Estimation for Wireless Communications – MIMO/OFDM Cellular and Sensor Networks
Professor Aditya K Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur
Lecture Number 39
Example-Sequential Estimation of Wireless Channel Co-efficient

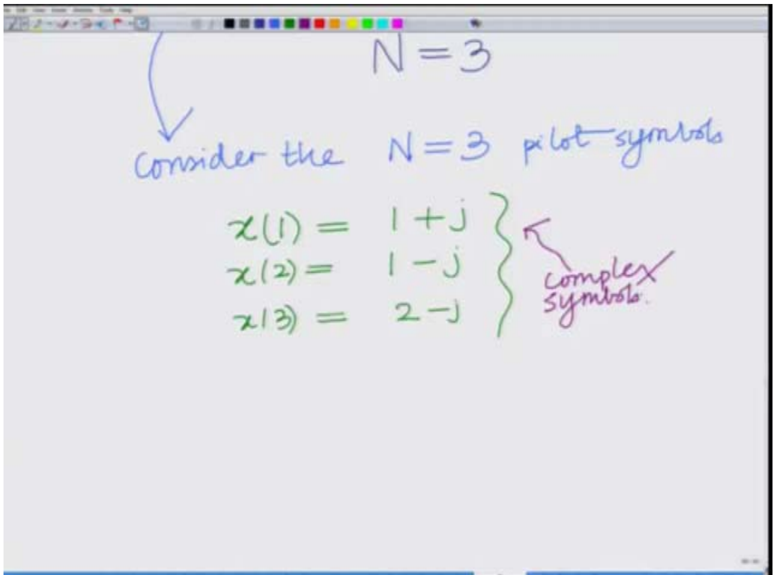
Hello! Welcome to another module in this massive online open course on 'Estimation for Wireless Communication Systems'. So, in the previous module, we have looked at sequential estimation, that is how to keep continuously updating the estimate based on the received observations, that is sequentially updating the estimate and also the variance of the estimate, alright? So today, let us look at an example to understand this paradigm better. So what we want to look at today, is an example for sequential; we want to look at an example for sequential estimation, alright?

So let us start by considering our, the same paradigm that we are considering before, that is 'Wireless Channel Estimation'. Let us start by considering N is equal to three samples. Remember, sequential estimation in involves, com, computing an initial estimated N, and then later updating it to the estimated time instant N plus one once the N plus oneth observation is derived. Or once the N plus oneth observation is made. So we have considered the same example that we've seen previously for 'Wireless Channel Estimation', except now, we will estimate the Wireless Channel sequentially.

So previously we considered example with N equal to four symbols. Now what we're going to do, we're going to modify the same example, to basically start with N equal to three symbols, that is a transmitted N equal to three pilot symbols, received N equal to three pilot observations. Compute the estimate at N equal to three, alright? And then consider the arrival of the N plus oneth observation at N equal, that is N plus one equal to four, and how, and then demonstrate how the update, or how the estimate H hat N plus one, that is H hat four is computed as an update of H hat three, the estimate at time N is equal to three. Okay?

So let us consider the three pilot symbols. Consider, consider the three pilot symbols. Rather consider the N equal to three pilot symbols. The N equal to three pilot symbols, let's say they are given as, similar to what we've considered before, that is x one equals one plus j, x two equals one minus j, and x three equals two minus j. Now one important observation that you can make here is that we're considering complex pilot symbols.

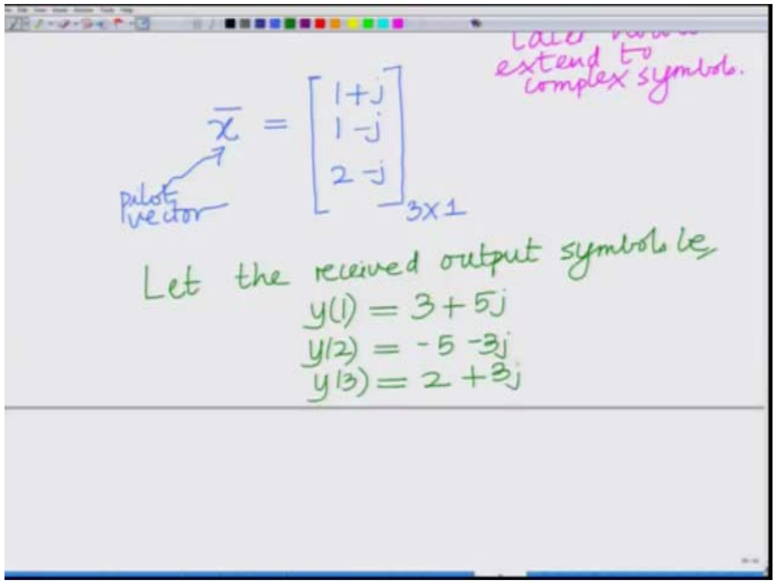
(Refer Slide Time: 03:49)



However, in the, in the framework for sequel sequential estimation, we considered only, um, real symbols and real parameters, okay? So what we're going to do now, is, is, um, during this example I'm going to illustrate how to extend the framework of sequential estimation to a complex parameter, and that is going to be fairly simple, alright? So, just to bring to your notice, we're considering complex symbols, so the framework of sequential estimation. So we will see how to extend; we will see later how to extend to complex symbols, okay?

