Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks

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Lecture Number - 36

Example - Frequency Domain Equalization (FDE) for Inter Symbol Interference (ISI) Removal in Wireless Systems - Estimation in Frequency/Time Domains

Hello, welcome to another module in this massive open online course on Estimation Theory for Wireless Communication. So in the previous model we have started looking at an example for FDE that is Frequency Domain Equalization and we said as the name implies equalisation that is removal of Internet Symbol Interference Is Done in the Frequency Domain.

This is similar to OFDM in the sense that before transmission of the block of symbol, one adds the cyclic prefix. However, at the same time it is different from OFDM in the sense that we are not transmitting the IFFT samples of the symbols rather; we are transmitting the symbols directly after adding the city prefix, okay. So we are looking at an example of FDE which stands for Frequency Domain Equalization.

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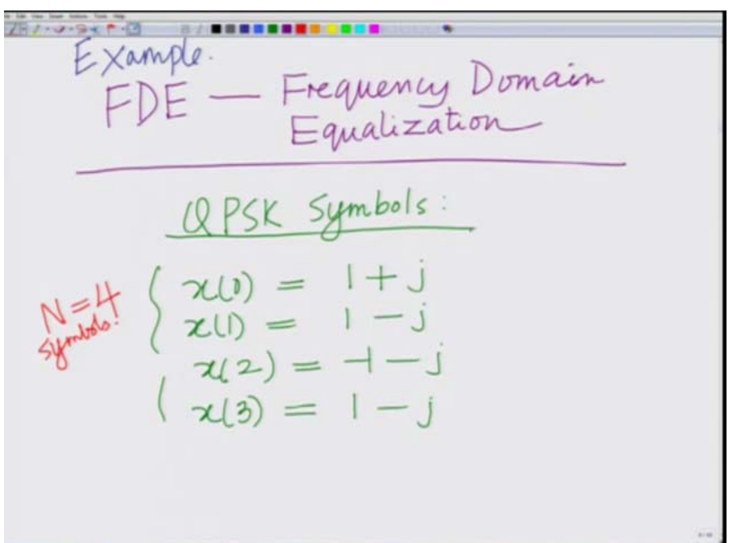

We are looking at an example for FDE that is Frequency Domain Equalization, all right. And we have said we are going to consider QPSK or Quadrature Phase Shift Keying QPSK symbols which are basically x 0 equals 1 + j, x 1 equals 1 - j, small x 2 equals - 1 - j and small x 3 equals basically your 1 - j, these are basically your N equals 4, these are basically your N equals 4 symbol.

And these are QPSK symbols that is, Quadrature Phase Shift Keying symbols. Remember QPSK is constellation in which basically there are 4 symbol; 1 + j, 1 - j, - 1 + j, - 1 - j, right. So these are the this is QPSK Quadrature Phase Shift Keying digital constellation with 4 symbol and we are also considering a system that is an FDE system with N equal to 4.

You can think of this because there are no subcarriers really speaking in an FDE system, so you can think of it as an FDE system in which the block size is capital N equal to 4, okay. And now what we do and as we said, these are directly the symbols in the time domain, not the samples because in OFDM, we transmit samples in the time domain.

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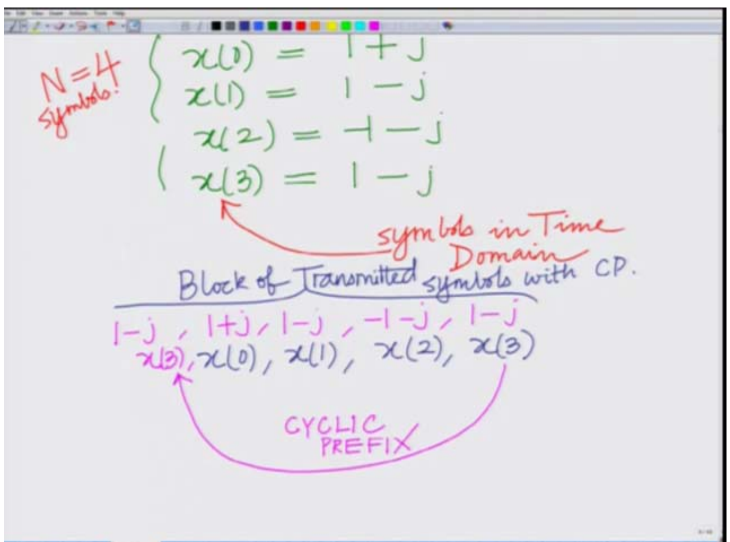

In FDE, we directly transmit the symbols in the time domain. And therefore what we have been basically we have x 0, x 1, x 2, x 3 and to this we add the cyclic prefix that is realise here the cyclic prefix is directly added to the symbols that is, there are no samples, you take the symbols, block of symbols, add the cyclic prefix.

