

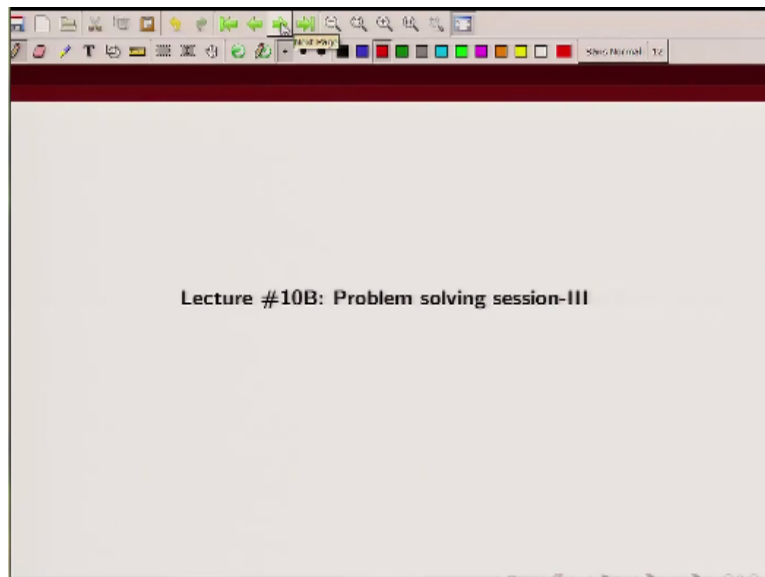
**Indian Institute of Technology Kanpur**  
**National Programme on Technology Enhanced Learning (NPTEL)**  
**Course Title**  
**Error Control Coding: An Introduction to Convolutional Codes**

**Lecture-10B**  
**Problem solving sessions-III**

by  
**Prof. Adrish Banerjee**  
**Dept. Electrical Engineering, IIT Kanpur**

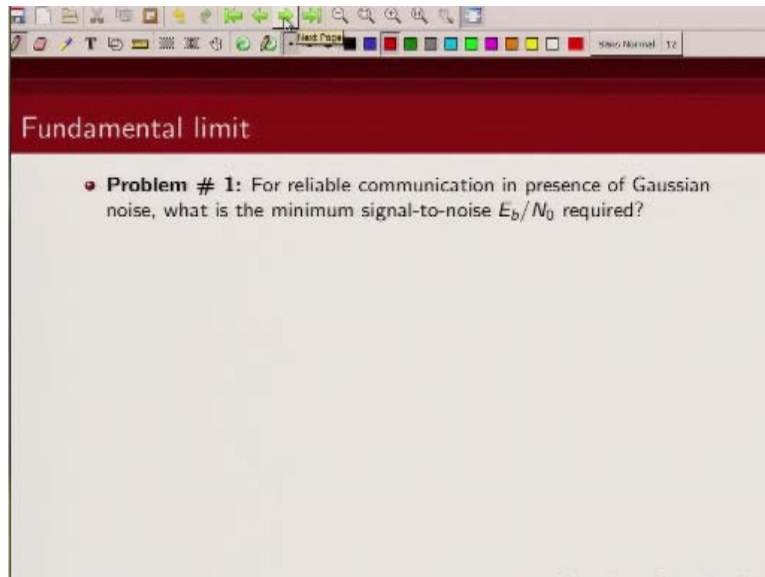
Welcome to the codes on error control coding, an introduction to convolutional code. So in this lecture we will try to solve some problems

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Related to convolutional code and in general.

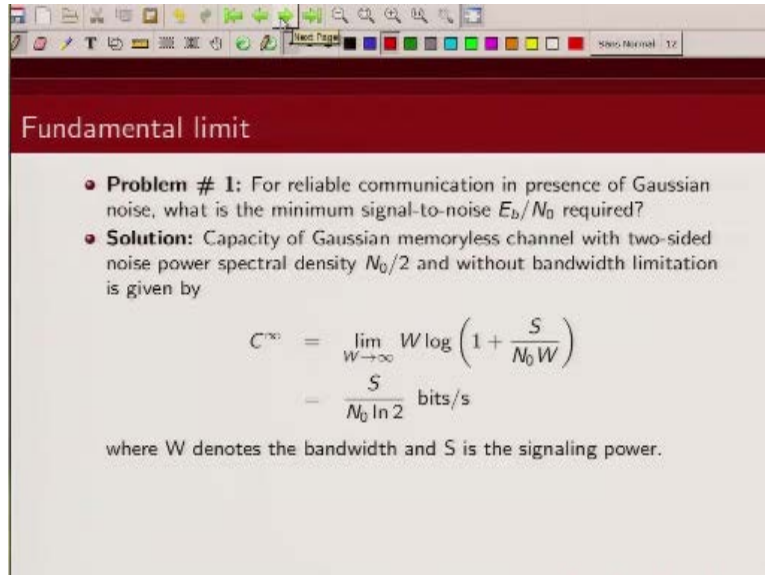
(Refer Slide Time: 00:25)



The image shows a screenshot of a presentation slide. At the top, there is a dark red header bar with the text "Fundamental limit" in white. Below the header, the slide content is on a light beige background. It features a bullet point labeled "Problem # 1:" followed by the text: "For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required?". The slide is displayed within a window that has a standard toolbar at the top with various icons for navigation and editing.

So the first question that we will try to answer is, if you want to do reliable communication in presence of an additive white Gaussian noise channel and of course we have infinite bandwidth, what is the minimum signal to noise ratio required?

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The image is a screenshot of a presentation slide. At the top, there is a dark red header bar with the title "Fundamental limit" in white text. Below the header, the slide content is on a light gray background. It starts with two bullet points: "Problem # 1: For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required?" and "Solution: Capacity of Gaussian memoryless channel with two-sided noise power spectral density  $N_0/2$  and without bandwidth limitation is given by". Below the text, the equation for channel capacity  $C^\infty$  is shown, involving a limit as bandwidth  $W$  approaches infinity. The equation simplifies to  $\frac{S}{N_0 \ln 2}$  bits/s. A final line of text explains that  $W$  is bandwidth and  $S$  is signaling power.

**Fundamental limit**

- **Problem # 1:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required?
- **Solution:** Capacity of Gaussian memoryless channel with two-sided noise power spectral density  $N_0/2$  and without bandwidth limitation is given by

$$C^\infty = \lim_{W \rightarrow \infty} W \log \left( 1 + \frac{S}{N_0 W} \right)$$
$$= \frac{S}{N_0 \ln 2} \text{ bits/s}$$

where  $W$  denotes the bandwidth and  $S$  is the signaling power.

So for that we first need the expression for capacity of additive white Gaussian noise channel.

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**Fundamental limit**

- **Problem # 1:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required?
- **Solution:** Capacity of Gaussian memoryless channel with two-sided noise power spectral density  $N_0/2$  and without bandwidth limitation is given by

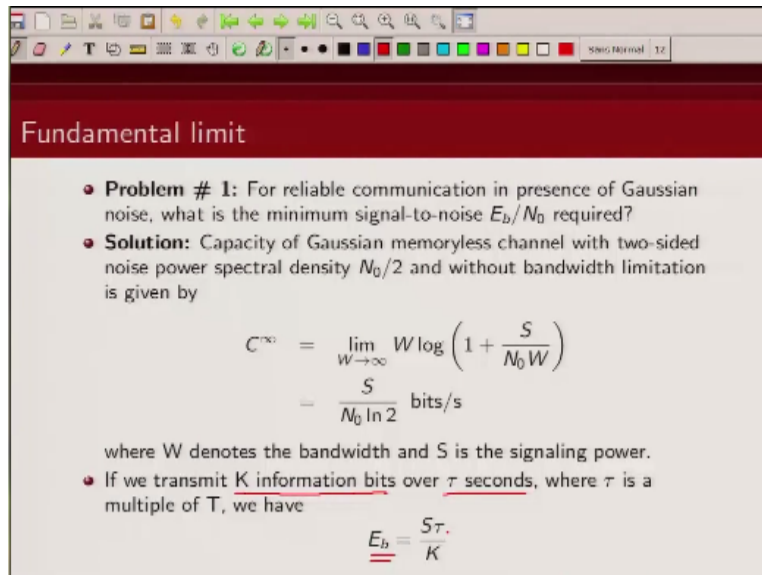
$$C^\infty = \lim_{W \rightarrow \infty} W \log \left( 1 + \frac{S}{N_0 W} \right)$$
$$= \frac{S}{N_0 \ln 2} \text{ bits/s}$$

where  $W$  denotes the bandwidth and  $S$  is the signaling power.

And the capacity of additive white Gaussian noise channel is given by this expression, where  $w$  is a bandwidth,  $s$  is my signaling power;  $N_0/2$  is two sided power spectral density. Now – so we are considering when bandwidth is infinite so this can be written, so this will be  $\log(1 + \frac{S}{N_0 W})$  when  $w$  is infinite then this will go to zero, so  $\log(1)$  will be zero and  $1/w$  will also go to zero, so it is  $0/0$  form.

So we will differentiate and we can find out that the capacity when bandwidth is infinite is given by this expression,  $S/N_0$  natural log of two bits per second.

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The image is a screenshot of a presentation slide. At the top, there is a red header bar with the text "Fundamental limit" in white. Below the header, the slide contains two bullet points. The first bullet point is "Problem # 1: For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required?". The second bullet point is "Solution: Capacity of Gaussian memoryless channel with two-sided noise power spectral density  $N_0/2$  and without bandwidth limitation is given by". Below the text, there is a mathematical equation for  $C^\infty$  as a limit of  $W \log(1 + \frac{S}{N_0 W})$  as  $W \rightarrow \infty$ , which simplifies to  $\frac{S}{N_0 \ln 2}$  bits/s. Below the equation, it says "where  $W$  denotes the bandwidth and  $S$  is the signaling power." The third bullet point is "If we transmit  $K$  information bits over  $\tau$  seconds, where  $\tau$  is a multiple of  $T$ , we have". Below this, the equation  $E_b = \frac{S\tau}{K}$  is shown, with the  $E_b$  and the fraction  $\frac{S\tau}{K}$  underlined.

**Fundamental limit**

- **Problem # 1:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required?
- **Solution:** Capacity of Gaussian memoryless channel with two-sided noise power spectral density  $N_0/2$  and without bandwidth limitation is given by

$$C^\infty = \lim_{W \rightarrow \infty} W \log \left( 1 + \frac{S}{N_0 W} \right)$$
$$= \frac{S}{N_0 \ln 2} \text{ bits/s}$$

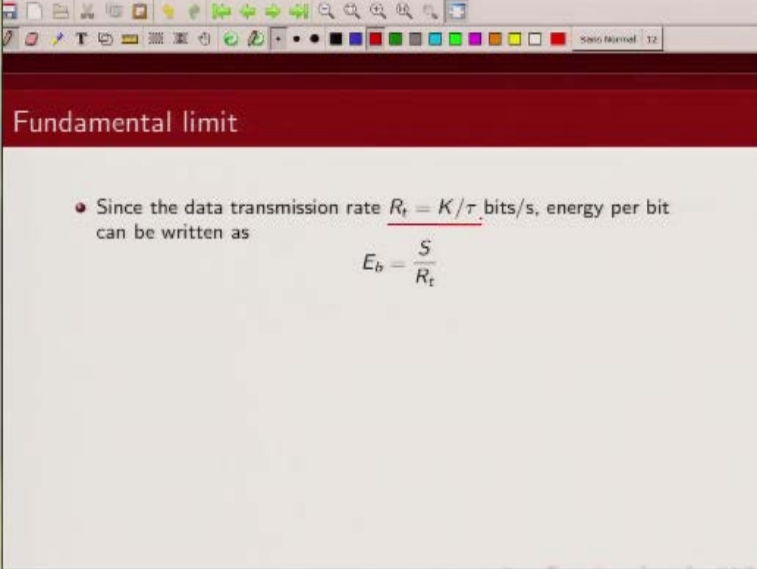
where  $W$  denotes the bandwidth and  $S$  is the signaling power.

- If we transmit  $K$  information bits over  $\tau$  seconds, where  $\tau$  is a multiple of  $T$ , we have

$$\underline{E_b} = \underline{\frac{S\tau}{K}}$$

So we are interested in transmitting  $k$  bits over  $\mathcal{T}$  seconds. So if we do that where  $\mathcal{T}$  is a multiple of time period  $T$ , so if we do that our energy per bit is given by  $S \mathcal{T}/K$  this  $S$  was my signaling power we are sending over time  $T$ ,  $\mathcal{T}$  and total number of information bits was  $K$ , so energy per bit is  $S \mathcal{T}/K$ .

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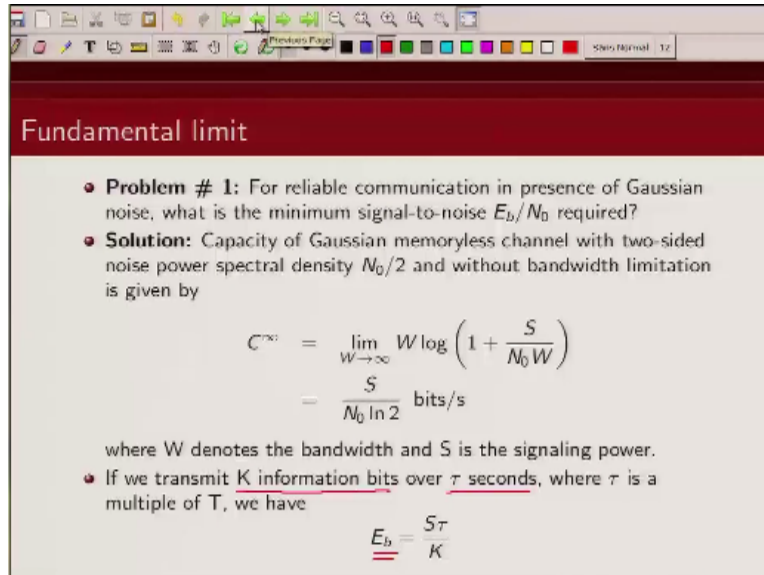
**Fundamental limit**

- Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as

$$E_b = \frac{S}{R_t}$$

Now what transmission because we are transmitting K bits what time  $T$  so our transmission rate is  $K/T$  bits per second.

(Refer Slide Time: 02:52)



The image is a screenshot of a presentation slide titled "Fundamental limit". The slide is displayed in a window with a standard toolbar at the top. The content of the slide is as follows:

**Fundamental limit**

- **Problem # 1:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required?
- **Solution:** Capacity of Gaussian memoryless channel with two-sided noise power spectral density  $N_0/2$  and without bandwidth limitation is given by

$$C^\infty = \lim_{W \rightarrow \infty} W \log \left( 1 + \frac{S}{N_0 W} \right)$$
$$= \frac{S}{N_0 \ln 2} \text{ bits/s}$$

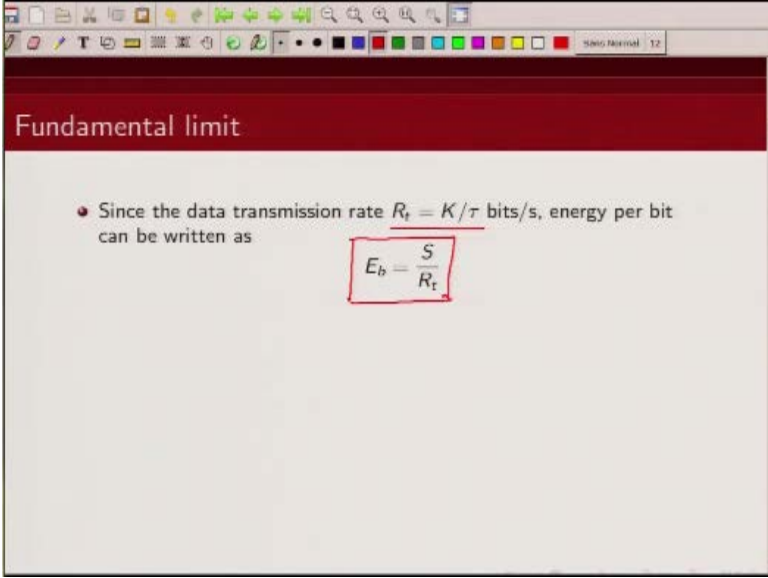
where  $W$  denotes the bandwidth and  $S$  is the signaling power.

- If we transmit  $K$  information bits over  $\tau$  seconds, where  $\tau$  is a multiple of  $T$ , we have

$$\underline{E_b} = \frac{S\tau}{K}$$

And our energy per bit that we wrote here is basically  $S T/K$ .

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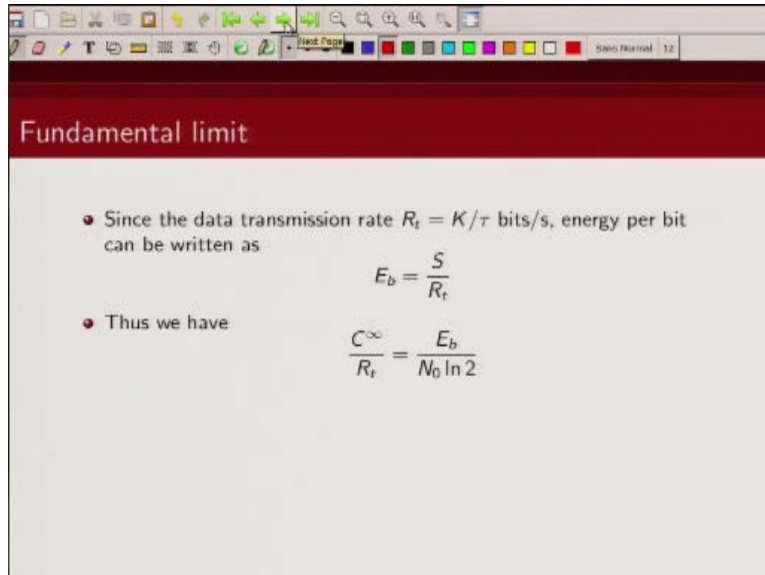


The image is a screenshot of a presentation slide. At the top, there is a red header bar with the text "Fundamental limit" in white. Below the header, the slide has a light gray background. A bullet point is present, stating: "Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as". To the right of this text, the equation  $E_b = \frac{S}{R_t}$  is displayed, enclosed in a red hand-drawn rectangular box. The presentation software's toolbar is visible at the top of the window.

And  $K/\tau$  is  $R_t$  so we can write energy per bit in terms of signaling power and transmission rate  $R_t$ .



