Estimation for Wireless Communications-MIMO/OFDM Cellular and Sensor Networks.
Professor Aditya K Jagannatham.
Department of Electrical Engineering.
Indian Institute of Technology Kanpur.
Lecture -07.
Wireless Fading Channel Estimation-Pilot Symbols and Likelihood Function.

Welcome to another module in this massive open online course on estimation for wireless communication. So, so far we have looked at the development of maximum likelihood estimation, the maximum likelihood estimation framework and the maximum likelihood estimate in the context of the wireless sensor network where sensor is trying to estimate a parameter such as the temperature or pressure from a basic set of observations. Let us now look at another interesting example in the context of a wireless communication scenario.

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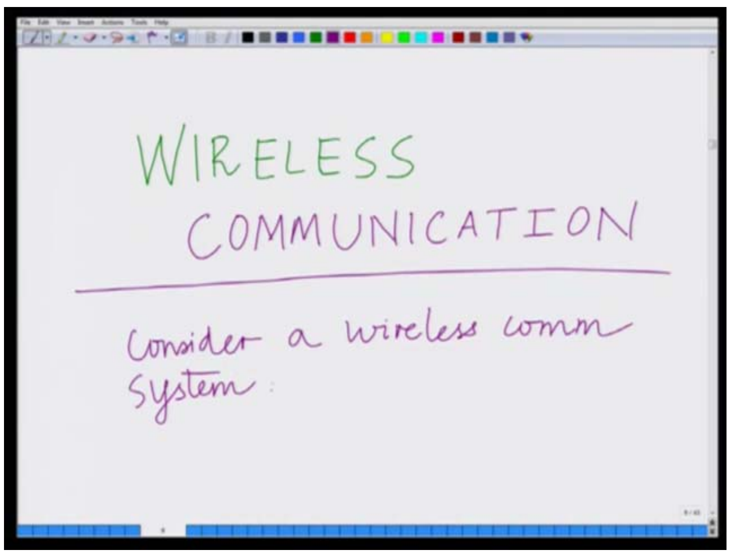

So, let us look at example of estimation in the context of a wireless communication scenario. We have looked that estimation in the context of a wireless sensor network, now let us look at the estimation in a wireless communications system, right. So, let us put it as a wireless communication system, so let us consider a wireless scenario, so consider now, let us say consider a wireless communication system, now naturally in wireless communication, wireless communication any communication takes place between a transmitter and a receiver.

So, in a wireless communication we have a transmitter and receiver which are communicating over a wireless channel.

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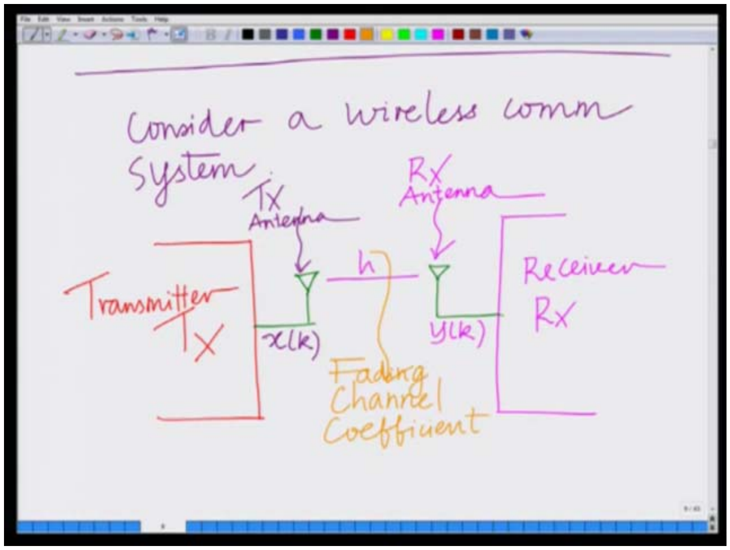

So, we have, basically what we have is we have a transmitter and we also have here, we also have a receiver,. And we have of course a transmit antenna from which the signal is transmitted in the wireless system, we have the receive antenna, so these are basically, this is your TX, that is your transmit antenna from which the symbol XK is transmitted at time instant K. You have your receive antenna, that this the RX antenna denoted by RX, the receive antenna, which receives the symbol YK.