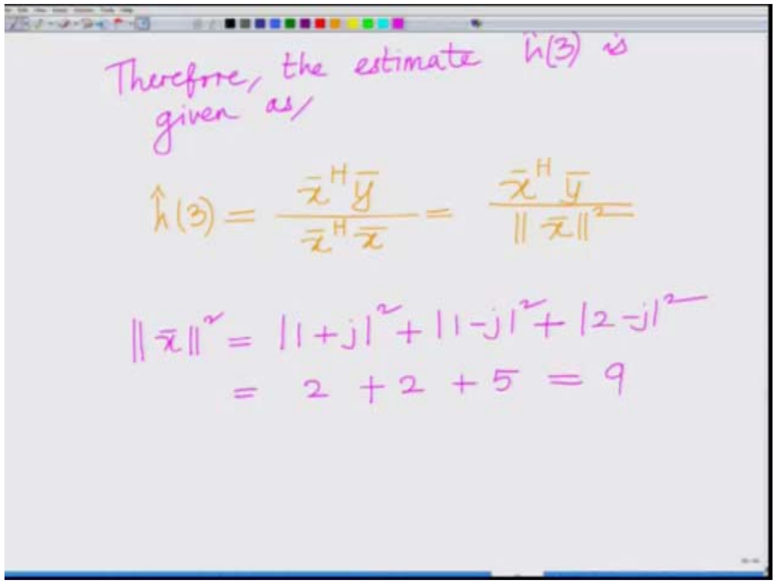
So therefore, our vector, pilot vector x bar, remember x bar, what is this x bar? This is the pilot vector, and this is given as x one, x two, x three, that is basically one plus j, one minus j, two minus j. This is your pilot vector, which is basically N cross one or basically three cross one. And similarly let the received symbols be; let the received output symbols be y one equals, well y one equals three plus five j, y two equals minus five minus three j, and y three equals two plus three j, alright? So we have, corresponding to the tree, three transmitted pilot symbols, x one, x two, x three, we have the three received pilot symbols y one, y two, y three; and therefore, we have the received vector y bar, which is formed from y one, y two, y three, and that is given as, y bar equals three plus five j, minus five minus three j, two plus three j.

(Refer Slide Time: 05:28)



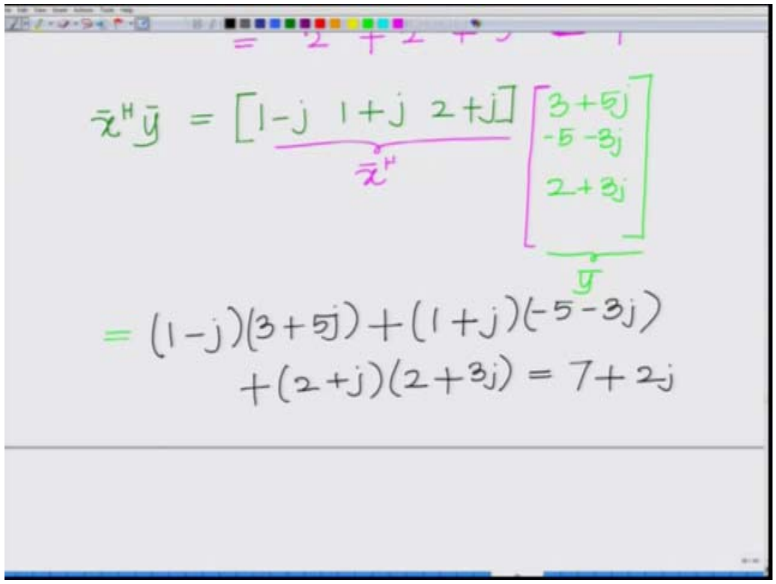
Therefore the estimate H hat, we already know how to compute this. Therefore the estimate H hat, is given as, we have, H hat of three is x bar hermitian y bar divided by x bar hermitian x bar, that is basically again, you can write this as x bar hermitian y bar divided by x bar hermitian x bar is basically, your norm of x bar square. Okay? And therefore now you can see norm of x bar square is simply the sum squared of the maxum magnitude squares of each of the components. One plus magnitude one plus j square plus magnitude one minus j square plus, magnitude two minus j square, that is, well two plus two plus five, which is equal to nine, okay?

(Refer Slide Time: 07:31)



So norm x square equals nine, and x bar hermitian y bar again, hermitian is nothing but the transpose and conjugate of each element, that is one minus j, one plus j, two plus j times your vector y bar, that is the same as before, that is a column vector y bar. So what is this? This is your x bar hermitian times y bar. y bar is basically three plus five j, minus five minus three j, and two plus three j. This is your vector y bar, and therefore this can be simplified as this, is equal to well, that is your, one minus j, times three plus five j plus one plus j times minus five minus three j, plus two plus j into two plus three j equals seven plus two j.