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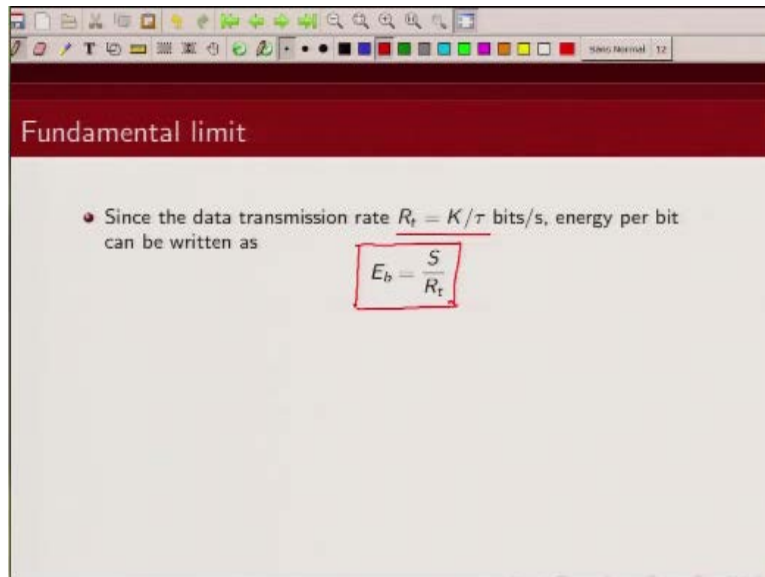


The image is a screenshot of a presentation slide. At the top, there is a dark red header bar with the title "Fundamental limit" in white text. Below the header, the slide content is on a light gray background. It features two bullet points, each preceded by a red circular icon. The first bullet point discusses the data transmission rate  $R_t = K/\tau$  bits/s and energy per bit. The second bullet point states "Thus we have" and is followed by a mathematical equation. The presentation software's toolbar is visible at the top of the slide area, and a status bar at the bottom right shows "Slide Number 12".

### Fundamental limit

- Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as
$$E_b = \frac{S}{R_t}$$
- Thus we have
$$\frac{C^\infty}{R_t} = \frac{E_b}{N_0 \ln 2}$$

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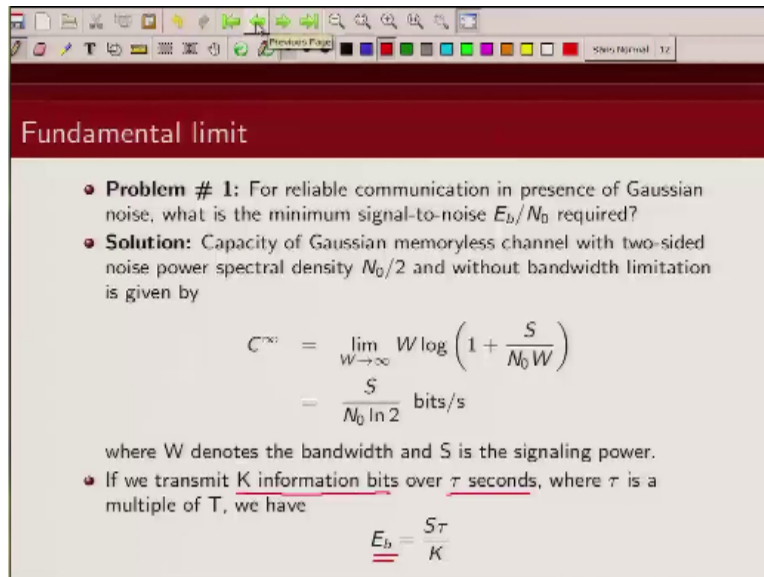


The image is a screenshot of a presentation slide. At the top, there is a dark red header bar with the text "Fundamental limit" in white. Below the header, the slide has a light beige background. A bullet point is present, stating: "Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as". To the right of this text, the equation  $E_b = \frac{S}{R_t}$  is displayed, enclosed in a red hand-drawn rectangular box. The presentation software's interface is visible at the top, showing a toolbar with various icons and a status bar at the bottom right indicating "Slide Number 12".

### Fundamental limit

- Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as  $E_b = \frac{S}{R_t}$

(Refer Slide Time: 03:12)



The image is a screenshot of a presentation slide. At the top, there is a dark red header bar with the text "Fundamental limit" in white. Below the header, the slide content is on a light beige background. It starts with two bullet points: "Problem # 1: For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required?" and "Solution: Capacity of Gaussian memoryless channel with two-sided noise power spectral density  $N_0/2$  and without bandwidth limitation is given by". This is followed by a mathematical equation for channel capacity  $C^\infty$  as a limit of  $W \log(1 + \frac{S}{N_0 W})$  as  $W \rightarrow \infty$ , which simplifies to  $\frac{S}{N_0 \ln 2}$  bits/s. A note explains that  $W$  is bandwidth and  $S$  is signaling power. Another bullet point states: "If we transmit K information bits over  $\tau$  seconds, where  $\tau$  is a multiple of T, we have". Finally, the equation  $\underline{E_b} = \frac{S\tau}{K}$  is shown, with  $E_b$  underlined.

## Fundamental limit

- **Problem # 1:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required?
- **Solution:** Capacity of Gaussian memoryless channel with two-sided noise power spectral density  $N_0/2$  and without bandwidth limitation is given by

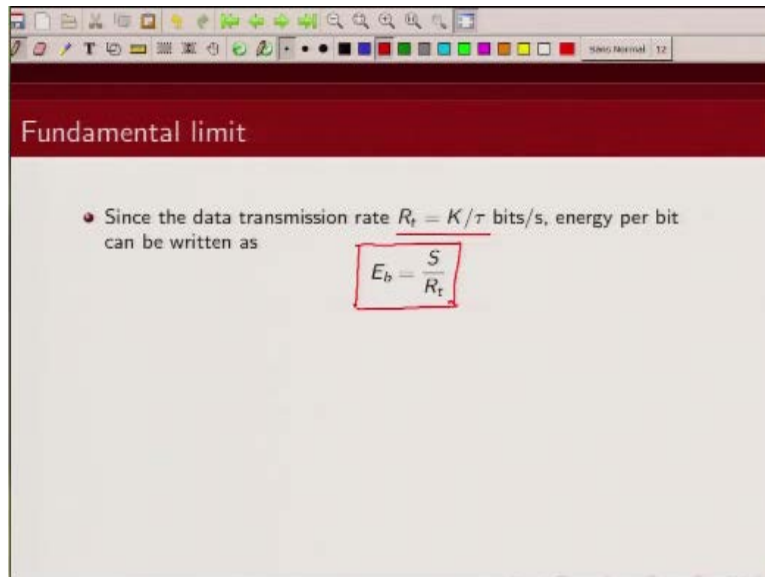
$$C^\infty = \lim_{W \rightarrow \infty} W \log \left( 1 + \frac{S}{N_0 W} \right)$$
$$= \frac{S}{N_0 \ln 2} \text{ bits/s}$$

where  $W$  denotes the bandwidth and  $S$  is the signaling power.

- If we transmit K information bits over  $\tau$  seconds, where  $\tau$  is a multiple of T, we have

$$\underline{E_b} = \frac{S\tau}{K}$$

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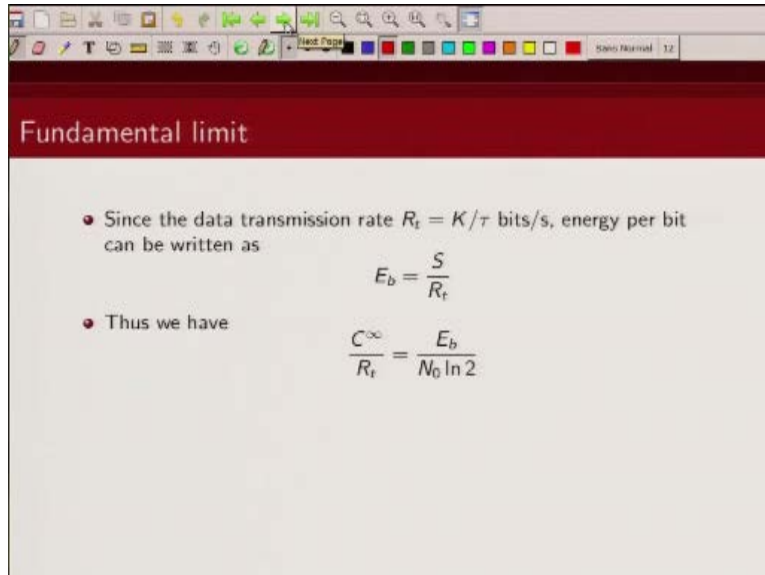


The image is a screenshot of a presentation slide. At the top, there is a dark red header bar with the text "Fundamental limit" in white. Below the header, the slide has a light gray background. A bullet point is present, stating: "Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as". To the right of this text, the equation  $E_b = \frac{S}{R_t}$  is displayed, enclosed in a red hand-drawn rectangular box. The top of the slide shows a standard software interface with various icons and a status bar at the bottom right indicating "Slide Number 12".

### Fundamental limit

- Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as  $E_b = \frac{S}{R_t}$

(Refer Slide Time: 03:18)

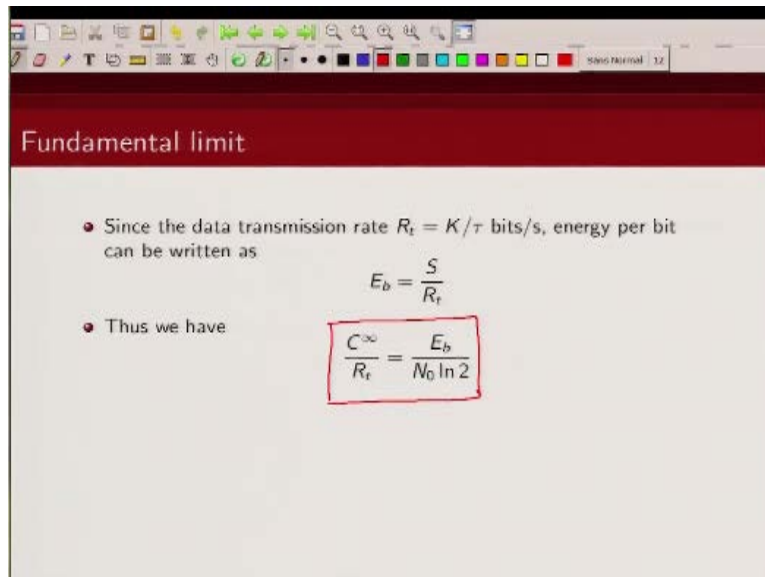


The image shows a presentation slide with a red header bar containing the title "Fundamental limit". The slide content is on a light gray background. It features two bullet points. The first bullet point states: "Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as". Below this text is the equation 
$$E_b = \frac{S}{R_t}$$
. The second bullet point states: "Thus we have". Below this text is the equation 
$$\frac{C^\infty}{R_t} = \frac{E_b}{N_0 \ln 2}$$
. The presentation software interface is visible at the top, showing a toolbar with various icons and a status bar at the bottom right indicating "Slide 10 of 12".

## Fundamental limit

- Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as
$$E_b = \frac{S}{R_t}$$
- Thus we have
$$\frac{C^\infty}{R_t} = \frac{E_b}{N_0 \ln 2}$$

(Refer Slide Time: 03:24)



The image is a screenshot of a presentation slide. At the top, there is a red header bar with the text "Fundamental limit" in white. Below the header, the slide has a light gray background. There are two bullet points: the first says "Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as" followed by the equation  $E_b = \frac{S}{R_t}$ ; the second says "Thus we have" followed by the equation  $\frac{C^\infty}{R_t} = \frac{E_b}{N_0 \ln 2}$ , which is enclosed in a red hand-drawn rectangular box. The top of the slide shows a software toolbar with various icons and a color palette.

Fundamental limit

- Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as
$$E_b = \frac{S}{R_t}$$
- Thus we have
$$\frac{C^\infty}{R_t} = \frac{E_b}{N_0 \ln 2}$$

So if we divide our expression for general capacity by  $R_t$  what we get is this expression. Now we know from Shannon noisy channel coding theorem that as long as transmission rate is less than channel capacity we can reliably communicate over the communication channel. So we want  $R_t$  to be less than this channel capacity.

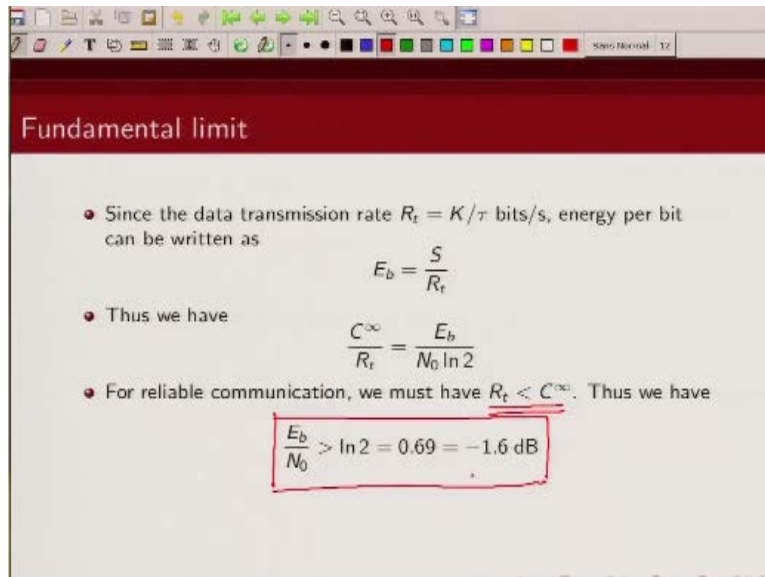
(Refer Slide Time: 03:46)

The slide is titled "Fundamental limit" and contains the following content:

- Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as
$$E_b = \frac{S}{R_t}$$
- Thus we have
$$\frac{C^\infty}{R_t} = \frac{E_b}{N_0 \ln 2}$$
- For reliable communication, we must have  $R_t < C^\infty$ . Thus we have
$$\frac{E_b}{N_0} > \ln 2 = 0.69 = -1.6 \text{ dB}$$

So if we want this to hold then we get a condition on signal to noise ratio, so we get this following condition on energy per bit, so snr per information bit we get this condition that snr per information bit should be greater than -1.6 dB. This is for the case when we have infinite bandwidth.

(Refer Slide Time: 04:19)



The image is a screenshot of a presentation slide titled "Fundamental limit". The slide has a dark red header with the title in white. The main content area is light gray. It contains three bullet points, each preceded by a red dot. The first bullet point discusses the data transmission rate  $R_t = K/\tau$  bits/s and energy per bit  $E_b$ . The second bullet point states "Thus we have" and is followed by the equation  $\frac{C^\infty}{R_t} = \frac{E_b}{N_0 \ln 2}$ . The third bullet point states "For reliable communication, we must have  $R_t < C^\infty$ . Thus we have" and is followed by a boxed equation  $\frac{E_b}{N_0} > \ln 2 = 0.69 = -1.6 \text{ dB}$ . The box is drawn with a red border. The slide is displayed in a window with a standard operating system taskbar at the top.

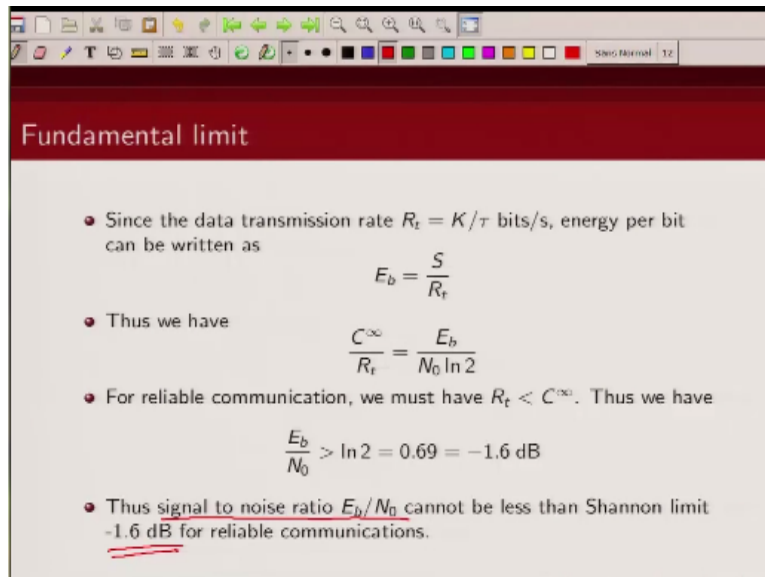
### Fundamental limit

- Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as
$$E_b = \frac{S}{R_t}$$
- Thus we have
$$\frac{C^\infty}{R_t} = \frac{E_b}{N_0 \ln 2}$$
- For reliable communication, we must have  $R_t < C^\infty$ . Thus we have
$$\frac{E_b}{N_0} > \ln 2 = 0.69 = -1.6 \text{ dB}$$

And our code rate can actually go to zero.



(Refer Slide Time: 04:23)

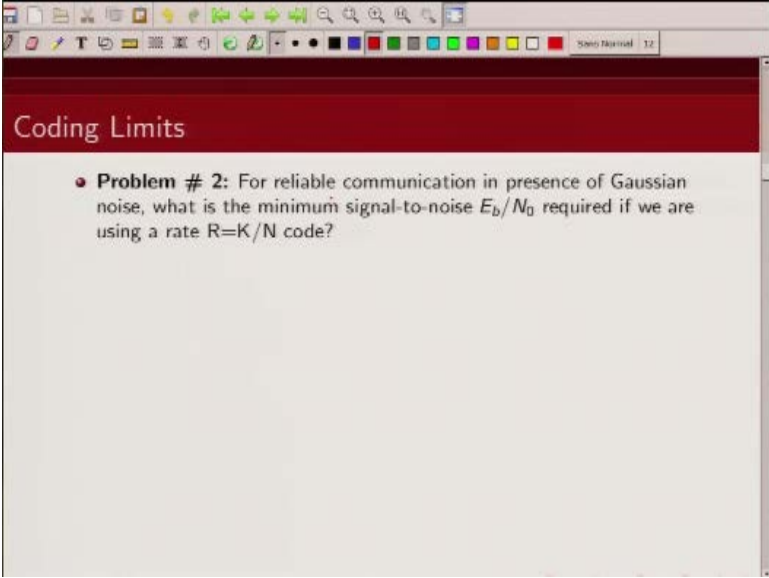


**Fundamental limit**

- Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as
$$E_b = \frac{S}{R_t}$$
- Thus we have
$$\frac{C^\infty}{R_t} = \frac{E_b}{N_0 \ln 2}$$
- For reliable communication, we must have  $R_t < C^\infty$ . Thus we have
$$\frac{E_b}{N_0} > \ln 2 = 0.69 = -1.6 \text{ dB}$$
- Thus signal to noise ratio  $E_b/N_0$  cannot be less than Shannon limit -1.6 dB for reliable communications.

So signal to noise ratio then cannot be less than this limit which is -1.6dB. Okay let us look at the next problem.

