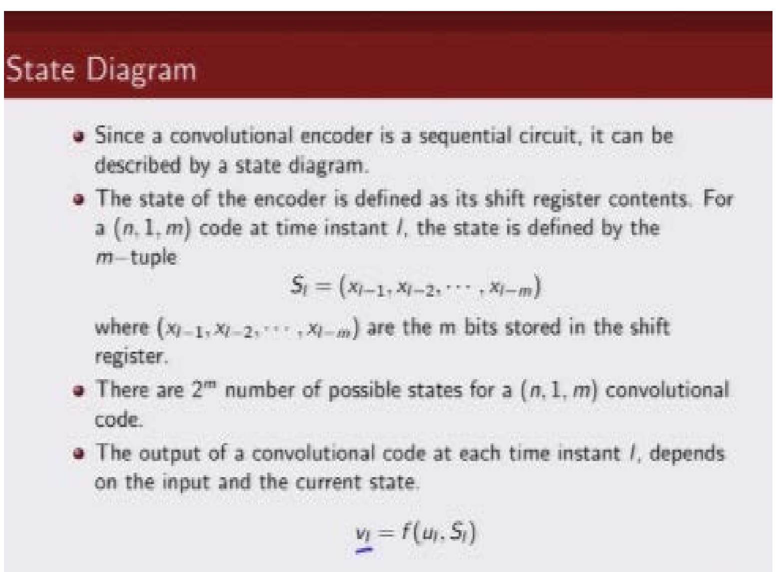

As we know is function of what is the input.

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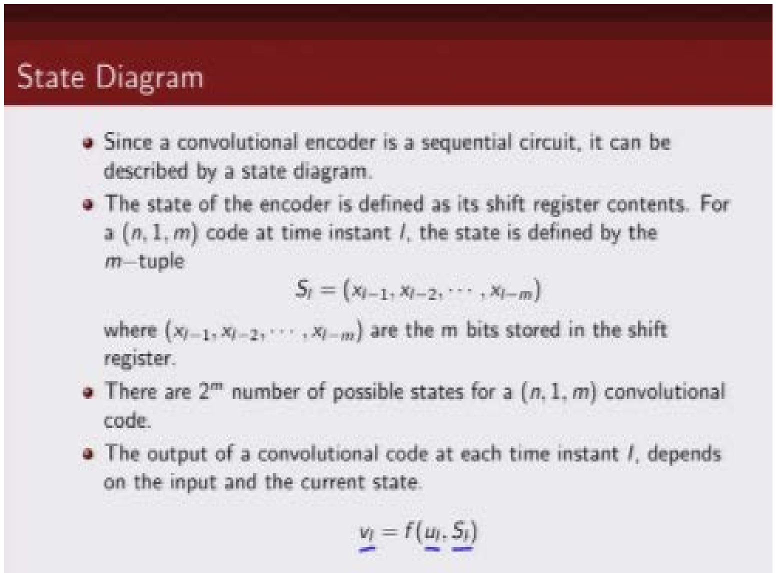

And what is the current state.

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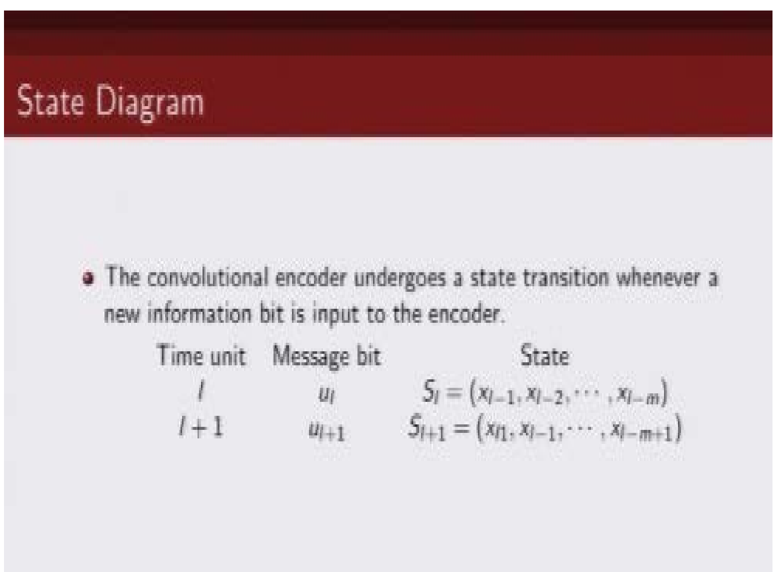

And what happens when we receive a new bit? When we receive a new bit the convolutional encoder is in a particular state let us say at time t, when it receives a new bit that new bit will transform it will take it to some other state and some output will be emitted. So we undergo what we call.

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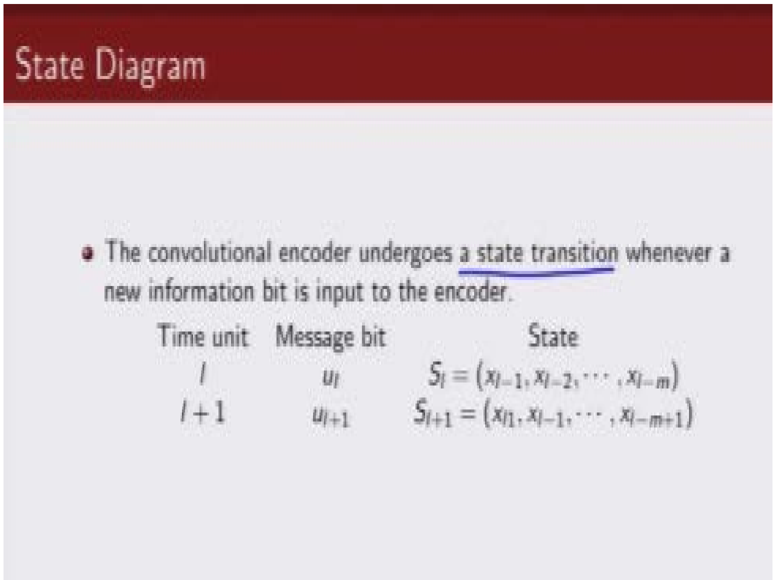

A state transition whenever a new information bit arise so what happens when the new information bit will arrive it will transform it from a given state let us call it S_l to another state S_{l+1}.

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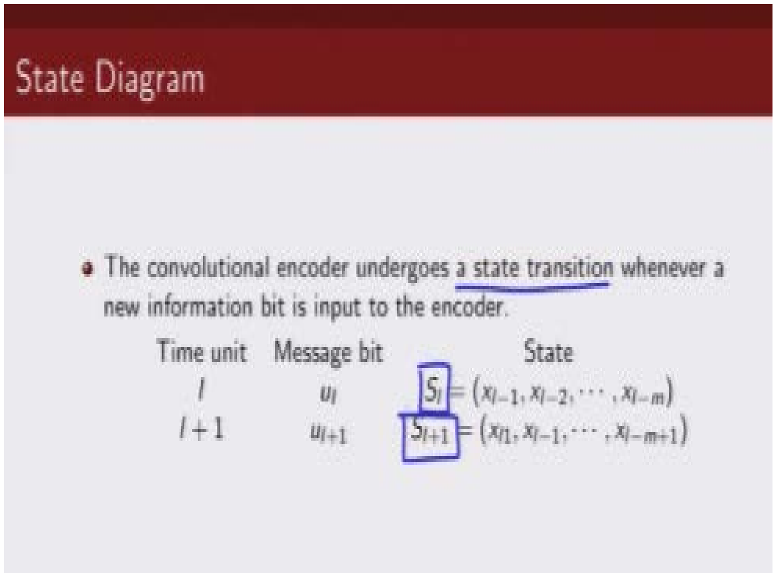

And this transition happens when you receive a new information message bit.

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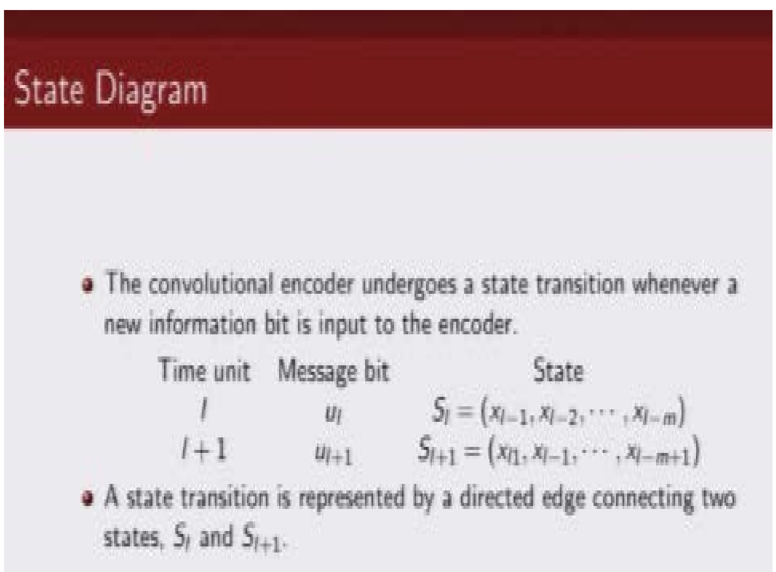

And these state transitions are represented in a state diagram by a directed edge. So if there is a directed edge from state S_l to state S_{l+1} That means from state S_l you will undergo state transition to state S_{l+1} when a particular new bit has arrived.

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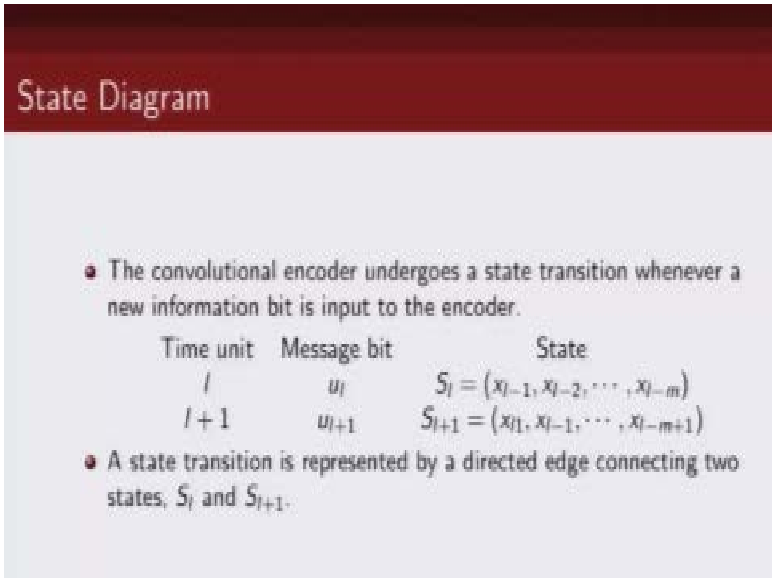

And normally what we do is.

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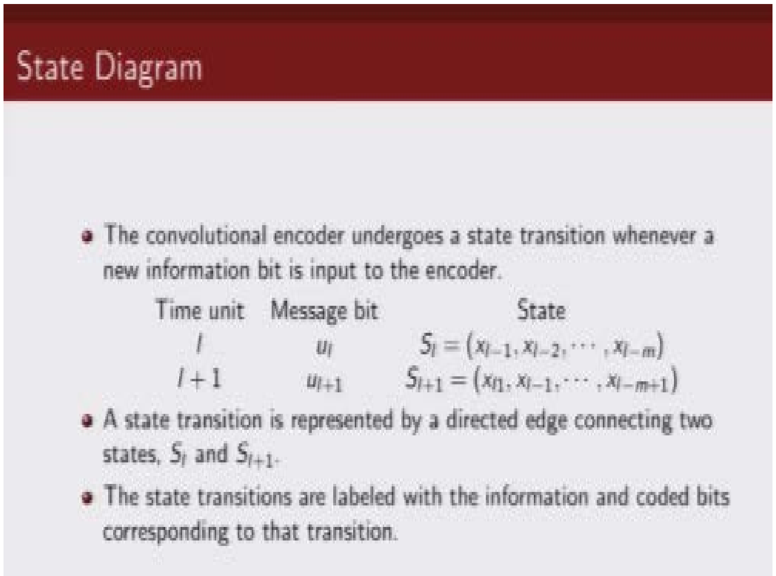

We label these state transitions with what is that information bit.

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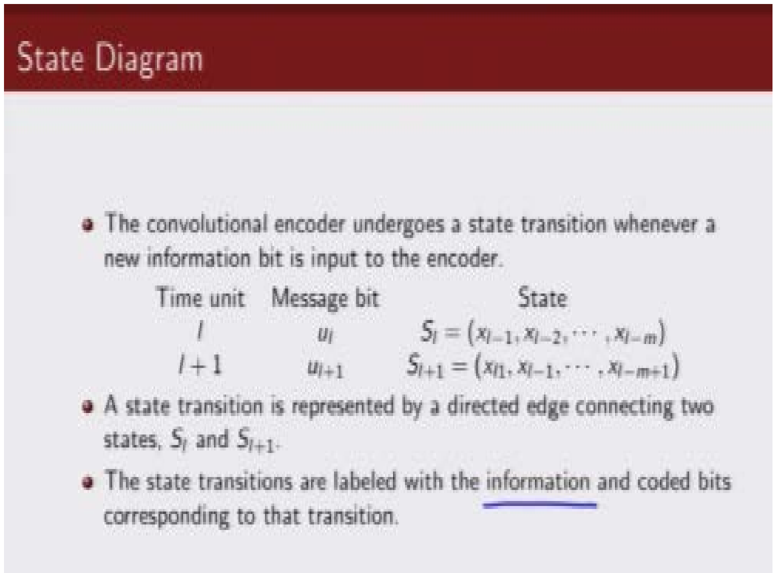

That cause that state transition and what are the coded bits.