And the channel between them is denoted by the channel coefficient by H. This H is an interesting object, this is known as a channel, it represents the channel, this is known as the channel coefficient, this can be fading in nature, where it is known as the fading channel coefficient. Typically this is a Raleigh fading, that it follows the Rayleigh distribution, that is the amplitude, the magnitude follows the Rayleigh distribution, this is known as the Rayleigh fading coefficient. Anyway, this does not matter for the purpose of estimation, what is important is frequently this channel coefficient H is unknown.

At the beginning of wireless communication, that is before the communication process can begin between the transmitter and receiver, one has to estimate this channel coefficient because this channel coefficient H is unknown and that process is known as channel estimation.

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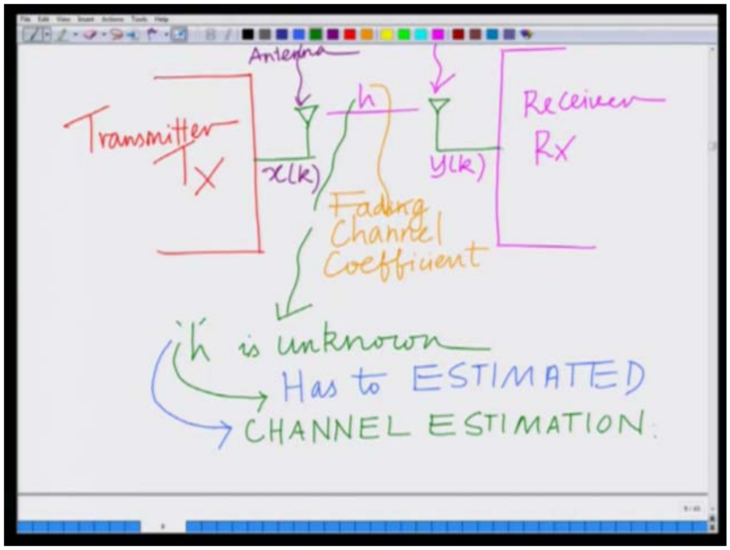

So, to begin with, at the beginning of communication Epoque, the H is basically, which is the channel coefficient is unknown. Therefore your H has to be this, has to be estimated, which is the central focus of this course on estimation. So, this H has to be estimated, which is an important, this has to be estimated, which is an important process in a wireless communication system. The process of estimation of H, this is termed as channel estimation.

Let me write it out clearly. The process of estimating this channel coefficient H, this is termed as channel estimation. That is this algorithm, that is the scheme for estimating this unknown channel coefficient H so that communication can take place between the transmitter and the receiver, this process is known as channel estimation. Now let us explore a scheme for this channel estimation based on the maximum likelihood estimation framework that we have developed sofa.

So, as I said before, I can write, represent this wireless communication system as follows.

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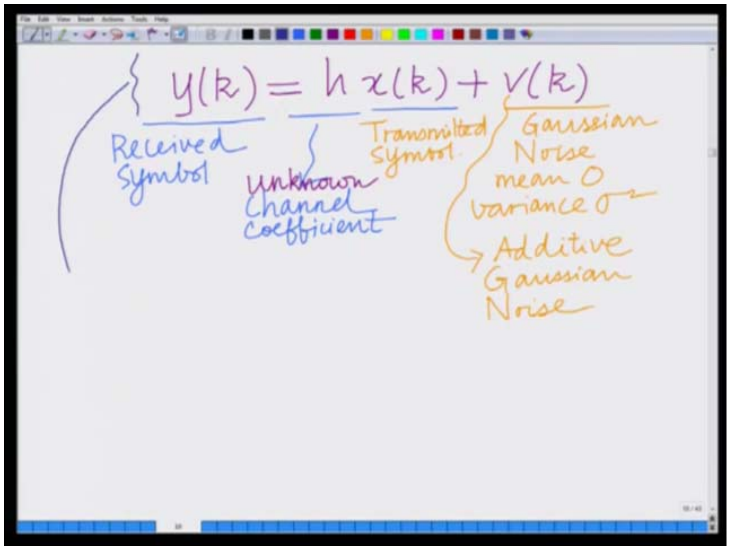

YK which is the symbol received on the receive antenna equals H times XK + VK. Let me describe the different components, this YK is the received symbol, your H is the channel coefficient, XK as we already said, this is the transmitted symbol. And this VK is the noise and this is Gaussian noise, similar to what we have seen before, this is Gaussian noise, 0 mean, variance or power equals Sigma square. So, we have YK equals H XK times H times XK + VK where YK is the symbol received on the transmitter and the received antenna, H is the unknown channel coefficient which has to be estimated, XK is the transmitted symbol, and VK is Gaussian noise at the receiver, this is also known as additive Gaussian noise because the noise… Look at this, this is additive in nature, so this letter VK is also known as additive, in fact that is a popular nomenclature, this is known as additive Gaussian noise.