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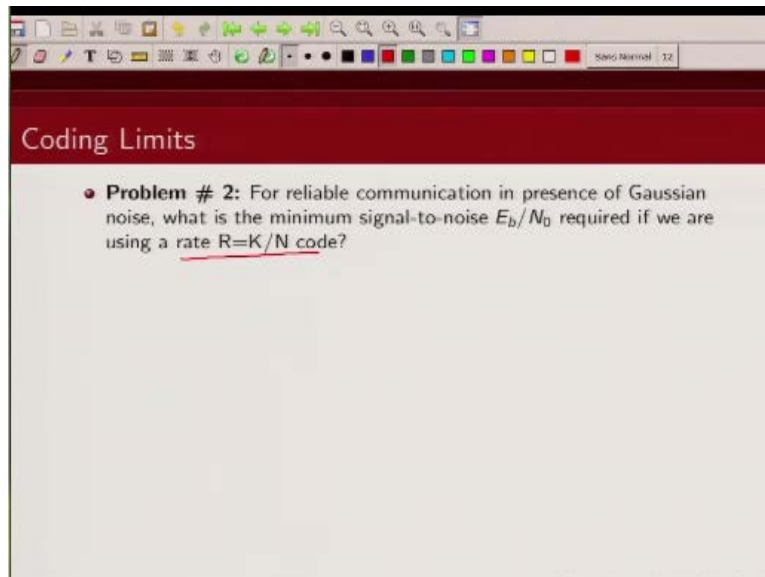


The image is a screenshot of a presentation slide. At the top, there is a dark red header bar with the title "Coding Limits" in white text. Below the header, the slide content is on a light gray background. A single bullet point is visible, starting with a red circular icon. The text of the bullet point is: "• **Problem # 2:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required if we are using a rate  $R=K/N$  code?" The presentation software's toolbar is visible at the top of the window, and a status bar at the bottom right shows "Slide Number 12".

## Coding Limits

- **Problem # 2:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required if we are using a rate  $R=K/N$  code?

(Refer Slide Time: 04:52)

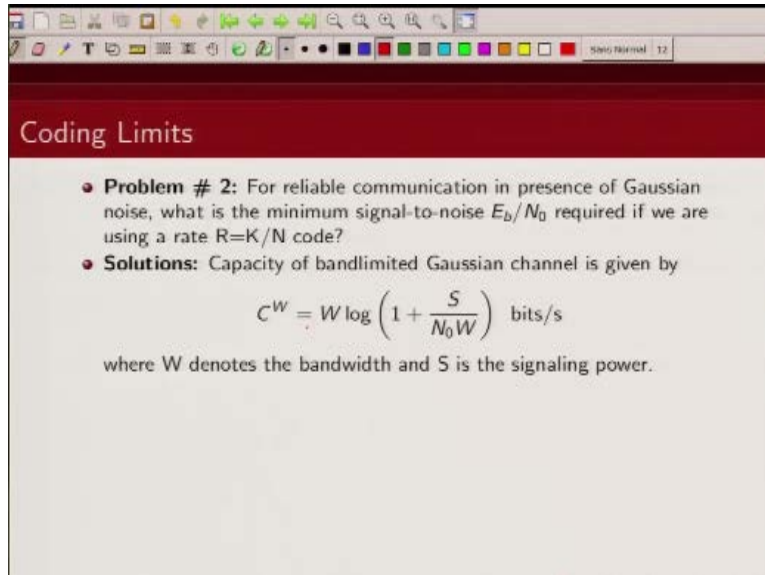


**Coding Limits**

- **Problem # 2:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required if we are using a rate  $R=K/N$  code?

So we are interested in reliable communication over additive white Gaussian noise channel and we are transmitting using a rate  $K/N$  code.

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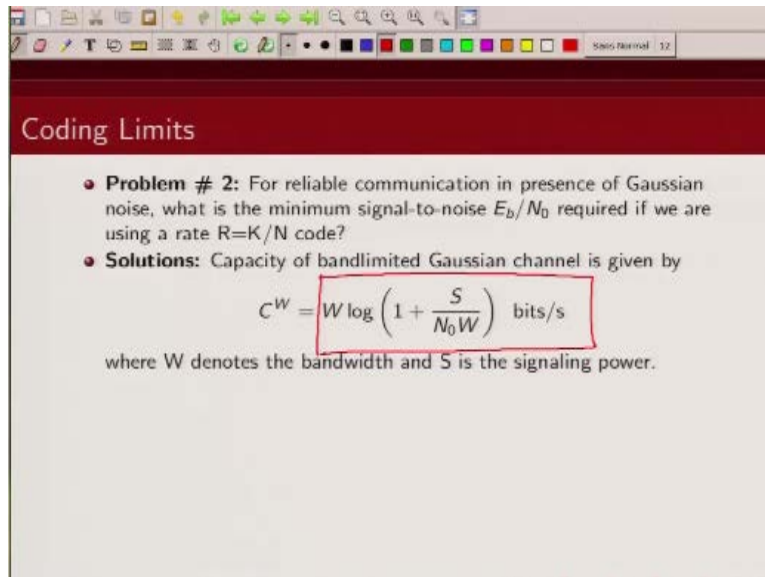


The image is a screenshot of a presentation slide. At the top, there is a dark red header bar with the title "Coding Limits" in white text. Below the header, the slide content is on a light gray background. It starts with a bullet point labeled "Problem # 2:" followed by a question about reliable communication in the presence of Gaussian noise. The second bullet point, labeled "Solutions:", states that the capacity of a bandlimited Gaussian channel is given by a specific formula. The formula is  $C^W = W \log \left( 1 + \frac{S}{N_0 W} \right)$  bits/s. Below the formula, a sentence explains that W denotes the bandwidth and S is the signaling power. The slide is framed by a standard presentation window with a toolbar at the top and a status bar at the bottom.

## Coding Limits

- **Problem # 2:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required if we are using a rate  $R=K/N$  code?
- **Solutions:** Capacity of bandlimited Gaussian channel is given by
$$C^W = W \log \left( 1 + \frac{S}{N_0 W} \right) \text{ bits/s}$$
where W denotes the bandwidth and S is the signaling power.

(Refer Slide Time: 05:05)



**Coding Limits**

- **Problem # 2:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required if we are using a rate  $R=K/N$  code?
- **Solutions:** Capacity of bandlimited Gaussian channel is given by
$$C^W = W \log \left( 1 + \frac{S}{N_0 W} \right) \text{ bits/s}$$
where  $W$  denotes the bandwidth and  $S$  is the signaling power.

So what is the effect of the bandwidth  $w$ , now we know for a band limited channel the capacity is given by this expression okay.

(Refer Slide Time: 05:09)

**Coding Limits**

- **Problem # 2:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required if we are using a rate  $R=K/N$  code?
- **Solutions:** Capacity of bandlimited Gaussian channel is given by
$$C^W = W \log \left( 1 + \frac{S}{N_0 W} \right) \text{ bits/s}$$
where  $W$  denotes the bandwidth and  $S$  is the signaling power.
- Assuming we are transmitting at a rate of  $2W$  samples per second and using a rate  $R=K/N$  block code. If we transmit  $K$  information bits during  $\tau$  seconds, we have
$$N = 2W\tau \text{ samples per codeword}$$

So let us say we are transmitting at Nyquist rate.

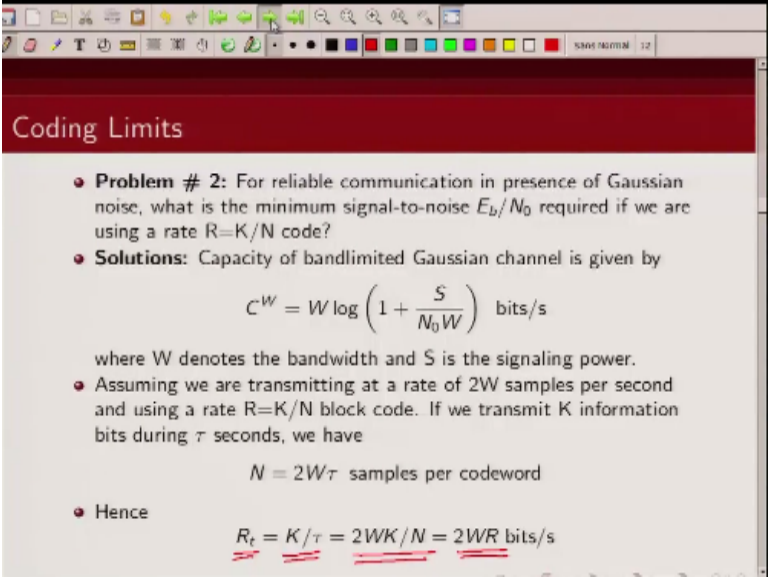
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**Coding Limits**

- **Problem # 2:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required if we are using a rate  $R=K/N$  code?
- **Solutions:** Capacity of bandlimited Gaussian channel is given by
$$C^W = W \log \left( 1 + \frac{S}{N_0 W} \right) \text{ bits/s}$$
where  $W$  denotes the bandwidth and  $S$  is the signaling power.
- Assuming we are transmitting at a rate of  $2W$  samples per second and using a rate  $R=K/N$  block code. If we transmit  $K$  information bits during  $\tau$  seconds, we have
$$\underline{N = 2W\tau} \text{ samples per codeword}$$

So we are transmitting at  $2W$  samples per second and we are using a rate  $K/N$  code. So if we transmit  $K$  information bits during time  $\tau$  then number of samples per code word that we are sending is  $2W \tau$ .

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The image is a screenshot of a presentation slide titled "Coding Limits". The slide contains a list of bullet points and mathematical formulas. The first bullet point is "Problem # 2: For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required if we are using a rate  $R=K/N$  code?". The second bullet point is "Solutions: Capacity of bandlimited Gaussian channel is given by" followed by the formula  $C^W = W \log \left( 1 + \frac{S}{N_0 W} \right)$  bits/s. Below this, it says "where W denotes the bandwidth and S is the signaling power." The third bullet point is "Assuming we are transmitting at a rate of  $2W$  samples per second and using a rate  $R=K/N$  block code. If we transmit K information bits during  $\tau$  seconds, we have" followed by the formula  $N = 2W\tau$  samples per codeword. The fourth bullet point is "Hence" followed by the formula  $R_t = K/\tau = 2WK/N = 2WR$  bits/s. The formula  $R_t = K/\tau = 2WK/N = 2WR$  is underlined in red.

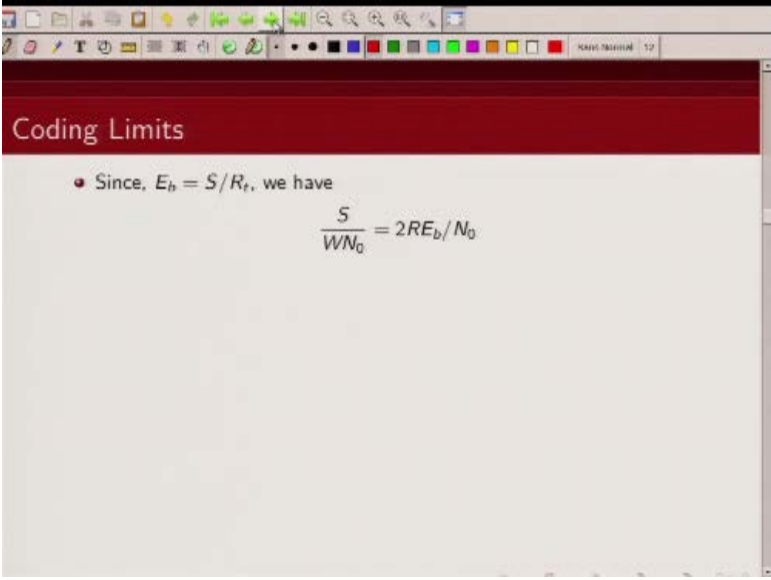
**Coding Limits**

- **Problem # 2:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required if we are using a rate  $R=K/N$  code?
- **Solutions:** Capacity of bandlimited Gaussian channel is given by
$$C^W = W \log \left( 1 + \frac{S}{N_0 W} \right) \text{ bits/s}$$
where W denotes the bandwidth and S is the signaling power.
- Assuming we are transmitting at a rate of  $2W$  samples per second and using a rate  $R=K/N$  block code. If we transmit K information bits during  $\tau$  seconds, we have
$$N = 2W\tau \text{ samples per codeword}$$
- Hence
$$R_t = K/\tau = 2WK/N = 2WR \text{ bits/s}$$

Now our transmission rate is K information bits over  $\tau$  time, so a transmission rate is  $K/\tau$  and this we can write in terms of these samples per code word n which is given by this expression and  $K/N$  is nothing but our code rate, so this transmission rate is given by  $2WR$  where R is my code rate, so this many bits per second is my transmission rate.



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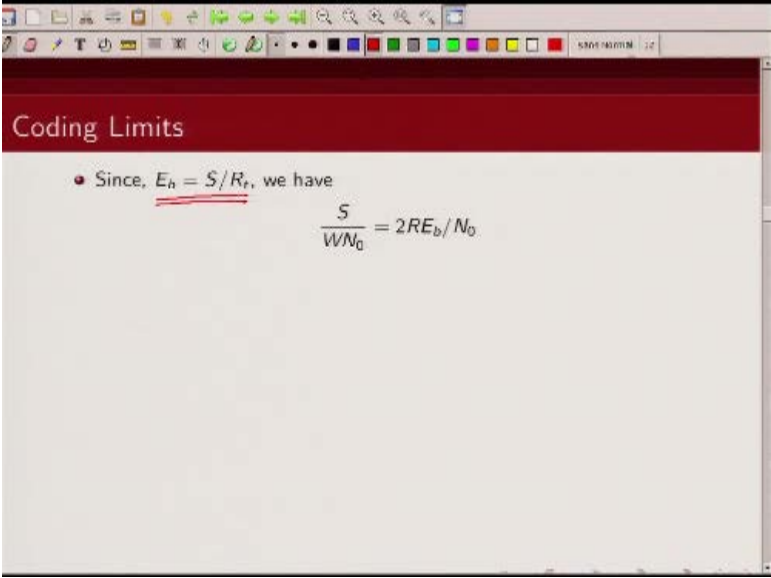
Coding Limits

- Since,  $E_b = S/R_t$ , we have

$$\frac{S}{WN_0} = 2RE_b/N_0$$

Now we know we that

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Coding Limits

- Since,  $E_b = S/R_t$ , we have

$$\frac{S}{WN_0} = 2RE_b/N_0$$

Energy per bit is given by signaling power by transmission rate, this we have done in the last example.

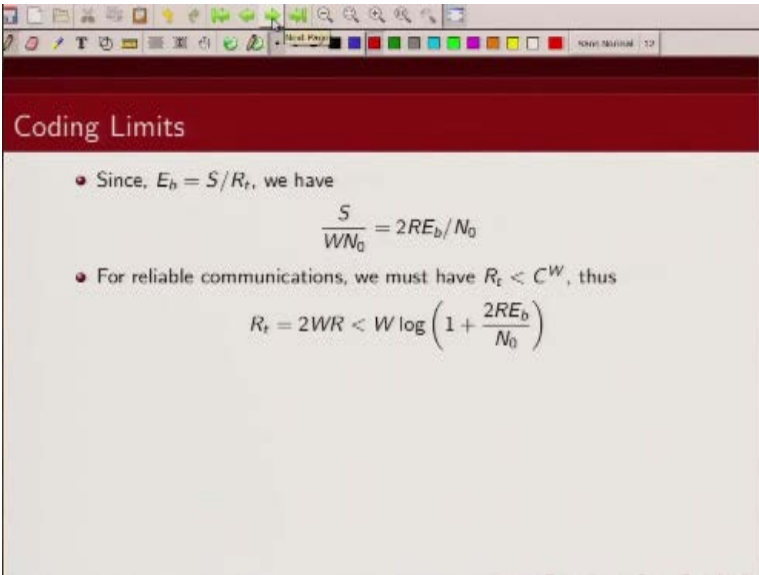
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**Fundamental limit**

- Since the data transmission rate  $R_t = K/\tau$  bits/s, energy per bit can be written as  $E_b = \frac{S}{R_t}$
- Thus we have  $\frac{C^\infty}{R_t} = \frac{E_b}{N_0 \ln 2}$
- For reliable communication, we must have  $R_t < C^\infty$ . Thus we have  $\frac{E_b}{N_0} > \ln 2 = 0.69 = -1.6 \text{ dB}$
- Thus signal to noise ratio  $E_b/N_0$  cannot be less than Shannon limit -1.6 dB for reliable communications.

We had this right so using this, then so if we do that

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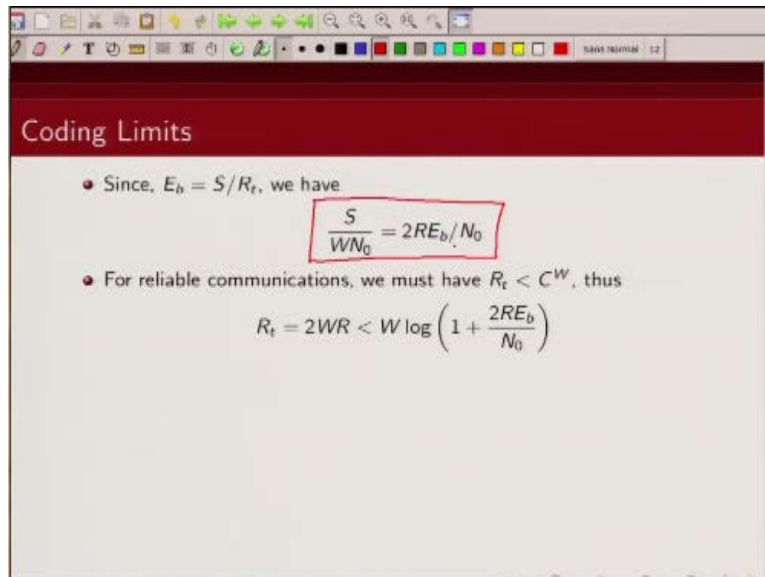
The image is a screenshot of a presentation slide titled "Coding Limits". The slide has a dark red header bar with the title in white. Below the header, there is a list of two bullet points. The first bullet point states "Since,  $E_b = S/R_t$ , we have" followed by the equation  $\frac{S}{WN_0} = 2RE_b/N_0$ . The second bullet point states "For reliable communications, we must have  $R_t < C^W$ , thus" followed by the equation  $R_t = 2WR < W \log \left( 1 + \frac{2RE_b}{N_0} \right)$ . The slide is displayed in a window with a standard toolbar at the top.

