# Indian Institute of Technology Kanpur
**National Programme on Technology Enhanced Learning (NPTEL)**
**Course Title**
**Error Control Coding: An Introduction to Convolutional Codes**

**Lecture – 1A**
**Introduction to Error Control Coding–I**

**by**
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Welcome to the course on Error control coding an Introduction to Convolutional codes, I am Adrish Banerjee from IIT Kanpur.

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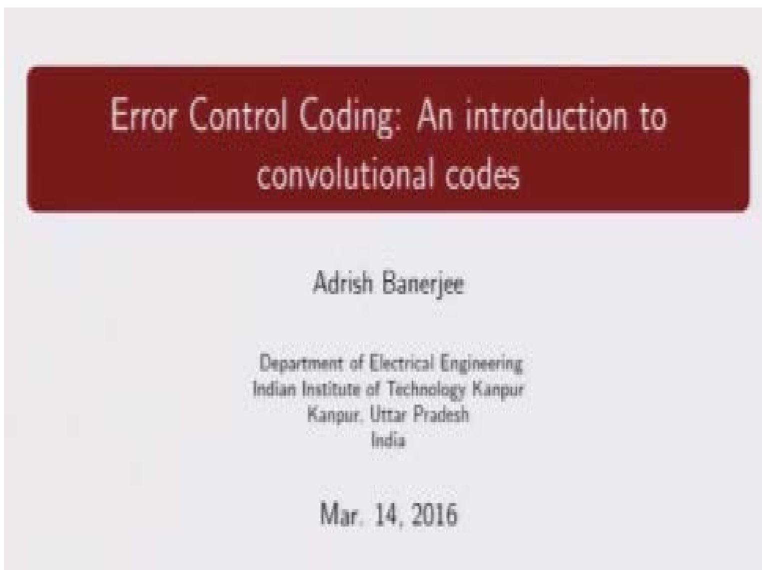


So in this lecture we are going to give a very brief Introduction on what coding theories all about, so we are going to motivate you with a very simple example on repetition codes and we are going to show how we can use simple error correcting codes for error deduction and error correction. So before I go into this lecture, I will first like to share with you.

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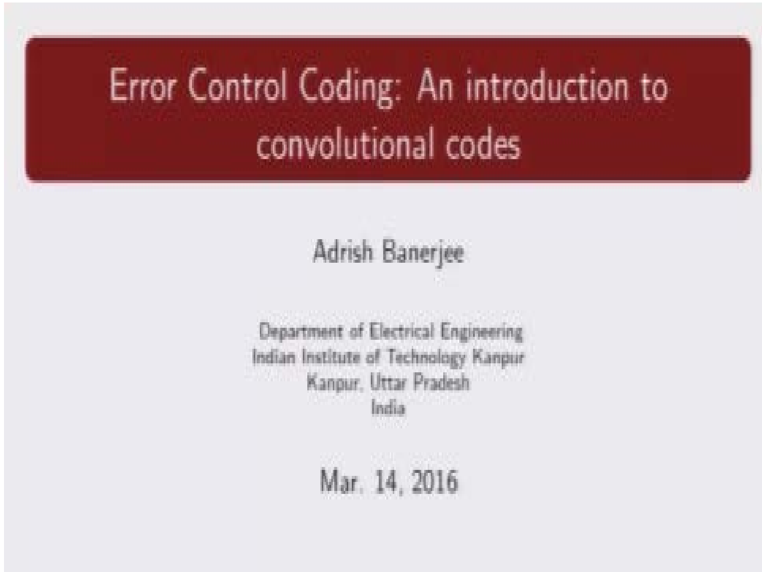

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The books that we are going to follow for this course.

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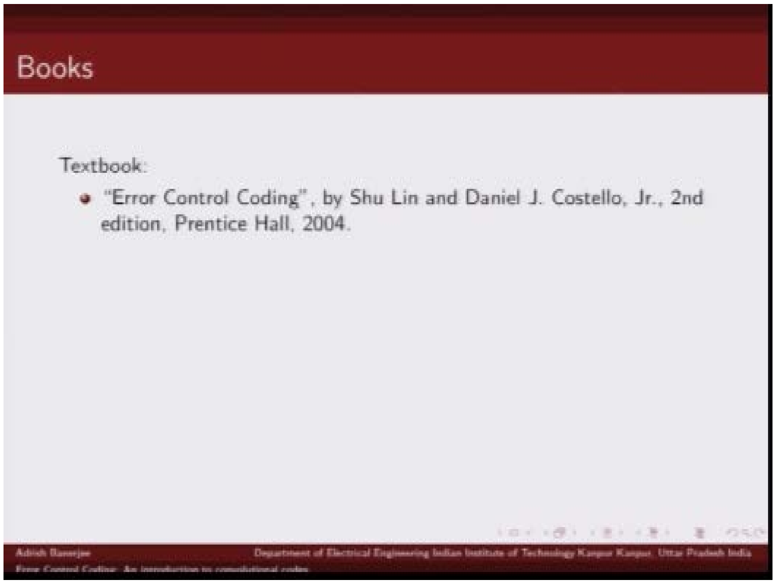


So as a text book we are going to use this book on error control coding by Lin and Costello, second edition, this will be our text book and there some nice books available as reference which we are going to use as references.

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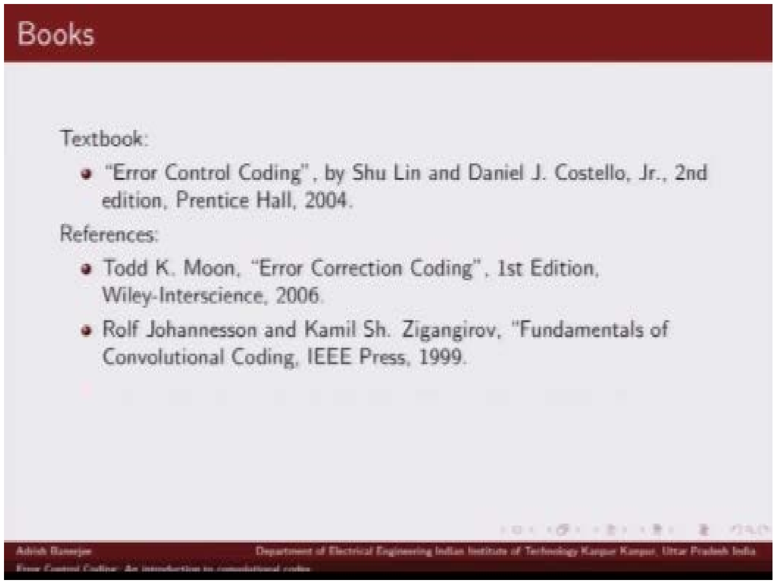


I am just going to list them, so this book by Todd K. Moon on Error Control Coding is a very simple introduction on error control coding, this book by Rolf Johannesson and Kamil Sh. Zigangirov this deals completely with convolutional code a very, very nice book if you want to read on convolutional codes.

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Another book devoted to convolutional codes is basically this book by Ajay Dholakia.

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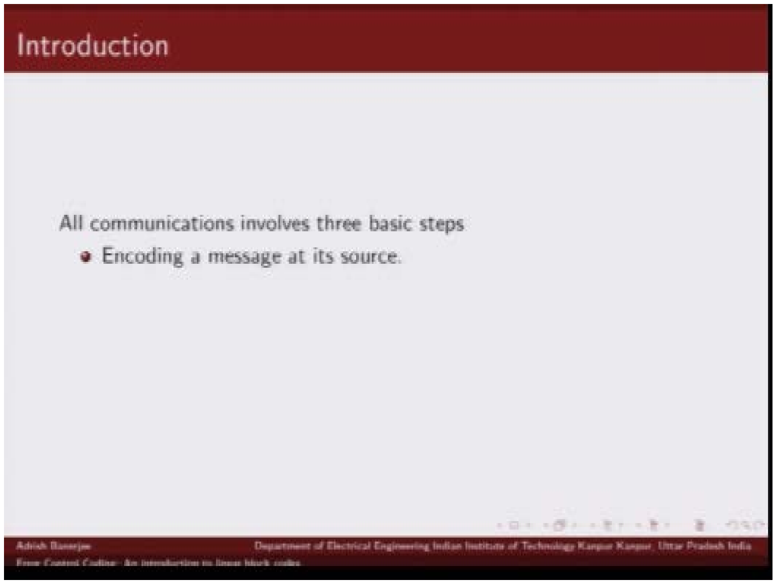


So, when we talk about communications, communications basically involves three basics, steps to first is encoding a message.