And this channel coefficient H, this is known to begin with, this is the channel coefficient which has to be estimated and another important point, subtle point that I would like to mention at this point is currently we are considering all the quantities are real.

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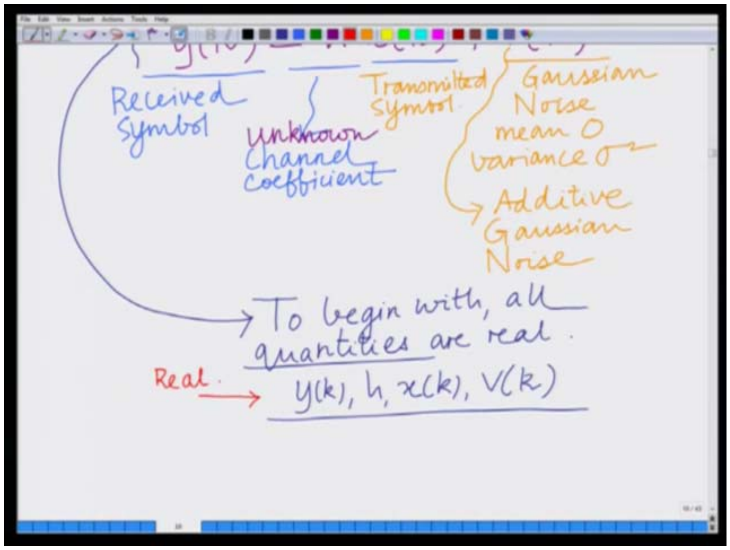

To begin with, assume all quantities, by all quantities we mean your YK, H, XK and VK, we are saying that all these quantities are real parameters. That is they are real numbers, not complex… And we have seen in the previous model how to extend, how to extend an estimation scheme developed for real parameters to complex scenario. And we will do the same for the scenario as well.

So, to begin with we consider aside a simplistic scenario where all the quantities, including the parameter H, that is the channel efficient H to be estimated is a real parameter. However this has to be extended to a scenario where H is complex because H is typically a complex baseband channel coefficient, that is what we also mentioned in the previous module when we develop the framework for complex parameter estimation.

However that will come slightly later, to begin with, let us understand how to estimate the real parameter, how to estimate this channel coefficient H when H is a real parameter. And one of the most popular schemes for channel estimation is what is known as pilot or training symbol-based channel estimation and that can be described as follows. So, let me rewrite this system model of this equation over here again.

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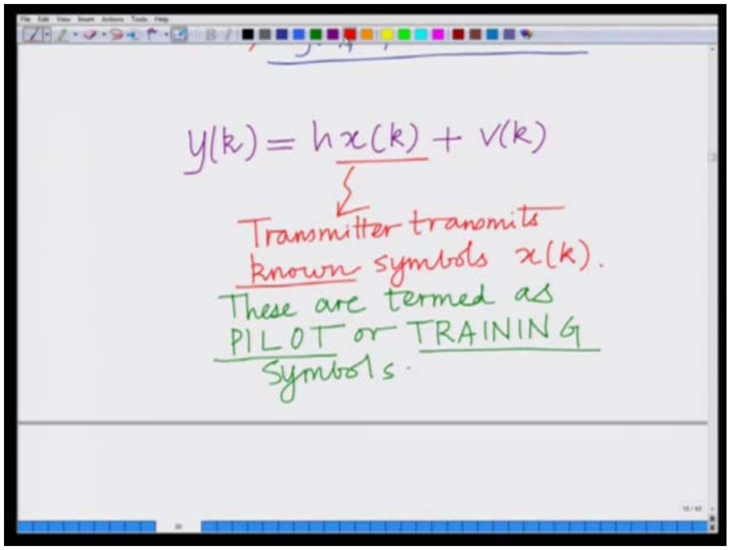

We have our YK equals H XK + VK. Now what we do is we transmit symbols X key, that is the transmitter transmits known symbols, that is previously agreed-upon or known or standard set of symbols, known symbols XK. And these have a name, these are termed as pilot symbols or training symbols. These are termed as pilots or training symbols, these are not information bearing symbols, these are pilot or basically your training symbols which are used for pilot or training symbols, these are exclusively meant for estimation, these are exclusively meant for channel estimation. So, the transmitter transmits the sequence of pilot symbols X1, X2… XN which are basically meant for the purpose of channel estimation.