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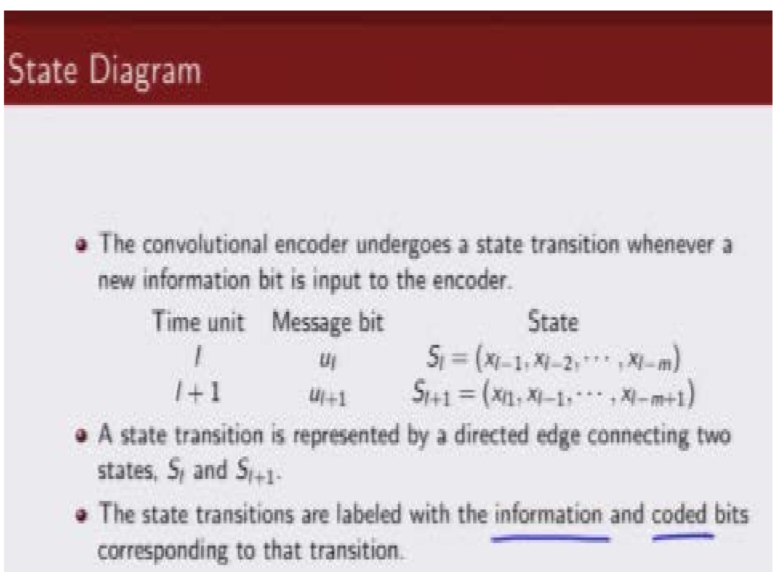

Corresponding to this state transition. So let us say there is a information bit zero which causes you to move from state 00 to state 01 then in this we will draw a directed graph, directed edge from state 00 to state 01 and we will label it by the input which is zero and we will also write what is the corresponding output.

So that is how we will basically label all these state consequence and in a state diagram we are essentially enumerating all possible state diagrams corresponding to all possible states.

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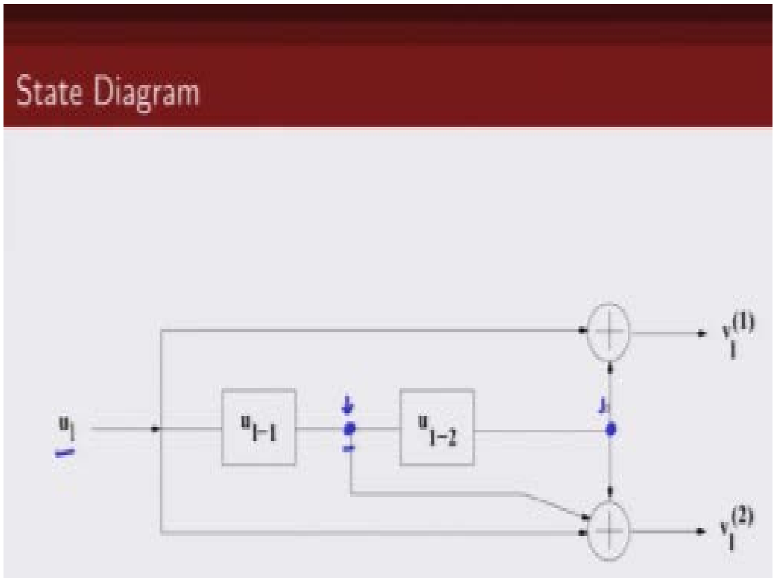

So let us take the same example that we have been dealing with so far this is a rate ½ convolutional code with memory two and what are the state of this. So state of this convolutional encoder is given by what is the content of these shift register which I am denoting by these blue dots. So what is the content here and what is the content there that is essentially the state of the convolutional encoder.

As you can see basically when you get a new information bit depending upon what the state is. You will get corresponding output and what happens? In this new information bit will cause the next cycle this information between move here and whatever was here will move here, so there will be a state transition, because this u_l will move to this location.

And what was stored here will move to this location. Okay, so let us try to draw the state diagram for this convolutional encoder.

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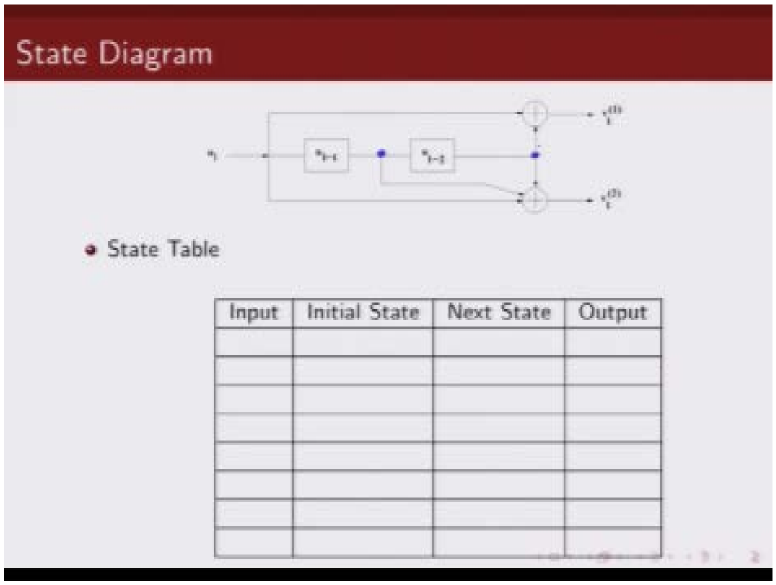

So as we said basically these are the states of the convolutional encoder so we have to. We first enumerate all possible state and all possible input so since the memory is two –

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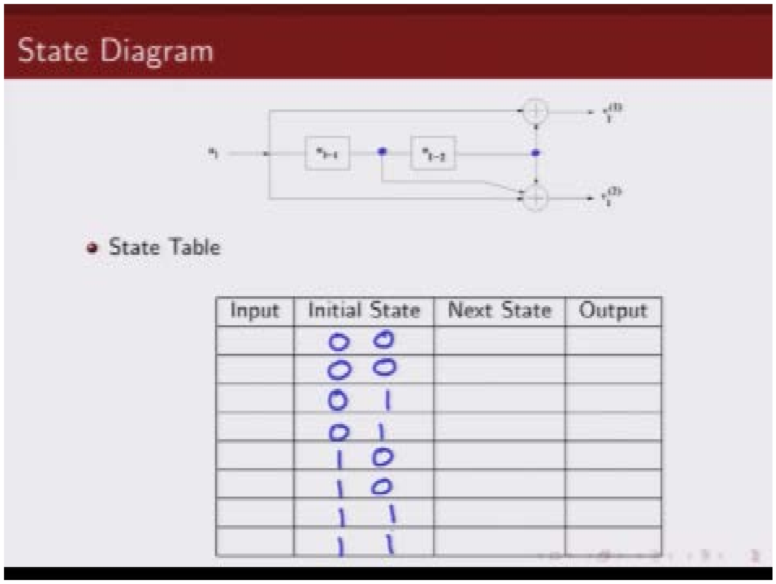

The all possible states are 00, 01, let us say 00, 01, 10 and 11.

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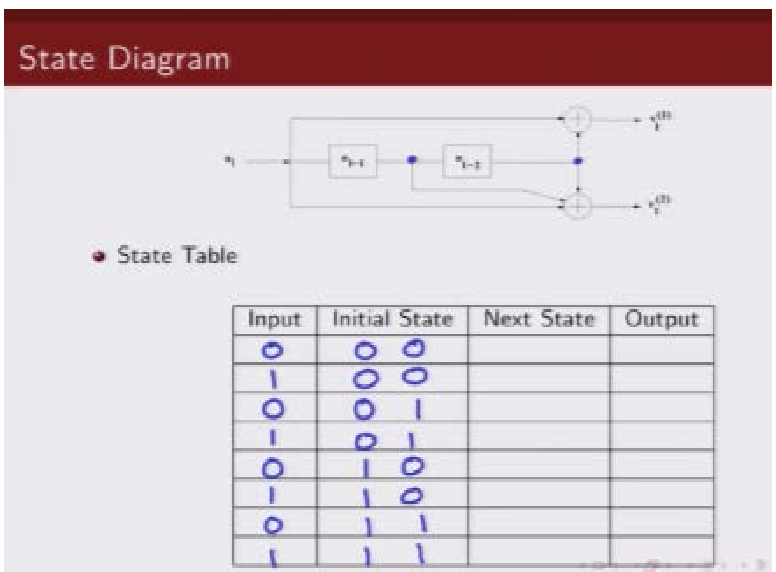

And corresponding to each state we can have input 0 or 1, input 0 or 1, input 0 or 1, input 0 or 1. Okay. So now let us look at each case separately so we have a input 0 and initial state 0. Now what is going to happen in the next state so when there is a transition this 0 will move in here and this is 00 so this 0 will move here. So when a 0 comes and if you are in initial state 00 this will move to state 00.

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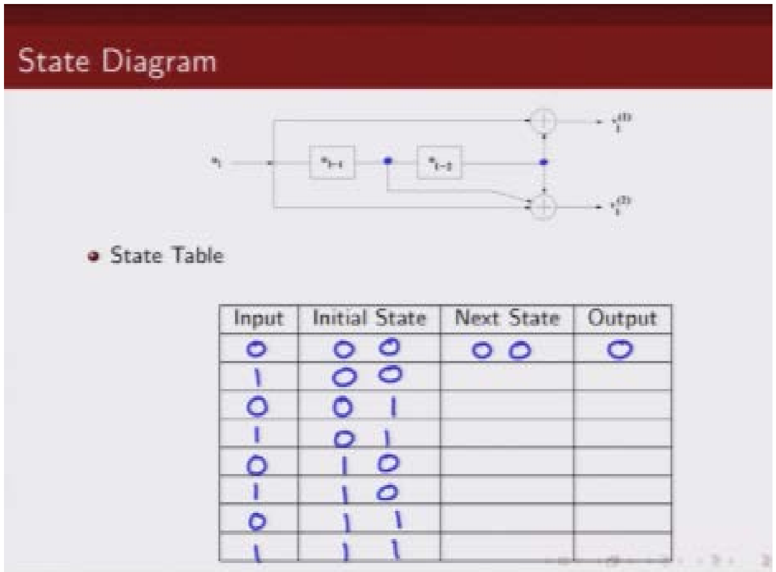

So it will remain in state 00 if the input is 0 and what is the corresponding output, since the input is 0 and these state is 0 what you will get here is 0+0 is 0 and 0+0+0 is 0 so output will be 0.

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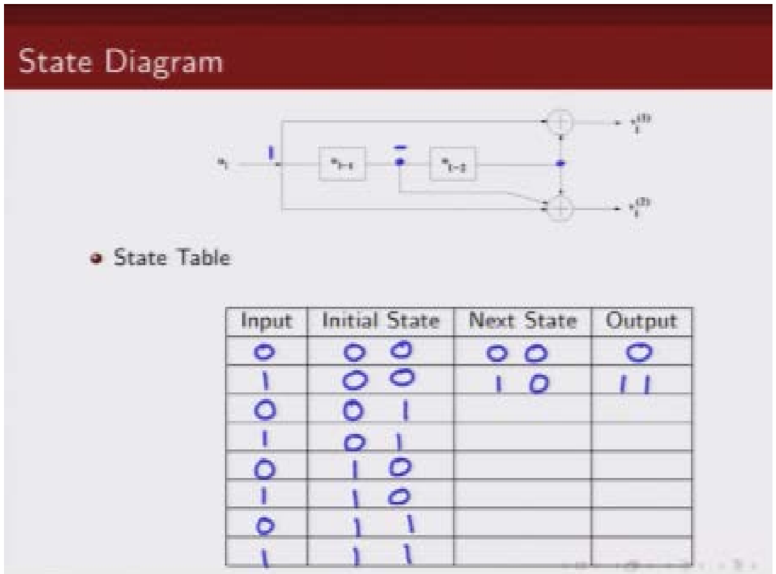

Now let us look at this if the input is 1 and initial state is 00 then this 1, the 1 which is here will now move to this location, right? So the next state would be 10 because 1 from this input will move here and 0 from here will move so the next state would be 10. And what will be the output since the input is 0 initial states are 0 so 1+0 will be 1. And 1+0+0 will be 1 so this output will be 1 and this output is 00 we have already mention this.

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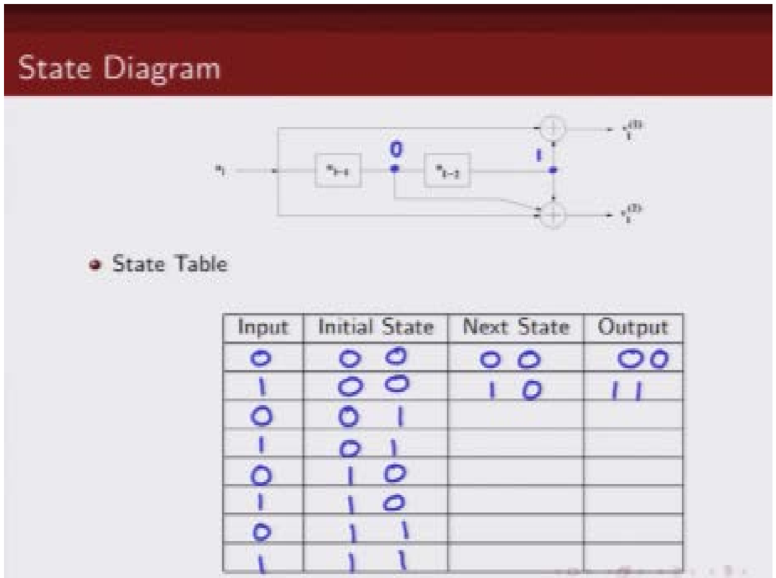

Let us look at next case initial state 01, so initially this is 0 and this is 1. If the input is 0, if the input is 0 what is going to happen? This 0 will move here and this 0 will move there so when you get a 0 and the initial state is 00 next state will be 00.

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Because this 0 which is here will move to here and this 0 will move to here so next state will be 00 and what is the output? This will be 1 and this will be 1 so output is 11.

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Next if the initial state is 01 but the input is 1 so in that case the next state would be this 0 will move here and this 1 will move here. So next state would be 10 and what is the corresponding output? This will be 1+1 that is 0 and this will be 1+0+1 that is again 0 okay.