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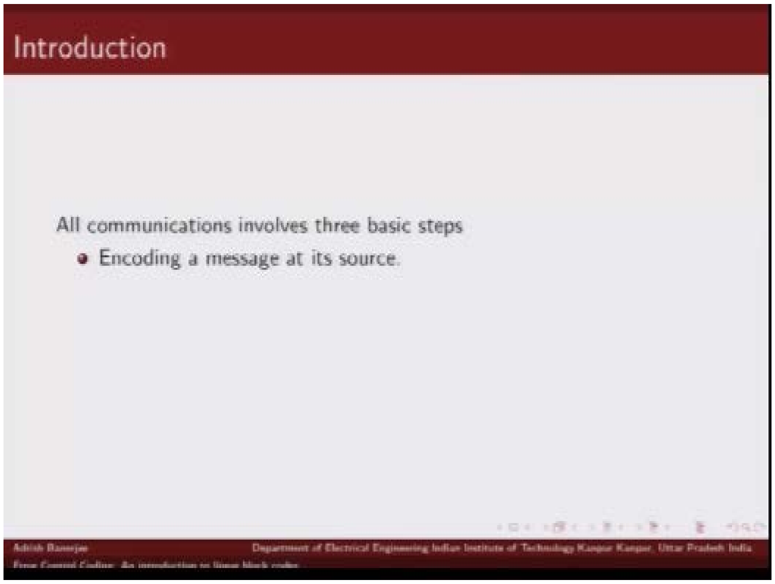


You have a message source that you want to represent efficiently, now you can for example consider a speech signal, now if you want to cross better speech signal you first have to convert your analog signal to digital signal and then you need to get rid of useless redundancies, why because we want to transmit basically useful information at the, we want a source inherently has lot of redundancy and when we try to represent a source we would like to represent a source efficiently in, be known number of possible bits, so first step involved in any communication.

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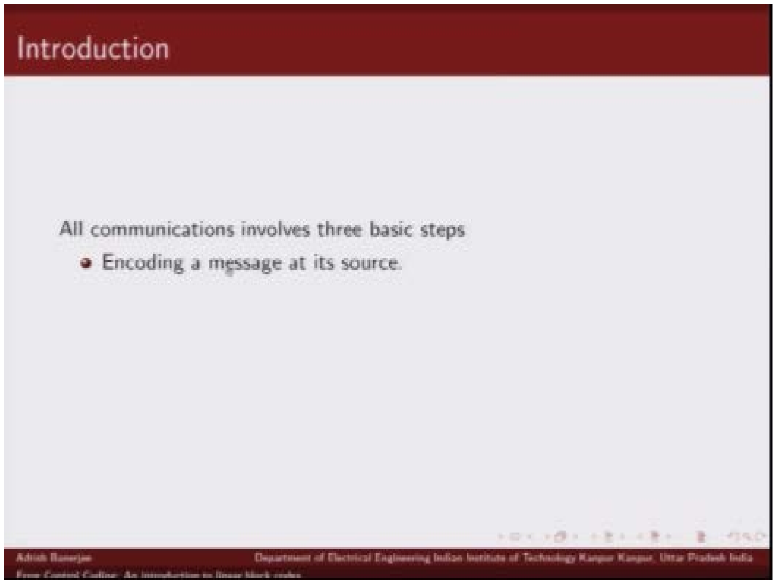


Is basically encoding a message.

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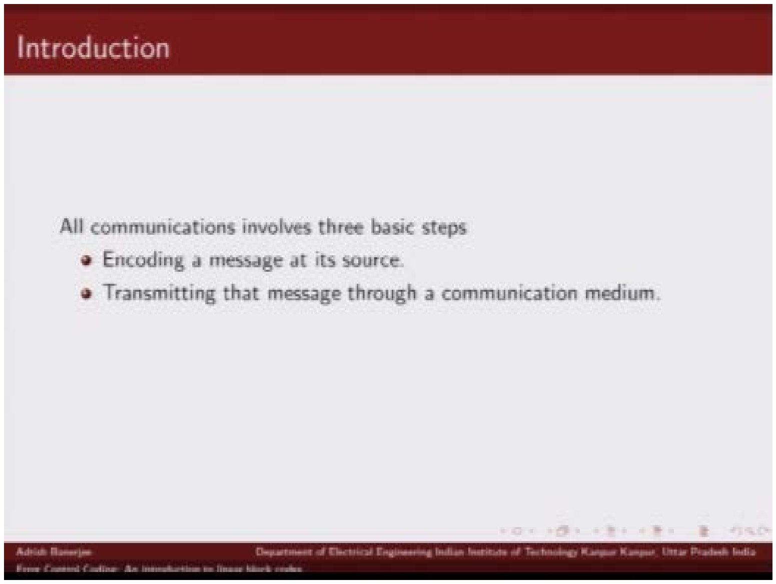


Second is once you have represented your source, you want to transmit that source over the communication channel.

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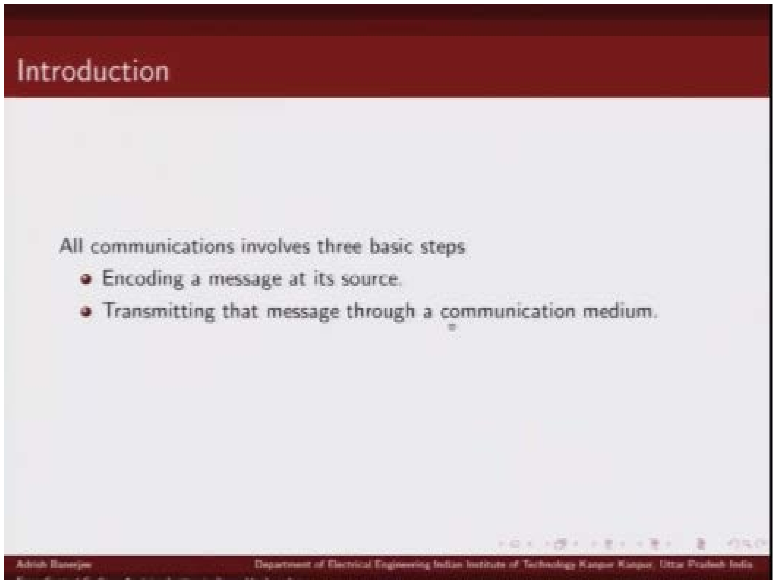


So the second thing is transmission of the message through a communication channel.

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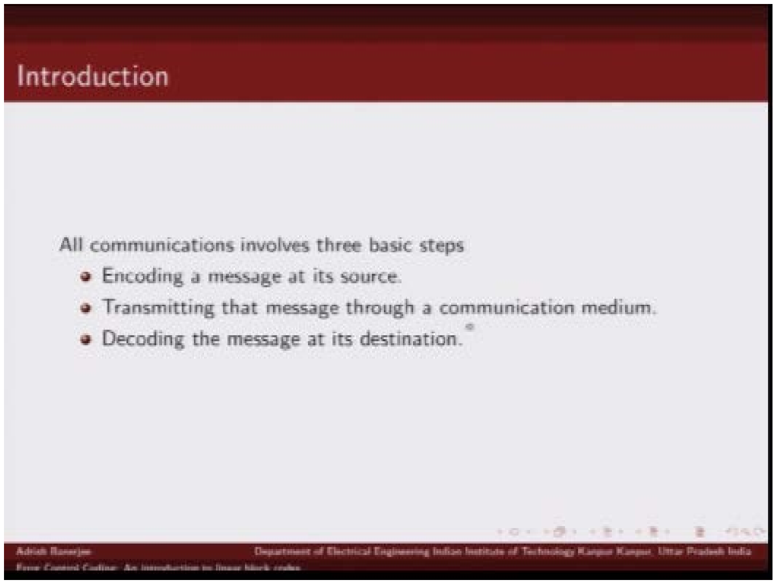


And finally once the receiver received the message it has to decode to find out what was the information that first transmits, so broadly there are three steps involved in communication, and coding, transmission, and then finally decoding.

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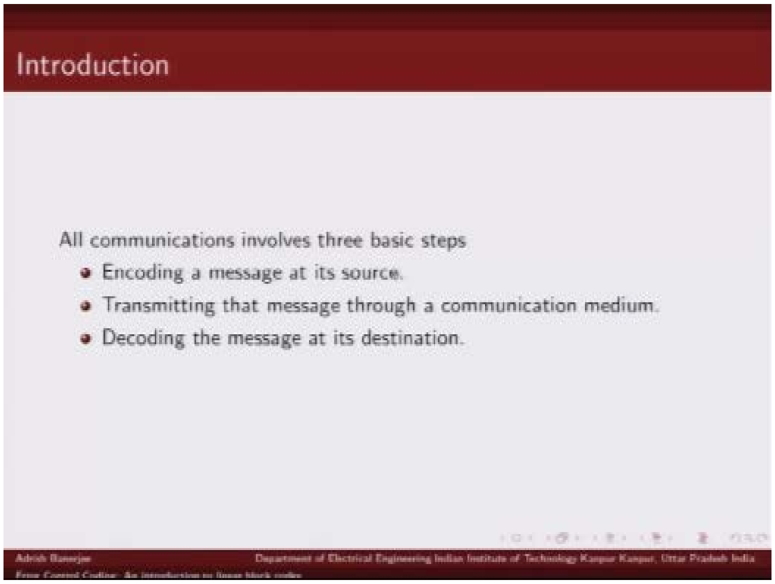


So, information theory basically gives us fundamental limits of what is the maximum limit of like what is the max best compression fundamental limit and compression that we can achieve, it also gives us fundamental limits on what is the maximum transmission rate possible over a communication channel.