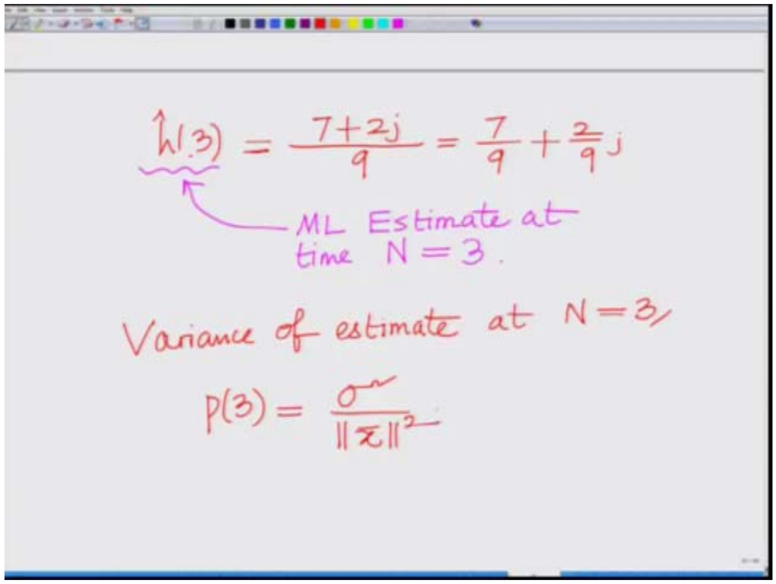
(Refer Slide Time: 08:56)



Therefore we have, um, evaluated x bar hermitian y bar, as seven plus two j, and we've also evaluated, um, norm x bar square or x bar hermitian x bar, which is nine, and therefore, the ML estimate at time instant three, that is H hat three is given as x bar hermitian y bar divided by norm x bar square, which is equal to so we have H hat of three, which is basically your ML estimate at time instant three, that is basically seven plus two j divided by nine, which is nothing but seven divided by nine plus two divided by nine j. So what is this? This is your ML estimate at time instant three. ML estimate at time instant three. ML estimate of the channel co-efficient of course, yeah?

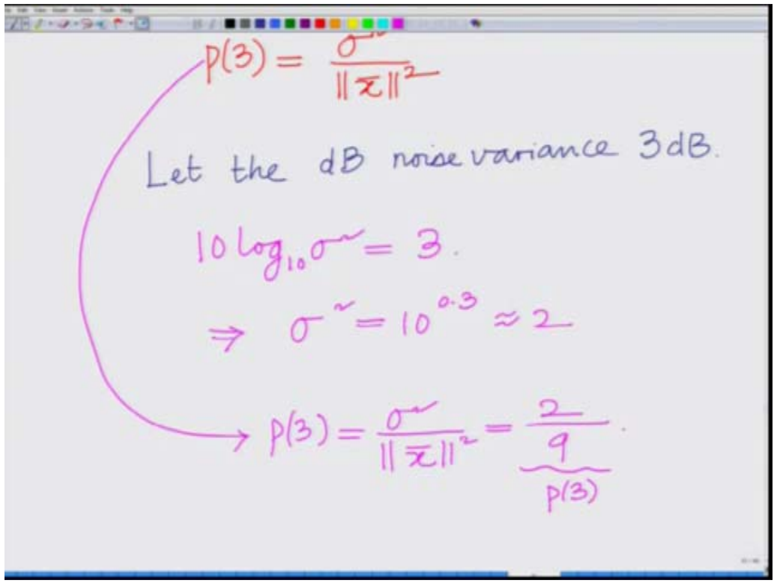
Now the other thing, remember the other thing that we've to compute in the sequential process, the sequential estimation process is basically the variance at time instant three. The variance P(N) at time instant three is sigma square divided by norm x bar square, alright? We know that also. So the variance P(N) at time three, that is P of three. Variance of the estimate, at N equal to three, that is basically P of three, equals sigma square divided by norm x bar square.

(Refer Slide Time: 10:46)



Let us consider similar to the previous example in the Wireless Channel Estimation, let sigma square, let the dB variance, let the dB noise variance be three dB, which implies ten log ten sigma square equals three, which basically implies your sigma square equals ten power point three, this is approximately two, okay? So three dB noise variance, right, three dB noise variance basically corresponds to a sigma square value of two. Okay? So now, the noise variance P three at time N equal to three is sigma square divided by norm x bar square, so this P three equals sigma square divided by norm x bar square, which is sigma square is two, divided by norm x bar square is nine, so this is basically your value of P three.