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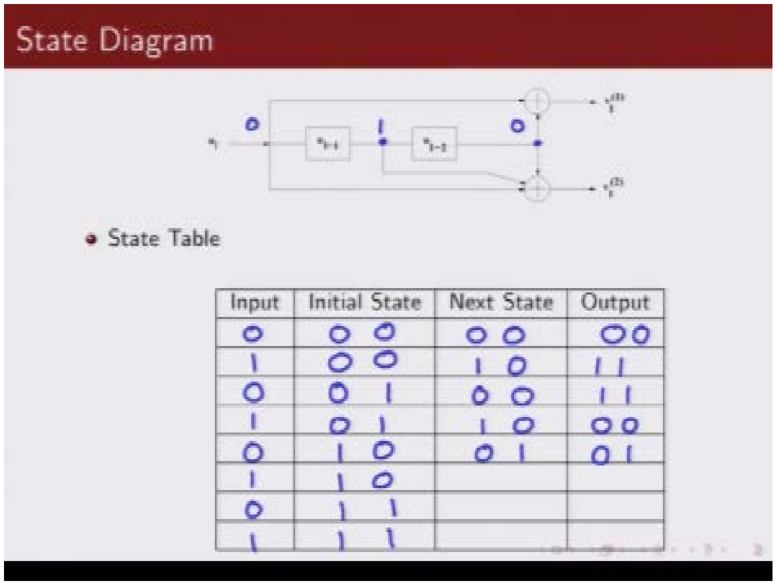

Now let us look at another case. So next you have initial state 10. So initial state 10 if the input is 0 then the next state will be 01 and what is the output if the input is 0 the output will be 0+0 is 0 and 0+1+0 is 1. So output is 01 and if the input is 1.

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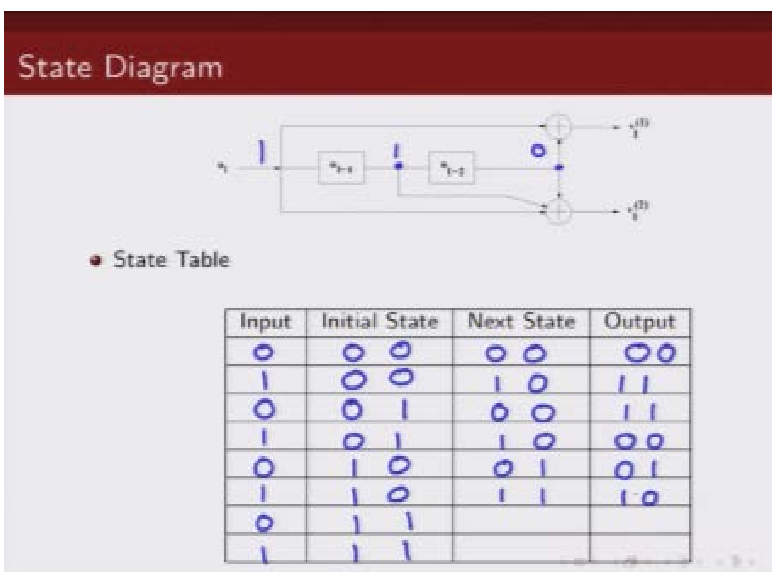

Then the next state would be 11 and the output will be 1+0 is 1 and 1+1+0 is 0.

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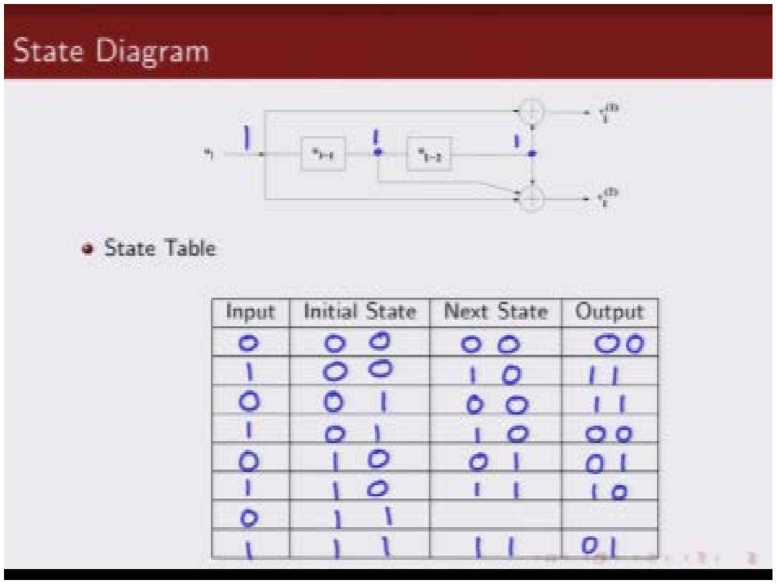

And finally if the next state is 11. Let us say if the input is 1 then the next state is 11 and output will be 1+1 is 0 and 1+1+1 is 1. And if this is instead of 1 if this is 0.

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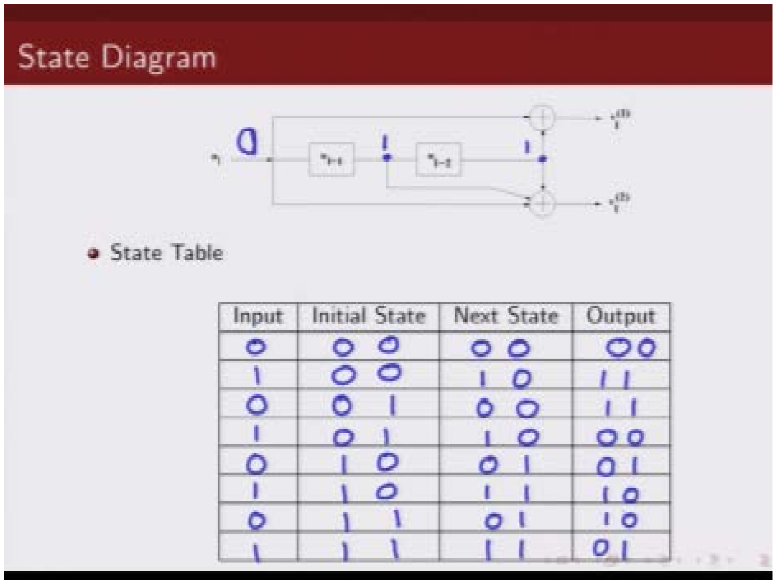

The next state would be 01 and output will be 0 + 1 we gets 1 and this is 0+1+1 this is 0. So note that we have drawn all possible state transitions corresponding to all possible inputs and all possible initial state, now what the state diagram does it, it represent the state table graphically and how do we show it graphically.

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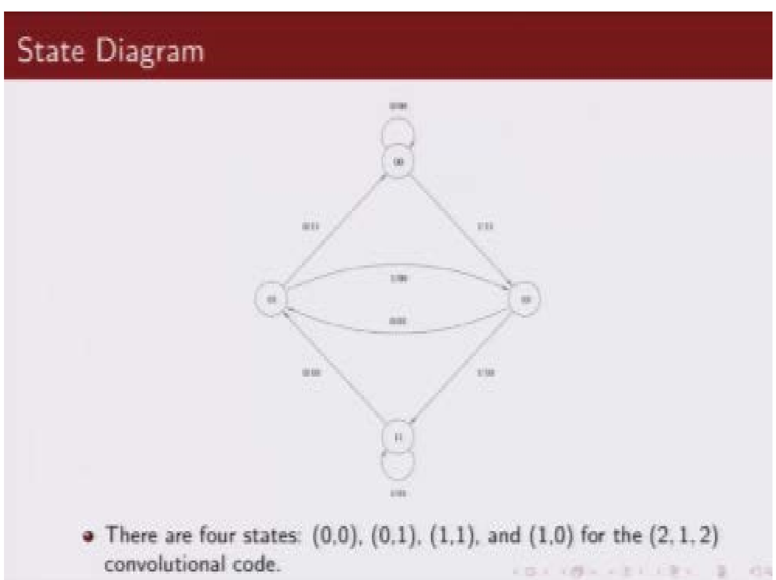

So will go back to this so note here so we four states (0, 0), (0, 1), (1, 0), (1, 1) and we are showing each transitions graphically so if you go back and see here.

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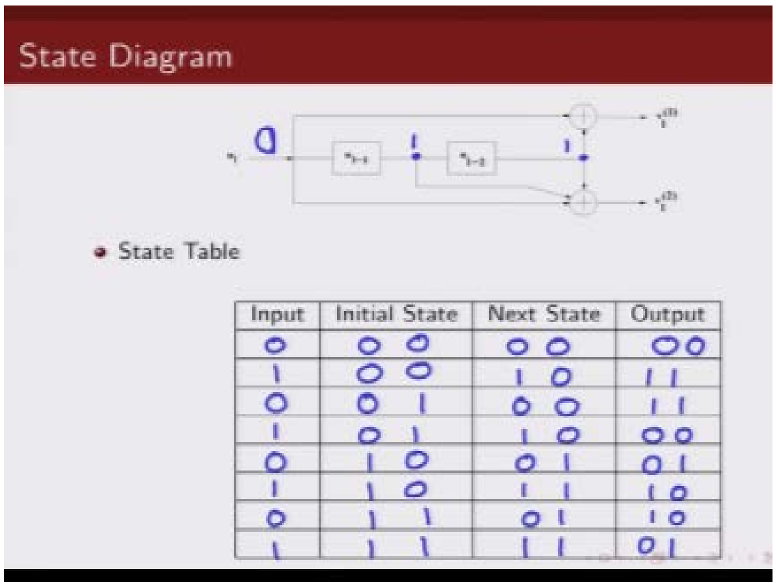

If the initial state is 0 and input is 0 it remains in 0 state and output is 00, so how do we denote this?

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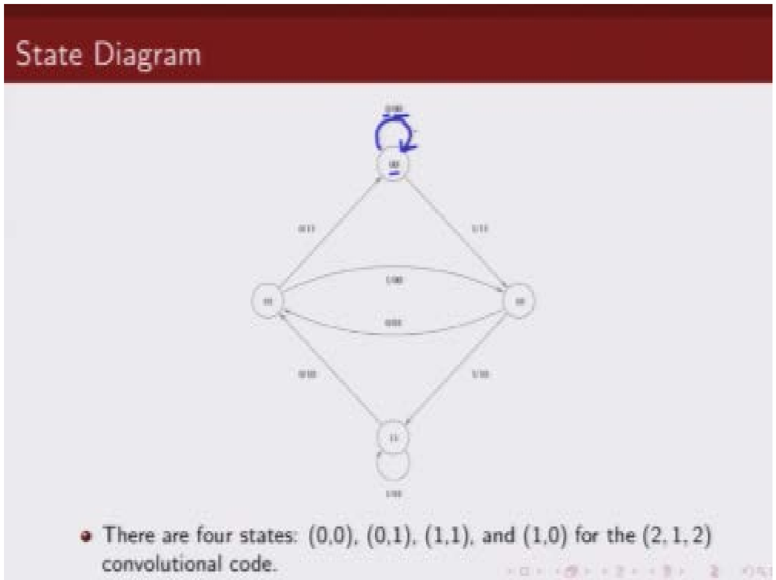

Initial state is 00. Input is 0. Corresponding output is 00. And it remains in all 0 state, so that we are denoting like this.

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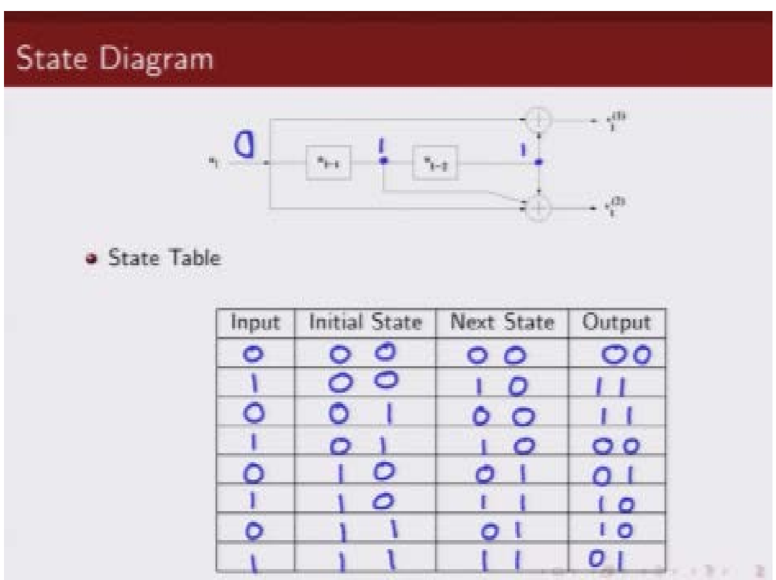

Now what happens when you are in 00 state and you receive a 1. If you are in 00 state and you receive a 1 you move to a next state 10 and the output is 11.

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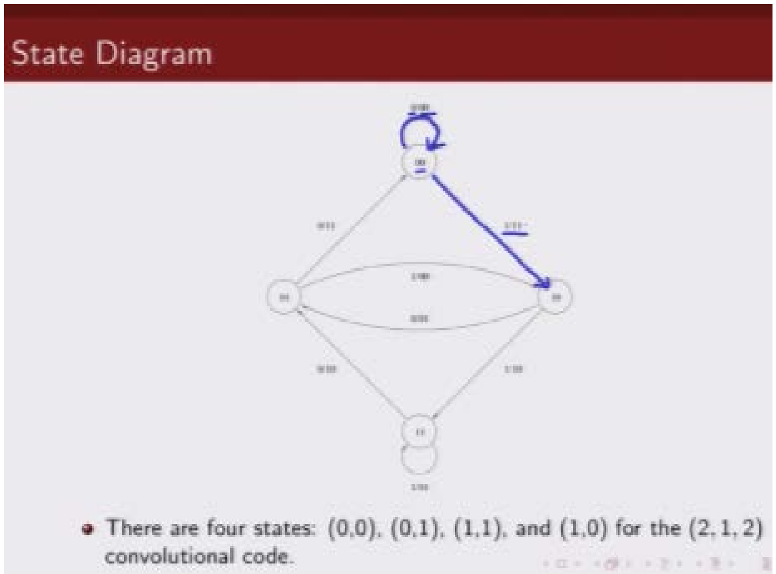

So when you get a 1 you move to next state which is 10 and the output is 11. So note how I'm writing this state transition graphically. So I show by a directed graph transition from state 00 to state 10 and I am labeling this state transition by the corresponding input and output. So 1 is the input and after slash I have the output.