### Coding Limits

- Since,  $E_b = S/R_t$ , we have
$$\frac{S}{WN_0} = 2RE_b/N_0$$
- For reliable communications, we must have  $R_t < C^W$ , thus
$$R_t = 2WR < W \log \left( 1 + \frac{2RE_b}{N_0} \right)$$

We can write

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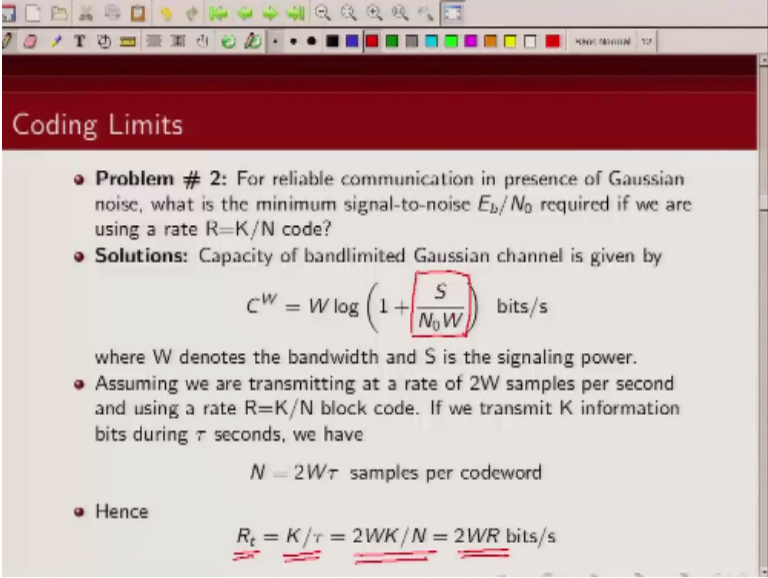


**Coding Limits**

- Since,  $E_b = S/R_t$ , we have
$$\frac{S}{WN_0} = 2RE_b/N_0$$
- For reliable communications, we must have  $R_t < C^W$ , thus
$$R_t = 2WR < W \log \left( 1 + \frac{2RE_b}{N_0} \right)$$

$S/WN_0$  in terms of  $E_b/N_0$ , this is equal to 2 times code rate by into SNR per information bit so in this expression of

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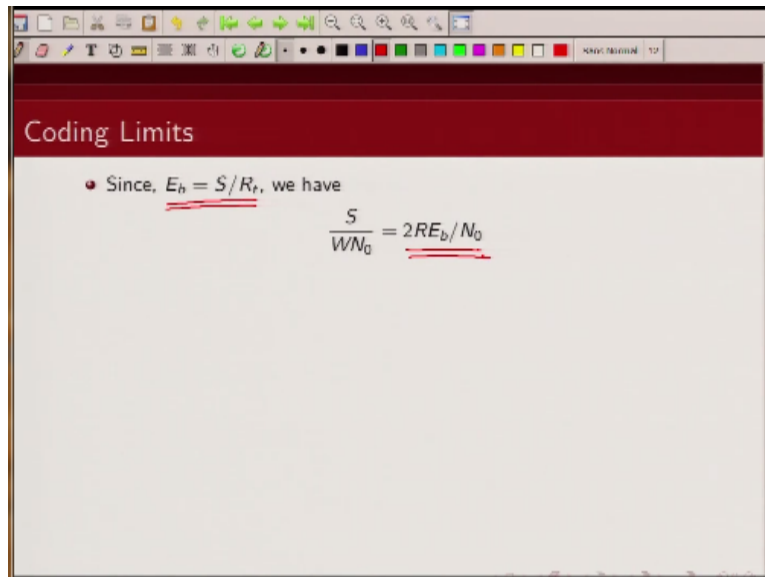


**Coding Limits**

- **Problem # 2:** For reliable communication in presence of Gaussian noise, what is the minimum signal-to-noise  $E_b/N_0$  required if we are using a rate  $R=K/N$  code?
- **Solutions:** Capacity of bandlimited Gaussian channel is given by
$$C^W = W \log \left( 1 + \frac{S}{N_0 W} \right) \text{ bits/s}$$
where  $W$  denotes the bandwidth and  $S$  is the signaling power.
- Assuming we are transmitting at a rate of  $2W$  samples per second and using a rate  $R=K/N$  block code. If we transmit  $K$  information bits during  $\tau$  seconds, we have
$$N = 2W\tau \text{ samples per codeword}$$
- Hence
$$R_t = K/\tau = 2WK/N = 2WR \text{ bits/s}$$

Capacity of band limited channel we can replace this expression by

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Coding Limits

- Since,  $E_b = \underline{S/R_t}$ , we have

$$\frac{S}{WN_0} = 2RE_b/\underline{N_0}$$

This, fine?

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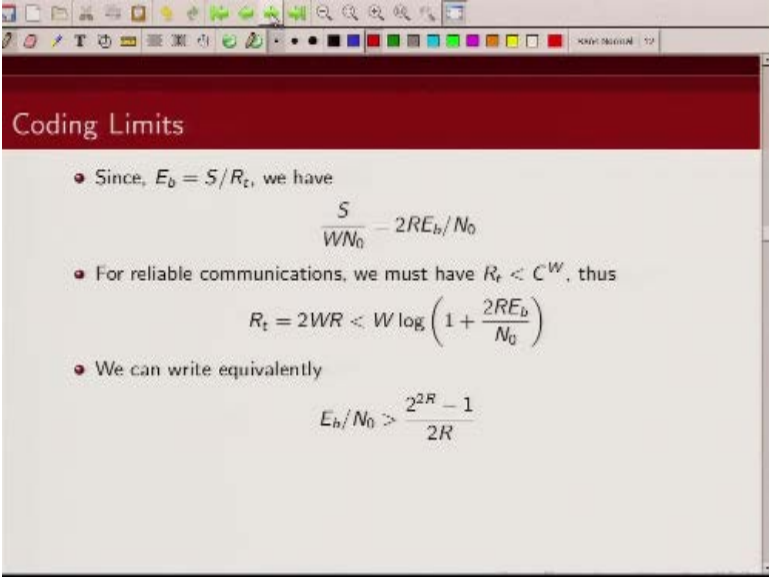
**Coding Limits**

- Since,  $E_b = S/R_t$ , we have  
$$\frac{S}{WN_0} = 2RE_b/N_0$$
- For reliable communications, we must have  $R_t < C^W$ , thus  
$$R_t = 2WR < W \log \left( 1 + \frac{2RE_b}{N_0} \right)$$

Now for reliable communication we know that our transmission rate should be less than channel capacity and what is channel capacity, that is for the band limited channels  $W \log (1+ S/WNN_0)$  which is basically given by this expression, so a transmission rate which is 2 times WR should be less then  $W \log (1+ 2RE_b/ N_0)$



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**Coding Limits**

- Since,  $E_b = S/R_t$ , we have
$$\frac{S}{WN_0} = 2RE_b/N_0$$
- For reliable communications, we must have  $R_t < C^W$ , thus
$$R_t = 2WR < W \log \left( 1 + \frac{2RE_b}{N_0} \right)$$
- We can write equivalently
$$E_b/N_0 > \frac{2^{2R} - 1}{2R}$$

So from here then we can write down

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**Coding Limits**

- Since,  $E_b = S/R_t$ , we have
$$\frac{S}{WN_0} = 2RE_b/N_0$$
- For reliable communications, we must have  $R_t < C^W$ , thus
$$R_t = 2WR < W \log \left( 1 + \frac{2RE_b}{N_0} \right)$$
- We can write equivalently
$$\underline{\underline{E_b/N_0 \geq \frac{2^{2R} - 1}{2R}}}$$

The expression for minimum SNR information bit is a energy per information bit by noise power [indiscernible][00:08:05] so SNR by information bit we can write it as this is should be greater than equal to  $2^{2R} - 1 / 2R$ , now this right hand side term is an increasing function of R

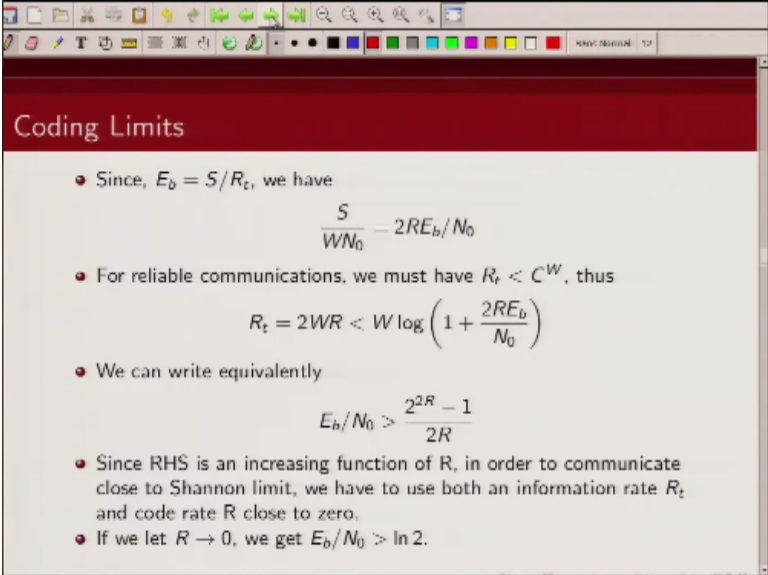
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**Coding Limits**

- Since,  $E_b = S/R_t$ , we have
$$\frac{S}{WN_0} = 2RE_b/N_0$$
- For reliable communications, we must have  $R_t < C^W$ , thus
$$R_t = 2WR < W \log \left( 1 + \frac{2RE_b}{N_0} \right)$$
- We can write equivalently
$$E_b/N_0 > \frac{2^{2R} - 1}{2R}$$
- Since RHS is an increasing function of R, in order to communicate close to Shannon limit, we have to use both an information rate  $R_t$  and code rate R close to zero.

So if you want to communicate close to the Shannon limit then your information rate  $R_t$  as well as code rate R should be close to zero. In fact

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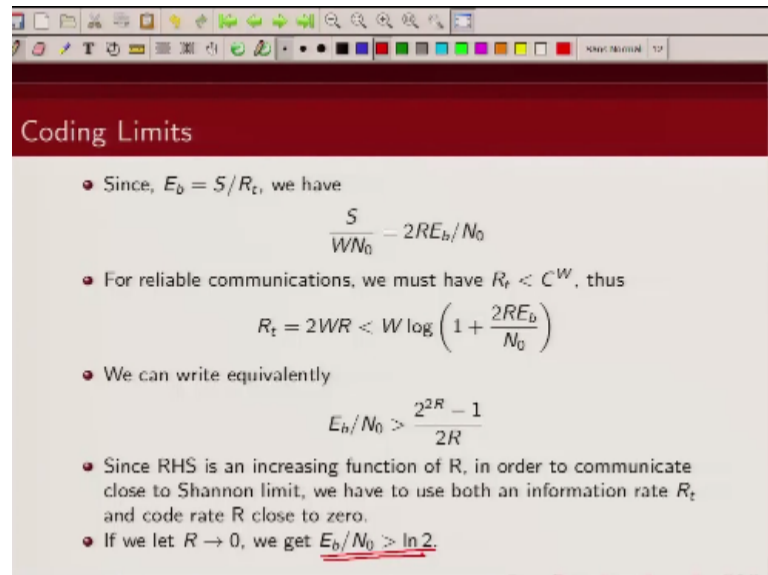


**Coding Limits**

- Since,  $E_b = S/R_t$ , we have
$$\frac{S}{WN_0} = 2RE_b/N_0$$
- For reliable communications, we must have  $R_t < C^W$ , thus
$$R_t = 2WR < W \log \left( 1 + \frac{2RE_b}{N_0} \right)$$
- We can write equivalently
$$E_b/N_0 > \frac{2^{2R} - 1}{2R}$$
- Since RHS is an increasing function of R, in order to communicate close to Shannon limit, we have to use both an information rate  $R_t$  and code rate R close to zero.
- If we let  $R \rightarrow 0$ , we get  $E_b/N_0 > \ln 2$ .

If you let  $R \rightarrow 0$  what you will see is

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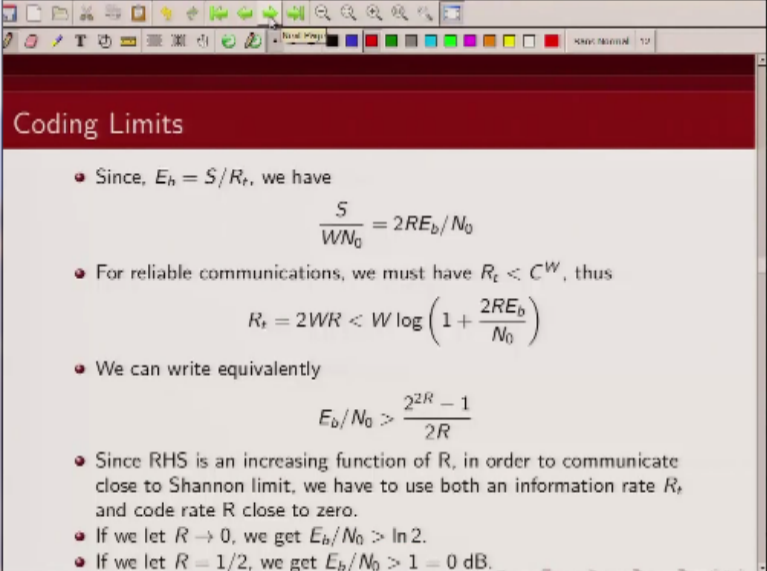


### Coding Limits

- Since,  $E_b = S/R_t$ , we have
$$\frac{S}{WN_0} = 2RE_b/N_0$$
- For reliable communications, we must have  $R_t < C^W$ , thus
$$R_t = 2WR < W \log \left( 1 + \frac{2RE_b}{N_0} \right)$$
- We can write equivalently
$$E_b/N_0 > \frac{2^{2R} - 1}{2R}$$
- Since RHS is an increasing function of R, in order to communicate close to Shannon limit, we have to use both an information rate  $R_t$  and code rate R close to zero.
- If we let  $R \rightarrow 0$ , we get  $E_b/N_0 > \ln 2$ .

You will get the limit that we had just talked about in the previous problem which was minus 1.6DP and if, let us plug in some practical values of R, let us say R is

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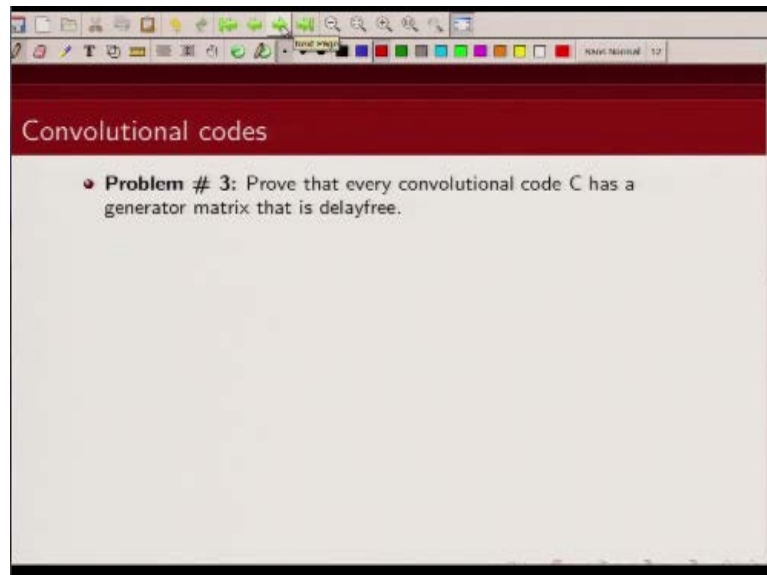


**Coding Limits**

- Since,  $E_b = S/R_t$ , we have
$$\frac{S}{WN_0} = 2RE_b/N_0$$
- For reliable communications, we must have  $R_t < C^W$ , thus
$$R_t = 2WR < W \log \left( 1 + \frac{2RE_b}{N_0} \right)$$
- We can write equivalently
$$E_b/N_0 \geq \frac{2^{2R} - 1}{2R}$$
- Since RHS is an increasing function of  $R$ , in order to communicate close to Shannon limit, we have to use both an information rate  $R_t$  and code rate  $R$  close to zero.
- If we let  $R \rightarrow 0$ , we get  $E_b/N_0 \geq \ln 2$ .
- If we let  $R = 1/2$ , we get  $E_b/N_0 \geq 1 = 0 \text{ dB}$ .

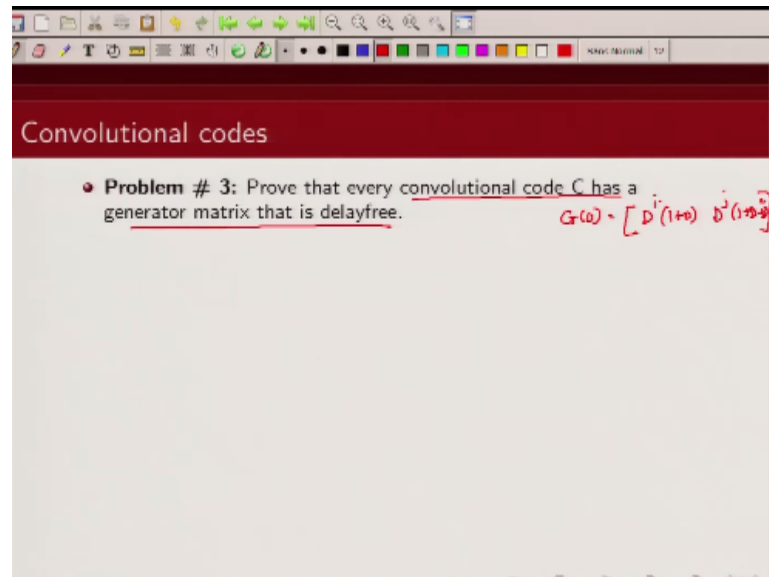
Half, we put value of  $R$  to be half then  $E_b$  by not should be more than 0 dB, so you can see for any rate which is away from 0 then this minimum SNR required for transmission is also more, okay.

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Now the next question that we will solve is to prove that

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Convolutional codes

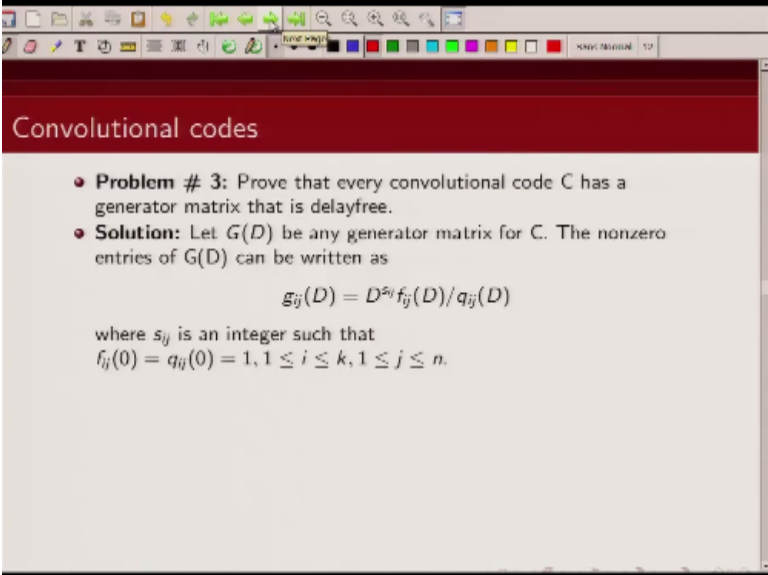
- **Problem # 3:** Prove that every convolutional code C has a generator matrix that is delayfree.

