Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks

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Lecture Number 37

Introduction to Sequential Estimation - Application in Wireless Channel Estimation

Hello, welcome to another module in this massive open online course on Estimation for Wireless Communications. So in this module we will start looking at another paradigm or a new technique of estimation which is termed as Sequential Estimation. So let us start looking at Sequential Estimation.

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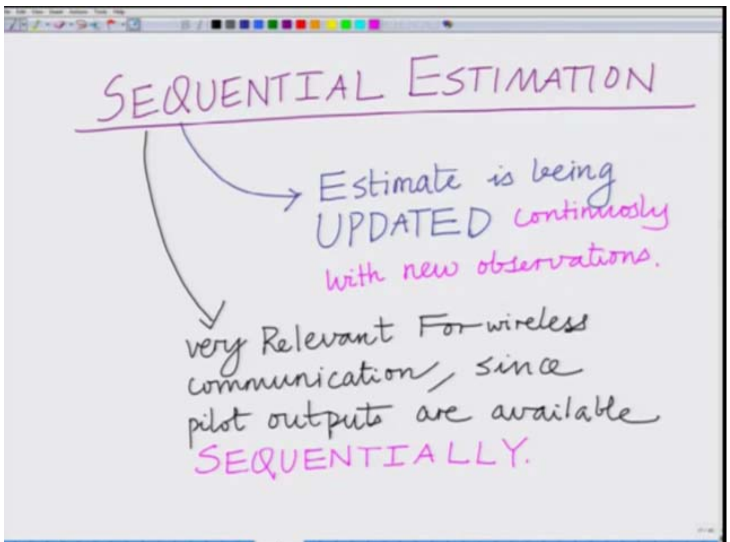

Basically Sequential Estimation is about how to continuously keep estimating, as the name implies it is the estimation is being carried out sequentially that is which means that the estimate is being continuously updated as and when the observations are arriving, all right. So basically this means that your estimate, you do not have a fix estimate, but the estimate is being updated and that is the keyword updating the estimate.

Updated estimate is being updated continuously with the new observations, yeah with the new observations. And this is very relevant in the context of wireless communication because all the time there are new observations that is new pilot symbols being transmitted and new received output symbols that is new output symbols being received.

So with every new output symbol, one can basically update the estimate of the channel coefficient, all right. So this paradigm of physical sequential estimation of continuously updating the estimate is very relevant for Wireless Communication, so basically this is very relevant for Wireless Communication. This is very relevant for Wireless Communication setups since the pilot symbols are arriving or the pilot outputs are arriving continuously.

Or pilot outputs are available let us say they are available sequentially, right. They are available sequentially and that is the key term here that is they are not available all at the same time, but they are available sequentially in the sense that there is a pilot symbol transmitted, there is a pilot output that is observed based on which you compute the estimate. And the next instant you have another pilot symbol that is transmitted.

And we have a corresponding pilot output, so one need not compute the fresh estimate or one need not recompute the entire estimate, rather one can simply um update the previous estimate. And that is really the keyword here; that is instead of recomputing the estimate at every time instant, one can simply update the estimate from the previous time instant.

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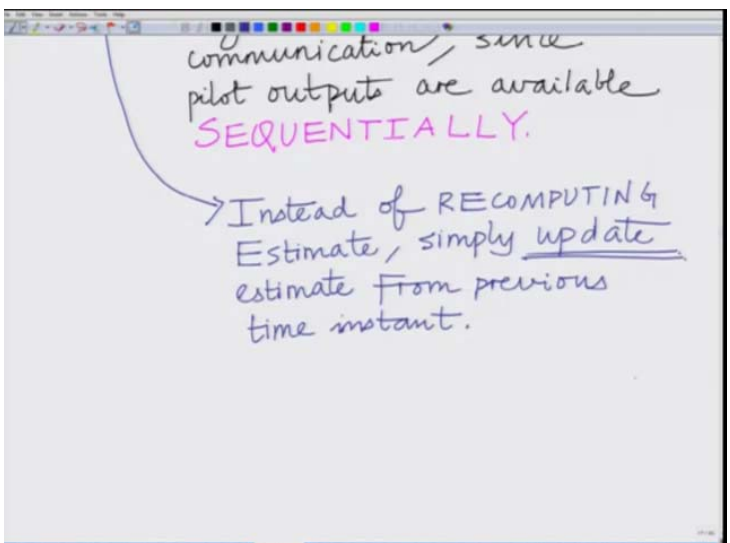