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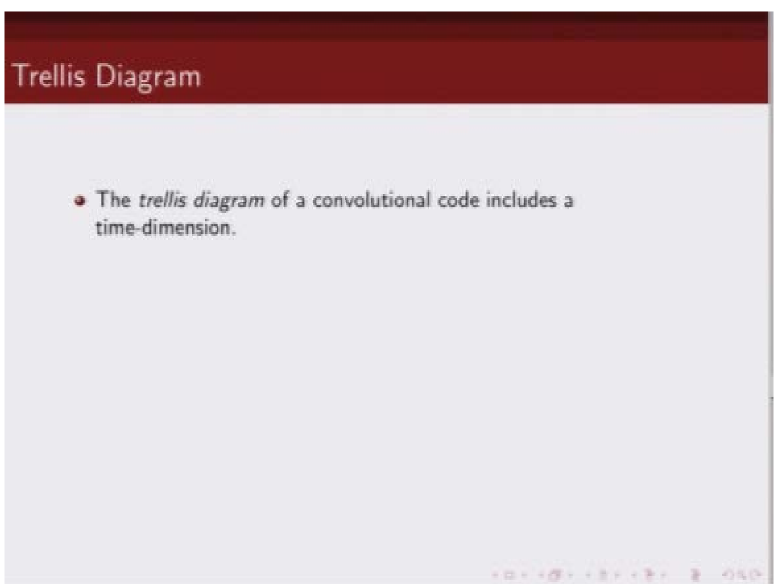

Similarly you did it for all the states.

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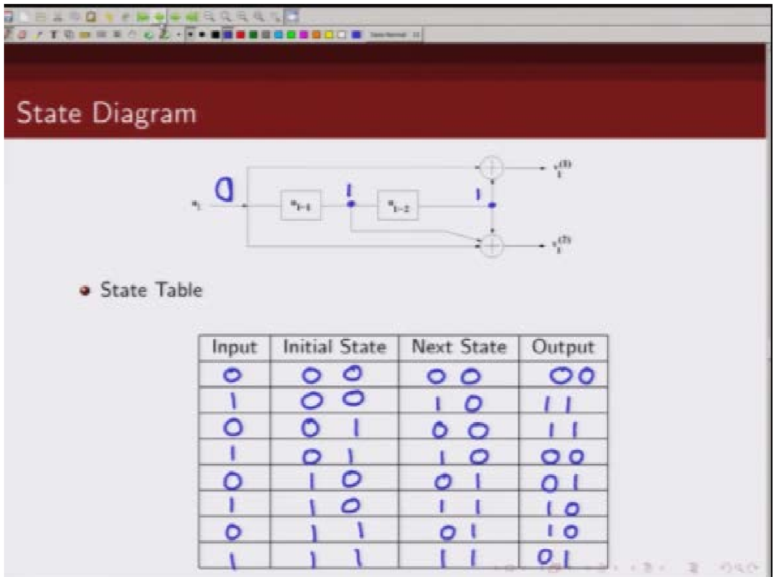

Input	Initial State	Next State	Output
0	0 0	0 0	0 0
1	0 0	1 0	1 1
0	0 1	0 0	1 1
1	0 1	1 0	0 0
0	1 0	0 1	0 1
1	1 0	1 1	1 0
0	1 1	0 1	1 0
1	1 1	1 1	0 1

So if the initial state is 0, 01 and you get a 0 you move to 00 state.

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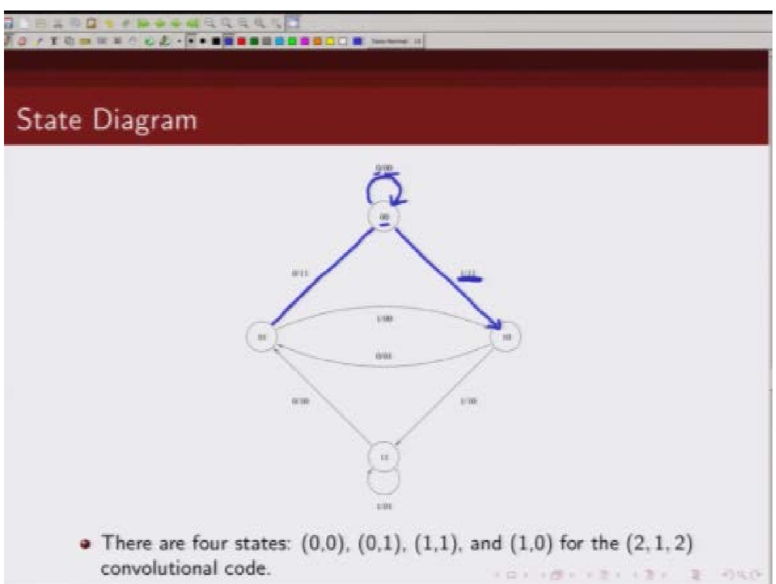

And output is 11. So if you are in initial state (0,1) you receive a 0, you go back to all zero state and output is 1,1

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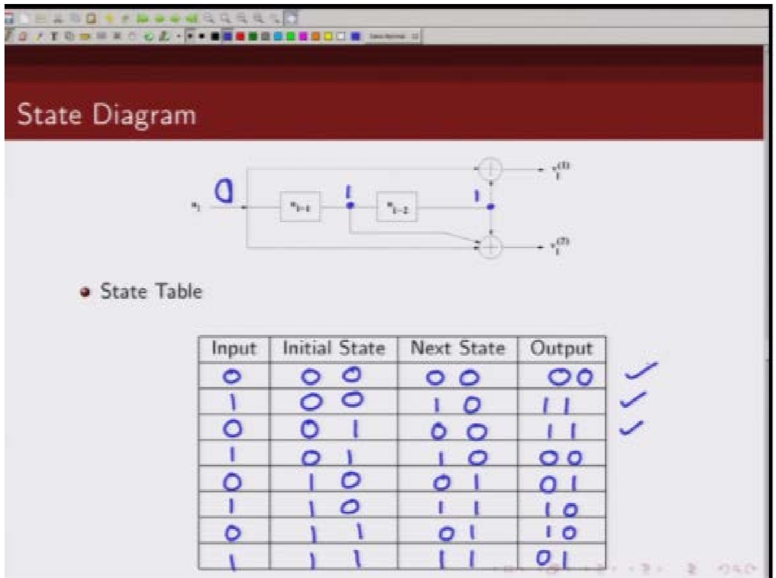

So this is taken care of, this is taken care of. If you are in initial state 0, 1 you receive a 1, you move to 1, 0 state and output is 0, 0.

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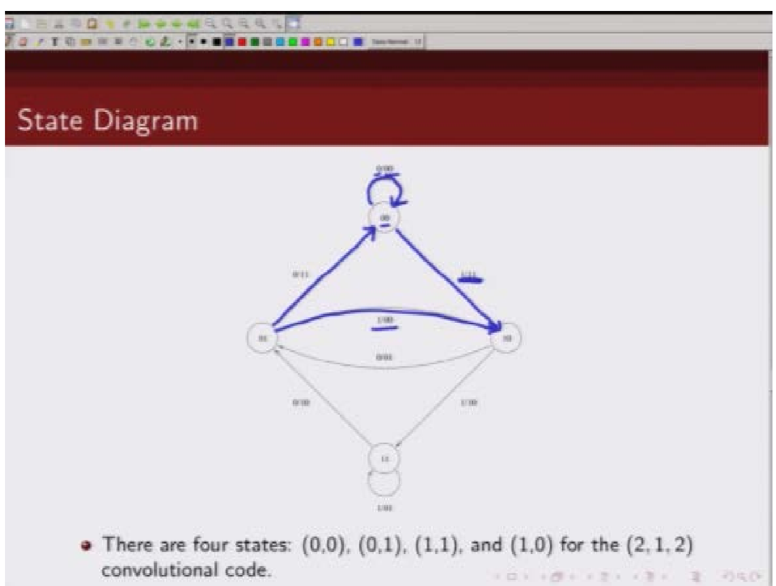

What is 0, 1 you receive a one. You move to state 1, 0 input is one output is 0, 0.

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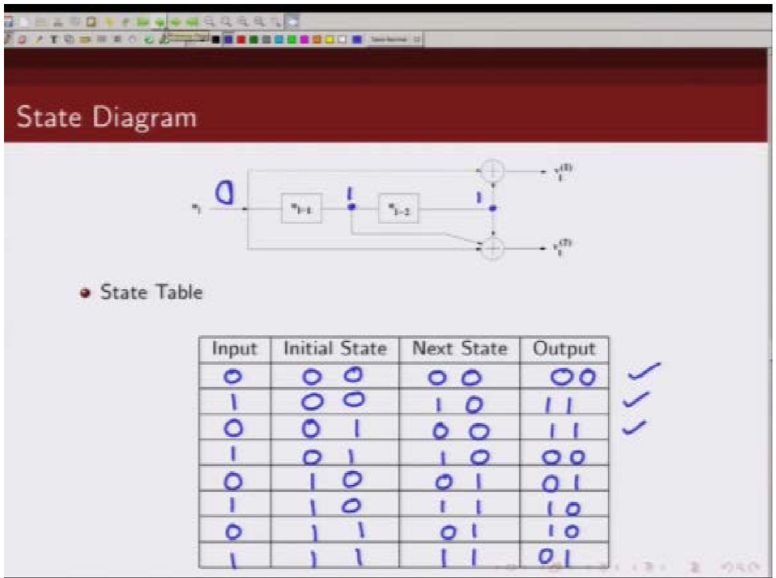

This is done state 1, 0 initials input is zero you move to state zero, one and output is zero, one.

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So you one is state one, zero you receive a zero you move to state zero, one and output is zero, one so that is this okay.

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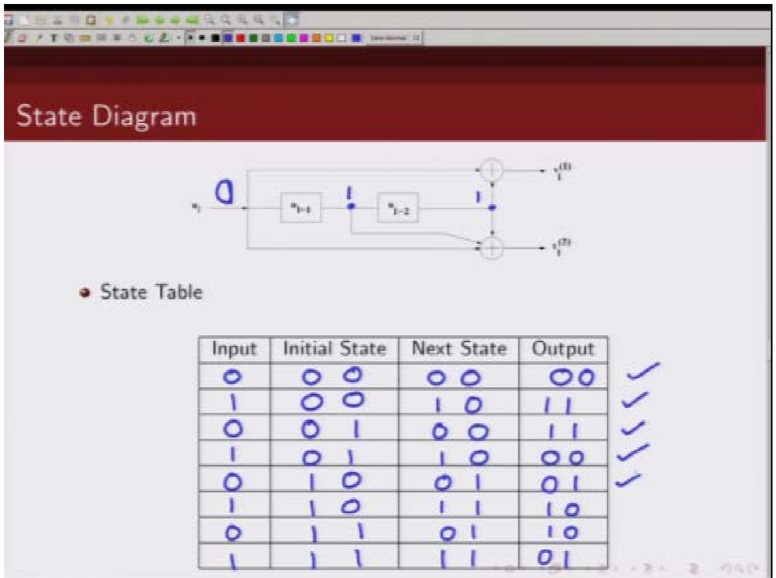

And so this is taken care of and what happens when you receive a one and you are in a state one, zero you move to next state one, one and output is one, zero.

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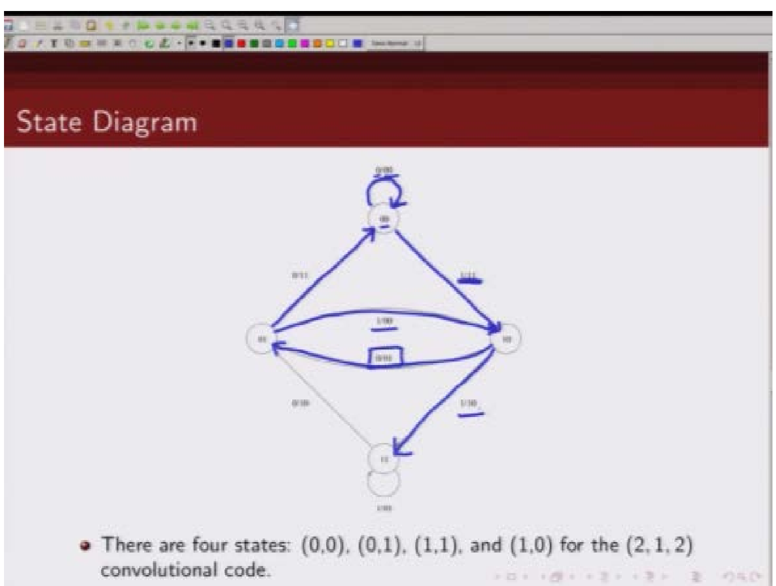

So you receive a one from, when you are initial state one, zero you move to state one, one which is this, this is the input and corresponding output is one, zero.

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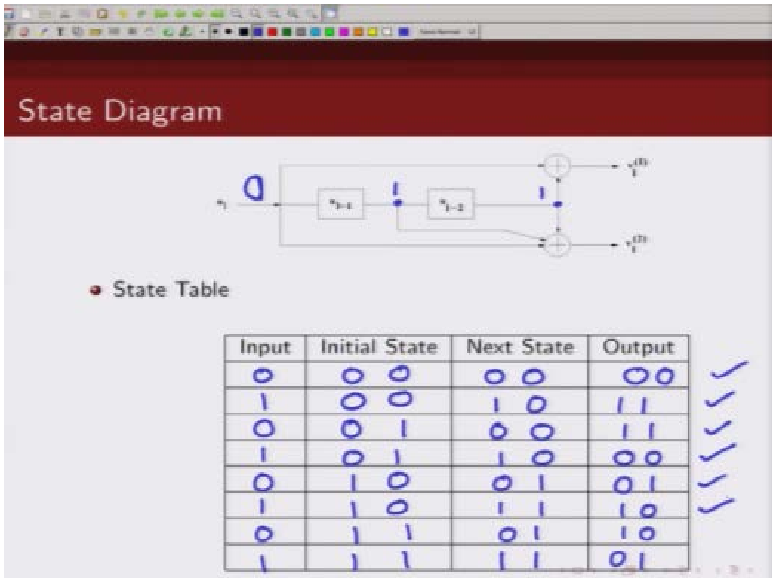

And finally when you are in state one, one if you receive a zero, you move to state zero, one and output is one, zero.