$G(z) = \begin{bmatrix} D^i(1+z) & D^j(1+z) \end{bmatrix}$

For every convolutional code has a generator matrix which is delay free. Now delay free meanings are basically if we have a generator matrix let us say of the form  $D^i$  and then some, something like  $1 + D$  and some  $D^j 1 + D + D^2$  or something like that we can always write an equivalent generator matrix which will be free of these like delay terms, I will talk about that.



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**Convolutional codes**

- **Problem # 3:** Prove that every convolutional code  $C$  has a generator matrix that is delayfree.
- **Solution:** Let  $G(D)$  be any generator matrix for  $C$ . The nonzero entries of  $G(D)$  can be written as
$$g_{ij}(D) = D^{s_{ij}} f_{ij}(D) / q_{ij}(D)$$
where  $s_{ij}$  is an integer such that
$$f_{ij}(0) = q_{ij}(0) = 1, 1 \leq i \leq k, 1 \leq j \leq n.$$

So let us see we have a generator matrix

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**Convolutional codes**

- **Problem # 3:** Prove that every convolutional code  $C$  has a generator matrix that is delayfree.
- **Solution:** Let  $G(D)$  be any generator matrix for  $C$ . The nonzero entries of  $G(D)$  can be written as
 
$$g_{ij}(D) = D^{s_{ij}} \frac{f_{ij}(D)}{q_{ij}(D)}$$
 where  $s_{ij}$  is an integer such that  $f_{ij}(0) = q_{ij}(0) = 1, 1 \leq i \leq k, 1 \leq j \leq n$ .
   

$$\frac{f_{ij}(D)}{q_{ij}(D)} = \frac{1 + f_0 D + \dots}{1 + q_0 D + \dots}$$

$G(D)$  okay and its non-zero entries can be written in this particular form, so there is some delay term we calling  $D^{s_{ij}}$  and then we have this rational function we have this rational function which is  $f_{ij}(D) / q_{ij}(D)$  and  $f_{ij}(D)$  and  $q_{ij}(D)$  is of the form  $1 + \text{some } f_0 D + \dots$ . This is  $1 + q_0 D + \text{something}$ , something okay, so that is what I meant if  $f_{ij}(0)$  and  $q_{ij}(0)$  is 1, so any entry in the generator matrix can be written in this form okay? Now

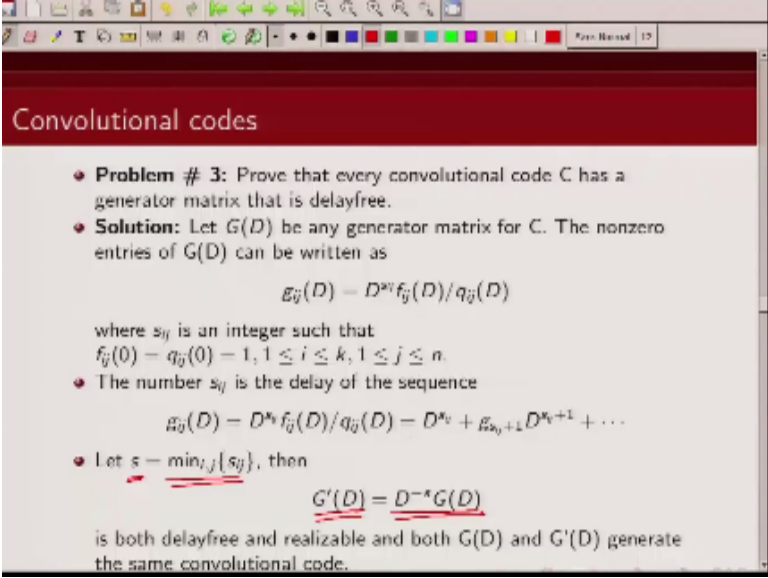
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**Convolutional codes**

- **Problem # 3:** Prove that every convolutional code  $C$  has a generator matrix that is delayfree.
- **Solution:** Let  $G(D)$  be any generator matrix for  $C$ . The nonzero entries of  $G(D)$  can be written as
 
$$g_{ij}(D) = D^{s_{ij}} f_{ij}(D)/q_{ij}(D)$$
 where  $s_{ij}$  is an integer such that
 
$$f_{ij}(0) = q_{ij}(0) = 1, 1 \leq i \leq k, 1 \leq j \leq n$$
- The number  $s_{ij}$  is the delay of the sequence
 
$$g_{ij}(D) = D^{s_{ij}} f_{ij}(D)/q_{ij}(D) = \underline{D^{s_{ij}} + g_{s_{ij}+1} D^{s_{ij}+1} + \dots}$$

What is this term  $s_{ij}$ ; it is essentially a delay term, so if we have some term like  $s_{ij}$  so what you will get is terms of this particular form.

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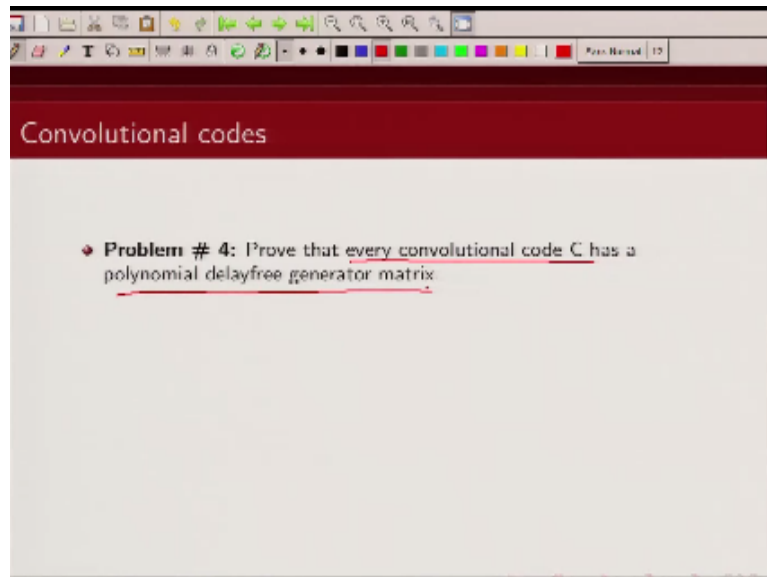


**Convolutional codes**

- **Problem # 3:** Prove that every convolutional code C has a generator matrix that is delayfree.
- **Solution:** Let  $G(D)$  be any generator matrix for C. The nonzero entries of  $G(D)$  can be written as
 
$$g_{ij}(D) = D^{s_{ij}} f_{ij}(D)/q_{ij}(D)$$
 where  $s_{ij}$  is an integer such that  $f_{ij}(0) = q_{ij}(0) = 1, 1 \leq i \leq k, 1 \leq j \leq n$ .
- The number  $s_{ij}$  is the delay of the sequence
 
$$g_{ij}(D) = D^{s_{ij}} f_{ij}(D)/q_{ij}(D) = D^{s_{ij}} + g_{s_{ij}+1} D^{s_{ij}+1} + \dots$$
- Let  $s = \min_{i,j} \{s_{ij}\}$ , then
 
$$\underline{G'(D)} = \underline{D^{-s} G(D)}$$
 is both delayfree and realizable and both  $G(D)$  and  $G'(D)$  generate the same convolutional code.

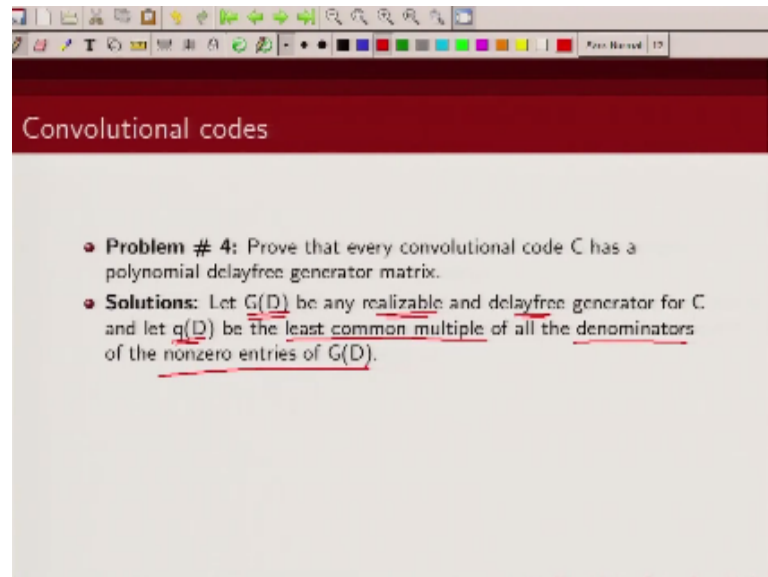
Now if we consider S which is the minimum  $S_{ij}$  over all I and J then we can find an equivalent generator matrix which would be  $G - of D$  which we can write as  $D-S GD$  and this  $GID$  will be delay free and it is of course realizable, and it also generate the same set of code words, so whenever we have a generator matrix which has the delay term we can always have - find an equivalent generator matrix which is delay free.

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Next question, prove that for every convolutional code has a polynomial delay free generator matrix. So for any convolutional code we have an equivalent polynomial delay free generator matrix.

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Convolutional codes

- **Problem # 4:** Prove that every convolutional code  $C$  has a polynomial delayfree generator matrix.
- **Solutions:** Let  $G(D)$  be any realizable and delayfree generator for  $C$  and let  $q(D)$  be the least common multiple of all the denominators of the nonzero entries of  $G(D)$ .

So let us say we have a generator matrix which is given by  $G(D)$  and this be any realizable and delay free generator for this convolutional code  $C$ , and let  $q(D)$  be the least common multiple of all the denominators of the nonzero entries of  $G(D)$ , so  $GD$  basically has a natural form and  $q(D)$  is the LCM of the denominator terms of the nonzero entries of  $G(D)$ .

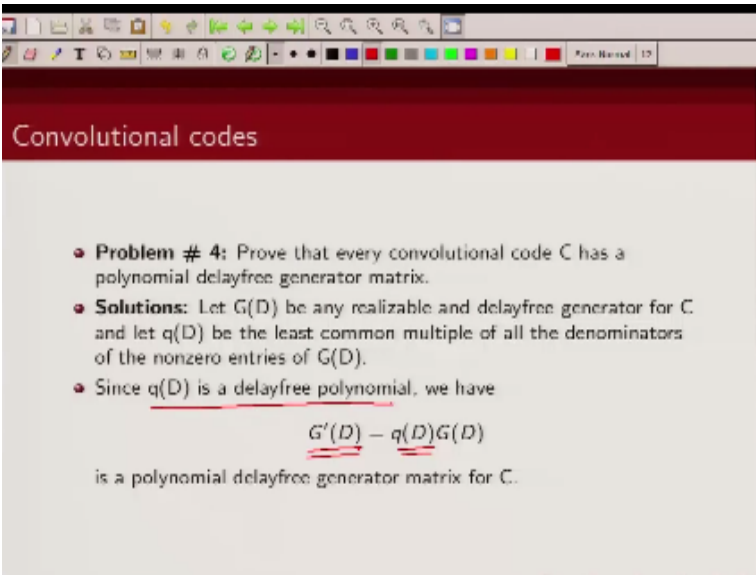
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Convolutional codes

- **Problem # 4:** Prove that every convolutional code  $C$  has a polynomial delayfree generator matrix.
- **Solutions:** Let  $G(D)$  be any realizable and delayfree generator for  $C$  and let  $q(D)$  be the least common multiple of all the denominators of the nonzero entries of  $G(D)$ .
- Since  $q(D)$  is a delayfree polynomial, we have
$$G'(D) = \underline{q(D)}G(D)$$
is a polynomial delayfree generator matrix for  $C$ .

Now if we have such and so we will have a  $q(D)$  which is basically delay free so if we multiply our original generator matrix by this  $q(D)$  we will end up with a

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The image is a screenshot of a presentation slide. At the top, there is a red header bar with the text "Convolutional codes" in white. Below the header, the slide content is on a light gray background. It contains three bullet points. The first bullet point is "Problem # 4: Prove that every convolutional code C has a polynomial delayfree generator matrix." The second bullet point is "Solutions: Let  $G(D)$  be any realizable and delayfree generator for C and let  $q(D)$  be the least common multiple of all the denominators of the nonzero entries of  $G(D)$ ." The third bullet point is "Since  $q(D)$  is a delayfree polynomial, we have". Below this, the equation  $\underline{G'(D)} = \underline{q(D)}G(D)$  is shown, with red underlines under  $G'(D)$  and  $q(D)$ . Below the equation, it says "is a polynomial delayfree generator matrix for C."

Convolutional codes

- **Problem # 4:** Prove that every convolutional code C has a polynomial delayfree generator matrix.
- **Solutions:** Let  $G(D)$  be any realizable and delayfree generator for C and let  $q(D)$  be the least common multiple of all the denominators of the nonzero entries of  $G(D)$ .
- Since  $q(D)$  is a delayfree polynomial, we have

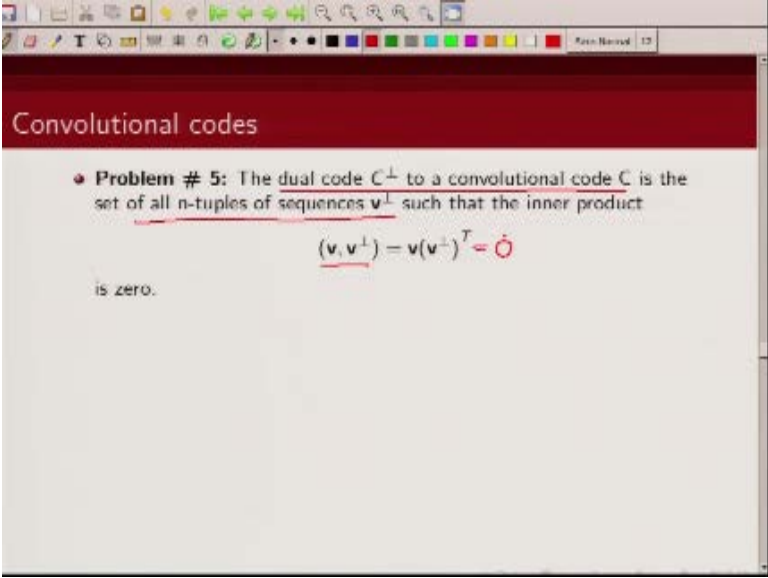
$$\underline{G'(D)} = \underline{q(D)}G(D)$$

is a polynomial delayfree generator matrix for C.

New generator matrix  $G'$  – of  $D$  which will be polynomial and delay free.



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**Convolutional codes**

- **Problem # 5:** The dual code  $C^\perp$  to a convolutional code  $C$  is the set of all  $n$ -tuples of sequences  $\mathbf{v}^\perp$  such that the inner product  $(\mathbf{v}, \mathbf{v}^\perp) = \mathbf{v}(\mathbf{v}^\perp)^T = \mathbf{0}$  is zero.

Okay next so let us first define a dual of convolutional code so we define a dual to convolutional code  $C$  as a set of all  $n$ -tuples of sequence  $V$  dual such that the inner product between this which is define as  $V$  and this  $V$  dual transpose this is essentially zero. So if you have a dual to the original convolutional code, then if you take set of code  $n$ -tuples sequence from the original code and the dual code, their inner product will be zero.

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**Convolutional codes**

- **Problem # 5:** The dual code  $C^\perp$  to a convolutional code  $C$  is the set of all  $n$ -tuples of sequences  $\mathbf{v}^\perp$  such that the inner product
 
$$(\mathbf{v}, \mathbf{v}^\perp) = \mathbf{v}(\mathbf{v}^\perp)^T$$
 is zero.
- Let rate  $k/n$  convolutional code be generated by the semi-infinite generator matrix  $\mathbf{G}$  and the rate  $R = (n-k)/n$  dual code  $C^\perp$  be generated by the semi-infinite generator matrix  $\mathbf{G}^\perp$ , where
 
$$\mathbf{G}^\perp = \begin{pmatrix} G_0^\perp & G_1^\perp & \cdots & G_m^\perp \\ G_0^\perp & G_1^\perp & \cdots & G_m^\perp \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

Then

$$\mathbf{G}(\mathbf{G}^\perp) = \mathbf{0}$$

So let us define a rate  $k/n$  convolutional code which is generated by a generator matrix  $\mathbf{G}$  and you know that we can write the generator matrix or convolutional code in a semi-infinite fashion.

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**Convolutional codes**

- **Problem # 5:** The dual code  $C^\perp$  to a convolutional code  $C$  is the set of all  $n$ -tuples of sequences  $\mathbf{v}^\perp$  such that the inner product
 
$$(\mathbf{v}, \mathbf{v}^\perp) = \mathbf{v}(\mathbf{v}^\perp)^T$$
 is zero.
- Let rate  $k/n$  convolutional code be generated by the semi-infinite generator matrix  $\mathbf{G}$  and the rate  $R = (n-k)/n$  dual code  $C^\perp$  be generated by the semi-infinite generator matrix  $\mathbf{G}^\perp$ , where
 
$$\mathbf{G}^\perp = \begin{pmatrix} G_0^\perp & G_1^\perp & \cdots & G_m^\perp \\ G_0^\perp & G_1^\perp & \cdots & G_m^\perp \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

Then  $\mathbf{G}(\mathbf{G}^\perp)^T = \mathbf{0}$

And let us see the dual code has is this, which has rate  $n - K / n$  and it is generated by this semi-infinite generator matrix, then show that  $\mathbf{G}, \mathbf{G}^T$  basically will be zero whereas  $\mathbf{G}$  is the generator matrix of the original code and this is the generator matrix of the dual code.