So let us summarise that, instead of recomputing so what is sequential estimation about instead of recomputing the estimates, simply update the estimate from the previous time instant. Simply update the estimate and that is really the key that is to update the estimate, how do you update the estimate from the previous time instant? And to understand this, let us go back to our example; let us look at this to illustrate this with the help of an example.

All right, or a certain estimation scenario that we have already considered, let us go back to Channel estimation that is where we were estimating the fading wireless channel coefficient. So let us illustrate the sequential estimation for Wireless for the wireless channels sequential or the Wireless fading channels. So let us call this a sequential Wireless sequential Wireless Channel estimation and remember.

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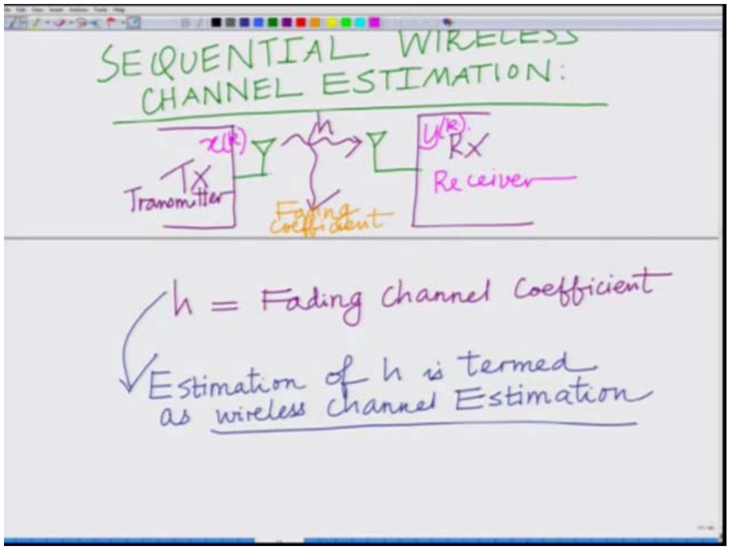

In Wireless Channel estimation something that we have described, we have the transmitter, this is your transmitter, you have your receiver, this is your receiver, let us consider the simple scenario where we have a single transmit antenna, where we have a single receive antenna, so these are basically the antenna and the channel between them, the transmitter and receiver is represented by the fading coefficient.

This h, if you might remember from the previous module, this is your fading coefficient or the fading channel coefficient. So h is the fading channel coefficient, h is basically your h is the fading channel coefficient and estimation of h estimation of this fading channel coefficient h, this is termed as Channel estimation that is the Wireless Channel estimation. So estimation of h is termed as estimation of h is termed as Wireless Channel estimation.

We transmit toward this end, we transmit the pilot symbol x k from this and receive the corresponding the pilot output symbol y k, okay. And the system model for this can be expressed as the system model for this.