At the receiver now, observe the corresponding outputs Y1, Y2… YN corresponding to the training symbols.

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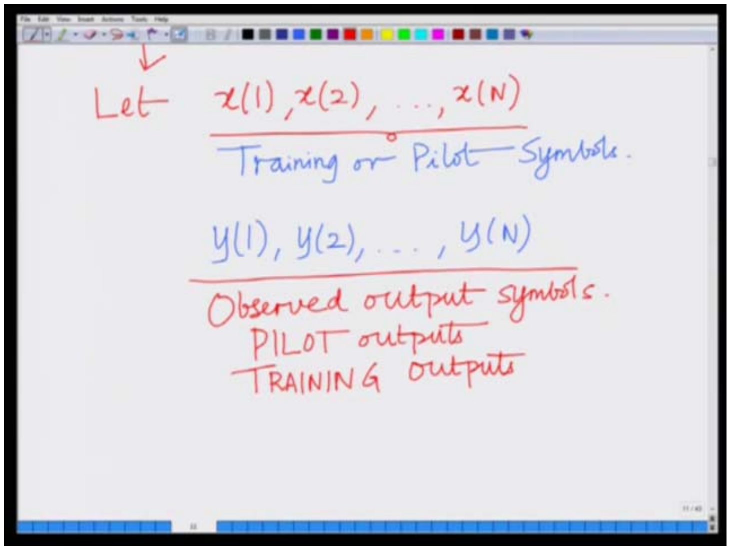

So, at the receiver, so what do we do, let X1, X2… XN, let these denote the training symbols, these denote the training or pilot symbols and let Y1, Y2,… YN now corresponding to these training symbols, let Y1, Y2… YN denotes the corresponding output symbols. So, these are known as the, these are the observed outputs**,** also known as the pilot outputs, or basically your training outputs, these are basically your pilot outputs or training outputs and corresponding to the transmitted pilot or basically training symbols.

So, what we are saying is we are going to transmit a bunch or a block of pilot symbols X1, X2,… XN which are known at the receiver and corresponding to this, the receiver observes the pilot output or the training output Y1, Y2… YN from the knowledge of X1, X2,… XN and the observed outputs Y1, Y2… YN, one can now estimate the unknown channel coefficient H as we are going to describe shortly. So, now one can describe these pilot outputs as follows.

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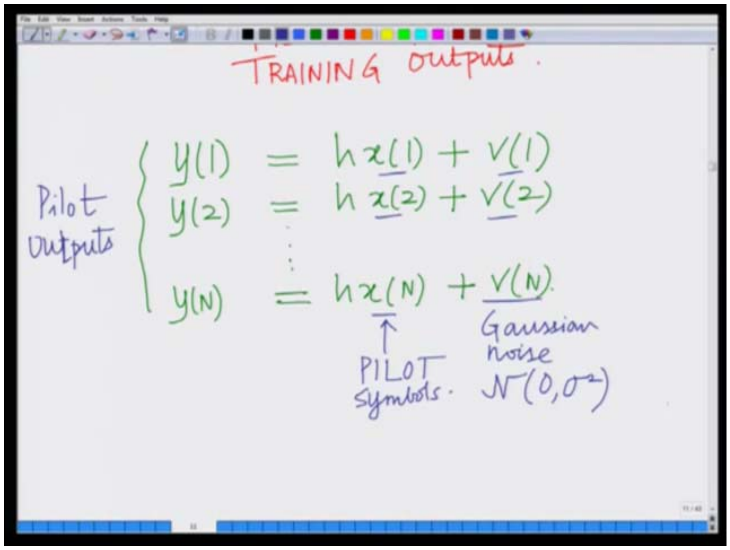

So, we have the pilot outputs Y1 equals H Times X1 + VI, Y2 equals H times X2 + V2, so 1 and so forth we have YN equals H times XN + VN, as we already said, these are basically your, your pilot your pilot outputs, X1, X2,… XN these are basically your pilot training symbols and V1, V2… VN, these are the Gaussian noise samples. These are 0 mean, so these are Gaussian noise samples with 0 mean and variance Sigma square.