(Refer Slide Time 15:11)

Convolutional codes

- Let  $\underline{v} = \underline{uG}$  and  $\underline{v}^\perp = \underline{u^\perp G^\perp}$ , where  $\underline{v}$  and  $\underline{v}^\perp$  are orthogonal. Then we have

$$\underline{v}(\underline{v}^\perp)^T = \underline{uG}(\underline{u^\perp G^\perp})^T = \underline{uG(G^\perp)^T}(\underline{u^\perp})^T = 0$$

So how does this follow so since  $V, V$  transposes zero so what is  $V, V$  is  $U$  times  $G$  and  $V$  and  $V$  dual is  $U$  dual,  $G$  dual. Now we know that they will be dual of each other if  $V, V$  dual basically is orthogonal so if, if this inner product is zero now if the inner product is zero, so then  $UG$  and this transpose should be zero, so this we can write as  $UG(G)$  transpose, you transpose. Now this term will be zero only if this term is zero for all  $u$  okay.

(Refer Slide Time: 16:04)

## Convolutional codes

- Let  $\mathbf{v} = \mathbf{uG}$  and  $\mathbf{v}^\perp = \mathbf{u}^\perp \mathbf{G}^\perp$ , where  $\mathbf{v}$  and  $\mathbf{v}^\perp$  are orthogonal. Then we have

$$\mathbf{v}(\mathbf{v}^\perp)^\mathsf{T} = \mathbf{uG}(\mathbf{u}^\perp \mathbf{G}^\perp)^\mathsf{T} = \mathbf{uG}(\mathbf{G}^\perp)^\mathsf{T}(\mathbf{u}^\perp)^\mathsf{T} = \mathbf{0}$$

- Thus we have

$$\mathbf{G}(\mathbf{G}^\perp) = \mathbf{0}$$

(Refer Slide Time: 16:06)

## Convolutional codes

- Let  $\mathbf{v} = \mathbf{uG}$  and  $\mathbf{v}^\perp = \mathbf{u}^\perp \mathbf{G}^\perp$ , where  $\mathbf{v}$  and  $\mathbf{v}^\perp$  are orthogonal. Then we have

$$\mathbf{v}(\mathbf{v}^\perp)^\mathsf{T} = \mathbf{uG}(\mathbf{u}^\perp \mathbf{G}^\perp)^\mathsf{T} = \mathbf{uG}(\mathbf{G}^\perp)^\mathsf{T}(\mathbf{u}^\perp)^\mathsf{T} = \mathbf{0}$$

- Thus we have

$$\underline{\underline{\mathbf{G}(\mathbf{G}^\perp)^\mathsf{T} = \mathbf{0}}}$$

And hence we get this condition that this should be zero, okay

(Refer Slide Time: 16:17)

## Convolutional codes

- Let  $\mathbf{v} = \mathbf{uG}$  and  $\mathbf{v}^\perp = \mathbf{u}^\perp \mathbf{G}^\perp$ , where  $\mathbf{v}$  and  $\mathbf{v}^\perp$  are orthogonal. Then we have

$$\mathbf{v}(\mathbf{v}^\perp)^\top = \mathbf{uG}(\mathbf{u}^\perp \mathbf{G}^\perp)^\top = \mathbf{uG}(\mathbf{G}^\perp)^\top (\mathbf{u}^\perp)^\top = \mathbf{0}$$

- Thus we have

$$\mathbf{G}(\mathbf{G}^\perp) = \mathbf{0}$$

- The convolutional dual code  $C^\perp$  to a convolutional code  $C$  which is encoded by the rate  $R = k/n$  generator matrix  $G(D)$  is the set of all codewords encoded by any rate  $R = (n - k)/n$  generator matrix  $G_\perp(D)$  such that

$$\mathbf{G(D)G_\perp^\top(D) = 0}$$

(Refer Slide Time: 16:18)

### Convolutional codes

- Let  $\mathbf{v} = \mathbf{u}\mathbf{G}$  and  $\mathbf{v}^\perp = \mathbf{u}^\perp\mathbf{G}^\perp$ , where  $\mathbf{v}$  and  $\mathbf{v}^\perp$  are orthogonal. Then we have
$$\mathbf{v}(\mathbf{v}^\perp)^\mathsf{T} = \mathbf{u}\mathbf{G}(\mathbf{u}^\perp\mathbf{G}^\perp)^\mathsf{T} = \mathbf{u}\mathbf{G}(\mathbf{G}^\perp)^\mathsf{T}(\mathbf{u}^\perp)^\mathsf{T} = \mathbf{0}$$
- Thus we have
$$\mathbf{G}(\mathbf{G}^\perp) = \mathbf{0}$$
- The convolutional dual code  $C^\perp$  to a convolutional code  $C$  which is encoded by the rate  $R = k/n$  generator matrix  $\mathbf{G}(D)$  is the set of all codewords encoded by any rate  $R = (n-k)/n$  generator matrix  $\mathbf{G}_\perp(D)$  such that
$$\mathbf{G}(D)\mathbf{G}_\perp^\mathsf{T}(D) = \mathbf{0}$$

So a convolutional dual to a convolutional code  $C$  which is encoded by a rate  $k/n$  generator matrix  $\mathbf{G}(D)$  is set of all code words encoded by rate  $n-k$  with generator matrix this such that  $\mathbf{G}(D)$  and the generator matrix of the dual transpose should be zero, so dual of the code is defined by this.

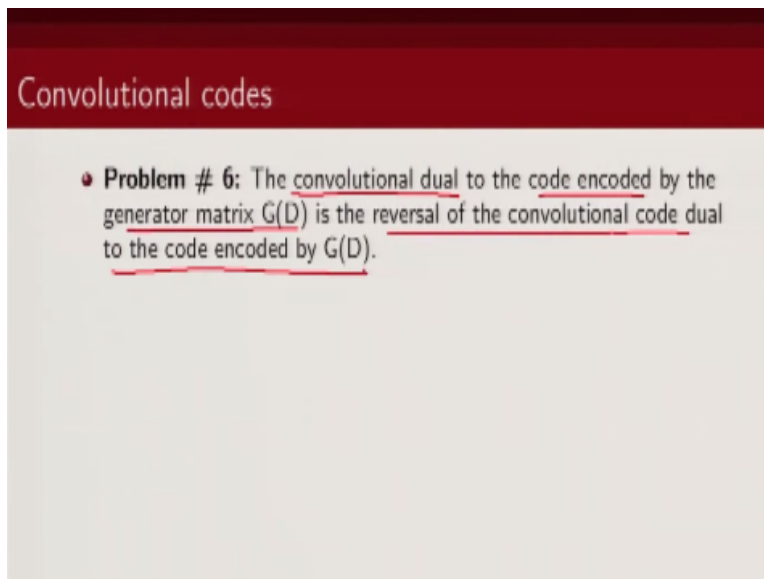


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## Convolutional codes

- **Problem # 6:** The convolutional dual to the code encoded by the generator matrix  $G(D)$  is the reversal of the convolutional code dual to the code encoded by  $G(D)$ .

(Refer Slide Time: 16:58)



### Convolutional codes

- **Problem # 6:** The convolutional dual to the code encoded by the generator matrix  $G(D)$  is the reversal of the convolutional code dual to the code encoded by  $G(D)$ .

Next, so here we will show that the convolutional dual to the code encoded by the generator matrix  $G(D)$  is nothing but reversal of the convolutional code dual to the code generated by  $G(D)$ .

(Refer Slide Time: 17:17)

## Convolutional codes

- **Problem # 6:** The convolutional dual to the code encoded by the generator matrix  $G(D)$  is the reversal of the convolutional code dual to the code encoded by  $G(D)$ .
- Let us consider a rate  $R = k/n$  convolutional code encoded by the polynomial generator matrix

$$G(D) = G_0 + G_1D + \cdots + G_mD^m$$

(Refer Slide Time: 17:17)

## Convolutional codes

- **Problem # 6:** The convolutional dual to the code encoded by the generator matrix  $G(D)$  is the reversal of the convolutional code dual to the code encoded by  $G(D)$ .
- Let us consider a rate  $R = k/n$  convolutional code encoded by the polynomial generator matrix

$$G(D) = G_0 + G_1D + \cdots + G_mD^m$$

So let us consider a rate  $k/n$  convolutional code which is encoded by a polynomial generator. We have already showed that we can find an equivalent polynomial delay free representation of any convolutional code right? So let us see this is our  $G(D)$  for our original code  $C$ .

(Refer Slide Time: 17:42)

## Convolutional codes

- **Problem # 6:** The convolutional dual to the code encoded by the generator matrix  $G(D)$  is the reversal of the convolutional code dual to the code encoded by  $G(D)$ .
- Let us consider a rate  $R = k/n$  convolutional code encoded by the polynomial generator matrix

$$G(D) = G_0 + G_1 D + \cdots + G_m D^m$$

(Refer Slide Time: 17:45)

## Convolutional codes

- **Problem # 6:** The convolutional dual to the code encoded by the generator matrix  $G(D)$  is the reversal of the convolutional code dual to the code encoded by  $G(D)$ .
- Let us consider a rate  $R = k/n$  convolutional code encoded by the polynomial generator matrix

$$G(D) = G_0 + G_1 D + \dots + G_m D^m$$

- Let  $\tilde{G}^\perp(D)$  denote the rate  $R=(n-k)/n$  polynomial generator matrix

$$\tilde{G}^\perp(D) = G_m^\perp + G_{m-1}^\perp D + \dots + G_0^\perp D^m$$

which is the reciprocal of the generator matrix

$$G^\perp(D) = G_0^\perp + G_1^\perp D + \dots + G_{m^\perp}^\perp$$

for the dual code  $C^\perp$ .

(Refer Slide Time: 17:50)

### Convolutional codes

- **Problem # 6:** The convolutional dual to the code encoded by the generator matrix  $G(D)$  is the reversal of the convolutional code dual to the code encoded by  $G(D)$ .
- Let us consider a rate  $R = k/n$  convolutional code encoded by the polynomial generator matrix  $\vec{G}^{\perp}(D) = D^m G^{\perp}(D^{-1})$ 

$$G(D) = G_0 + G_1 D + \dots + G_m D^m$$
- Let  $\vec{G}^{\perp}(D)$  denote the rate  $R = (n - k)/n$  polynomial generator matrix
 

$$\vec{G}^{\perp}(D) = G_m^{\perp} + G_{m-1}^{\perp} D + \dots + G_0^{\perp} D^{m-1}$$

 which is the reciprocal of the generator matrix
 

$$G^{\perp}(D) = G_0^{\perp} + G_1^{\perp} D + \dots + G_{m-1}^{\perp} D^{m-1}$$

 for the dual code  $C^{\perp}$ .

Now we define this as the reciprocal of the generator matrix now how do we come up with the reciprocal, so we do  $G(D^{-1})$  and we multiply by the maximum degree which is in this case  $m$ , so this is how we, we get the reciprocal, this how we get the reciprocal, okay. Now what I have shown you here is the reciprocal of the generator matrix for the dual, this is a generator matrix of the dual code and this is the reciprocal of the generator metric of the dual code, now note we have to show that the convolutional dual to the code encoded by  $G(D)$  is nothing but reversal of the convolutional code dual coded by  $G(D)$ . So if that is the case then  $G(D)$  dot product of  $G(D)$  with this should be zero.

(Refer Slide Time: 19:13)

## Convolutional codes

• Then we have

$$\begin{aligned}
 G(D)(\tilde{G}(D))^T &= G_0(G_{m+1}^T)^T + G_0(G_{m+1-1}^T)^T + G_1(G_{m+1}^T)^T D \\
 &\quad + \dots + G_m(G_0^T)^T D^{m+m+1} \\
 &= \left( \sum_{j=-m}^{m+1} \left( \sum_{i=0}^m G_i(G_{i+j}^T)^T \right) \right) D^{m+j} = 0
 \end{aligned}$$



(Refer Slide Time: 19:16)

### Convolutional codes

• Then we have

$$\begin{aligned}
 \underline{G(D)(\tilde{G}(D))^T} &= G_0(G_{m+1}^\perp)^T + G_0(G_{m+1-1}^\perp)^T + G_1(G_{m+1}^\perp)D \\
 &\quad + \dots + G_m(G_0^\perp)^T D^{m+m+1} \\
 &= \left( \sum_{j=-m}^{m+1} \left( \sum_{l=0}^m G_l(G_{l+j}^\perp)^T \right) \right) D^{m+j} = 0
 \end{aligned}$$

So let us try to find  $G(D)$  and transpose of this reversal.

(Refer Slide Time: 19:26)

### Convolutional codes

- **Problem # 6:** The convolutional dual to the code encoded by the generator matrix  $G(D)$  is the reversal of the convolutional code dual to the code encoded by  $G(D)$ .
- Let us consider a rate  $R = k/n$  convolutional code encoded by the polynomial generator matrix  $\tilde{G}^{\perp}(D) = D^m G^{\perp}(D^{-1})$ 

$$G(D) = G_0 + G_1 D + \dots + G_m D^m$$
- Let  $\tilde{G}^{\perp}(D)$  denote the rate  $R = (n - k)/n$  polynomial generator matrix
 

$$\tilde{G}^{\perp}(D) = G_m^{\perp} + G_{m-1}^{\perp} D + \dots + G_0^{\perp} D^m$$

 which is the reciprocal of the generator matrix
 

$$G^{\perp}(D) = G_0^{\perp} + G_1^{\perp} D + \dots + G_m^{\perp} D^m$$

 for the dual code  $C^{\perp}$ .

Of the dual matrix, see again pay attention to the question, basically what we are saying is the dual of the code encoded by the generator matrix is given by the reversal of the convolutional code dual so reversal of the convolutional code dual is this right? So if the dual of the original code is given by reversal of the convolutional code then what is the property the this generator matrix should satisfy, this and this transpose should be zero, so we have to show that  $G(D)$  and this generator matrix transpose is zero.

(Refer Slide Time: 20:13)

### Convolutional codes

- Then we have

$$\begin{aligned}
 \underline{G(D)(\tilde{G}(D))^T} &= G_0(G_{m-1}^\perp)^T + G_0(G_{m-2}^\perp)^T + G_1(G_{m-1}^\perp)^T D \\
 &\quad + \dots + G_m(G_0^\perp)^T D^{m+m^\perp} \\
 &= \left( \sum_{j=-m}^{m^\perp} \left( \sum_{i=0}^m G_i(G_{i+j}^\perp)^T \right) \right) D^{m+j} = 0
 \end{aligned}$$

So we do this so this can be written as this term, okay.

(Refer Slide Time: 20:18)

Convolutional codes

• Then we have

$$\begin{aligned}
 G(D) \tilde{G}(D)^T &= G_0(G_{m-1}^\perp)^T + G_0(G_{m-2}^\perp)^T + G_1(G_{m-1}^\perp)^T D \\
 &\quad + \dots + G_m(G_0^\perp)^T D^{m+m^\perp} \\
 &= \left( \sum_{j=-m}^{m^\perp} \left( \sum_{i=0}^m G_i (G_{i+j}^\perp)^T \right) \right) D^{m+j} = 0
 \end{aligned}$$

Now further we can write this as double  $\Sigma$  and what are, what is this? This is the dual of generator matrix is the dual of this so this, this  $G(D)$   $G$  transpose should be zero, right? So this whole  $\Sigma$  would be also zero, so what we have shown is then the dual of the dual of this code which has generator matrix  $G$ .

(Refer Slide Time: 20:59)

## Convolutional codes

- **Problem # 6:** The convolutional dual to the code encoded by the generator matrix  $G(D)$  is the reversal of the convolutional code dual to the code encoded by  $G(D)$ .

- Let us consider a rate  $R = k/n$  convolutional code encoded by the polynomial generator matrix

$$\tilde{G}^{\perp}(D) = D^m G^{\perp}(D^{-1})$$

$$G(D) = G_0 + G_1 D + \dots + G_m D^m$$

- Let  $\tilde{G}^{\perp}(D)$  denote the rate  $R = (n - k)/n$  polynomial generator matrix

$$\tilde{G}^{\perp}(D) = G_m^{\perp} + G_{m-1}^{\perp} D + \dots + G_0^{\perp} D^{m-1}$$

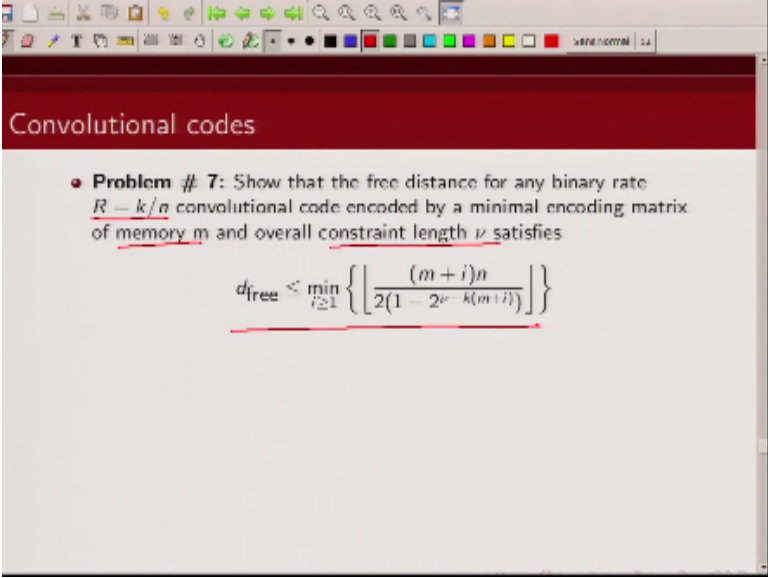
which is the reciprocal of the generator matrix

$$G^{\perp}(D) = G_0^{\perp} + G_1^{\perp} D + \dots + G_{m-1}^{\perp} D^{m-1}$$

for the dual code  $C^{\perp}$ .

Is nothing but reversal of the convolutional code dual because this generator matrix is nothing but it is the reversal of the generator matrix of the dual code of  $c$  okay?

(Refer Slide Time: 21:19)



The image is a screenshot of a presentation slide. At the top, there is a red header bar with the text "Convolutional codes" in white. Below the header, the slide contains a bullet point labeled "Problem # 7:" followed by the text "Show that the free distance for any binary rate  $R = k/n$  convolutional code encoded by a minimal encoding matrix of memory  $m$  and overall constraint length  $\nu$  satisfies". Below this text is a mathematical formula for the free distance  $d_{\text{free}}$ . The formula is 
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \left\lfloor \frac{(m+i)n}{2(1 - 2^{v-k(m+i)})} \right\rfloor \right\}$$
 The formula is underlined in red.