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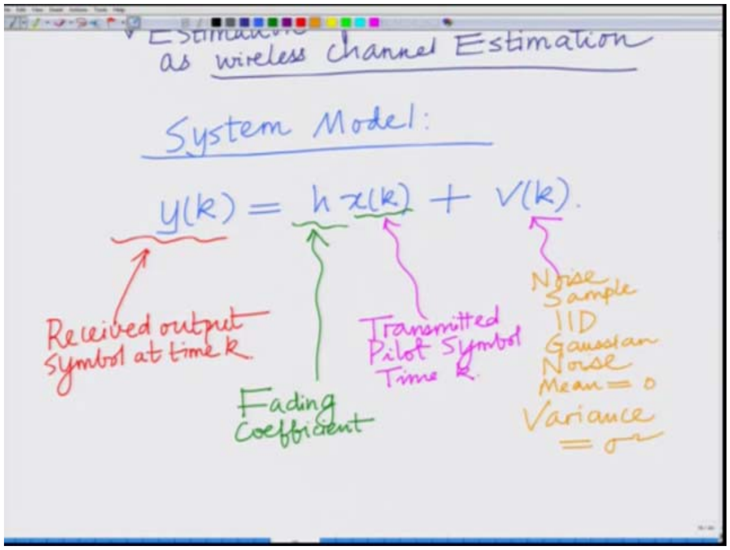

This is something that we have already seen, which is y k equals h that is your fading channel coefficient h times x k + v k. And what are the different quantities here, y k is the received output symbol at time instant k.

This is the received output symbol at time k, h is the fading coefficient, x k is the transmitted pilot symbol at time instant k and v k as we are already familiar, this is the IID Gaussian noise mean 0, so this is the noise sample and we are assuming IID Gaussian noise with mean equal to 0 and variance equal to sigma square, okay. So we have the same familiar model that is y k equals h times x k + v k, y k is the output symbol, and h is the channel coefficient.

X k is the kth transmitted pilot symbol and v k is the voice sample, all right. And now let us consider the transmission of N pilot symbols capital N pilot symbols similar to before and then we will and then we will adapt this or we will modify this model for the sequential estimation scenario, okay. So similar to before, this part is similar to before, consider the transmission of N pilot symbols.

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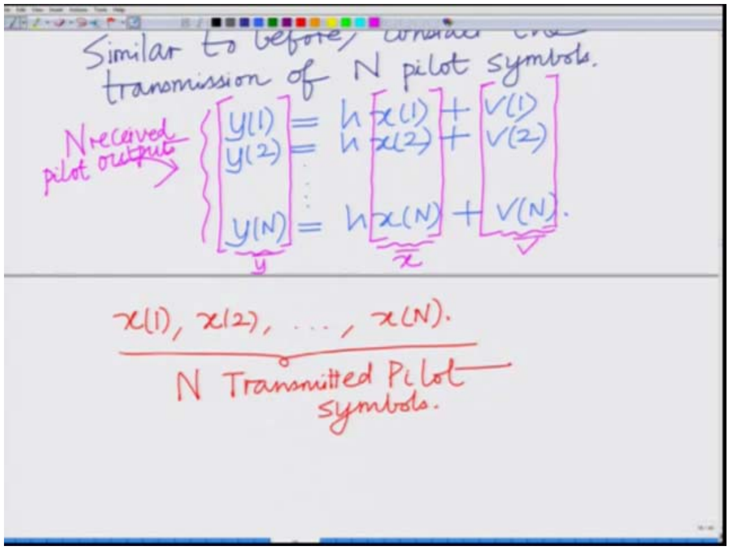

Consider the transmission of capital N pilot symbols and therefore we can write the received symbol y 1 equals h times x 1 + v 1, y 2 equals h times x 2 + v 2, y N equals h times x N + v N, all right. These are the N received outputs, these are the N received pilot outputs, x 1, x 2, x N are the transmitted x 1, x 2, x N are the N transmitted pilot symbols. What are these?

These are your these are the N transmitted pilot symbols, now we can represent this by a vector, I can write this as your vector y bar, y 1, y 2, y N is your vector y bar, this is your N dimensional vector. x 1, x 2, x N is the pilot vector, this is x bar which is your pilot vector + v 1, v 2, v N which is the noise factor v bar. So now I can write this as and this is also something we have seen before subsequently using vector notation.