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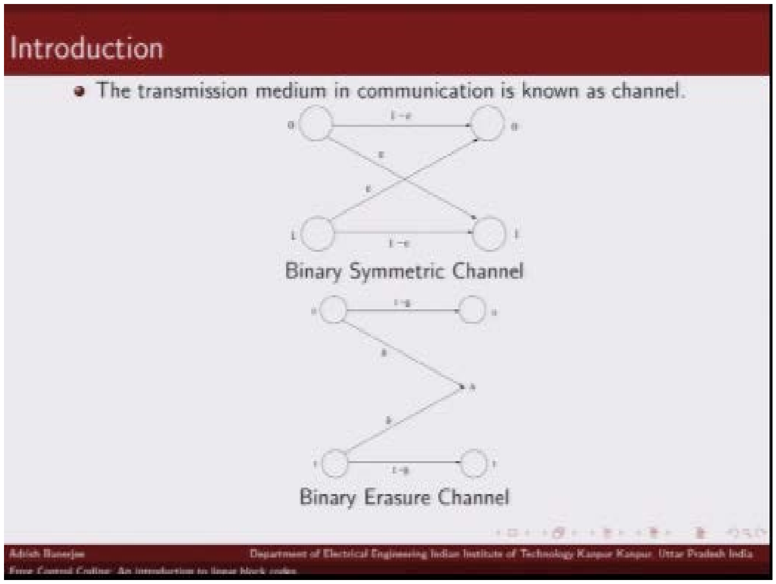


So let us spend some time on what is our transmission medium, so the transmission medium over which we want to send a packet that is known as channel and here I have illustrated.

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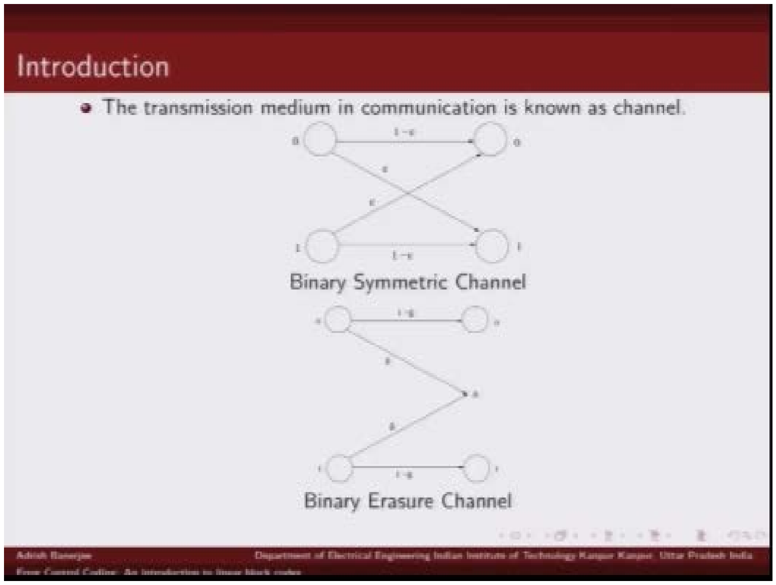


Two very simple channel models, the first which is binary symmetric channel, now you can see there are it has binary inputs, 0's and 1 and similarly it has binary output 0 and 1 now with probability 1-ε basically whatever you transmit is received correctly at the receiver, so this is a transmitter and this is the receiver side, so if you transmit 0 with probability 1-ε you will receive it correctly, similarly if you transmit 1 with probability 1-ε you will receive it correctly and this cross symbol probability of error is basically given by ε .

So this is basically a symmetric channel and this the binary channel because the binary input binary output is known as binary symmetric channel. Another channel which basically is very commonly used to model packet data networks is what is known as Binary Erasure Channel, so there are binary inputs 0's and 1's and the outputs are, either you receive whatever has been transmitted you receive it correctly or whatever you have transmitted is basically erased, so this Δ that you see basically we are denoting an erased bit using this symbol, so with probability 1-Δ you receive the bit correctly and with probability Δ the bit is erased or lost.

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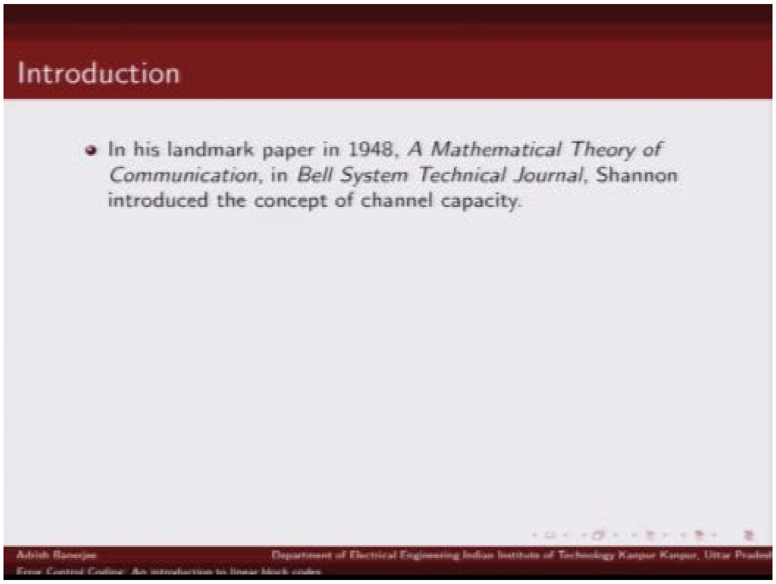


So in his landmark paper in 1948 Shannon introduced his concept of channel capacity that, what is the maximum rate at which we can communicate over a communication link.

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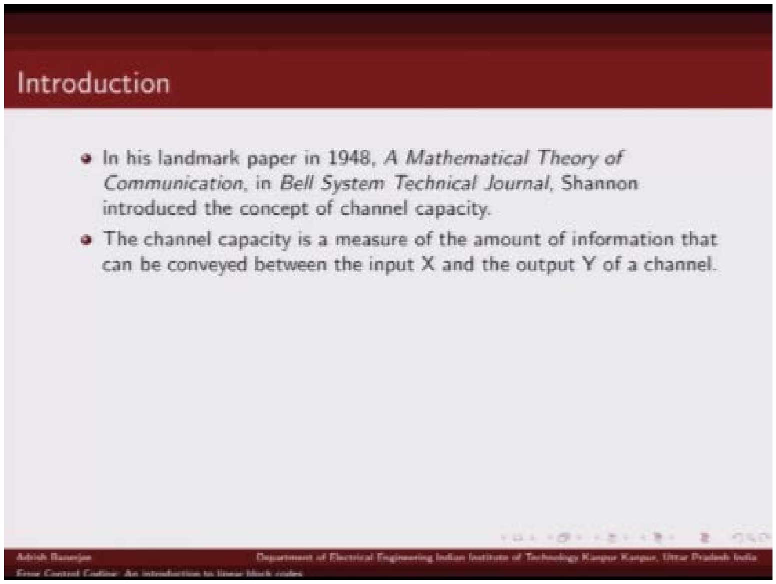


So a channel capacity is defined as the maximum amount of information that can be conveyed from the input to the output of a channel.

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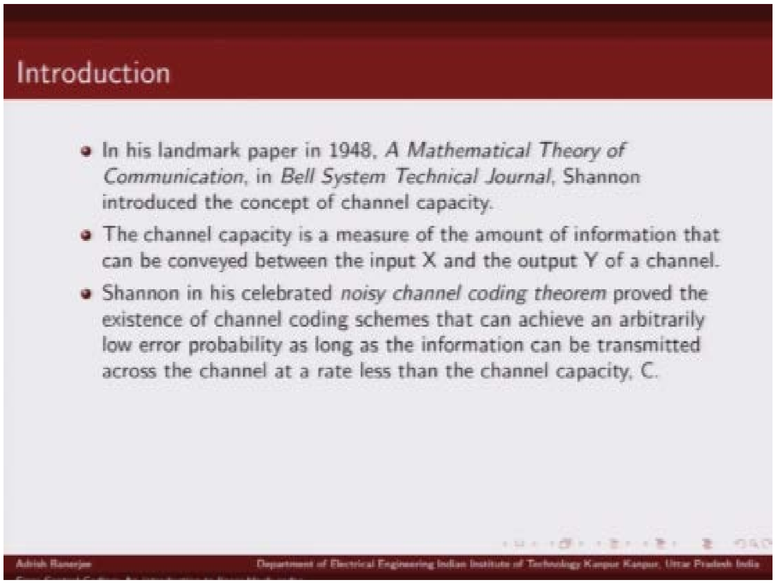


Shannon in his theorem also proved that there exist channel codings schemes that can achieve very low arbitrary, very low probability of error as long as the transmission rate is below channel capacity. So Shannon showed that there exists good channel codes as long as the transmission rate is below channel capacity we can achieve arbitrary low probability of error.