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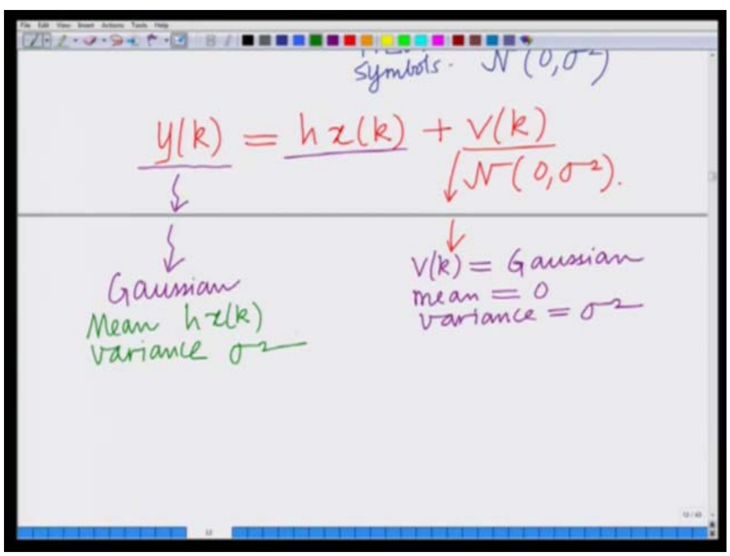

So, basically now we can write the Kth as we have written before YK is basically, YK equals H times the Kth pilot symbol XK + VK and we know VK is Gaussian with mean 0, variance Sigma square, so VK is so, let us write it over here. We have VK equals Gaussian, mean equals 0, variance equals Sigma square, now that is added to H XK, therefore now we see that YK is also Gaussian, similar to what we have seen in the scenario of sensor networks that is this is also a Gaussian. So, that is what are we saying, we have VK, that is the 0 mean noise, to which H XK is being added, so the resulted output YK is also Gaussian, except its mean is shifted by H XK and its variance is going to be the same, that is Sigma square.

So, similar to what we have seen in the context of sensor networks, this is going to be a Gaussian with mean H XK and the variance Sigma square.

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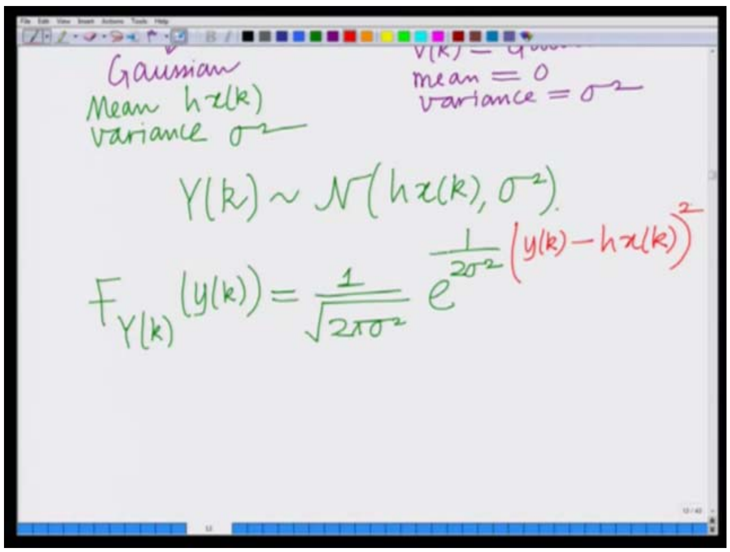

So, therefore YK that is observed output is Gaussian with mean H XK and variance Sigma square, similar to what we have seen in the context of sensor network. And therefore I have the probability density function, therefore YK of YK, this is basically equal to 1 over under root to pie Sigma square E raised to -1 over 2 Sigma square YK - H XK square.

Previously we had seen in the context of sensor networks that this is simply YK, the observation YK is Gaussian with mean H and variance Sigma square. How it is slightly different, in the context of this wireless channel estimation, YK is a Gaussian random variable with mean H times XK where XK is the transmitted kth pilot symbol and the variance of course again remains the same, it is Sigma square, which is the variance of the additive noise, that is the additive Gaussian noise at the receiver of this wireless communication system.

Now, similar, again, once again to the wireless sensor scenario, we are going to assume that these noise samples V1, V2… VN are IID, that is independent identically distributed, so let me also mention that.