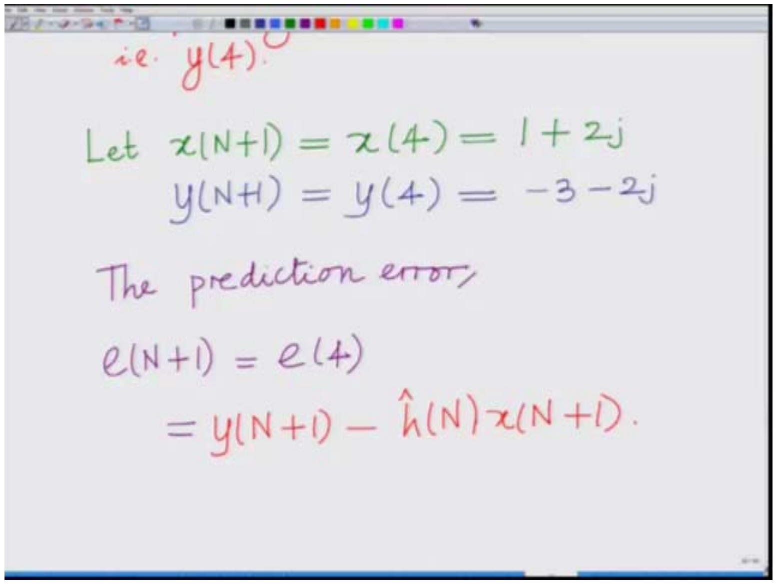
(Refer Slide Time: 12:01)



So, we have calculated H hat of three, which is a estimate at time instant N equal to three and P three which is the variance at time N equal to three. Now, let us assume that now, consider the received transmitted pilot symbol x N plus one at time instant N plus one, that is N equals three, therefore N plus one is four. Let us consider the transmission of x four, and the reception of the pilot sa, output pilot symbol y four that is y N plus one and now, basically using H hat of three, we would like to estimate it to H hat of four, okay, at time instant N plus one. So consider now, consider now the transmission of pilot symbol x N plus one that is, x of four.

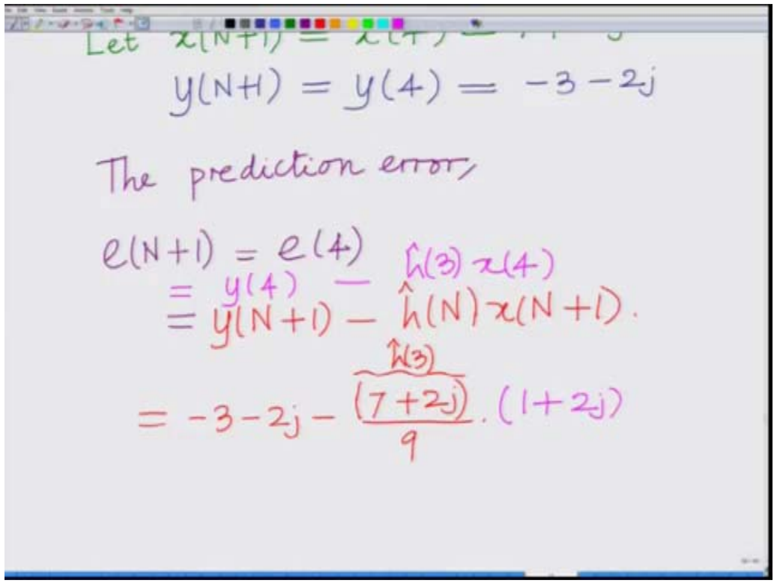
Remember, N equal to three, so we're considering the transmission of x of four. Corresponding received symbol is, corresponding received symbol is y N plus one, that is, y of four. Okay? So let x N plus one equals, remember, let x N plus one equals x of four. Let this be equal to one plus two j, and let the corresponding output symbol y N plus one equals y of four, and let this be equal to minus three minus two j, and therefore the prediction error; therefore the prediction error N plus one equals e four, equals y N plus one minus remember, we said H hat N times x N plus one. Remember, for the sequential estimation process, we have to compute the prediction error, e N plus one at time instant N plus one that is e four, which is basically, that tells us how accurate is the estimate H hat N computed at time instant N.

(Refer Slide Time: 14:57)



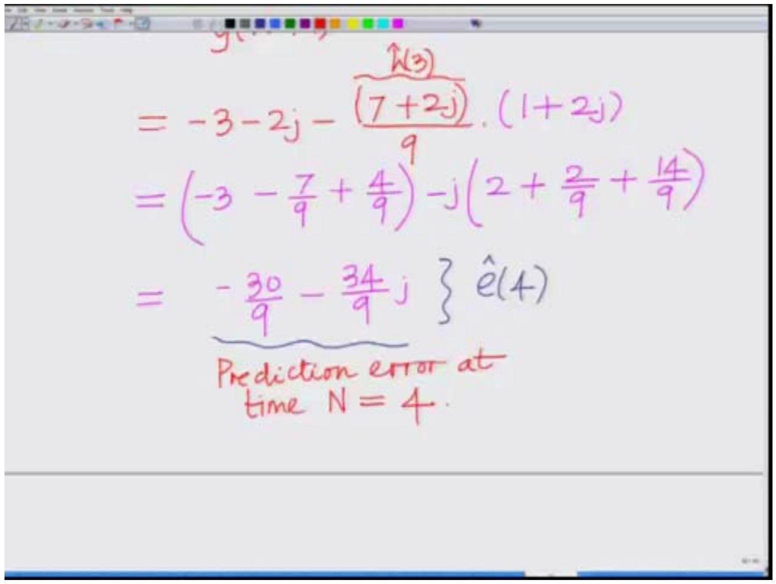
And it’s defined as y N plus one minus H hat N times x N plus one, okay? And therefore now, I’m going to substitute y N plus one. y N plus one we know, y N plus one is given as, well, that is already given, that is minus three minus two j minus H hat of N, H hat of N is seven plus two j divided by nine, that is what we already computed, remember this is H hat of three. Just to write it a little bit more explicitly, this is basically your y four minus H hat of three times x four. So seven plus two j over nine times one plus two j, and this is equal to, this is equal to well I can simplify this. This is minus three minus seven by nine plus four by nine minus j times two plus two by nine plus fourteen by nine, which is equal to minus thirty by nine minus thirty four by nine times j, and what is this?