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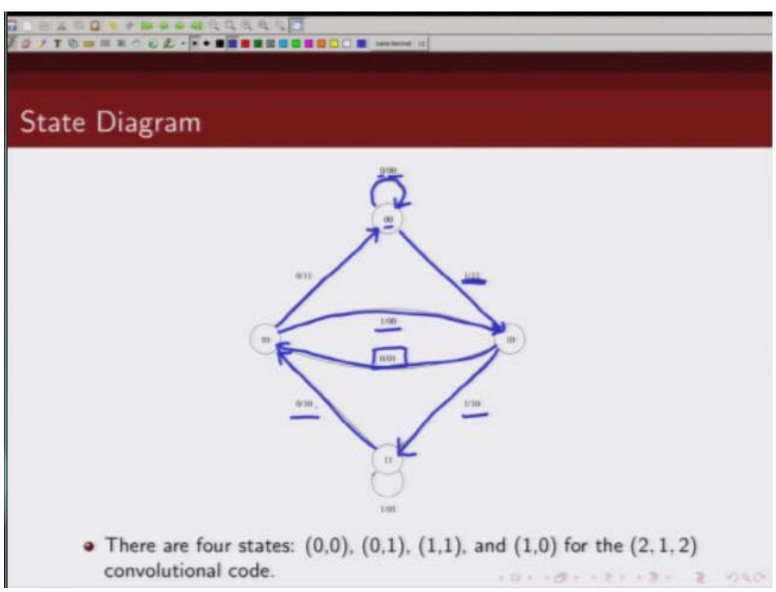

So you are in a state one, one you receive a zero you move to state zero, one input is zero, output is one, zero.

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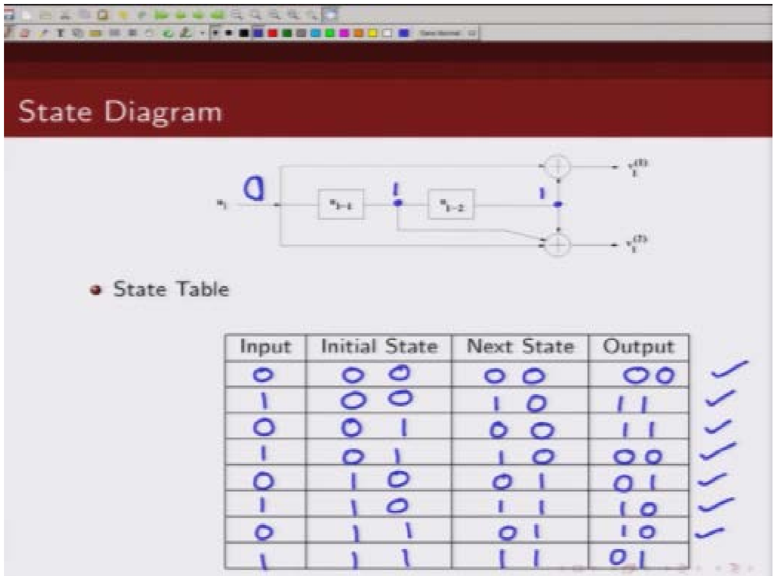

And finally state one, one input one you remain in state one, one and your output is zero, one.

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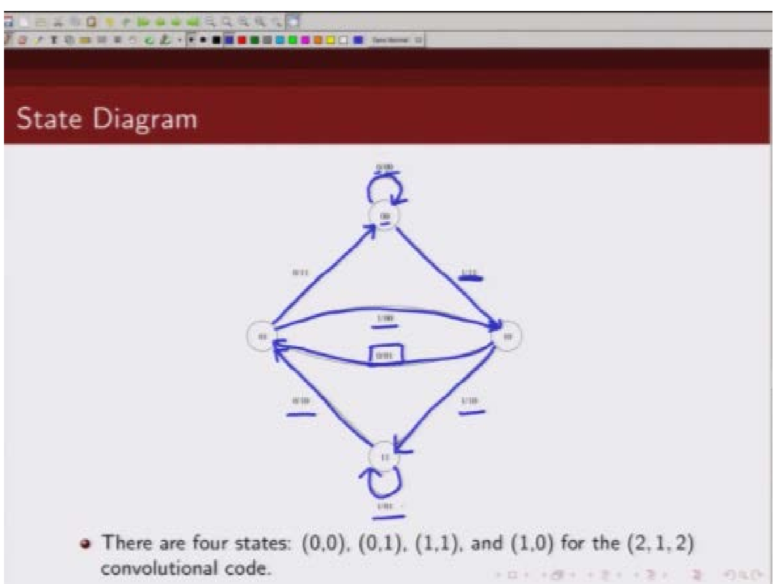

You receive a one you remain is state one, one input is one, output is zero, one. So note that here I have graphically represented all possible state transitions corresponding to the Convolutional encoder that we have considered. And this is my state diagram okay.

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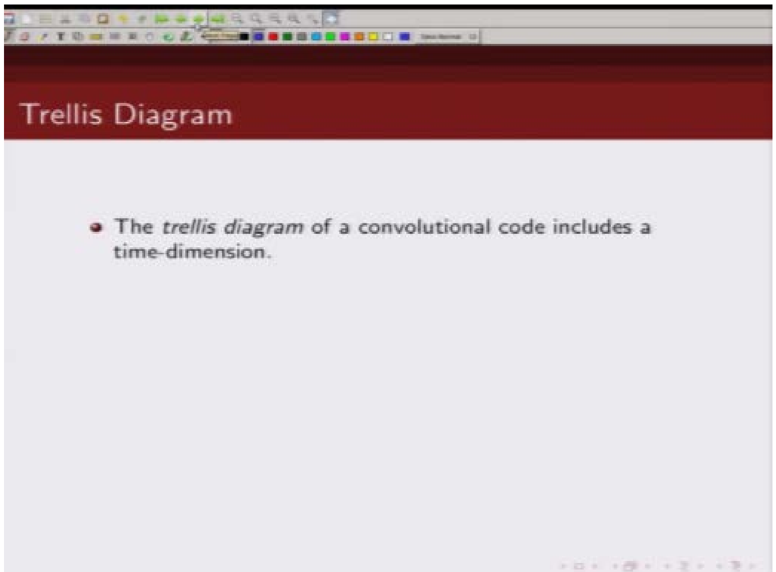

So next let us talk about trellis diagram so in trellis diagram we include another dimension which is the time dimension.

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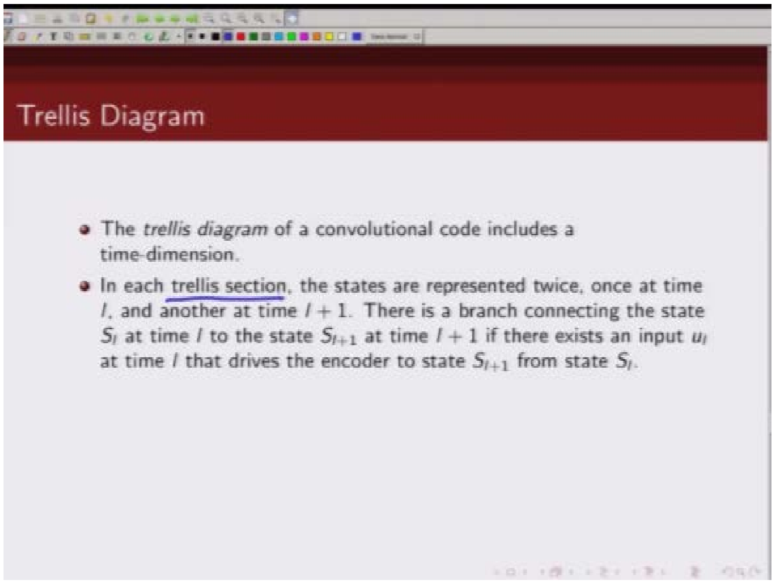

And how do we show that time dimension so let us first talk about what is a trellis section. So in a trellis section we represent each state twice. How do we represent each state let us take this example.

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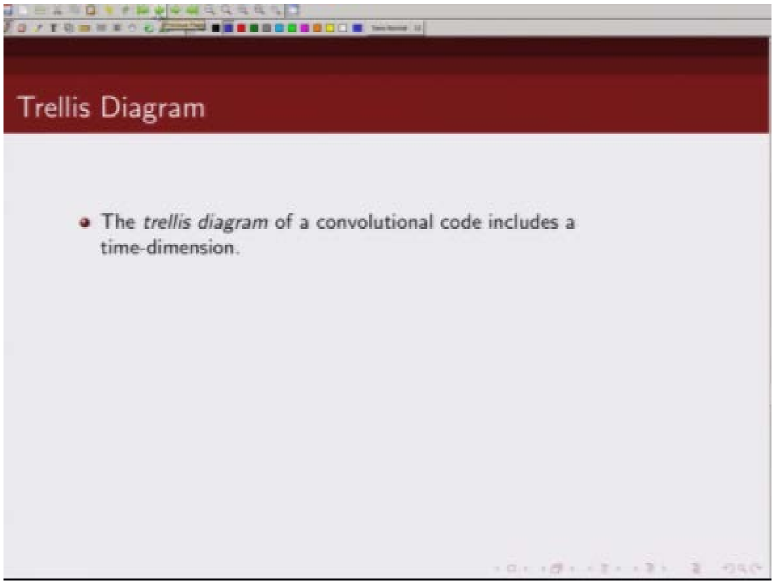

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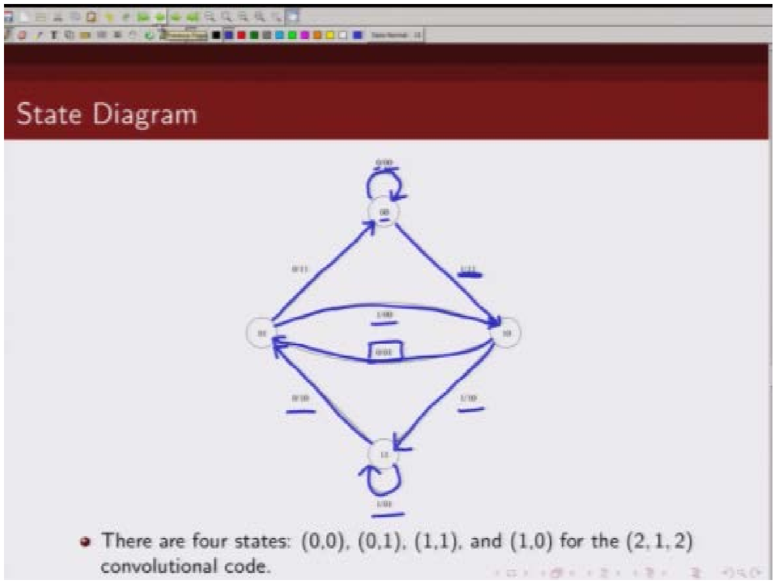

So here we have four states right? So if I have to draw a trellis section corresponding to this encoder how will I do it.

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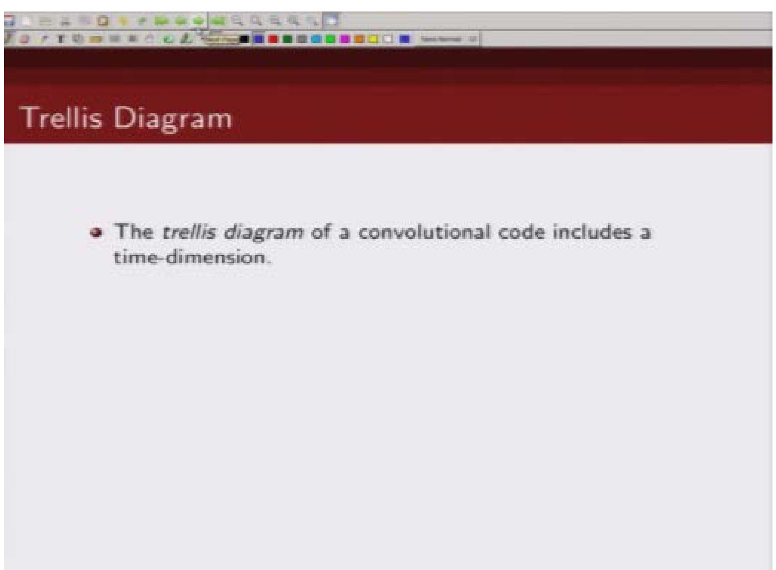

What I am going to do is.

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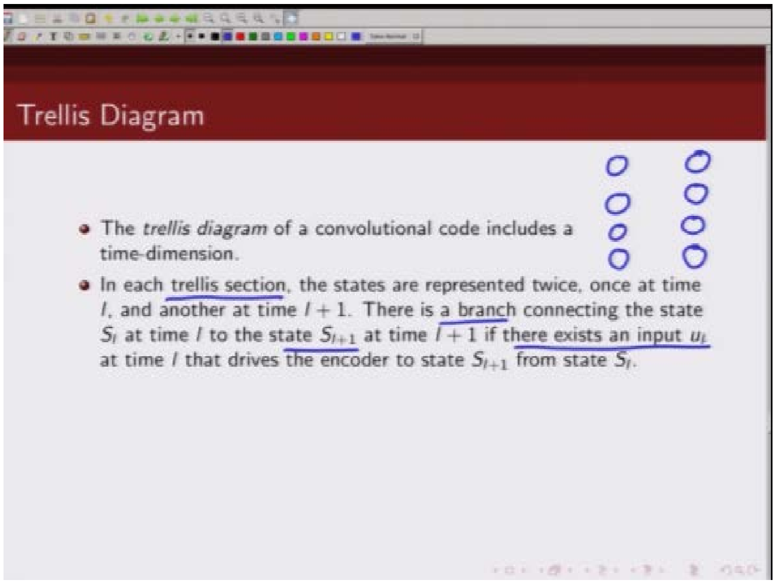

I am going to write all these four states here and I will write all those four states here. Next what I am going to do, I am going to add a branch. Connecting a state at this time instance a time l to a state at time l+1. If there exists an input that will drive the encoder to state Sl+1 from state Sl.

So what we do is we a time let say l these are the states correspond to at time l and these are states corresponding to time l+1. So there will be a valid transition from a state at time l to state at time l+1. Only if there exist an input that will drive from this state to some other state the next time is just for example.