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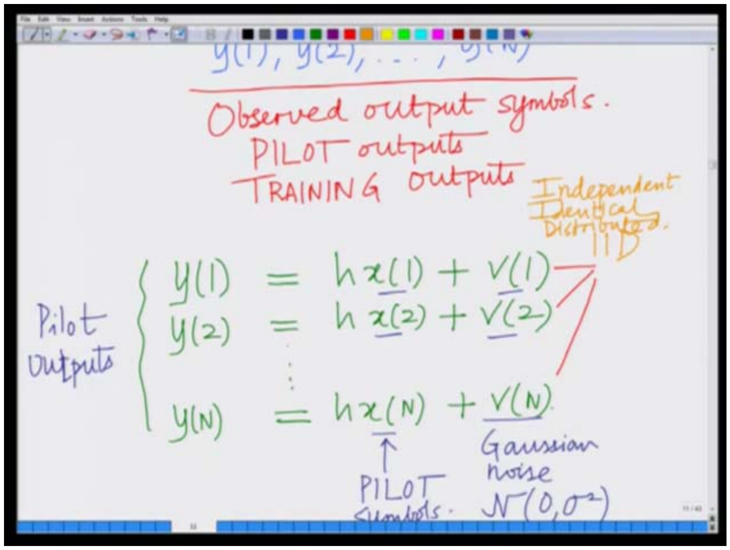

Now also assume that this V1, V2… VN, these are IID, these are that is… We know what is the meaning of IID, independent identical, these are independent identically distributed.

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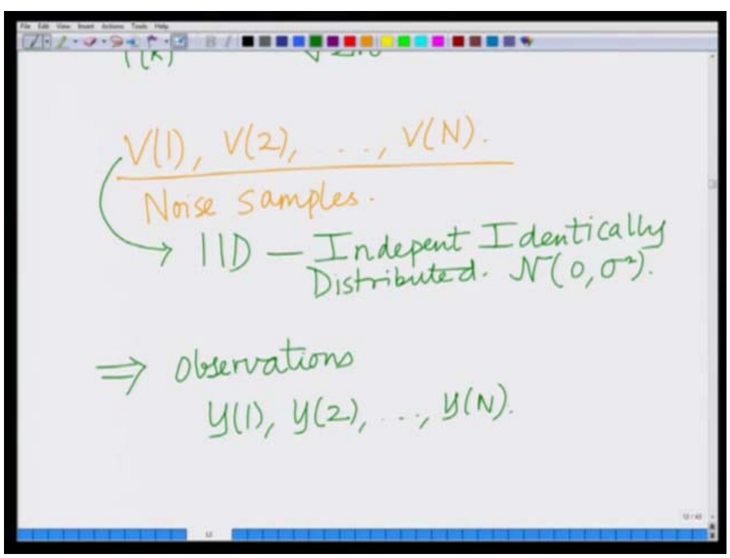

Now since these are independent and identically distributed, therefore we have V1, V2… VN which are the noise samples, these are your noise samples and we are assuming these to be

IID, that is independent identically distributed Gaussian random variables with mean 0, variance Sigma square.

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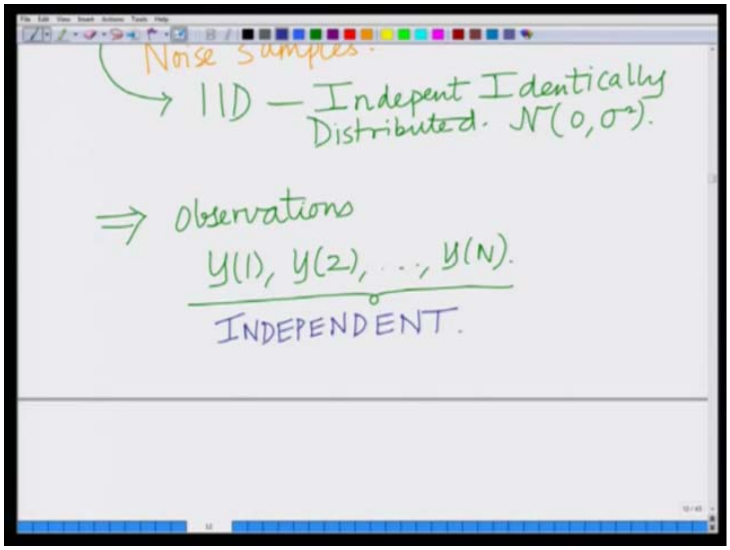

Now this naturally this implies that the observations or the outputs, the observations or the outputs Y1, Y2… YN, these are also naturally, the observations are also naturally, these are, the observations are independent. These are independent, the observations are. The observations are, basically these observations are independent, so that naturally means that these outputs Y1, Y2… YN are independent and therefore the joint PDF of the observation is given as the product of the individual probability density functions.