(Refer slide time: 16:02)



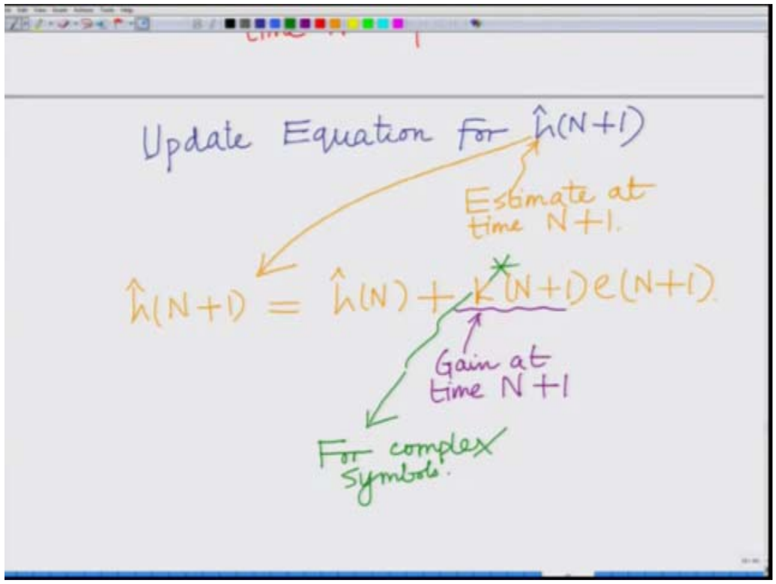
This is your prediction error e hat N plus one, that is e hat of four. This is your prediction error at time N equal to four. Prediction error at time; this is your prediction error at time N equal to four. So for the the complex quantities, now we have to come up with a update equation. Remember the update equation which we said previously, let me first write the update equation for the real symbols. Right? The update equation. This is the update equation for H hat N plus one, which is the estimate at time N plus one.

(Refer Slide Time: 17:09)



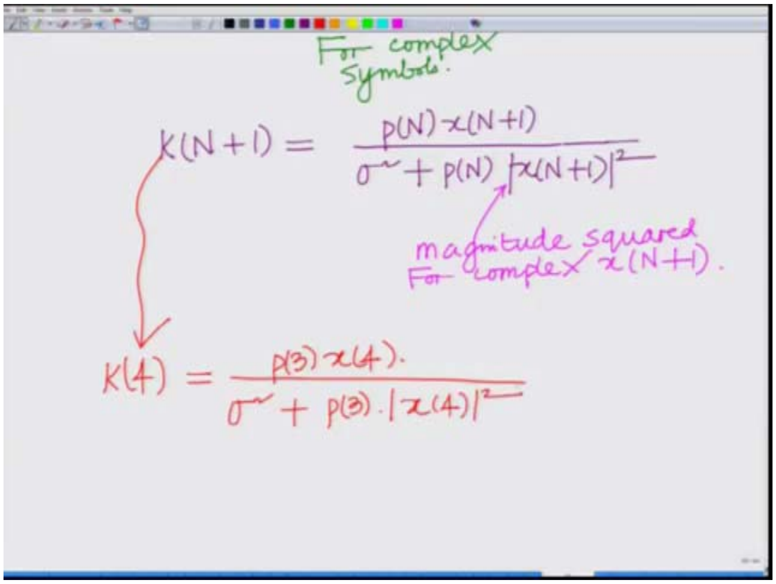
Which is the estimate at time N plus one and the update equation remember, is given as, H hat N plus one equals H hat N plus k N plus one, where k is the 'Gain' times e N plus one, where e N plus one, this is the prediction error. So k N plus one remember k N plus one is the, gain at time. So naturally for complex symbols what I'm going to do is, I'm going to do a simple modification. I'm going to simply replace this k N plus one by k conjugate, for complex symbols, where the conjugate. That's it. That is a simple modification that we have to do, that is, replace the gain k N plus one by k N plus one conjugate, because we're considering complex symbols x N, and complex received output symbol y N.