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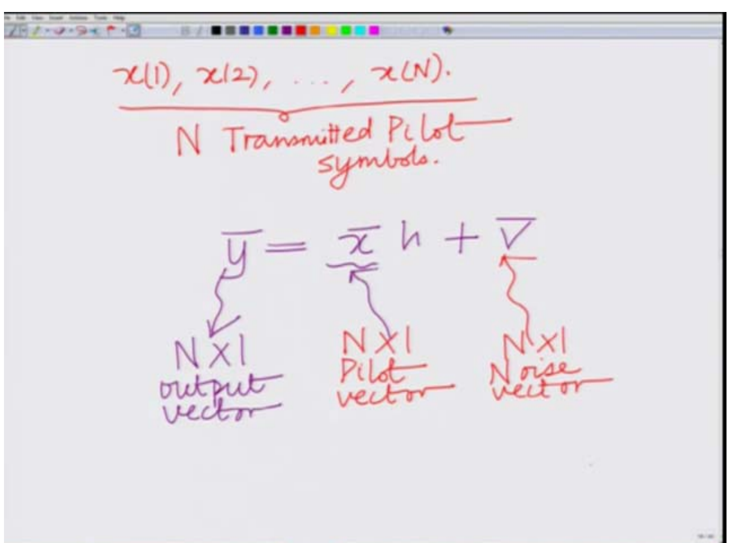

Or let me just right x bar times h + v bar, so this y bar is the n cross 1 output vector, x bar, what is this? This is your N cross 1 pilot vector, h is of course the channel coefficient, v bar is the N cross 1 point vector and this is also something this is also something that we have already seen before that is you have y bar equals x bar times h, the channel coefficient h + v bar, where x bar is the N dimensional pilot vector, okay.

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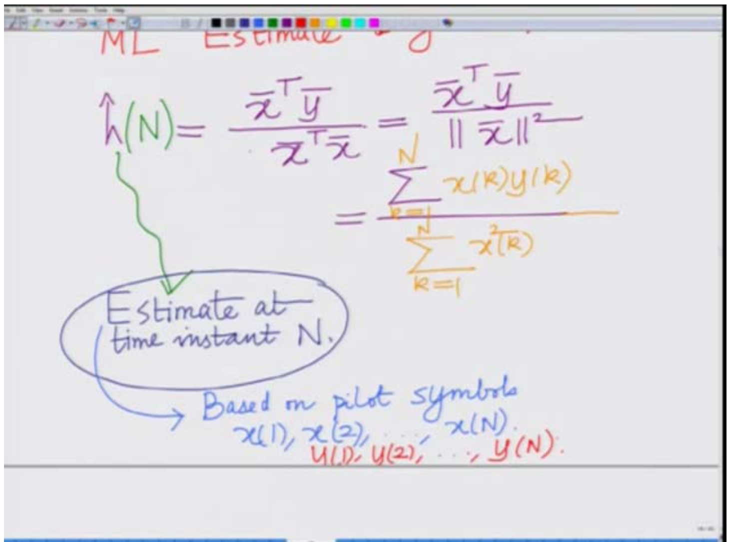

And then we computed the maximum likelihood estimate, okay. The maximum likelihood estimate the maximum likelihood estimate which maximises the likelihood that is the estimate which maximises the likelihood is h hat equals x bar transpose y bar divided by x bar transpose x bar equal to x bar transpose y bar divided by norm x bar square which is equal to.

I can write it now in the form of the submission, v have also seen it before k equals 1 to N x k, y of k divided by submission equal to 1 to N x k or x square k, okay. Norm x bar square, the norm of the vector square is basically submission k equal to 1 to N x k square, okay. Now we are going to make a small modification, not exactly a modification, a simple change in the notation.

Instead of calling this the estimate h hat, we will be calling this the estimate h hat of N that is the estimate at time instant capital N based on the pilot symbols x 1, x 2 up to x N all right. So in the paradigm of sequential estimation, because we are computing not one estimate, but we are updating the estimate at every times instant, the time instant so the estimate is also now characterised by time instant.