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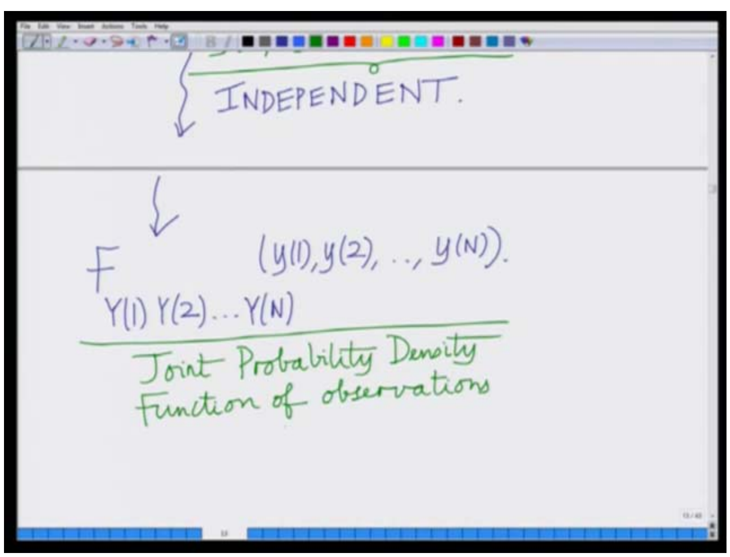

Therefore the joint probability density function, that is if I look at the joint PDF, that is F of Y1, Y2… Up to YN of Y1, Y2, that is your Y2 up to YN, what is this, this is the joint PDF of observations. The joint, the joint probability density function, this is equal to the product of the individual probability density functions,

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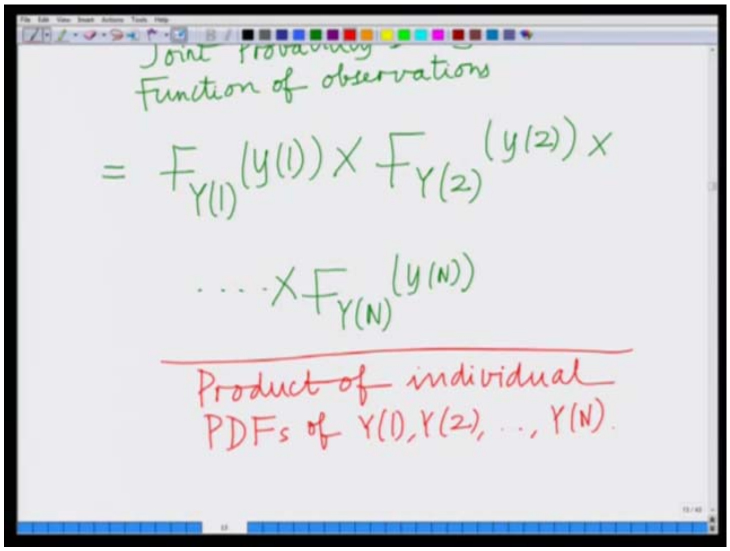

this is equal to the product of the, since these are independent, this is equal to the product of the individual probability density, this joint probability density function is equal to the product of the individual probability density functions.

This is the product, the individual probability density function of Y1, Y2… YN. So, what we have seen so far is basically we have developed a model for a simple wireless communication system across a channel or a fading channel with the transmitted signal symbol is XK, receive symbol is YK, the channel coefficient is H and the noise is VK and we have said that this channel coefficient H is unknown which has to be estimated, this process is known as the channel estimation, we have developed, we have said also that we are going to use a scheme which is basically pilot based channel estimation well-known symbols X1, X2,… Up to XN, which are also known as the pilot symbols or the training symbols are transmitted, corresponding to these we have the received output symbols or the observations Y1, Y2… YN, from the observations Y1, Y2… YN and the transmitted pilot symbols X1, X2,… XN, one can estimate this unknown channel coefficient H.

So, we will stop this module here, where we have basically developed an expression for the joint likelihood function, we will, we will continue with this process of this scheme in the next module and complete this procedure or develop this scheme for channel estimation completely, where the unknown the unknown channel coefficient H. Thank you very much.