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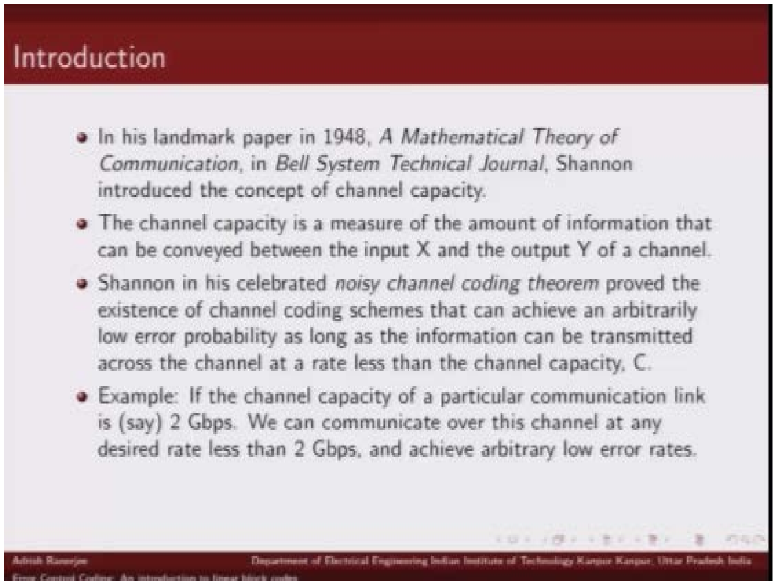


For example, if we talk about channel capacity of a particular link to be 2 Gbps then basically we should be able to communicate at rate any rate up to 2 Gbps over this communication link without basically and it can achieve very low probability of error at the decoder.

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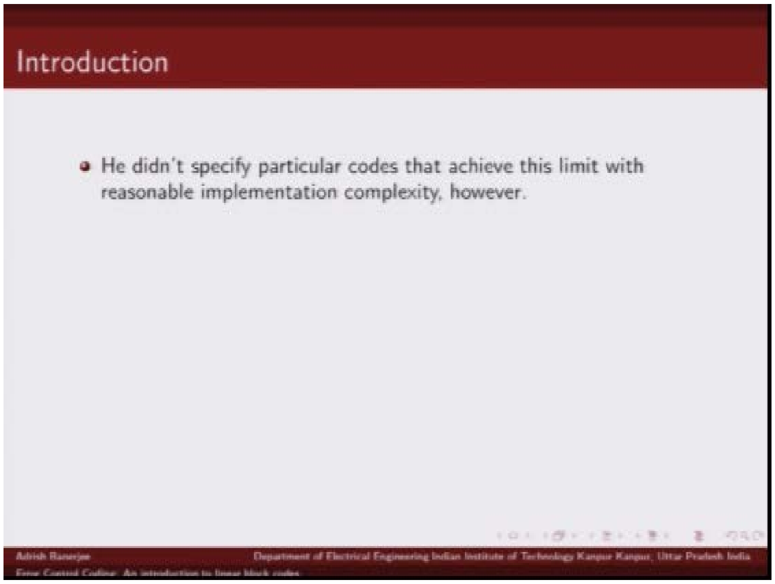


Now in his theorem Shannon did not specify how to design such codes which have rate close to capacity and that is where basically error control coding comes into picture, so the goal of error correcting coding theory is.

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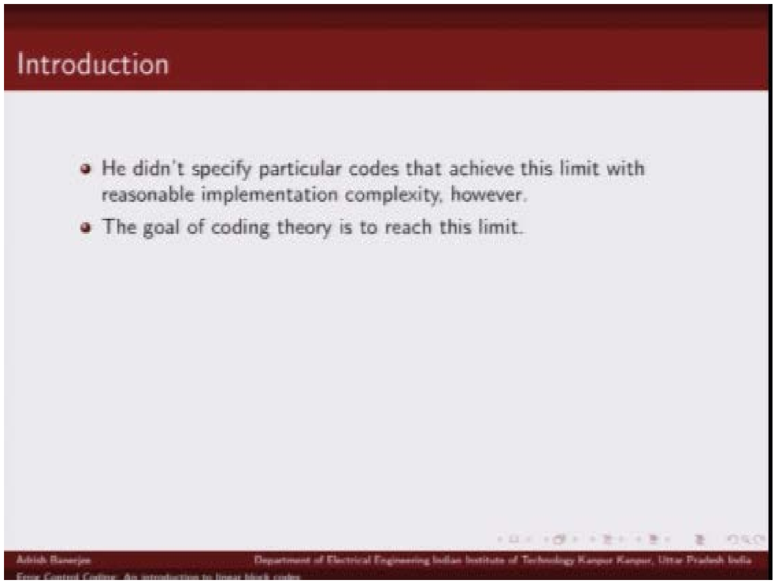


To achieve this, to design codes which can achieve this limit so basically Shannon has mentioned that we could transmit, we could design as long as we design error correcting codes which have rate less in channel capacity we can achieve arbitrary low probability of error, so the goal of the coding theory are our error control coding is to design such error correcting codes with rates as close to capacity which can achieve arbitrary low probability of error.

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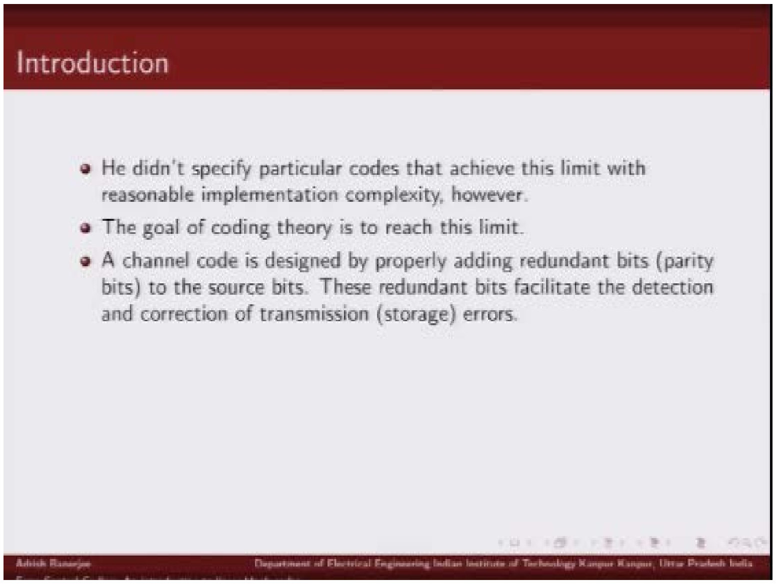


And Shannon did not specify how to design such codes so basically where coding theorists come into picture, so how do we design an error correcting code? An error correcting code is designed by adding some redundant bits to your message bits, the message bits we called them information bits and those additional redundant bits that we add those are known as parity bits. So error correcting code is designed by properly adding some redundant bits to your message bit and then send this coded message over a communication link. Now we use this additional redundant bits to detect error and to correct error.

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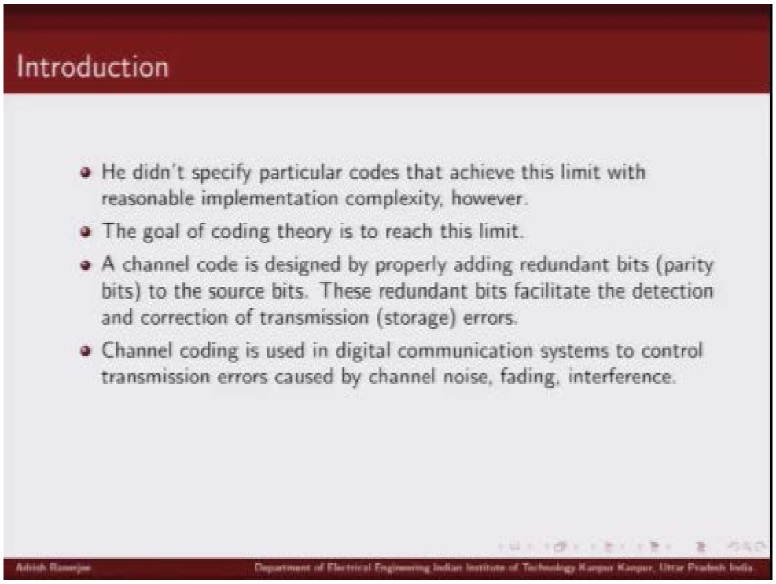


Error correcting codes has wide range of applications in digital communication and storage, I have listed basically of few of the uses for example, when we send a signal over communication link its get corrupted by noise, fading, interference so to combat the effect of all these basically we use error correcting codes to correct the errors.