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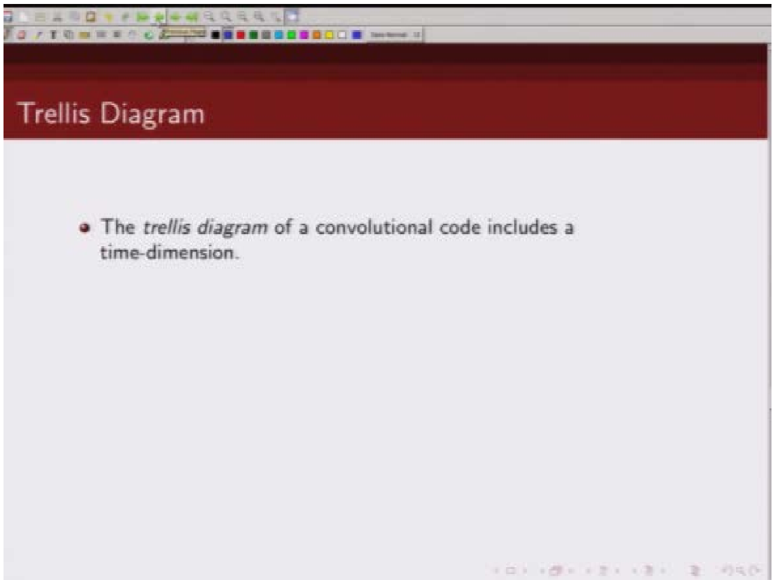

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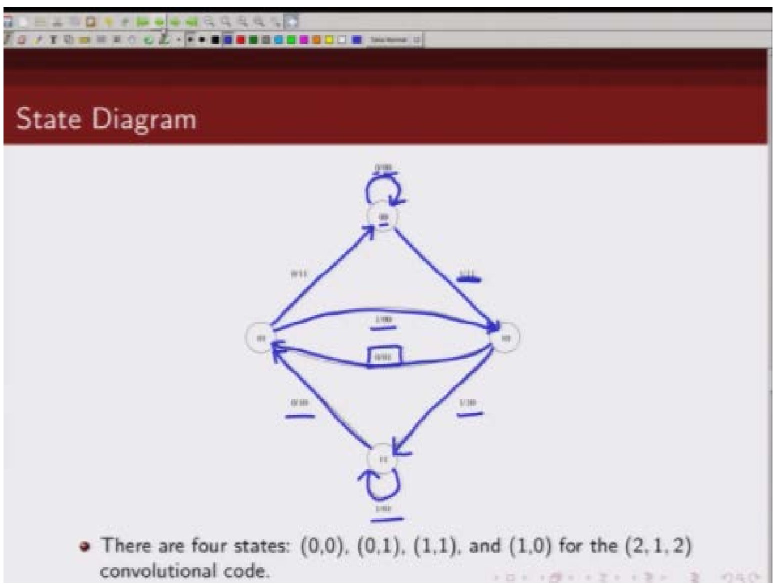

Let us go back to this. If you are in state zero and you get a zero you remain in all zero state.

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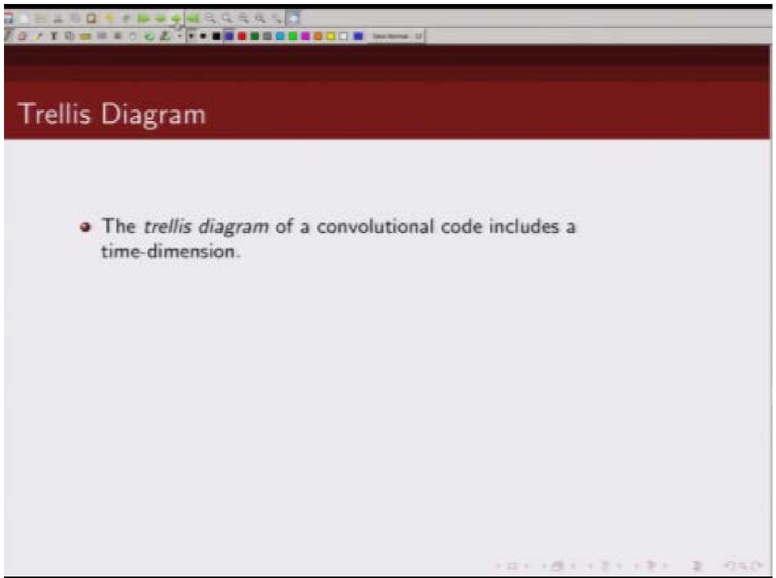

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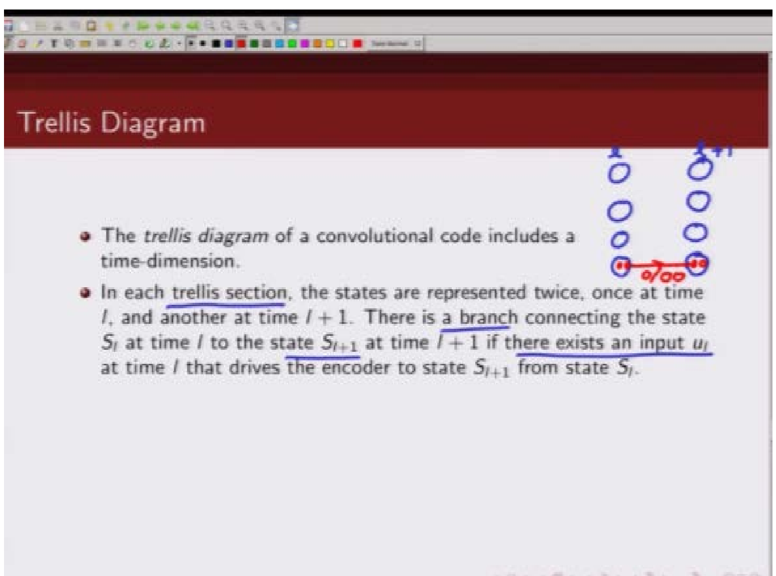

So this can be represented here. Let say this is my all zero state. Let says zero, zero state. And this also my zero, zero state. So there is a transition from here to here. And what is that input that causes this transition that input is zero.

And what is the output corresponding to that it is zero, zero. So I will label all those transitions which are happening from state at time Sl, I mean a state at time l to any state a time Sl+1 provided there is a valid transition okay.

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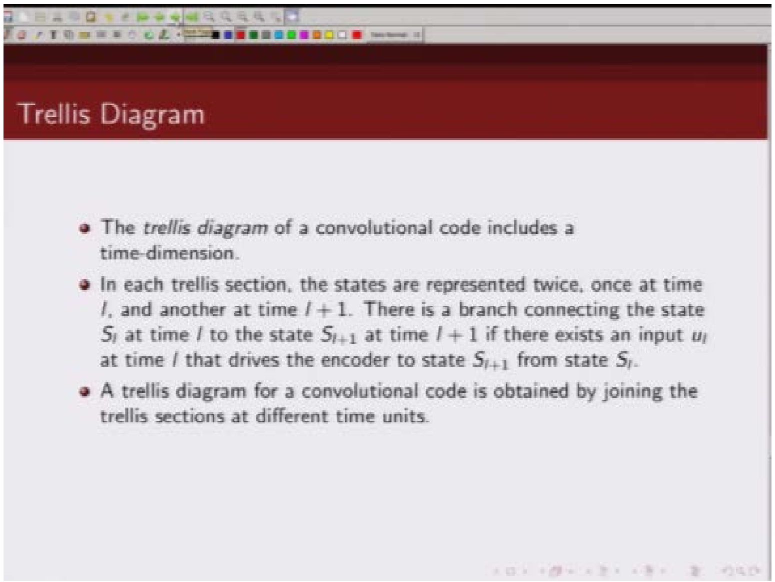

And a trellis diagram is obtained by joining trellis section at different time instance. So what I am going to do is to draw the complete state diagram I am going to join trellis section a time t for various times I will join them together, and that's how my trellis section will be -- trellis diagram will be drawn.

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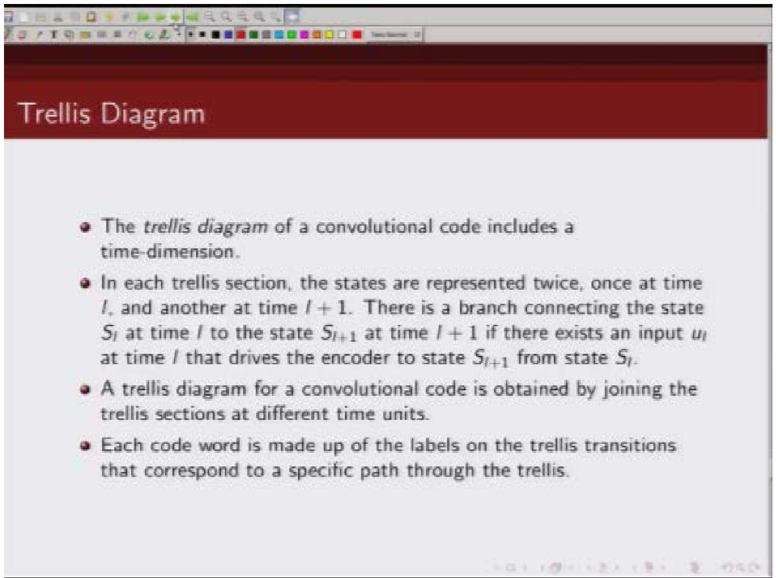

The interesting thing is in a trellis and we can see all possible code with any valid transitions basically is a valid code word, so let say initially encoder is all zero state and it goes back to all zero state then all possible parts from all zero state to the final all zero state will enumerate all possible code words that we have.

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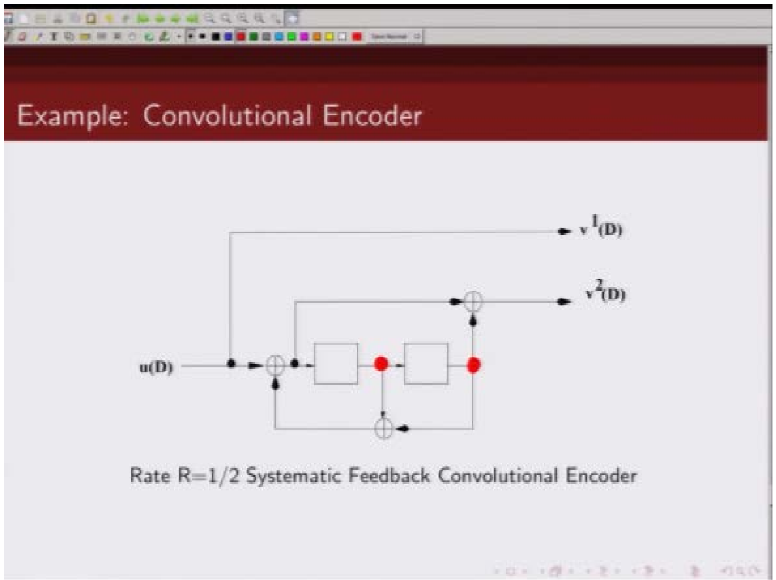

So let us take another example, this is an encoder now following the same procedure as we explained earlier we can draw the state diagram for this. So where are the states so this is one state, this is another state and similarly we can find out for all possible initial state and all possible inputs what are the next state and the corresponding output.

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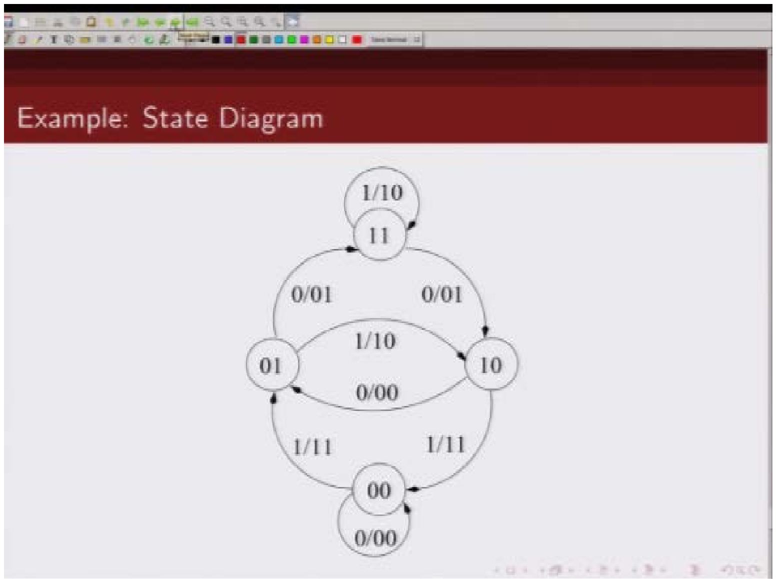

So this in this case turns out to be this. So when you are all zero state if you get a zero the output is zero, zero and you remain in all zero state. But if you get a one your output is one, one and you move to state zero, one. So this is the state diagram for the convolutional encoder I have just shown.

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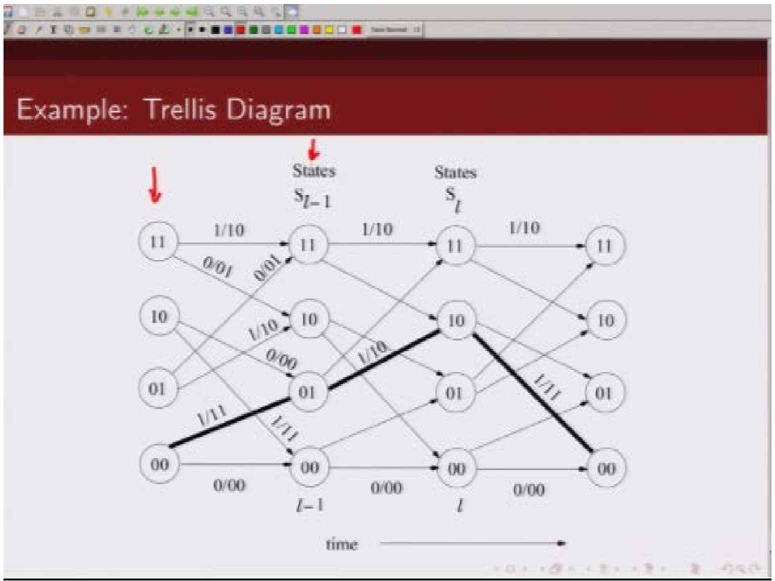

Now how do we draw the trellis diagram for this convolutional encoder? So as we said, at each time instance, let us consider this time instance this. And this so this is one particular time instance. So what we are going to do is we are first going to enumerate all possible encoders and we write them twice.