Therefore, the transmitted symbols are given as, now all I have to do is basically x 3 equals 1 - j, x 0 equals 1 + j, x 1 equals 1 - j, x 2 equals - 1 - j and x 3 is again equal to 1 - j. And what is this; this is basically your block of transmitted symbols with cyclic prefix. This is the block of your transmitted symbols with city prefix. And this is transmitted across the 2 tap ISI channel.

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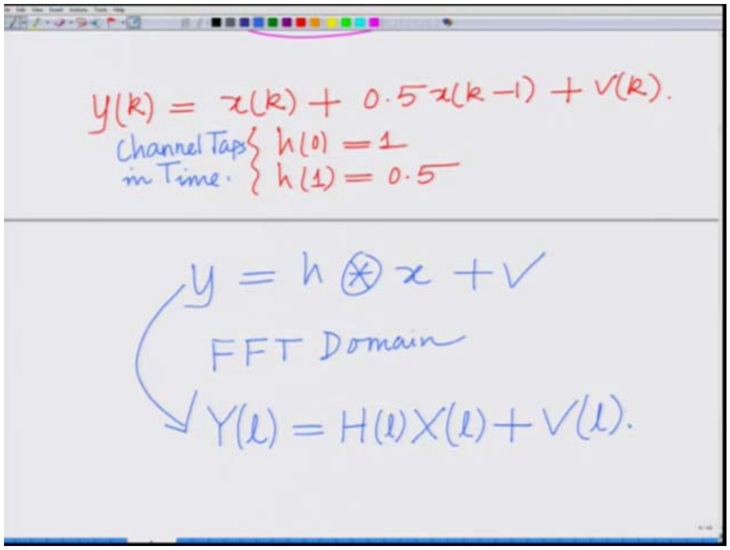

y k equals x k + 0.5 x k - 1 + v k, these are the, this is the 2 tap ISI channel with the tap h 0 equals 1, h 1 equals 0.5, these are the channel taps in the time domain. I hope everyone remembers that right. And therefore now once you transmit the cyclic prefix added block of symbols, right in the time domain. What you get is basically circular convolution of the channel with the transmitted symbols.

And once you take the FFT across the Lth FFT point you get capital Y L equals capital H L time capital X L + V L, all right. So in time we have y let me just write this, we have h circularly convolved with x + v, you take the FFT that is in FFT domain you have capital Y L equals capital H L times capital X L plus V L. Where this is Y L are basically the Lth FFT points of the output samples, yeah.

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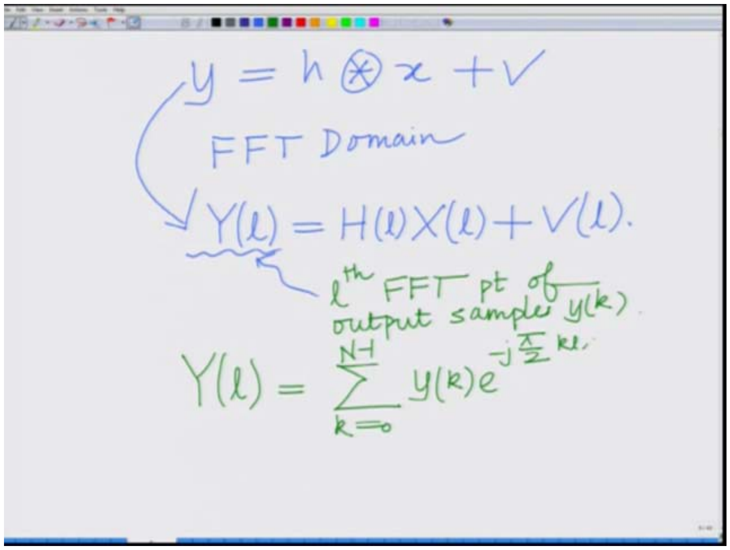

Lth FFT point of output samples, your small y k. Therefore, capital Y of L we have seen yesterday that is given as small L equal to 0 to N - 1 or small rather small k equal to 0 to N - 1 y k e power - j pie by 2 times k L, okay. Now we had also consider for the purpose of this example, the received samples y 0 equals 1, y 1 equals half, y 2 equals half, y 3 equals 1 and what we do now is we take the N equal to 4 point.