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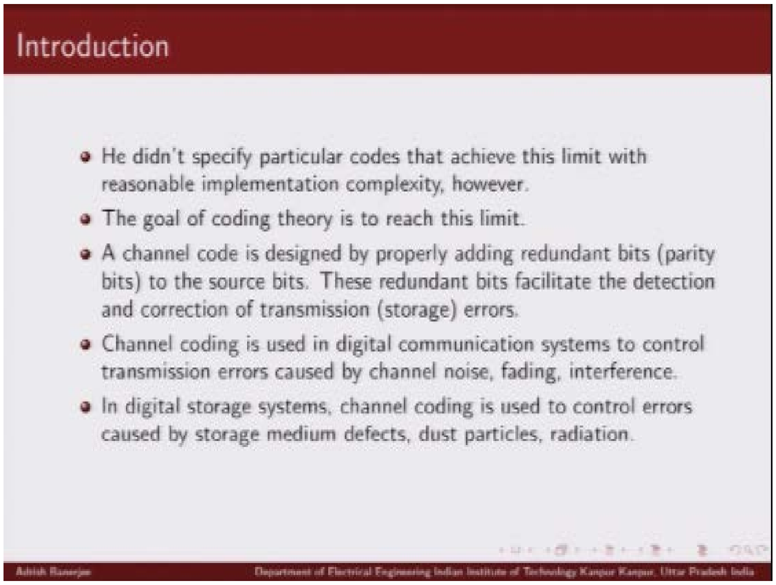


Similarly in digital storage system you want to correct the error caused you to storage media defects, dust particles, radiations we use error correcting codes there.

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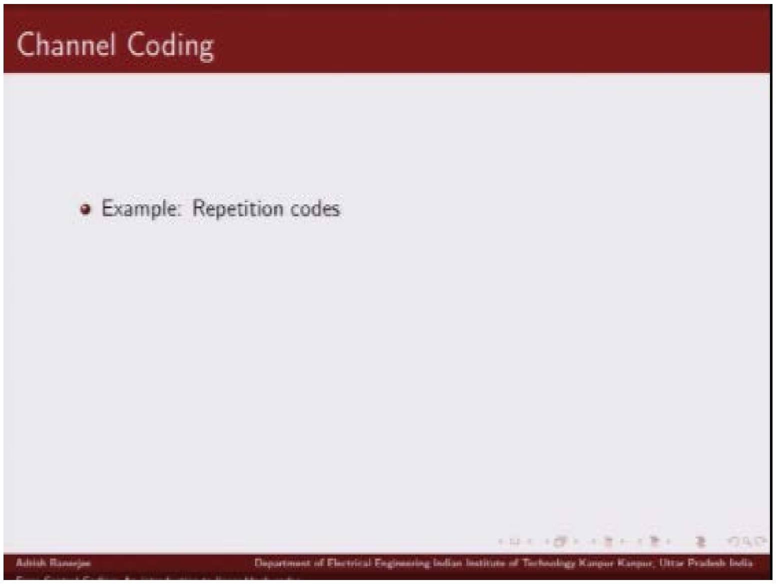


So let us take a very simple example of error correcting codes and illustrate.

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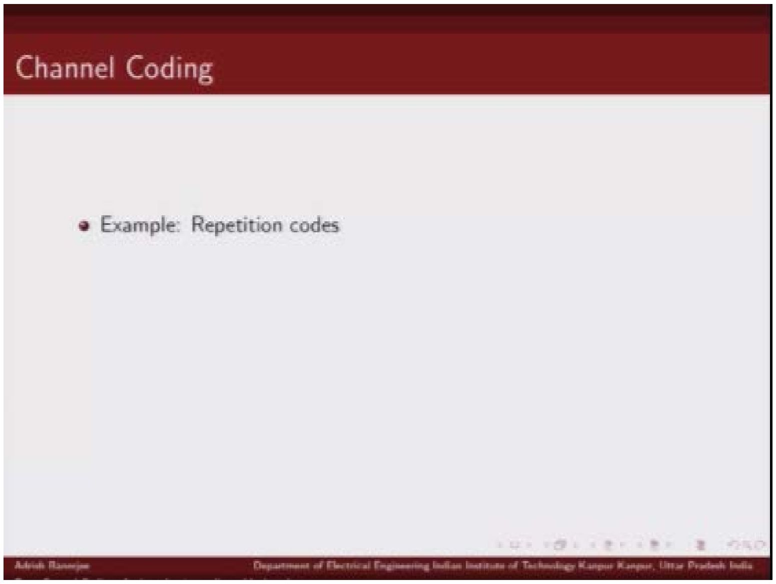


How we can use error correcting codes to detect and correct errors, so example I am going to show you right now is of what is known as repetition code.

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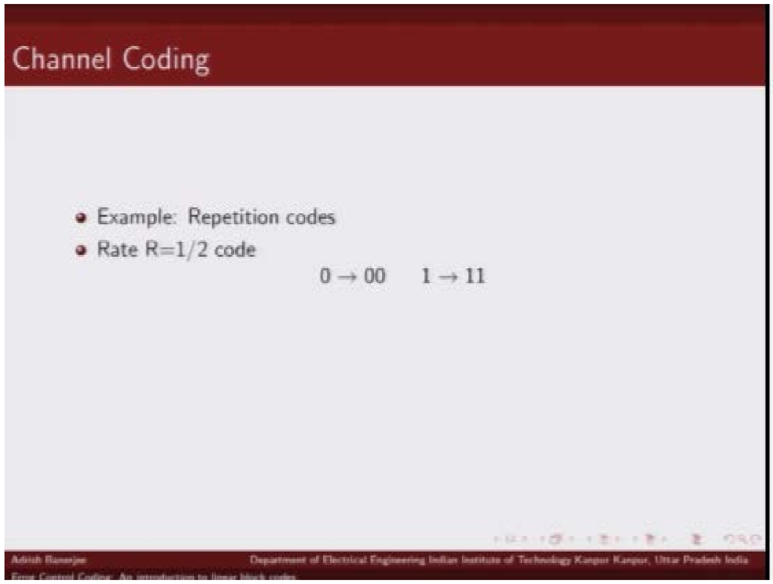


So the rate is defined as the ratio of number of information bits to number of coded bits, so when I say rate one ½ code I mean there is one information bit or one message bit, and there are two coded bits.

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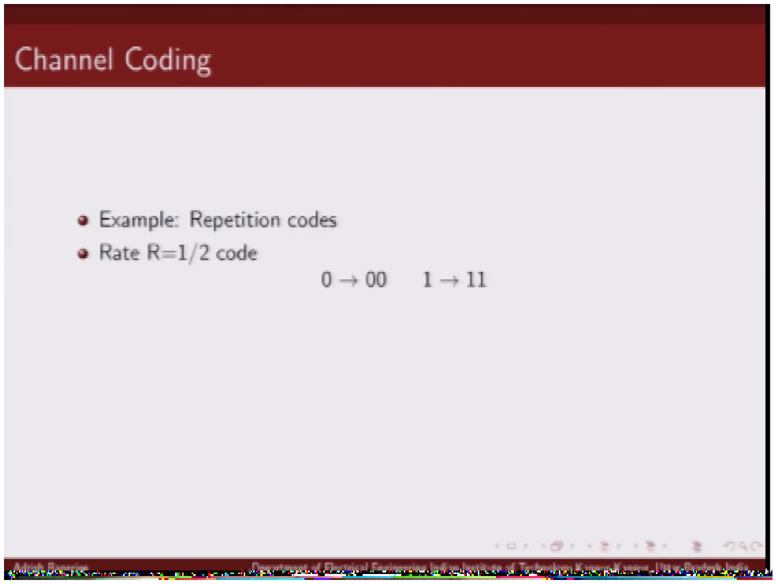


For example in a repetition code, in a binary repetition code basically we repeat whatever is the information bit is, so a rate ½ repetition code would be look like this a binary rate ½ repetition code would look something like this, so for 0 we would be transmitting 00 and for 1 we would be transmitting 11.

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Similarly for a rate 1/3 repetition code, for 0 we will be transmitting 000 and for 1 we will be transmitting 111, so you can see here in this in this, in rate one ½ case we are adding one additional redundant bit and in case of rate 1/3 code basically we are adding two additional redundant bits, now how we are going to make use of these redundant bits for error correction and a error deduction that will be explained in the next slide.

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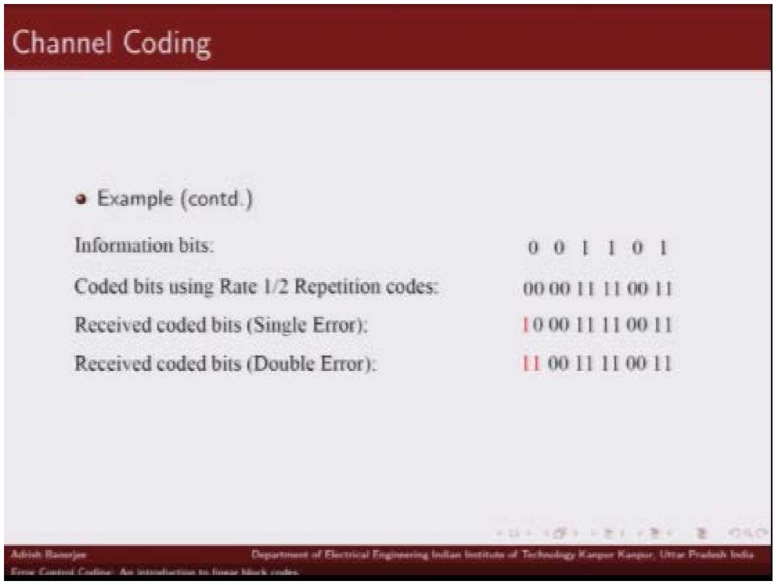


So let us take this example, let us say I want to transmit these set of bits, so I want to transmit 00110, now if I use a rate one ½ repetition code what would be my coded bits for 0 I will be transmitting 00, for 0 I will be encoding them as 00, for 1 I will be encoding them as 11 and for 1 I will be sending them as 11 for 0 I will be sending as 00 and for 1 I will be sending as 11, so this will be my coded sequence okay.