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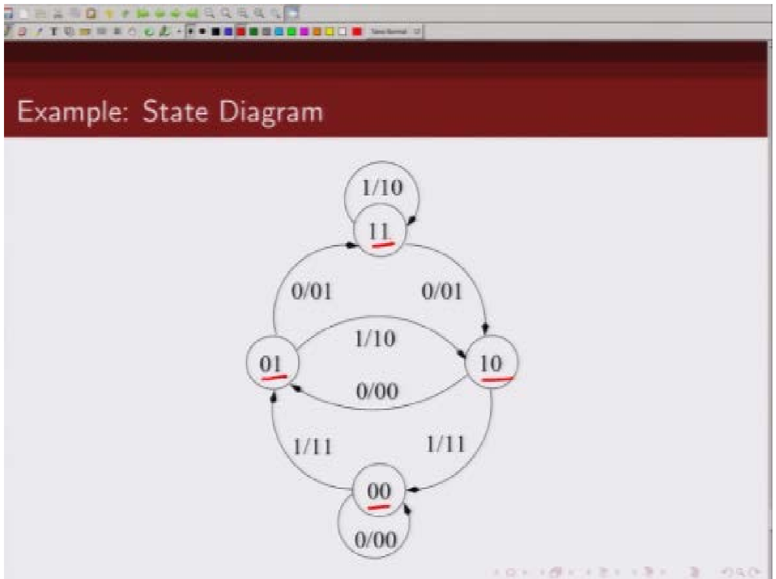

So in this example, we have four states zero, zero; zero, one; one, zero; one, one.

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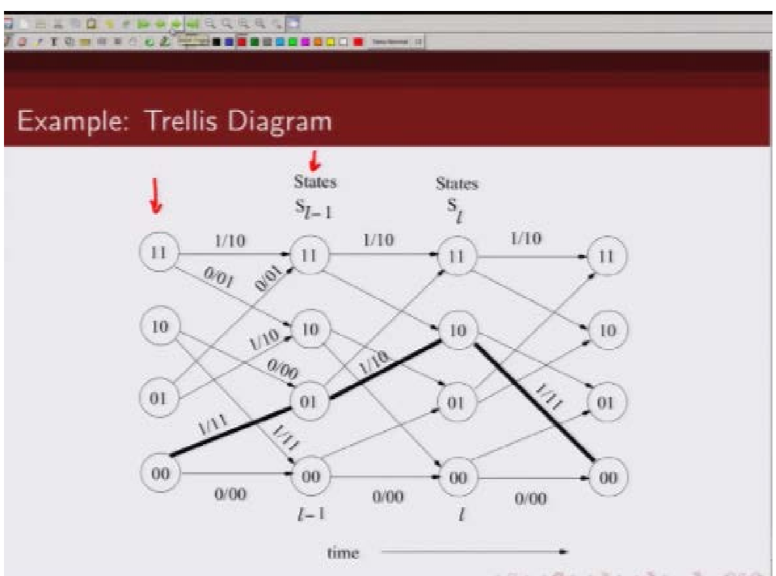

So what we do is we write these states twice zero, zero; zero, one; one, zero; one, one. Next we draw a state transition from each state to a next state provided there is a valid transition. So what we are going to do is.

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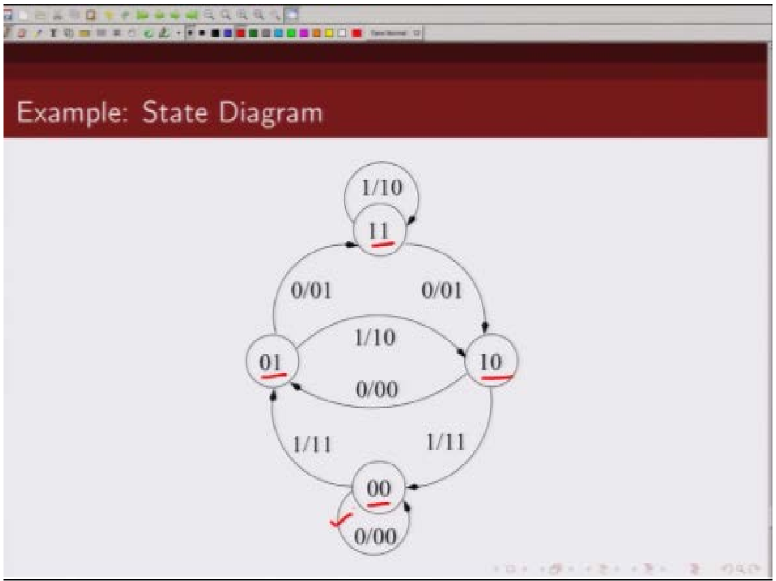

We are going to map all these state transitions into the diagram that we assured so let us take this example so we are in zero, zero state and we get a zero. The output is zero, zero and we remain in state zero, zero.

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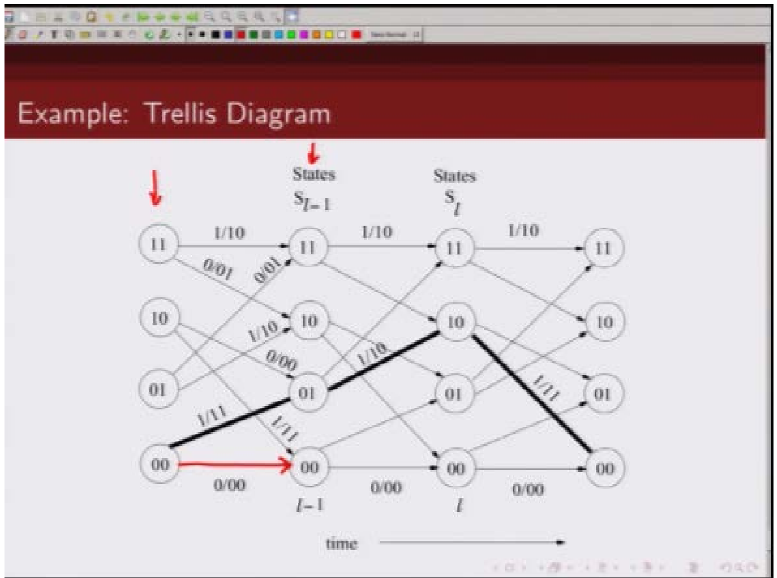

This is shown by this transition. You get a zero output is zero, zero and next state is zero, zero.

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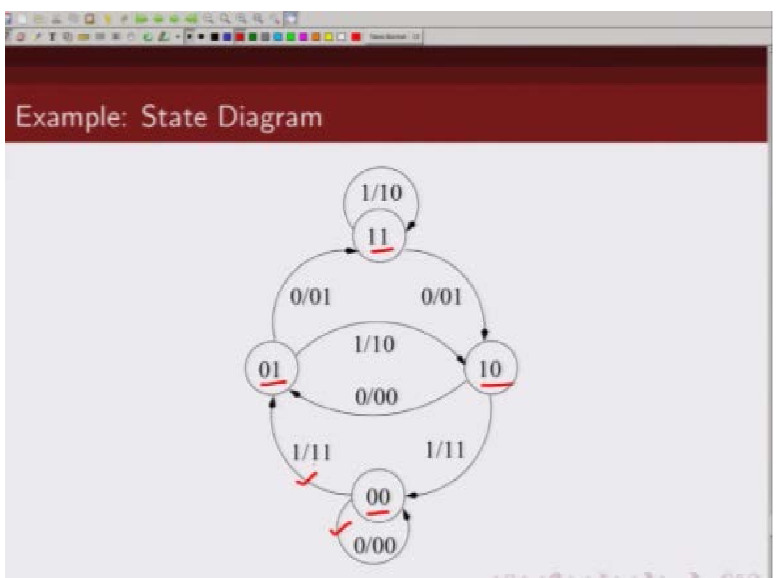

Now next if you get a one you move to state zero, one output is one, one.

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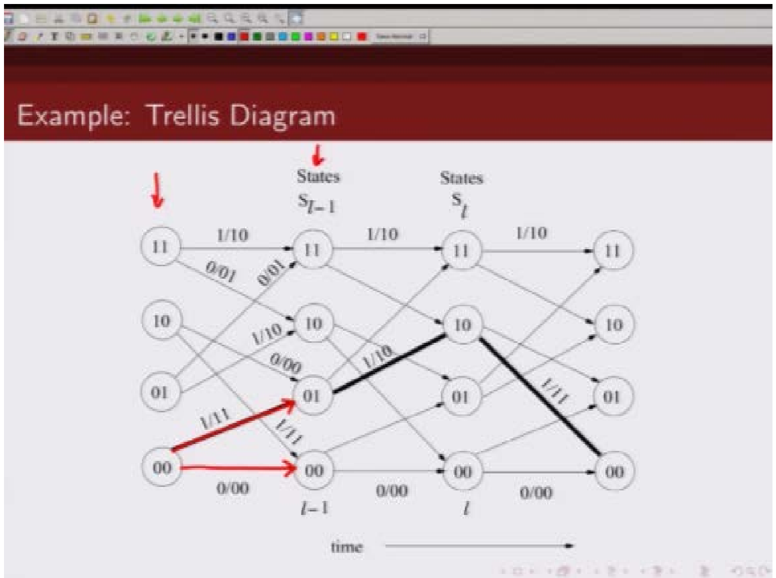

You get a one, you move to state zero, one and output is one, one.

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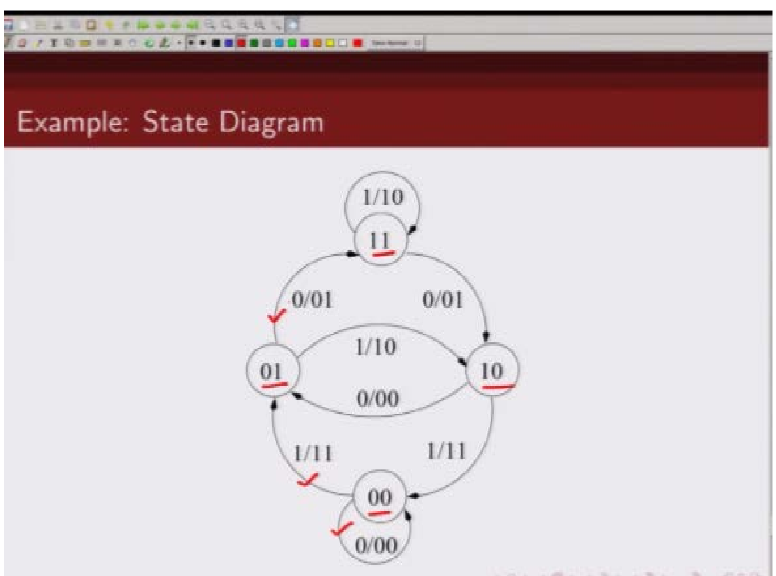

Let say you are in state zero, one and you receive a zero Then you move to state one, one and your output is zero, one. So how is that denoted?

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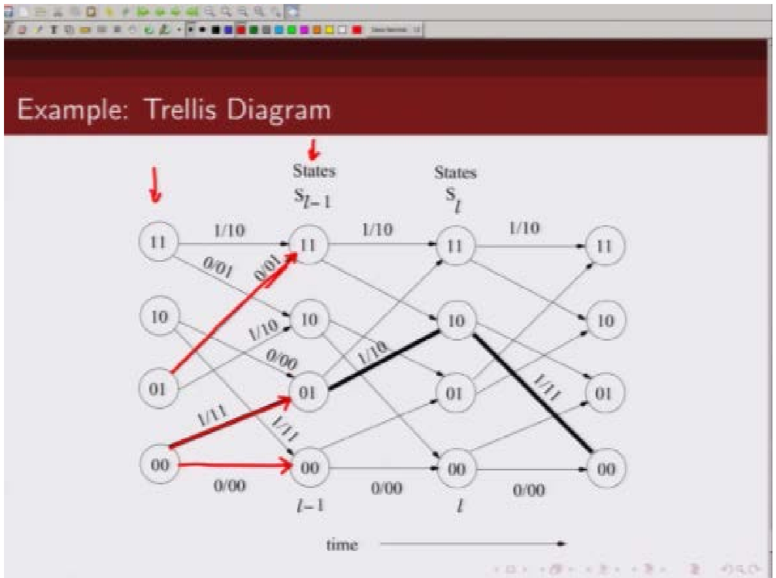

You are in a state zero, one you receive a zero you move to state one, one the output is zero, one.

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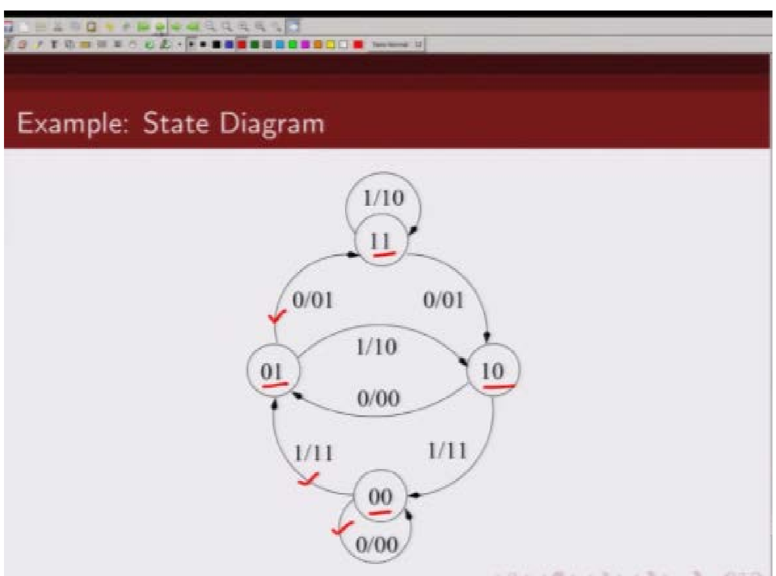

Similarly when you are in state 01 and you get a 1 you move to state 10 and the output is 10 so this is denoted by.