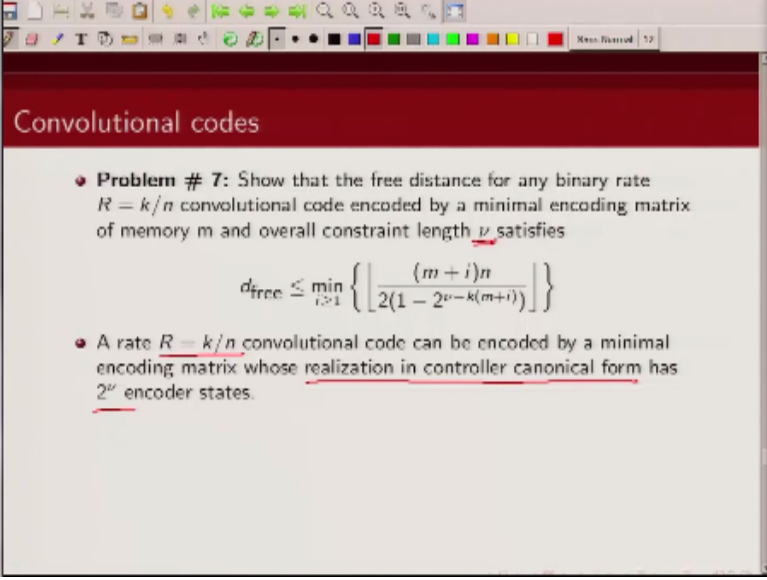
Convolutional codes

- **Problem # 7:** Show that the free distance for any binary rate  $R = k/n$  convolutional code encoded by a minimal encoding matrix of memory  $m$  and overall constraint length  $\nu$  satisfies

$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \left\lfloor \frac{(m+i)n}{2(1 - 2^{v-k(m+i)})} \right\rfloor \right\}$$

Next we are going to show a bound on free distance of convolutional code, so show that if you have a rate key by n convolutional code whose encoding matrix as memory m and over all constraint length  $\nu$ . Then free distances upper bounded by this.

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### Convolutional codes

- **Problem # 7:** Show that the free distance for any binary rate  $R = k/n$  convolutional code encoded by a minimal encoding matrix of memory  $m$  and overall constraint length  $\nu$  satisfies
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \left\lfloor \frac{(m+i)n}{2(1 - 2^{v-k(m+i)})} \right\rfloor \right\}$$
- A rate  $R = k/n$  convolutional code can be encoded by a minimal encoding matrix whose realization in controller canonical form has  $2^\nu$  encoder states.

So if you have a rate  $k/n$  code we can realize it using controller canonical form using  $2^\nu$  encoder state because this is over all constraint length is  $\nu$ .

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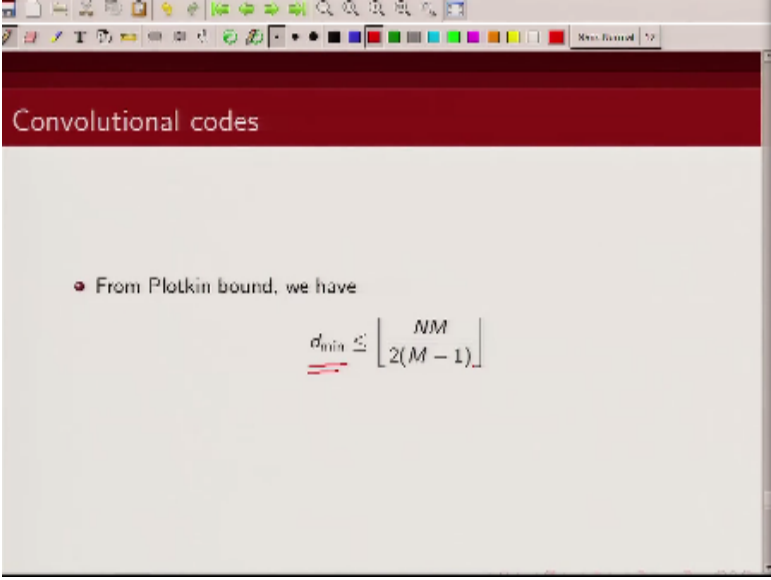
**Convolutional codes**

- **Problem # 7:** Show that the free distance for any binary rate  $R = k/n$  convolutional code encoded by a minimal encoding matrix of memory  $m$  and overall constraint length  $\nu$  satisfies
 
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \left\lfloor \frac{(m+i)n}{2(1-2^{-(m+i)})} \right\rfloor \right\}$$
- A rate  $R = k/n$  convolutional code can be encoded by a minimal encoding matrix whose realization in controller canonical form has  $2^\nu$  encoder states.
- Consider  $2^{k(m+i)}$ ,  $i = 1, 2, \dots$  information sequences.
- There exist  $2^{k(m+i)}/2^\nu$  information sequences starting in the zero state leading to the zero state.
- Corresponding code sequences constitute a block code with  $M = 2^{k(m+i)-\nu}$  codewords and blocklength  $N = (m+i)n$  for  $i = 1, 2, \dots$ .

Now if we consider  $2^{k(m+i)}$  information sequences. Then there exist  $2^{k(m+i)}/2^\nu$  information sequence starting at zero state leading to all zero state. So if we count number of such code words then number of such code words  $M$  is given by  $2^{k(m+i)-\nu}$  and what is the length of this code word, length of this code word would be  $(m+i)n$  because this is our rate  $k/n$  code right? Next we are going to use



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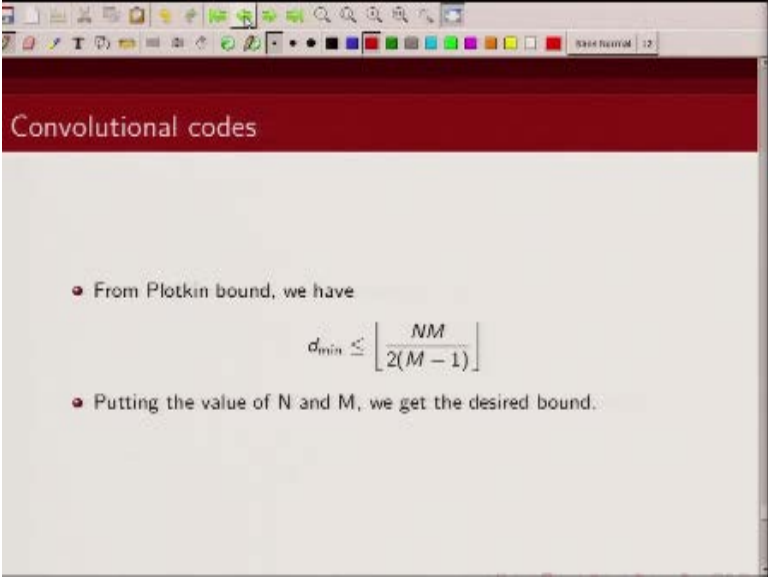
Convolutional codes

- From Plotkin bound, we have

$$\underline{d_{\min}} \leq \left\lfloor \frac{NM}{2(M-1)} \right\rfloor$$

Plotkin's bound, now Plotkin bound says it upper bounds the minimum distance as floor of this is code word length, number of code words 2 into number of code words -1, so in this example

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The image is a screenshot of a presentation slide. At the top, there is a dark red header bar with the text "Convolutional codes" in white. Below the header, the slide has a light gray background. There are two bullet points. The first bullet point says "From Plotkin bound, we have" followed by a mathematical equation. The second bullet point says "Putting the value of N and M, we get the desired bound." The presentation window's toolbar is visible at the top, and the status bar at the bottom shows "Slide Number 12".

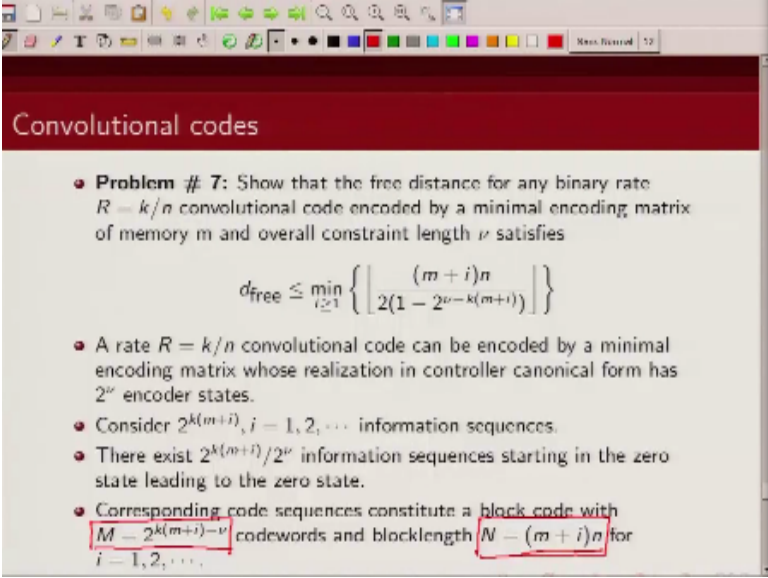
Convolutional codes

- From Plotkin bound, we have

$$d_{\min} \leq \left\lfloor \frac{NM}{2(M-1)} \right\rfloor$$

- Putting the value of N and M, we get the desired bound.

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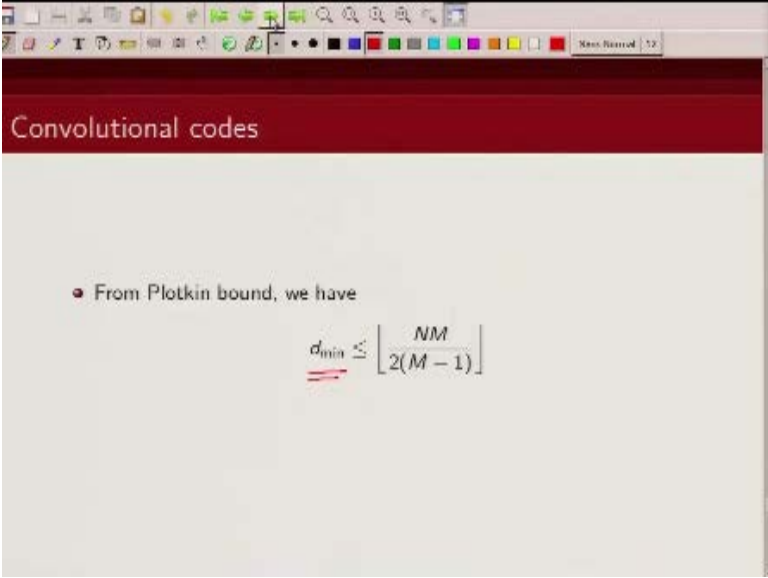


**Convolutional codes**

- **Problem # 7:** Show that the free distance for any binary rate  $R = k/n$  convolutional code encoded by a minimal encoding matrix of memory  $m$  and overall constraint length  $\nu$  satisfies
 
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \left\lfloor \frac{(m+i)n}{2(1-2^{\nu-k(m+i)})} \right\rfloor \right\}$$
- A rate  $R = k/n$  convolutional code can be encoded by a minimal encoding matrix whose realization in controller canonical form has  $2^m$  encoder states.
- Consider  $2^{k(m+i)}$ ,  $i = 1, 2, \dots$  information sequences.
- There exist  $2^{k(m+i)}/2^m$  information sequences starting in the zero state leading to the zero state.
- Corresponding code sequences constitute a block code with  $M = 2^{k(m+i)-m}$  codewords and blocklength  $N = (m+i)n$  for  $i = 1, 2, \dots$ .

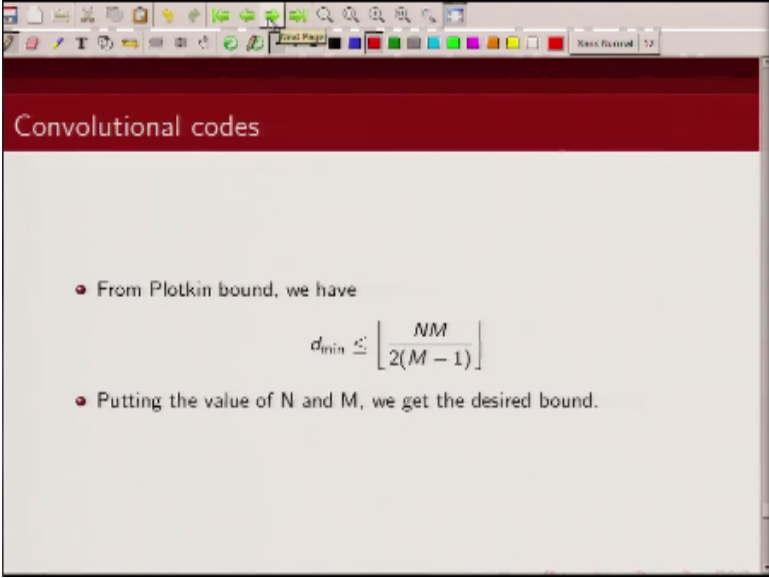
What is our N, our N is this, okay. This is our N and what is our M, our M is this. So if you plug in this value of M and N in our Plotkin bounds.

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The image is a screenshot of a presentation slide. At the top, there is a red header bar with the text "Convolutional codes" in white. Below the header, the slide has a light gray background. A bullet point is visible, stating "From Plotkin bound, we have". Below this, a mathematical inequality is displayed: 
$$\underline{d_{min}} \leq \left\lfloor \frac{NM}{2(M-1)} \right\rfloor$$
 The text "From Plotkin bound, we have" is preceded by a small red circle bullet point. The mathematical expression features a double underline under  $d_{min}$  and a floor function symbol  $\lfloor$  around the fraction  $\frac{NM}{2(M-1)}$ .

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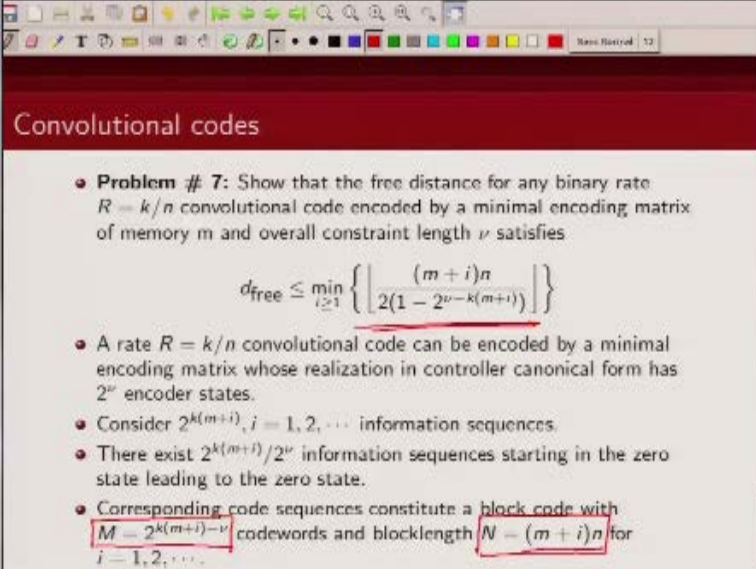


Convolutional codes

- From Plotkin bound, we have
$$d_{\min} \geq \left\lfloor \frac{NM}{2(M-1)} \right\rfloor$$
- Putting the value of N and M, we get the desired bound.

What we will get is our desired bound.

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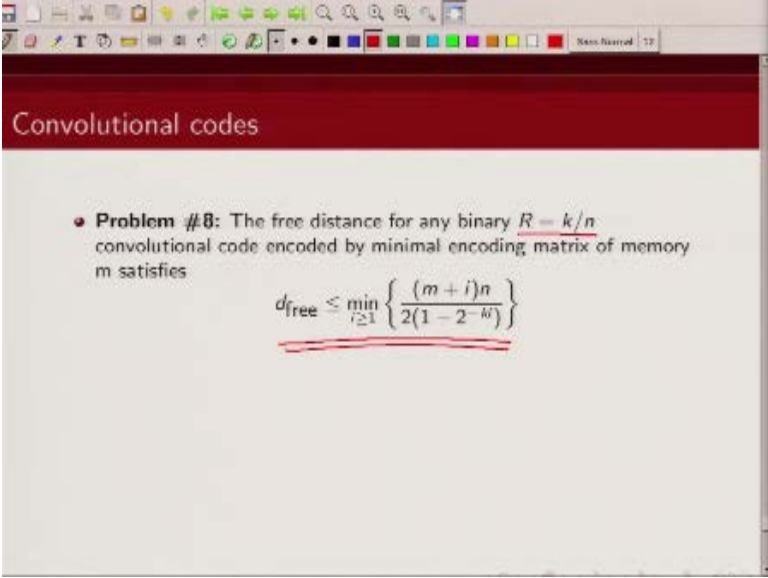
The image is a screenshot of a presentation slide titled "Convolutional codes". The slide contains a list of bullet points and a mathematical formula. The first bullet point is "Problem # 7: Show that the free distance for any binary rate  $R = k/n$  convolutional code encoded by a minimal encoding matrix of memory  $m$  and overall constraint length  $\nu$  satisfies". Below this is the formula  $d_{\text{free}} \leq \min_{i \geq 1} \left\{ \left\lfloor \frac{(m+i)n}{2(1-2^{\nu-k(m+i)})} \right\rfloor \right\}$ . The second bullet point states that a rate  $R = k/n$  convolutional code can be encoded by a minimal encoding matrix whose realization in controller canonical form has  $2^\nu$  encoder states. The third bullet point says to consider  $2^{k(m+i)}$ ,  $i = 1, 2, \dots$  information sequences. The fourth bullet point says there exist  $2^{k(m+i)}/2^\nu$  information sequences starting in the zero state leading to the zero state. The fifth bullet point says corresponding code sequences constitute a block code with  $M = 2^{k(m+i)-\nu}$  codewords and blocklength  $N = (m+i)n$  for  $i = 1, 2, \dots$ . The slide has a red header bar with the title "Convolutional codes". The background is light gray. The text is black. The formula is centered. The bullet points are on the left. The slide is framed by a window border with a toolbar at the top.

### Convolutional codes

- **Problem # 7:** Show that the free distance for any binary rate  $R = k/n$  convolutional code encoded by a minimal encoding matrix of memory  $m$  and overall constraint length  $\nu$  satisfies
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \left\lfloor \frac{(m+i)n}{2(1-2^{\nu-k(m+i)})} \right\rfloor \right\}$$
- A rate  $R = k/n$  convolutional code can be encoded by a minimal encoding matrix whose realization in controller canonical form has  $2^\nu$  encoder states.
- Consider  $2^{k(m+i)}$ ,  $i = 1, 2, \dots$  information sequences.
- There exist  $2^{k(m+i)}/2^\nu$  information sequences starting in the zero state leading to the zero state.
- Corresponding code sequences constitute a block code with  $M = 2^{k(m+i)-\nu}$  codewords and blocklength  $N = (m+i)n$  for  $i = 1, 2, \dots$ .

Which is this result, okay?