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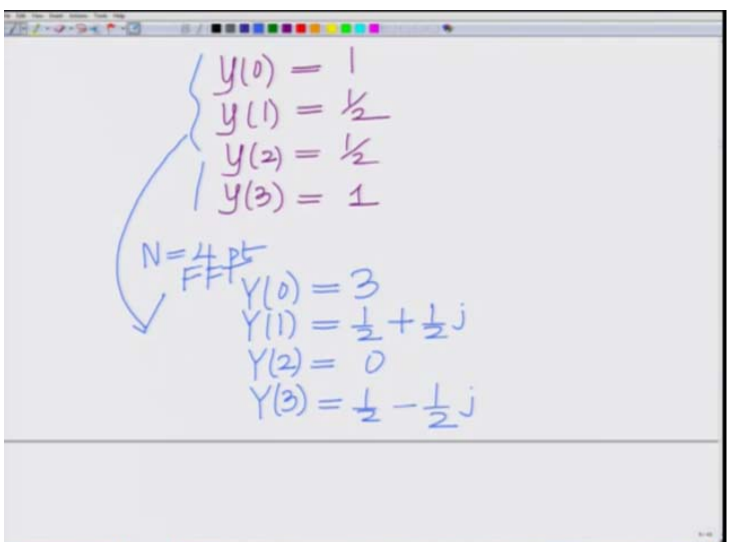

We take the N equal to 4 point FFT or basically DFT, FFT is a fast algorithm to perform the DFT, which is the Discrete Fourier Transform and we said that the samples across the various FFT points are Y 0 equals 3, Y 1 equals half + half j, Y 2 equals 0 and Y 3 equals half - half j, okay. So we have the small y, which are the output symbols in the time domain.

You take the FFT, the N point that is N equal to 4 point FFT, you get the output samples across the various FFT points or you can think of this as a across the various channels in the frequency domain. However, since it is not OFDM, we really do not have the notion of subcarriers, so it is better to simply think of these things as sub channels or simply your FFT points in the frequency domain, all right.

And the capital Hs, they are the channel coefficients across these various sub channels and they are given by the 0 pared FFT of the time domain channel, okay.

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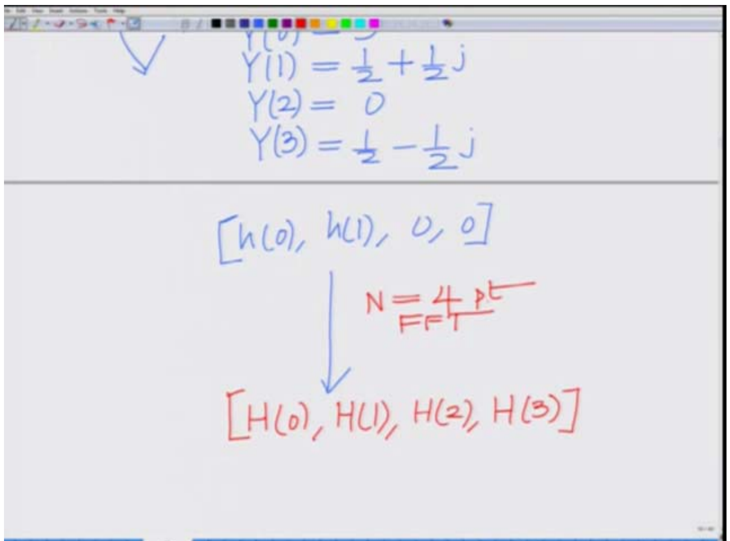

So I have your h 0, h 1, you pad them with 0s and you take the N equal to 4 point N equal to 4 point FFT and you get the channel coefficients in the frequency domain. You get the channel coefficients in the frequency domain, the channel taps are basically your h 0 equals 1, h 1 equals 0.5.