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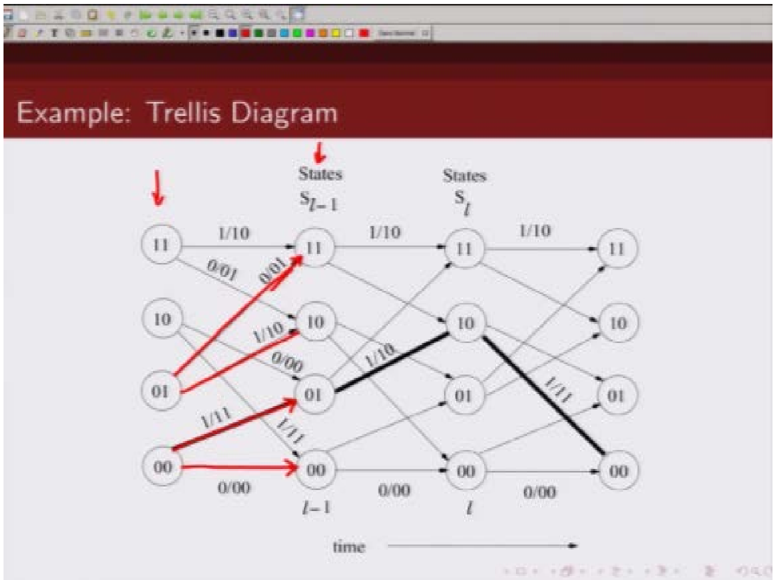

So you get a one you move to state one, zero and output is one, zero.

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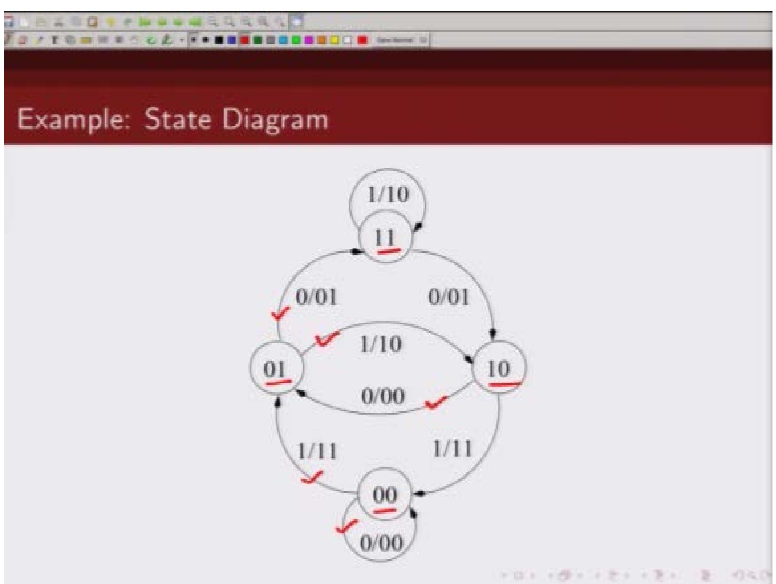

We repeat the same for other states. So let us say you are in state one, zero. When you get a zero the output is zero, zero and you move to state zero, one.

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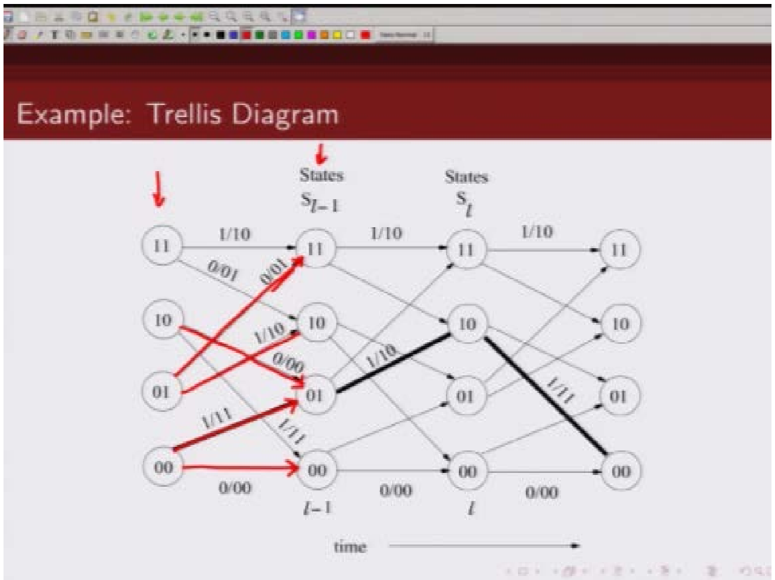

When you are at one, zero you get a zero you move to state zero one that this transition. And the output is zero, zero.

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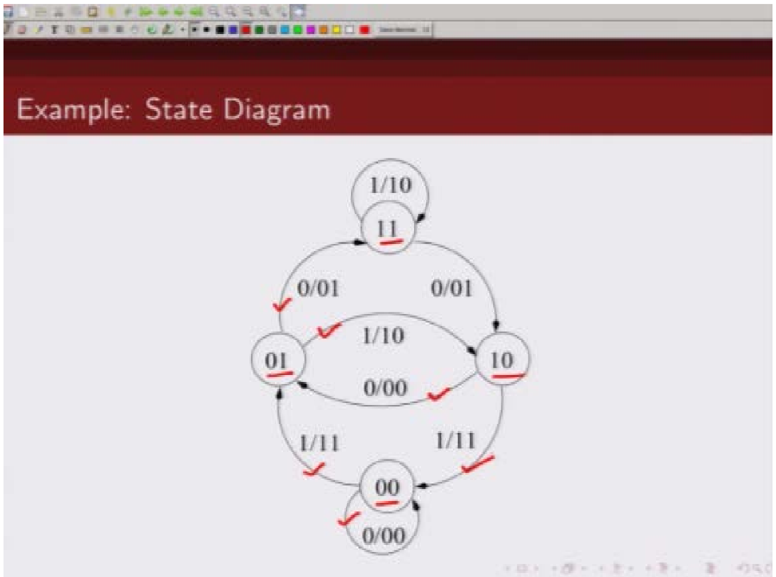

And when you get a one the output is one, one next state is zero, zero.

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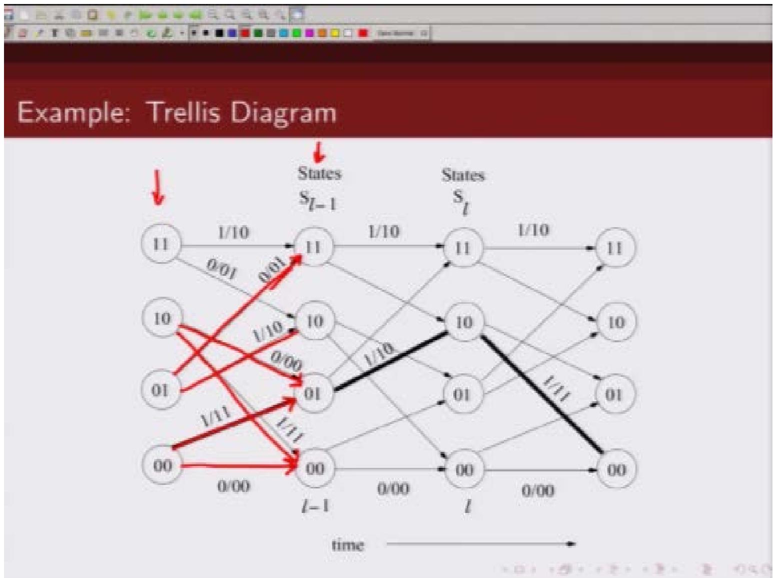

So when you are at one, zero and you get a one the state is zero, zero and the output is one, zero.

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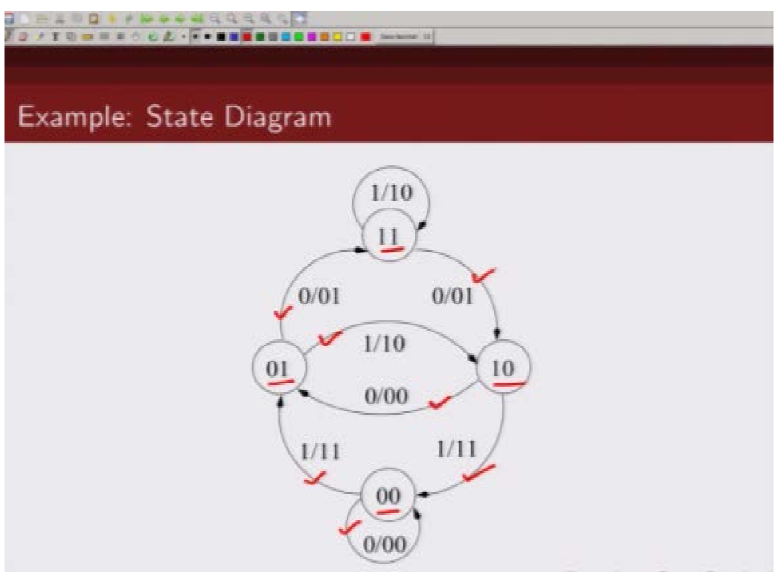

And let us -- so this we have all enumerated so only thing left is when you are in state one, one. When you are in a state one, one you get a zero you move to state one, zero and output is zero, one.

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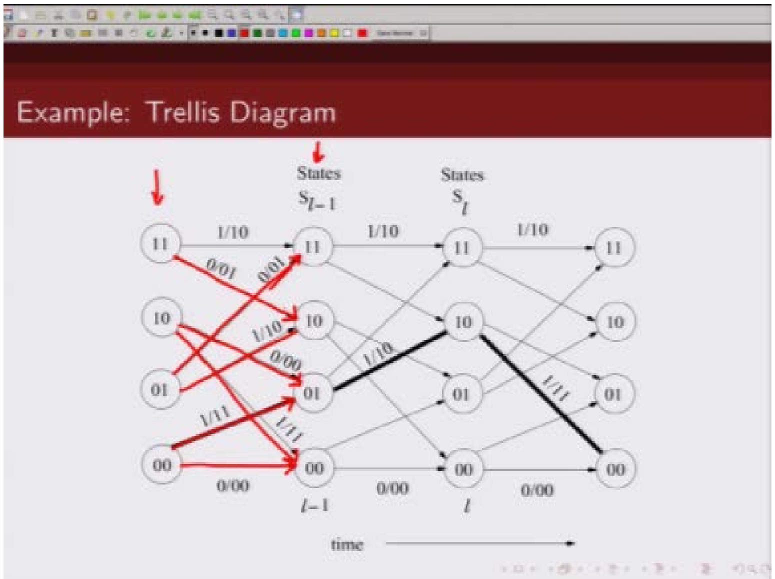

So state one, one you get a zero you move to state one, zero and the output is zero, one.

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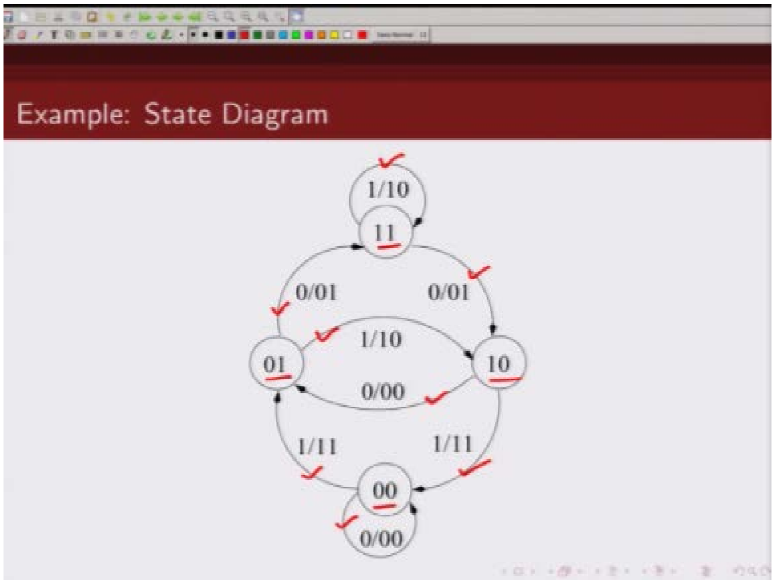

And finally when you are in state one, one and you get a one you remain is state one, one and the output is one, zero.

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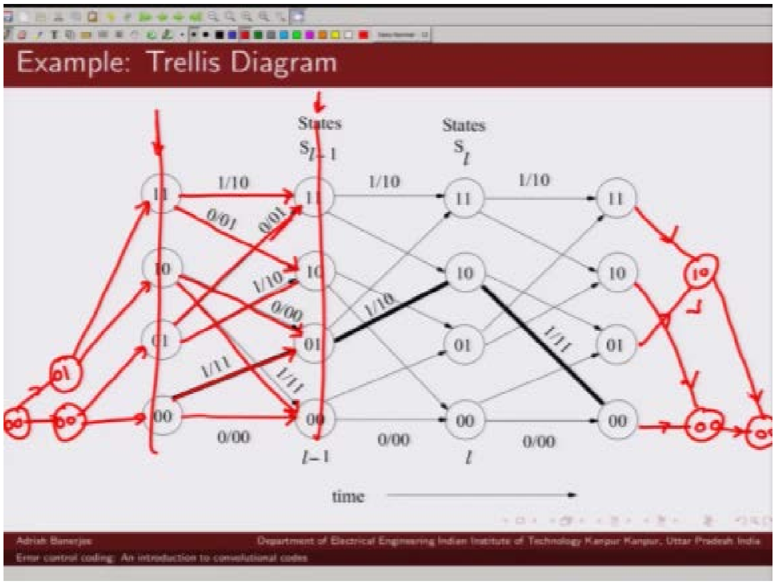

So that is this line. So note that here this trellis section. I have drawn the same information as I had in my state diagram, only thing now you have this so in trellis diagram what you do, you just combine all these trellis sections. So let say this is trellis section at time l this is trellis section at time l+1, this l+2.

So that is how we add this time dimension to the trellis diagram. So for example, initially let us see your Convolutional code or it is all zero state. So in that case what is going to happen you are in all zero state. Let say all zero state is zero, zero. So if you get a zero you move to zero, zero.

And if you get a one you move to zero, one. And next time instance if you get a -- if you are in a zero, zero state you move here. And if you get a one you move here. Similarly if you are in zero, one and you get a zero you move to this state. And if you get a one you move to this state. And let say what to terminate this encoder, I want to determinate meaning I want to bring this encoder back into all zero state.

So then in this state what I am going to do is, I am going to bring it back to state zero, zero and one zero so from state one, one. If I get a zero I move to this state and from state zero, one if get a one I move to this state. Similarly from one, zero I can move to zero, zero state if I get a one and from zero, zero I can move to zero, zero state.

If I get a zero and finally from zero, zero state if I get a zero I move to all zero state. And from one, zero I can move to zero state if I get a one. So you can see this how the states are evolving with time that I can see in the trellis diagram. With this I will end my discussion on state diagram and trellis diagram. Thank you.

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