(Refer Slide Time: 18:52)



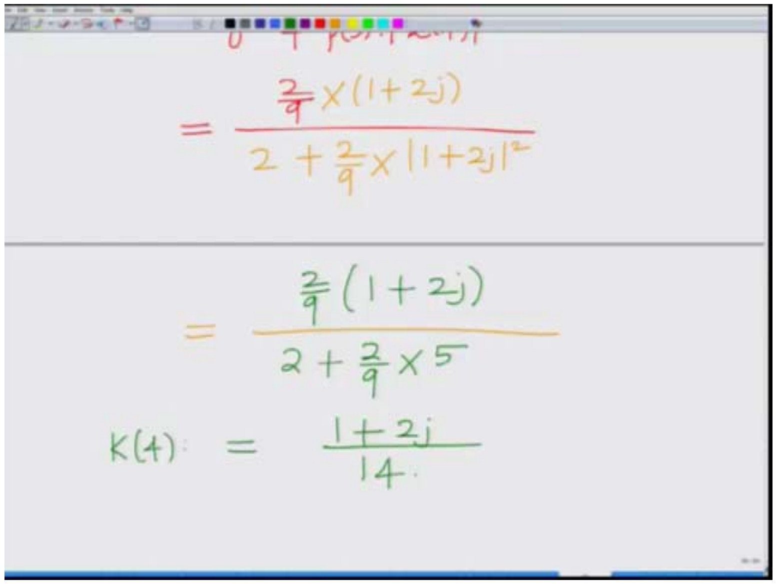
And similarly complex symbols x N plus one, and received output symbol y N plus one. So I'm replacing the gain k N plus one by the complex conjugate, okay? And one more small modification you'll if you look at the expression for k N plus one, we have k N plus one equals remember, we had defined k N plus one as P(N) the variance at time instant N divided by x N plus one, divided by sigma square, the noise variance, plus, P(N) times x square N plus one. Now I'm going to replace this by the magnitude square. All I'm going to do, is I'm going to replace by the magnitude square, that is magnitude square; replace this by the, replace this by the magnitude square for your complex symbol x N plus one, alright?

(Refer Slide Time: 20:56)



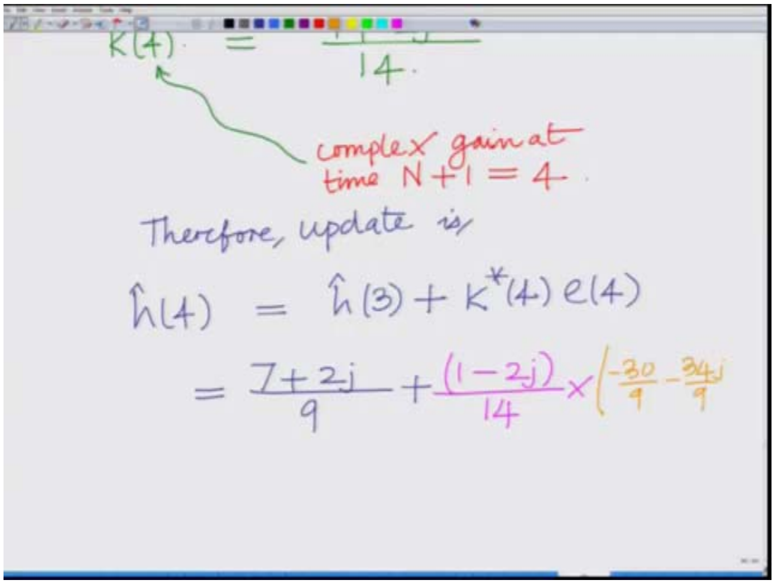
So there are two changes basically, one is k N plus one is replaced by the complex conjugate, alright? That is k N plus one conjugate in the update equation, and in the expression for k N plus one, instead of x square N plus one I have magnitude x N plus one whole square, okay? So now using this, let us compute k N plus one, the gain at time four set at, that is N equal to three, so k N plus one equals k four, so k four equals basically P three times x four divided by sigma square, plus P three times magnitude x four square, okay? We have computed P three before. P three is the variance at time three, that is two over nine times x four, that is one plus two j divided by sigma square which is two plus P three which is two over nine times magnitude, one plus two j whole square, which is equal to, which is equal to two over nine times one plus two j over two plus two over nine times magnitude one plus two j square is five.

(Refer Slide Time: 21:53)



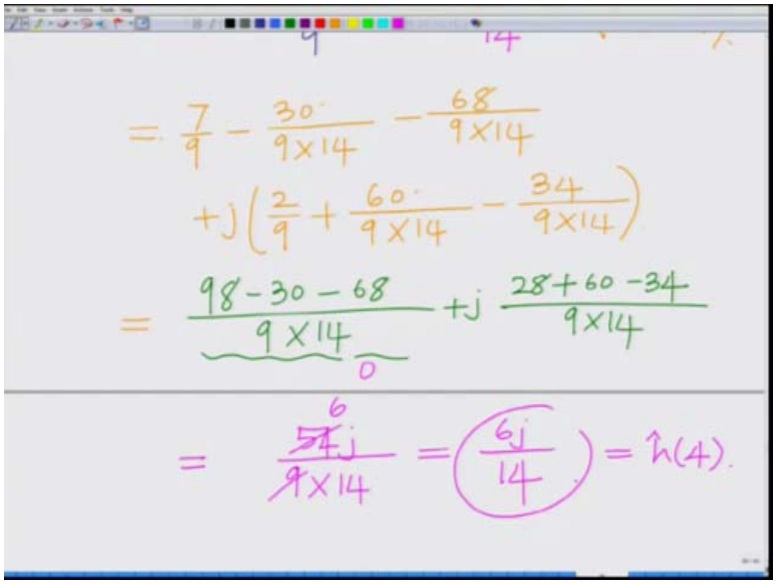
This is basically one plus two j divided by fourteen, okay? And what is this this? This is your k four, that is gain, complex gain at time N equal to four, n plus one equal to four. Okay? Complex gain at time N plus one equal to four. And now the update equation therefore is H hat, therefore, the update is H hat four equals H hat three plus, a conjugate four into e four, we know H hat three, H hat three is seven plus two j divided by nine, plus k four we've already calculated that here, so I've to take k conjugate four, that is one minus two j divided by fourteen times e four, which is basically, the prediction error minus thirty by nine minus thirty four j divided by nine.