So h hat becomes h hat of N, which denotes this is the estimate at time instant N. This is the estimate at time instant N, okay. And this is computed based on pilot symbols x 1, x 2 up to x N transmitted up to time instant N, okay. So what we are saying is this is estimation, now in the sequential estimation scenario, this is the continuous process, right.

So at each time instant, we are going to compute, we are going to update the estimate at each corresponding to each time instant. Basically as the pilot symbols are arriving as the pilot outputs are arriving, we are going to continuously keep updating our estimation. Hence naturally there is no one estimate, but there is an estimate for each time instant and this estimate h hat at time instant capital N, we are denoting it by h hat of H.

Which is basically computed from the transmitted pilot symbols small x 1, small x 2 up to small x N and the corresponding of course received pilot outputs, observation which is which are basically the small y 1, small y 2, small y N. So let me also mention that corresponding pilot symbols and naturally it goes without saying also the corresponding observations because the observations are also have to be used to compute the estimate okay.

So also now the variance of the estimate at time instant N. Remember, we have also calculated the variance, this equal to, let us denote this by P of N, this is the variance at time instant N let us denote this by the quantity P of N. And we know that this variance at time instant N this is P of N, this is equal to sigma square divided by norm x bar square which is basically equal to.

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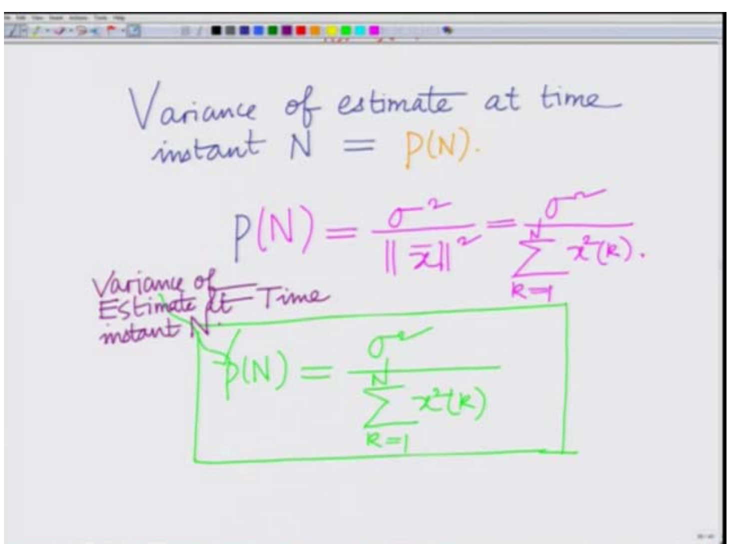

Sigma square divided by submission k equal to 1 to N x square of k. So basically you're P of N, the variance at time instant N equals sigma square divided by submission k equal to 1 to N x square of k. And what is this? This is the variance of the estimate at time instant N. Variance of estimate at time instant N. So we have 2 things, 1 is we h hat of N which is the estimate computed at time instant N.

And then we have P of N which is basically the variance of the estimate at time instant N. Now based on these 2, we will update the estimate at time instant N + 1 that is at the next time instant, so for the next time instant, now consider the transmission of pilot symbol x + 1.

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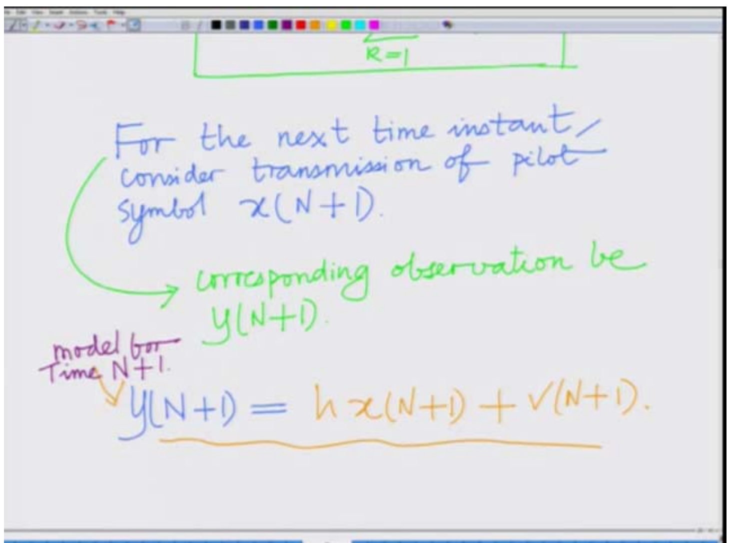