Now here I have illustrated one case where there is a single error so this was basically the sequence which was transmitted think of it that this base sequence as we transmit over a binary symmetric channel and this is what the receive sequence I received, so you can see here this is a case of a single error, the first bit which was transmitted 0 was received as 1, now how can I use error correcting codes to detect error?

So since is a rate one ½ code for each information bit I am sending two coded bits, so at the receiver I will look at two bits at the time so I will look at first I look at this 10, now since it is a repetition code what do you expect, I expect that both the bits should be same right? But here in this case first bit is 1; second bit is 0, which means there is a transmission error so I am able to detect single error, how?

Because these bits were encoded using rate ½ repetition code I expect these two bits to be same, so I know there is an error in the first bit but I do not know whether this is bit 0 or bit 1, let us look at other receive bits 00 this will be decoded as 0, 11 this will be decoded as 1, 11 this will be decoded as 1 there is no ambiguity, 00 this will be decoded as 0, again there is no ambiguity and 11 this will be decoded as 1, so we can see that using one additional redundant bit we are able to detect single error.

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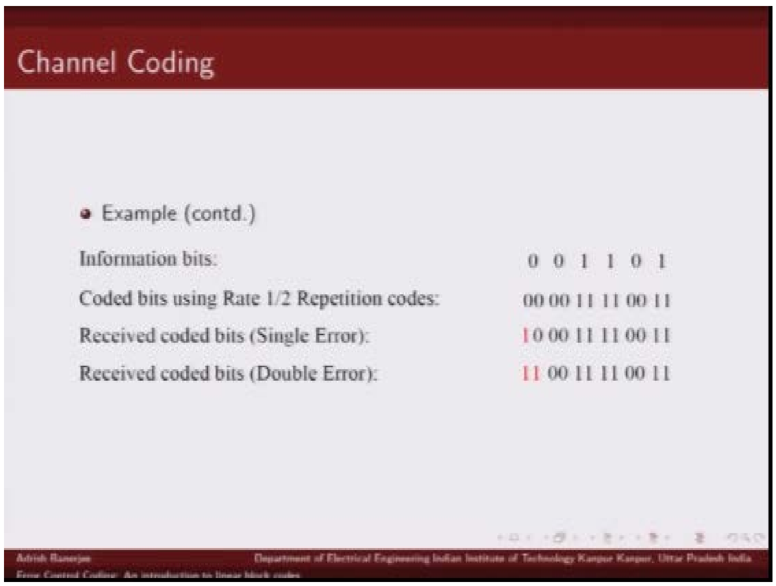


Now let us look at example for double error so let us say the first and second bit are received in error so what we have received is basically 11 00 11, 11 00 11 so the first two received bits are in error 11 now let us see whether we can detect error, using this rate 1/2 repetition code, so again we will follow the same logic for decoding we will look at two bits at a time, so first two bits are 11.

Now since these bits are same we will decode them as 1, but what was transmitted? It was 0, so we can see that this is a case of undetected error, even though these two bits were received in error the decoder is not able to detect this error, so this kind of thing happens when the error pattern is such that it transforms one code word into some other code word.

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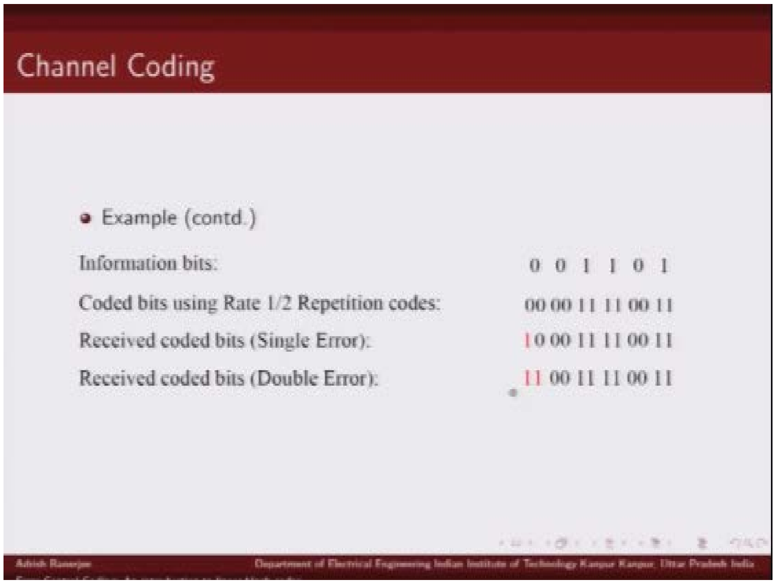


So since 11 is a valid code word for 1 the decoder is not able to detect this error, so this rate one ½ repetition code is able to detect single error, but it is not able to detect double errors, now let us look at whether it can correct any errors so let us look at.

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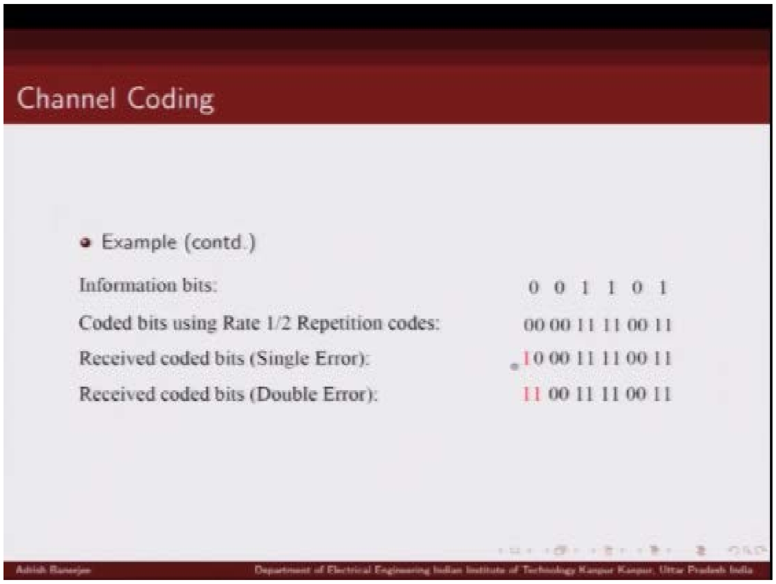


This example when we had single error, so note here, so what we received was 1 0 so we were able to detect error that there was an error but can we correct it?  No we cannot, why? It is equally likely.

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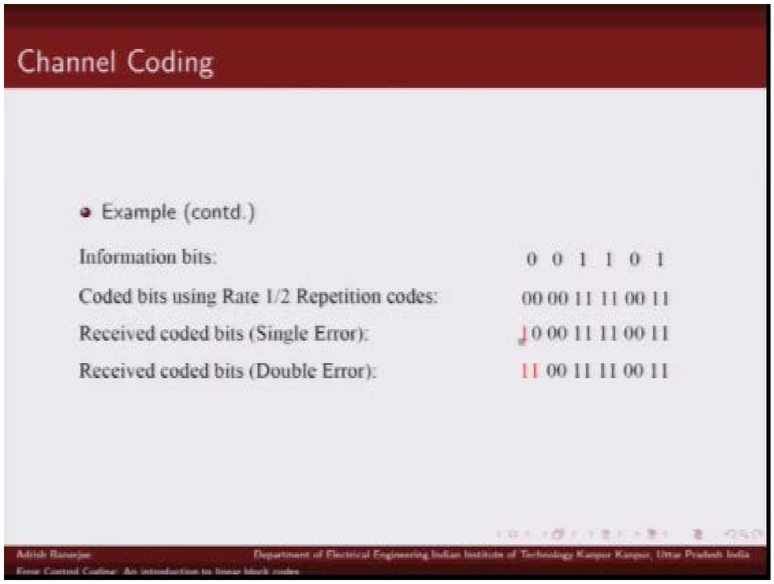


That this 1 that we received was 0 or this 0 that we received was 1, if we are talking about us binary symmetric channel, right? So we do not know whether the first bit got flipped to 0, first bit got flipped to 1 instead of 0, or the second bit got flipped to 0 instead of being 1, so this particular rate ½ repetition code cannot correct any errors, it can only detect single errors.