(Refer Slide Time: 23:10)



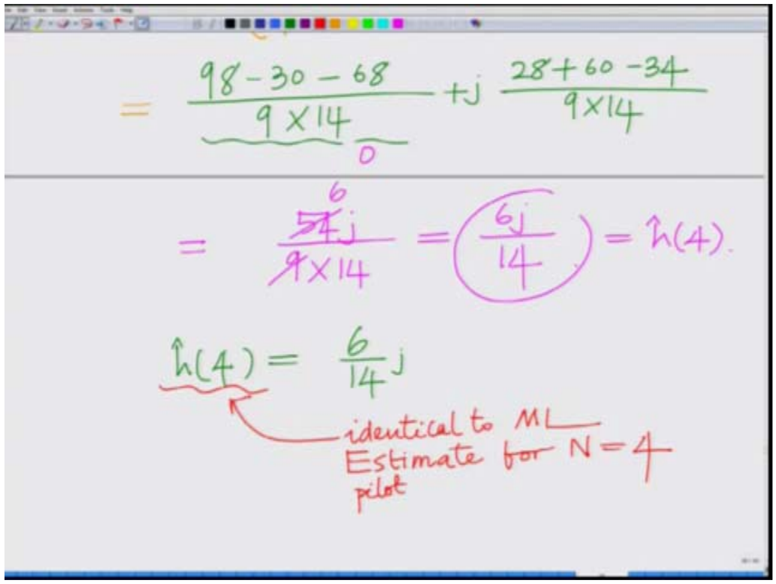
And this, is basically equal to, now I'm going to simplify this seven by nine minus thirty divided by nine into fourteen, you can check the calculation minus sixty eight divided by nine into fourteen plus j times two by nine plus sixty divided by nine into fourteen minus thirty four divided by nine into fourteen. This can be simplified as, this is equal to, well, this is ninety eight minus thirty, minus sixty eight divided by nine into fourteen, um, plus j times twenty eight plus sixty minus thirty four, divided by nine into fourteen. This you can see, this quantity zero, so this is basically, and this reduces to fifty four times j divided by nine into fourteen, which is basically six j divided by fourteen and what is this? This is the, estimate at time instant four.

(Refer Slide Time: 24:46)



And this is, similar, and what you observe interestingly is it is exactly identical to the maximum likelyhood estimate we calculated in the previous example, um, in, in the um, previous example for Wireless Fading Channel Estimation, considering four transmitted pilot symbols that is N equal to four transmitted pilot symbols, and N equal to four received output pilot symbols, and what we, what we observe, is that using this sequential estimation procedure, we basically get back the same estimate H hat four, alright? So let me, just highlight that H hat four equals, what is that, six j or six divided by fourteen times j, which is basically ML estimate, identical to ML estimate, over N equal to four.

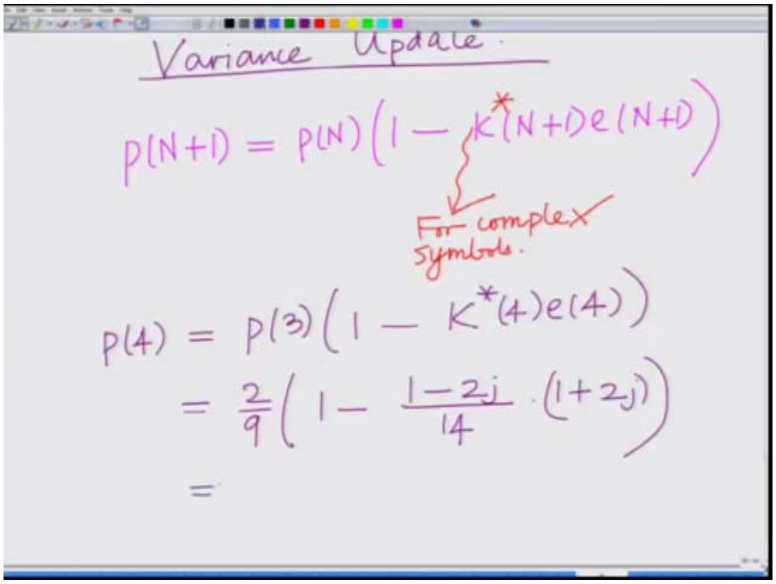
(Refer Slide Time: 25:56)