For the next time instant, transmission consider the transmission of pilot symbols x N + 1, that is pilot symbol at time instant N + 1.

And corresponding received symbol observation be y N + 1, so let corresponding observation, let the corresponding observation be y N + 1, which basically implies that your y N + 1 equals what does that mean, y N + 1 equals h times x N + 1 + v N + 1. This is the model at times N + 1. Your model for next time instant N + 1, okay.

So we have y N + 1 which is the pilot symbol received output symbol at time instant capital N + 1 corresponding to the pilot symbol x N + 1 transmitted at time instant N + 1 by the transmitter, all right. So now we have to update this, so now we have new observation y N + 1 and therefore, based on this observation we have to update the estimate h hat of N to h hat of N + 1 at time instant N + 1.

So based on this new observation, so now the task in sequential estimation is now we need to update h hat of N to h hat of N + 1 based on x N + 1 and y N + 1 okay.

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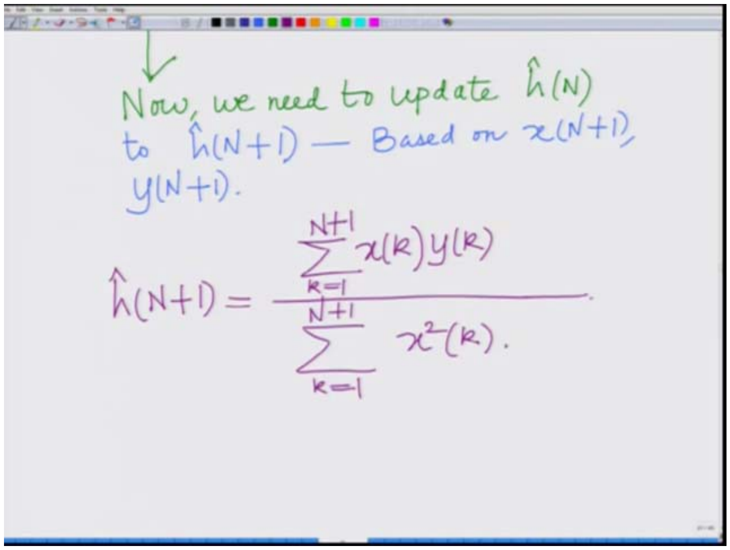

And now you can see basically, it is very simple to see that the pilot estimate at time instant N + 1. Let us first write it in this way, h hat of N + 1 equals submission k equal to 1 to N x k + 1, it is very similar to what we have written at time instant k submission k equal to 1 to N instead of.

I am sorry k equal to 1 to N x k times y k, k equal to 1 to N + 1 k equal to 1 to N + 1 x square of k and in fact this is something that we have it is very similar to what we have written here.

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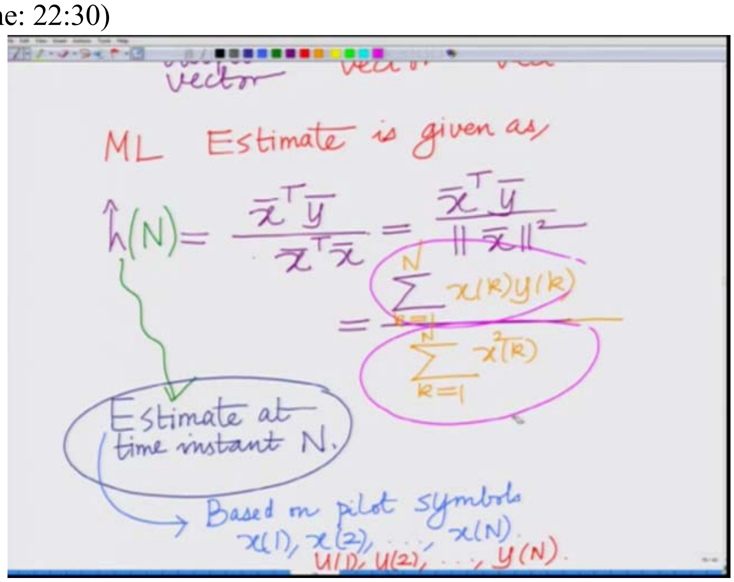