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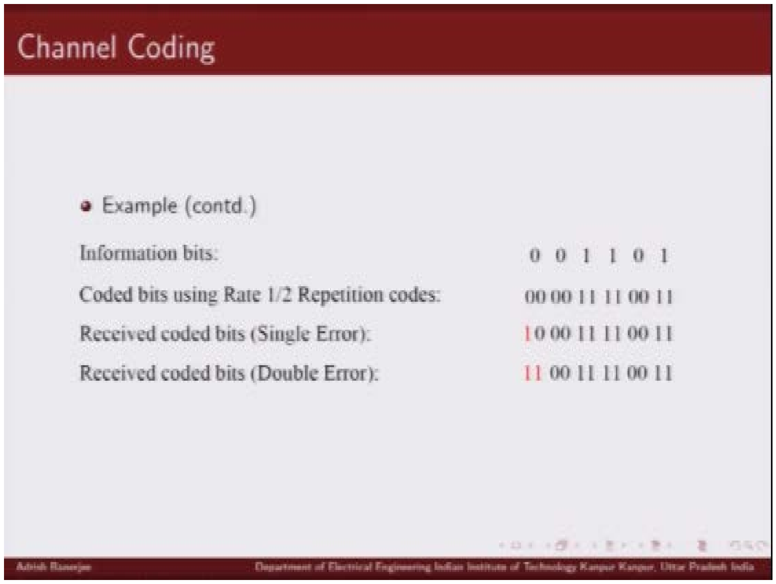


Now let us look at another example, this time we are considering a rate 1/3 repetition code, so what does rate 1/3 repetition code means for each and again we are considering binary code so for each bit we are adding so 2 parity bits and we are repeating the same bit, so for 0 we will be transmitting you will be coding it as 000, for 1 we will be coding it at 111, so again we consider the same example of transmitting 001101.

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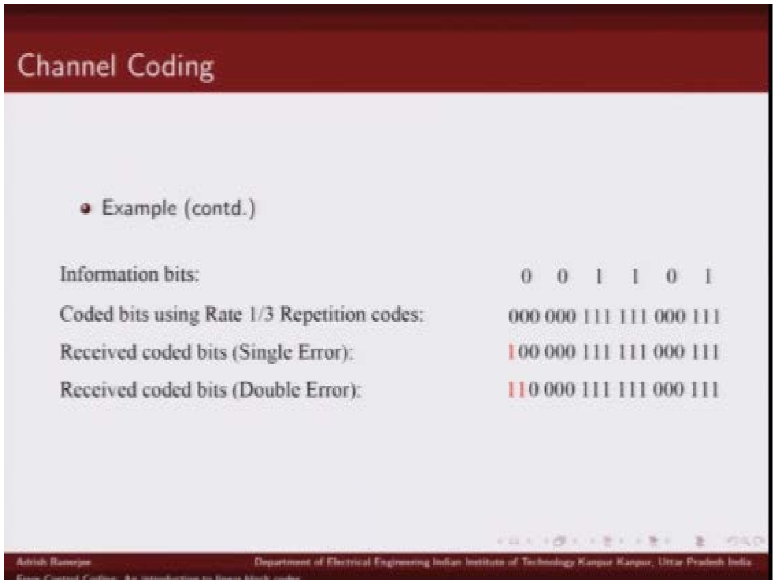


So we are transmitting the same information sequence, this time we are encoding them using rate 1/3 repetition code so this 0 will be encoded as 000, similarly 1 will be encoded as 111 so we will be transmitting this so this information sequence will be coded in this particular way, now we will again look at what happens when there are errors at the receive sequence like we did for rate one ½ repetition code.

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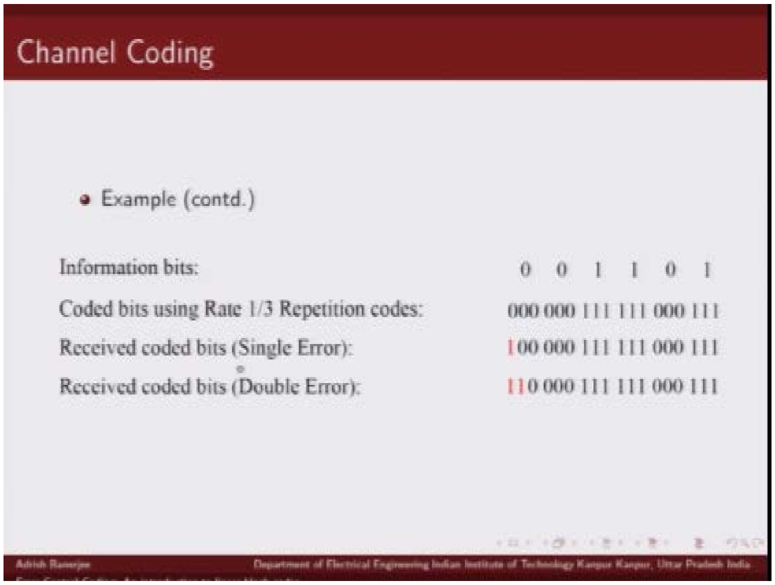


So let us again look at example for single error scenario so let us say the first bit was received in error so instead of 0 we received a 1, now let us see whether our rate 1/3 repetition code can detect single error, so since it is a rate 1/3 code for each information bit we are sending three coded bits, so we are going at the receiver we are going to look at three bits at a time, at the decoder we are going to look at three bits at a time.

So we will first look at these three bits 100, now what do we expect we expect since we are using a repetition code we expect all these three bits to be same, but here in this case they are not because what we have received is 100, now what does that mean, that means there is a transmission error, so we are able to detect single error using a rate 1/3 repetition code.

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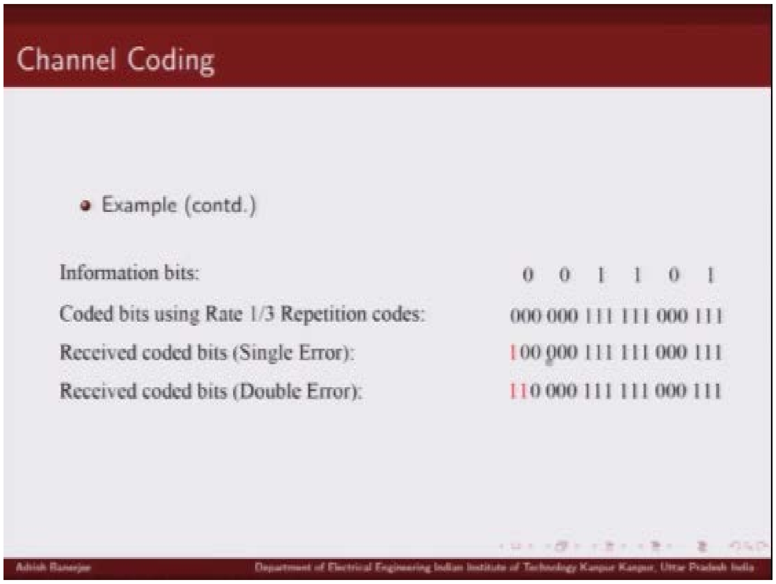


If we look at other sets of bits 000 there is no error here, 111 no error again no error here, no error here, no error here so create one ½ repetition code was able to detect single error, even rate 1/3 code is also able to detect single error, now let us look at double error, so let us consider scenario where the first two bits are received in error so we have 11 and rest of the sequences this okay, now can we detect double error?

So let us look at, we again look at three bits at a time so if you look at three bits at a time the first three bits are 110, now we could see that there is an error why? Because either this should have been 000 or 111 but we received 110 so we are able to detect using rate 1/3.

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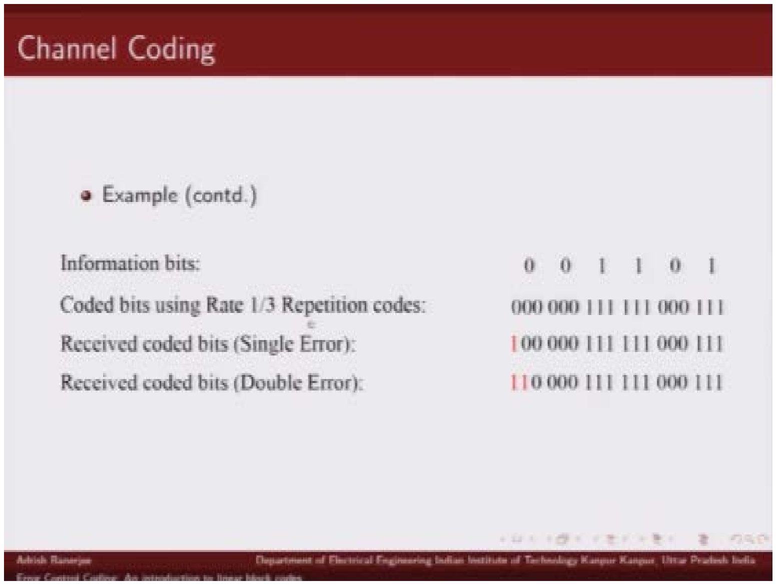


Repetition code we are able to detect double errors as well which we were not able to detect using rate one ½ repetition code, now let us look at the error correcting capability of this code so let us go back again.