This is identical to the ML estimate for N equal to four pilots computed previously. In previous example. By previous example I mean the previous example for maximum likelyhood estimation of the Wireless Fading Channel co-efficient, okay? Alright? So now, we have to compute the variance, remember we have to compute the variance. At every step, we have to compute both the um, estimate, and the variance in the sequential estimation procedure, okay? And the variance update, you remember variance update for the real case, is given as P(N) plus one equals, P(N) into one minus k N plus one into e N plus one.

k N plus one is again e N plus one is a prediction error. Now for the complex scenario, naturally all I have to do is replace this gain by the conjugate, for, for your complex symbols. For complex symbols, k N plus one is replaced by k N plus one conjugate, and that's the only change, and therefore, P of four equals P of three, one minus k conjugate four into prediction error e four, which is P of three, we know

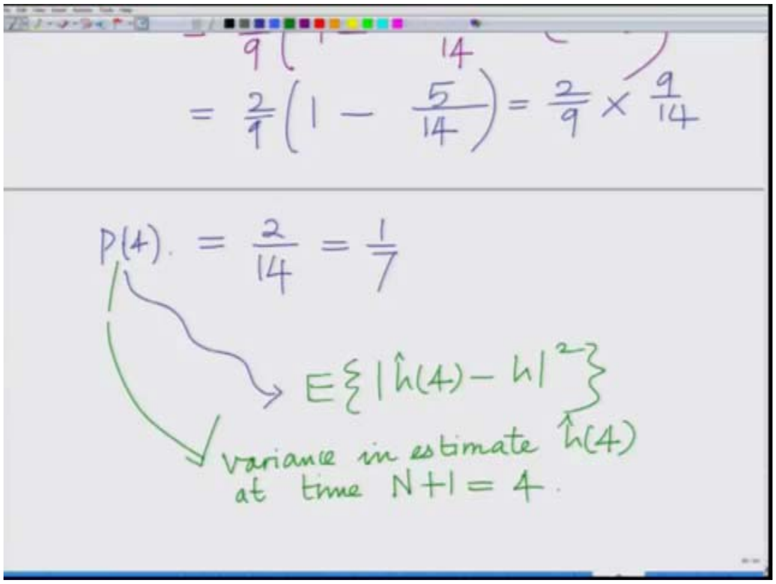
(Refer Slide Time: 27:54)



this is two over nine times one minus k conjugate four, which is one minus two j divided by fourteen times one plus two j, which is equal to, well two over nine into one minus, one minus two j into one plus two j is magnitude one plus two j square which is five divided by fourteen equals two over nine times, well fourteen minus five, that is nine divided by fourteen equals two over fourteen equals one by seven. And what is this? This is P four, prediction error at time N equal to four, and this is basically the prediction, I'm sorry, this is the variance at time N is equal to four.

That is basically the variance in the estimate H hat of four, that is this quantity is basically nothing but, you all know, this is the variance in the estimate, that is expected value of H hat four minus x square. That is variance in estimate computed at, at time N equal to four or N plus one equal to four, basically N plus one is the next time instant. Correct? So what we've calculated, what we've shown in this example is basically we have calculated the estimate H hat four, which is six j by fourteen, which is identical to the previous ML estimate, and in fact, you can also check, the variance P of four equals one by seven is also identical to the variance computed for the previous maximum likelyhood estimate. And in fact, we had proceeded further to derive that the variance of that, the errors of the real and imaginary parts are uncorrelated, and the variance of the real and imaginary parts of the estimates, um, of the estimate is basically half of the total variance.

(Refer slide time: 29:13)



That is the variance of the real part of the estimate is one over fourteen, and variance of the imaginary part is also one over fourteen, and that also holds again, um, it is natural that that also holds basically in this sequential estimation procedure. Alright? So, what we've described here, what we've shown in this module is a detailed example of basically, the paradigm of sequential estimation illustrated in the context of estimation of a Wireless Fading Channel co-efficient. Naturally this can be extended to other scenarios also, for instance, the Wireless Sensor Network, very easily. It may also be extended the vector estimation scenario, that is the Downlink Multiple Antenna Channel estimation, OFDM and other scenarios, although, extension to the vector case is not very straight forward, but it has a similar structure, which we will defer to a slightly later course, probably, because it's slightly more complicated than this.

But basically, what we've shown is we've shown, described this paradigm of sequential estimation which is very interesting, because we don't need to recompute the estimate at every time instant; rather simply update the latest or the previous estimate. Alright? So we've shown how to compute, update the estimate, the update equation in terms of the gain, the prediction error how to compute the updated estimate, and also how to update the variance at every time instant. So this, and basically we've also shown that this yields exactly identical results, um, as that of the ML estimate, which is basically, the one shot estimate computed at every time instant.

And naturally, because the sequential estimation procedure itself was derived from the ML estimate, alright? So we'll stop this module. Yeah. Thank you.