For instant at time instant N, we have submission of k equal to 1 to N x k y k guided by submission k equal to 1 to N x square of k. Now instead of N basically all we are doing at time instant N + 1 is we are changing the index.

Changing the limit on the submission to N + 1 that is we are including now N + 1 transmitted pilot symbols and N + 1 received output symbols. However, this is not the right approach to that because we are computing again the entire estimate remember the whole idea was to use h hat of N that is the estimate at time instant N and P of N, the variance of the estimate at time instant of N, rather than recomputing the entire estimate at time instant N + 1.

And that is what we are slowly going to do, that is what we want to do is basically we want to re, we do not want to recompute this estimate at time instant N + 1, we want to intelligently use the already existing the already computed estimate that is h hat of N at time instant N and that is what we would like to do, what we would like to do is rather than recompute the entire estimate update, we would like to update estimate based on your h hat of N and P of N.

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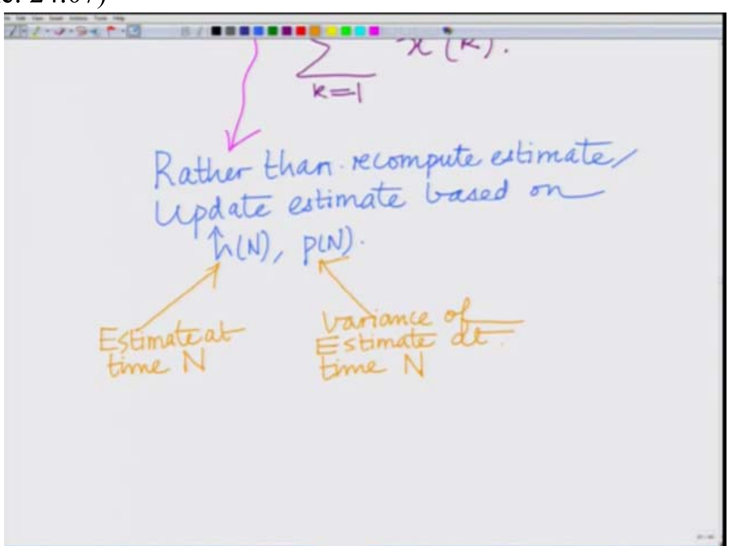

H hat of N is the estimated time instant N, this is your estimate at time instant N and P of N is the variance of the estimate at time instant N. So what we are doing is basically we have we have computed the straightforward right estimate, which is basically trying to recomputing the entire estimate at time instant N + 1, but that is not we want to do, we want to in fact intelligently update the previous estimate.

That is h hat of N at time instant N and using the variance P of N at time instant alright. So in this module what we have done is we have introduced this new concept this new framework of sequential estimation where we are not computing a single estimate, but rather we are updating the estimate at every time instant.

We are looking at it, we have introduced the model for this in the context of wireless fading channel estimation, when you consider the time instant h hat of N or the fading channel coefficient h at time instant N and the variance P of n at time instant N and subsequently we have also considered physically the transmission of a pilot symbol x of N + 1 and the corresponding received observation or measurement y of N + 1.

And using this we would like to now basically compute the sequential estimate h hat N + 1 at time instant N + 1, alright. This we will do in the subsequent module, thank you very much.