(Refer Slide Time: 23:40)



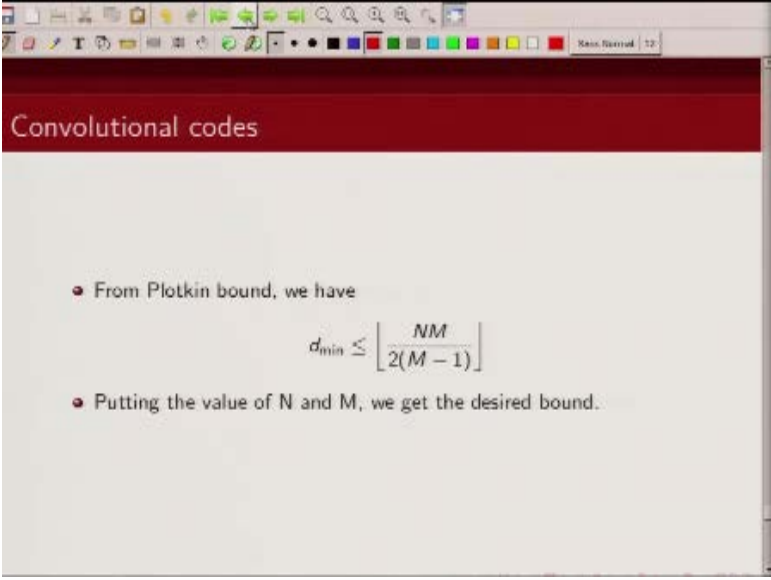
Convolutional codes

- **Problem #8:** The free distance for any binary  $R = k/n$  convolutional code encoded by minimal encoding matrix of memory  $m$  satisfies

$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \frac{(m+i)n}{2(1-2^{-i})} \right\}$$

So the next problem that we will solve is to show that free distance of a rate  $k/n$  convolutional code is upper bounded by this. Now this result can be obtained from

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## Convolutional codes

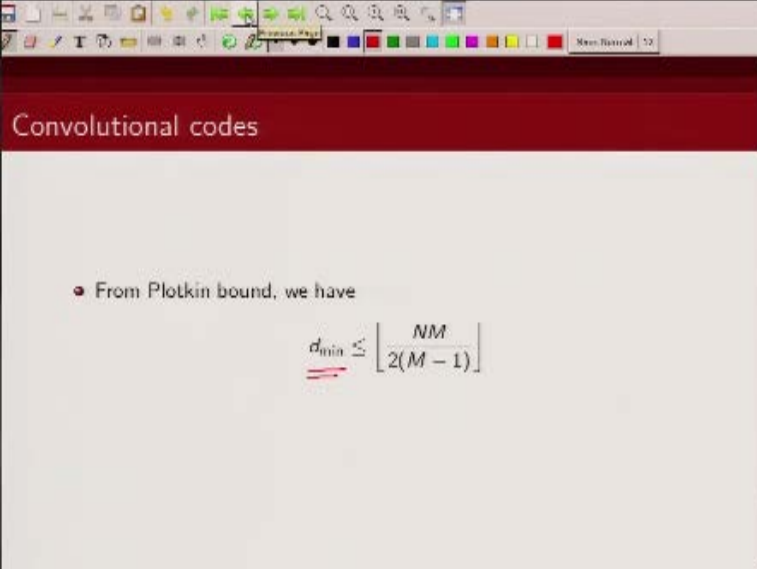
- From Plotkin bound, we have

$$d_{\min} \leq \left\lfloor \frac{NM}{2(M-1)} \right\rfloor$$

- Putting the value of N and M, we get the desired bound.



(Refer Slide Time: 24:01)



The image is a screenshot of a presentation slide. At the top, there is a red header bar with the text "Convolutional codes" in white. Below the header, the slide has a light gray background. A bullet point is visible, stating "From Plotkin bound, we have". Below this text, a mathematical equation is displayed: 
$$\underline{d_{min}} \leq \left\lfloor \frac{NM}{2(M-1)} \right\rfloor$$
. The equation is underlined with a red line. The presentation software's toolbar is visible at the top of the window, and a status bar at the bottom right shows "Slide Number: 12".

## Convolutional codes

- From Plotkin bound, we have

$$\underline{d_{min}} \leq \left\lfloor \frac{NM}{2(M-1)} \right\rfloor$$

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**Convolutional codes**

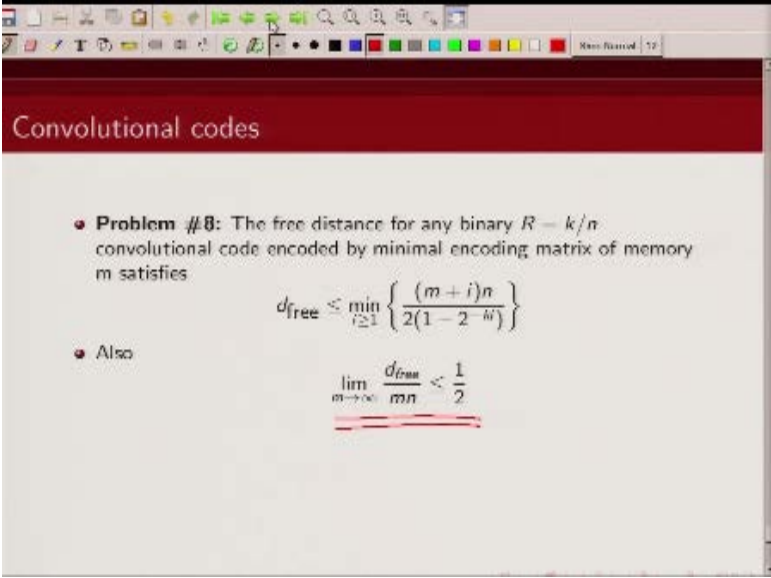
- **Problem # 7:** Show that the free distance for any binary rate  $R = k/n$  convolutional code encoded by a minimal encoding matrix of memory  $m$  and overall constraint length  $\nu$  satisfies

$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \left\lfloor \frac{(m+i)n}{2(1-2^{\nu-k(m+i)})} \right\rfloor \right\}$$

- A rate  $R = k/n$  convolutional code can be encoded by a minimal encoding matrix whose realization in controller canonical form has  $2^\nu$  encoder states.
- Consider  $2^{k(m+i)}$ ,  $i = 1, 2, \dots$  information sequences.
- There exist  $2^{k(m+i)}/2^\nu$  information sequences starting in the zero state leading to the zero state.
- Corresponding code sequences constitute a block code with  $M = 2^{k(m+i)-\nu}$  codewords and blocklength  $N = (m+i)n$  for  $i = 1, 2, \dots$ .

The bound that we have derived earlier, this bound okay?

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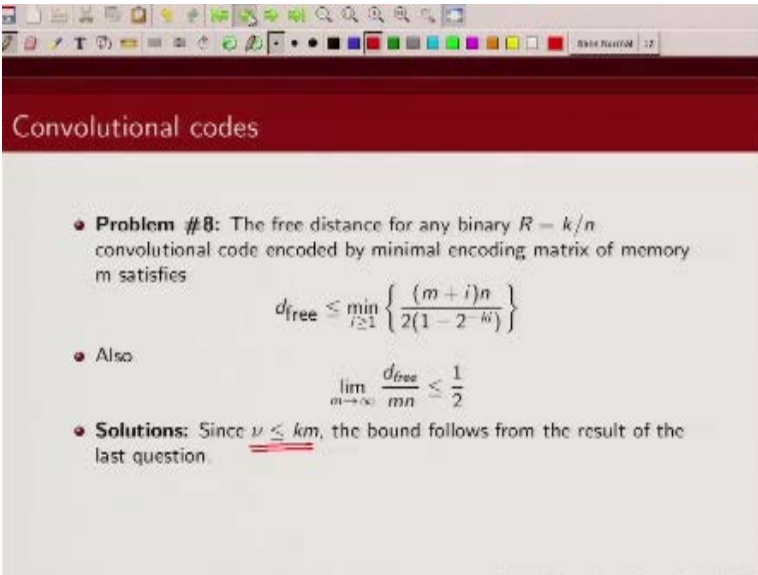


Convolutional codes

- **Problem #8:** The free distance for any binary  $R = k/n$  convolutional code encoded by minimal encoding matrix of memory  $m$  satisfies
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \frac{(m+i)n}{2(1-2^{-i})} \right\}$$
- Also
$$\lim_{m \rightarrow \infty} \frac{d_{\text{free}}}{mn} \leq \frac{1}{2}$$

And also we will show that this relation holds.

(Refer Slide Time: 24:14)



The image is a screenshot of a presentation slide titled "Convolutional codes". The slide has a dark red header with the title in white. Below the header, the text is on a light gray background. It contains a bullet point for a problem, a mathematical formula for free distance, another bullet point for a limit, and a final bullet point for a solution. The presentation software's toolbar is visible at the top.

**Convolutional codes**

- **Problem #8:** The free distance for any binary  $R = k/n$  convolutional code encoded by minimal encoding matrix of memory  $m$  satisfies
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \frac{(m+i)n}{2(1-2^{-i})} \right\}$$
- Also,
$$\lim_{m \rightarrow \infty} \frac{d_{\text{free}}}{mn} \leq \frac{1}{2}$$
- **Solutions:** Since  $\nu \leq km$ , the bound follows from the result of the last question.

So since overall constraint length is less than  $k$  times  $m$ , then if you go back to this expression.

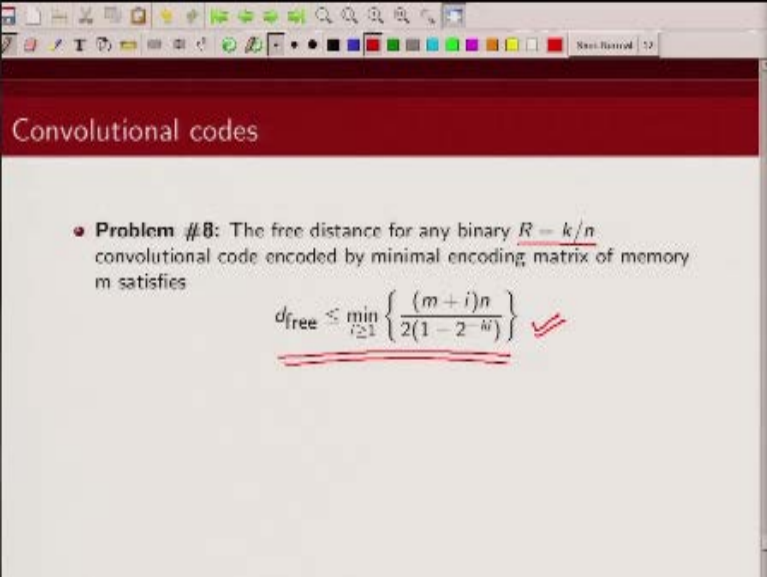
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**Convolutional codes**

- **Problem # 7:** Show that the free distance for any binary rate  $R = k/n$  convolutional code encoded by a minimal encoding matrix of memory  $m$  and overall constraint length  $\nu$  satisfies
 
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \left\lfloor \frac{(m+i)n}{2(1-2^{\nu-k(m+i)})} \right\rfloor \right\}$$
- A rate  $R = k/n$  convolutional code can be encoded by a minimal encoding matrix whose realization in controller canonical form has  $2^m$  encoder states.
- Consider  $2^{k(m+i)}$ ,  $i = 1, 2, \dots$  information sequences.
- There exist  $2^{k(m+i)}/2^m$  information sequences starting in the zero state leading to the zero state.
- Corresponding code sequences constitute a block code with  $M = 2^{k(m+i)-m}$  codewords and blocklength  $N = (m+i)n$  for  $i = 1, 2, \dots$

$\nu - km$  is actually less than zero so we can then upper bound this by just to restore  $-ki$ . So if we do that we get this expression okay?

(Refer Slide Time: 24:40)



The image is a screenshot of a presentation slide. At the top, there is a dark red header bar with the text "Convolutional codes" in white. Below the header, the slide content is on a light gray background. It starts with a bullet point labeled "Problem #8:" followed by text describing the free distance for a binary convolutional code. The text mentions a minimal encoding matrix of memory m. Below this text is a mathematical formula for d\_free, which is underlined in red. To the right of the formula is a red checkmark. The formula is: 
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \frac{(m+i)n}{2(1-2^{-hi})} \right\}$$

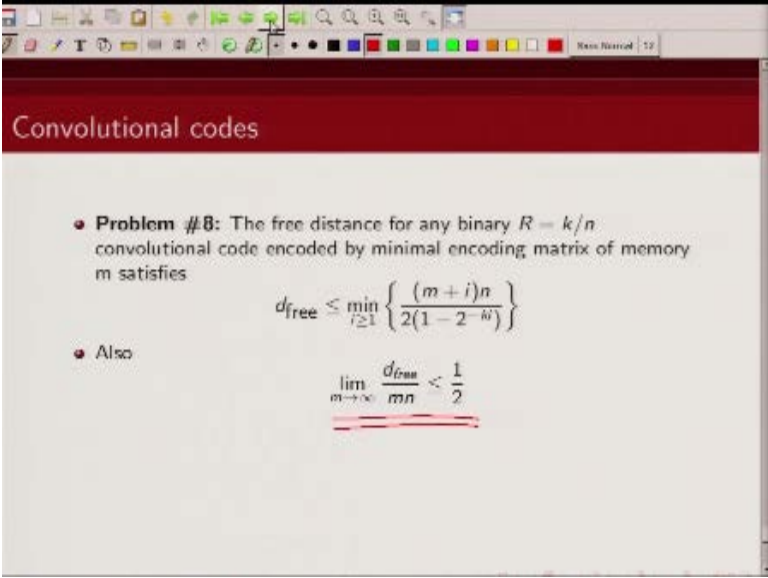
Convolutional codes

- **Problem #8:** The free distance for any binary  $R = k/n$  convolutional code encoded by minimal encoding matrix of memory m satisfies

$$\underline{d_{\text{free}} \leq \min_{i \geq 1} \left\{ \frac{(m+i)n}{2(1-2^{-hi})} \right\}}$$

✓

(Refer Slide Time: 24:43)



The image shows a presentation slide titled "Convolutional codes". The slide contains a problem statement and two mathematical formulas. The first formula is an inequality for the free distance  $d_{\text{free}}$  in terms of a minimum over  $i$  of a fraction involving  $m$ ,  $i$ , and  $n$ . The second formula is a limit as  $m$  approaches infinity of the ratio  $d_{\text{free}}/mn$  being less than  $1/2$ , which is underlined in red.

### Convolutional codes

- **Problem #8:** The free distance for any binary  $R = k/n$  convolutional code encoded by minimal encoding matrix of memory  $m$  satisfies
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \frac{(m+i)n}{2(1-2^{-i})} \right\}$$
- Also
$$\lim_{m \rightarrow \infty} \frac{d_{\text{free}}}{mn} < \frac{1}{2}$$

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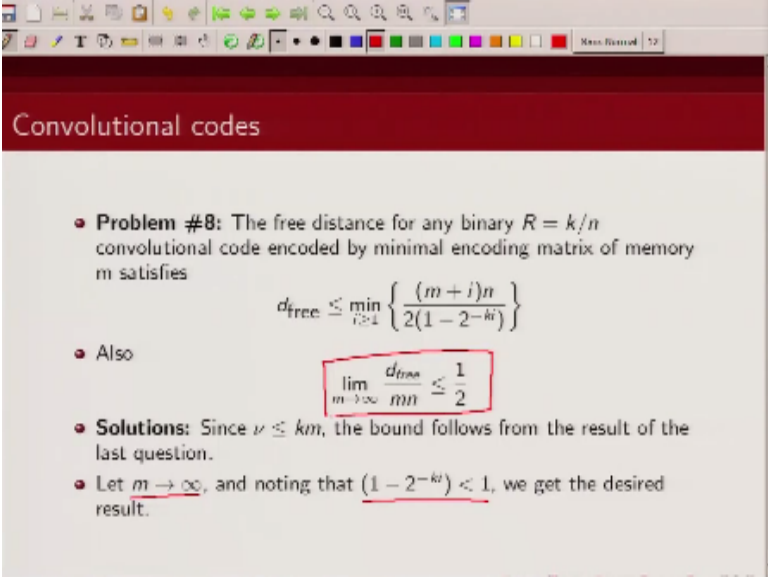
Convolutional codes

- **Problem #8:** The free distance for any binary  $R = k/n$  convolutional code encoded by minimal encoding matrix of memory  $m$  satisfies
 
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \frac{(m+i)n}{2(1-2^{-ki})} \right\} \leq \frac{(m+i)n}{2} \leq \frac{mn(1+i/m)}{2}$$
- Also
 
$$\lim_{m \rightarrow \infty} \frac{d_{\text{free}}}{mn} \leq \frac{1}{2}$$
- **Solutions:** Since  $\nu \leq km$ , the bound follows from the result of the last question.
 
$$\frac{d_{\text{free}}}{mn} \leq \frac{1+i/m}{2}$$

Now how do we get this expression, so we know that this term is less than 1 and so this we can just upper bound by  $(m+i)n/2$  and if I take  $m$  and  $n$  out this will be  $(1+i/m)/2$ . So  $d_{\text{free}}$  by  $m$  of  $n$  will then be so if I do further  $d_{\text{free}}$  of  $/mn$  would be upper bounded by  $1+i/m/2$  and if we let  $m$  go to infinity this will go to 0 so that will be upper bounded by half. So that is the proof.



(Refer Slide Time: 25:40)



The screenshot shows a presentation slide with a red header bar containing the title "Convolutional codes". Below the header, there is a list of bullet points and a mathematical equation. The first bullet point is "Problem #8: The free distance for any binary  $R = k/n$  convolutional code encoded by minimal encoding matrix of memory  $m$  satisfies". This is followed by the equation 
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \frac{(m+i)n}{2(1-2^{-ki})} \right\}$$
. The second bullet point is "Also", followed by a boxed equation 
$$\lim_{m \rightarrow \infty} \frac{d_{\text{free}}}{mn} \leq \frac{1}{2}$$
. The third bullet point is "Solutions: Since  $\nu \leq km$ , the bound follows from the result of the last question." The fourth bullet point is "Let  $m \rightarrow \infty$ , and noting that  $(1 - 2^{-ki}) < 1$ , we get the desired result."

**Convolutional codes**

- **Problem #8:** The free distance for any binary  $R = k/n$  convolutional code encoded by minimal encoding matrix of memory  $m$  satisfies
$$d_{\text{free}} \leq \min_{i \geq 1} \left\{ \frac{(m+i)n}{2(1-2^{-ki})} \right\}$$
- Also
$$\lim_{m \rightarrow \infty} \frac{d_{\text{free}}}{mn} \leq \frac{1}{2}$$
- **Solutions:** Since  $\nu \leq km$ , the bound follows from the result of the last question.
- Let  $m \rightarrow \infty$ , and noting that  $(1 - 2^{-ki}) < 1$ , we get the desired result.

So you let  $m$  go to infinity and since this is less than 1 what you will get is this, thank you.

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