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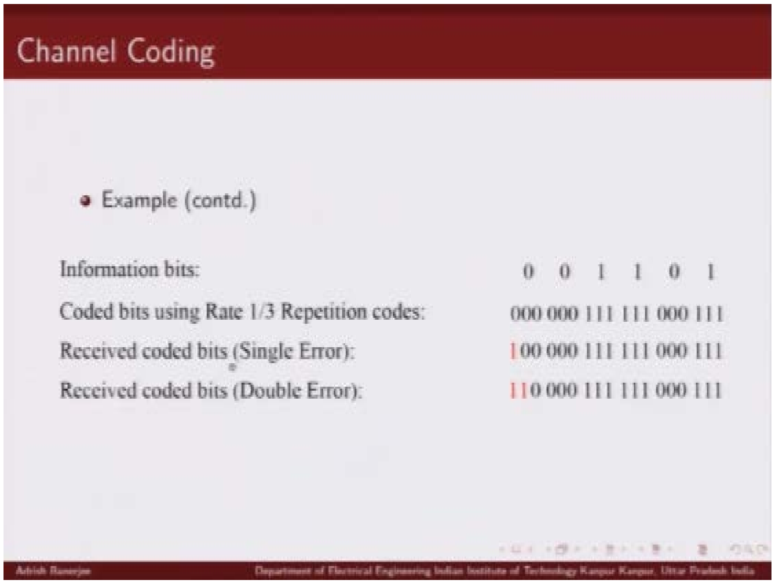


And look at single error situation now when the single error happens so something like this let us say one of the bit got flipped 100, now can we correct single errors? And the answer in this case is yes, why? Because if you look at these three bits two bits are already 0 and one bit is 1, so it is and what, what are the possible outcomes, this could be either 000 or 111 and it is more likely that one bit got flipped, it is more likely that 0 got flipped to 1 rather than two 0's, two 1's getting flipped to 0.

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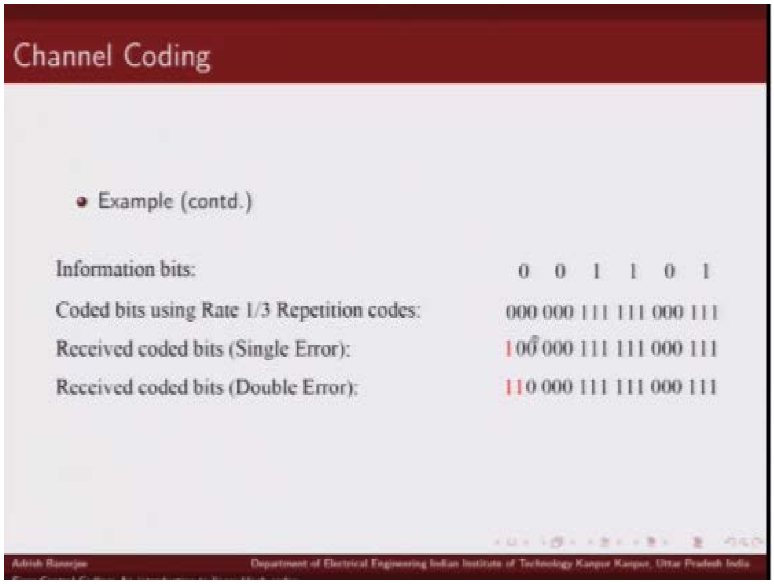


So it is more likely that this bit got flipped from 0 to 1 instead of these two bits getting flipped from 1 to 0, so using majority logic this majority of the bits are 0 we will decode this as 0, so we can see this rate 1/3 repetition code can correct single error this was not possible for rate one ½ repetition code, now can it correct double errors?

Now if we look at this 110, it will think that this particular bit got flipped from 1 to 0 so it will decode this as 1, so this cannot correct double errors, so to summarize we saw that rate one ½ repetition code can detect single error but cannot correct single error, it cannot detect double errors, whereas rate 1/3 repetition code can.

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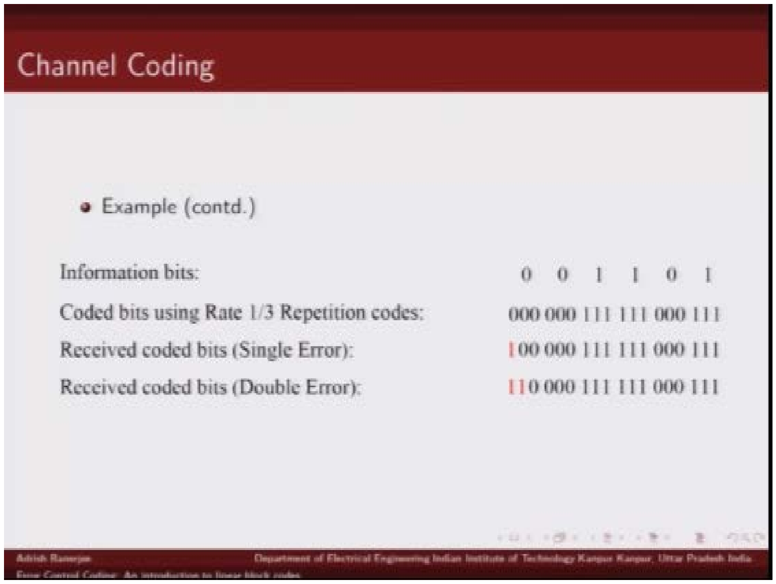


Correct single error and can detect single error, it can detect double error but it cannot correct double errors so why is this code better than the first code? It has certainly has better error detecting capability than the rate one ½ code, this we will discuss in subsequent lecture, it has to do with separation between this is a separation between the code words and you can see basically in this but to the code we are using two redundant bits.

And in the previous case we were you just choosing one redundant bits so the error correcting capability and error detecting capability of the code is depending, is dependent on the distance properties of the code and that we will talk about in subsequent lectures so to summarize it.

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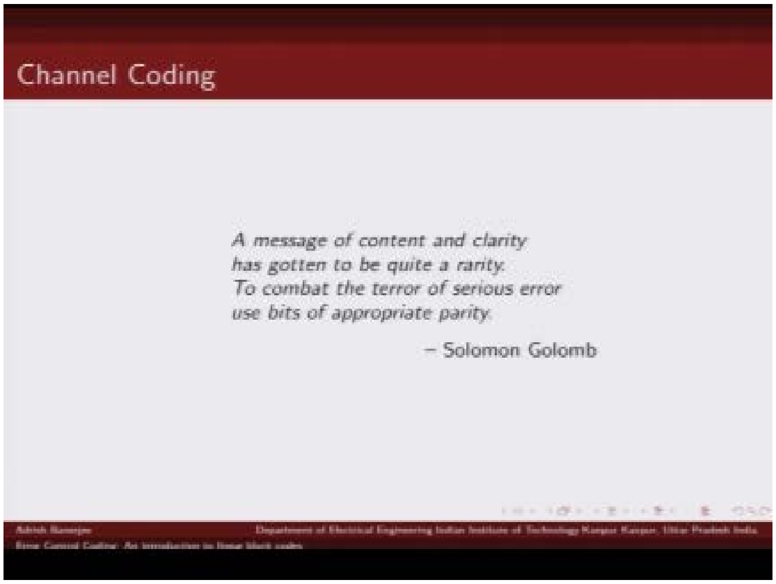
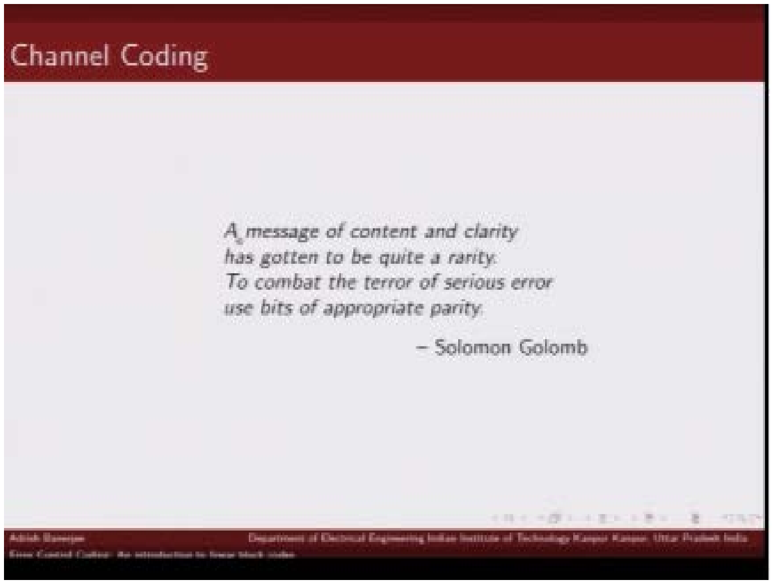



I think this quotation by Solomon Golomb rightly captures what error correcting code is all about so I will read it.

(Refer Slide Time: 23:20)




A message of content and clarity has gotten to be quite a rarity, to combat the terror of serious error use bits of appropriate parity, thank you.

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