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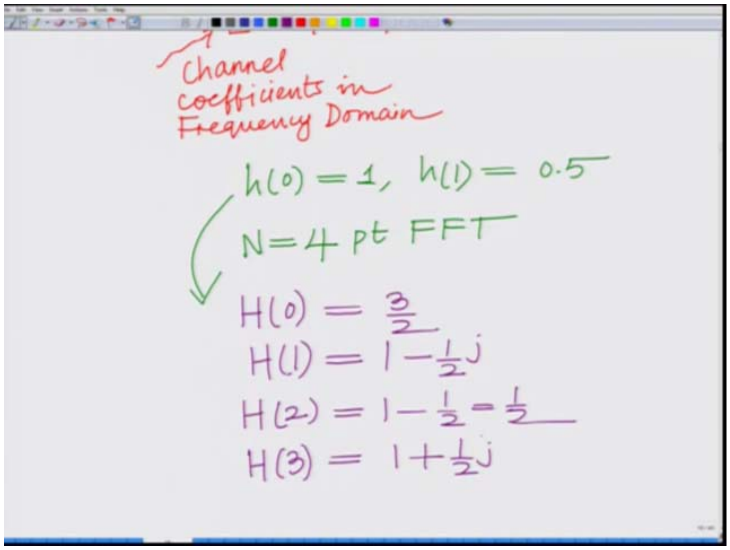

And you take the FFT of this you take the N point FFT of this after padding to, after padding with 0s and what you get is the channel coefficients in the frequency domain across the sub channels are well 3 by 2, H 0 equals coefficients across the 0th FFT channel or 0th FFT point, H 1 equals 1 - half j, H 2 equals 1 - half equals half and H 3 equals 1 + half j.

And now what we said is basically in that was the Lth FFT channel or the across the Lth sub channel we have well we have Y L equals H L X L + V L.

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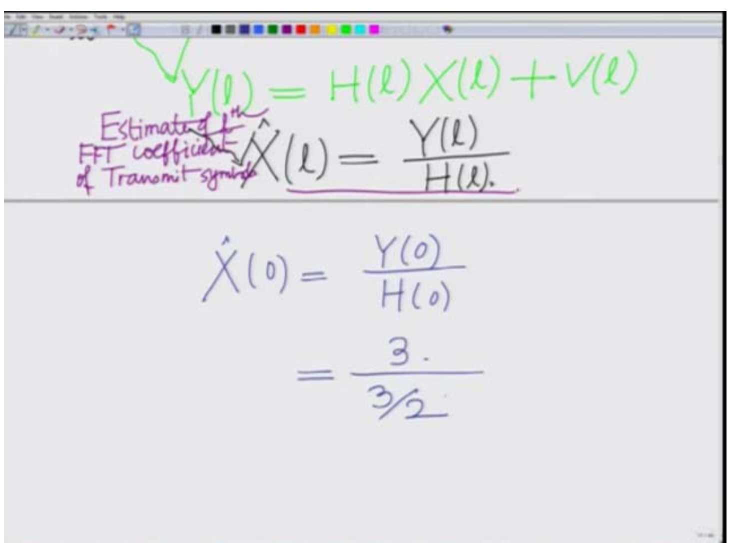

This is across your Lth FFT channel, across let say across Lth sub channel. Therefore, estimate X hat of the Lth FFT coefficient across the Lth sub channel is basically simply Y L divided by Y L divided by H L.

This is estimate what is this; this is basically your estimate of FFT coefficient of transmitted symbols which are the small x, this is estimate of the Lth FFT coefficients, so what if capital X hat L? Right remember, X L is basically the capital X L is basically the Lth FFT coefficient of the transmitted symbols which are the small x small x 0, small x 1, small x 2, small x 3, okay.

So now in the frequency domain, you are estimating capital X hat of L, which is basically an estimate of capital X of L that is the Lth FFT coefficient of the transmitted symbols, okay. So therefore, capital X hat L we have said equals Y L divided by X L, this is the estimate of the Lth FFT coefficient of the transmitted symbols of the transmitted symbols, and we can now calculate the various estimates as follow.

Capital X hat 0 equals Y of 0 divided by capital H of 0 which is equal to well, this is equal to capital Y of 3 that is something that we have already calculated, capital H of 0 equals 3 by 2, you can see this from here, you can see that Y of 0 equals 3, capital H 0 that is equal to 3 by 2 basically therefore, capital X hat of 0 equals Y 0 divided by capital H 0 that is 3 divided by 3 by 2 that is 2.

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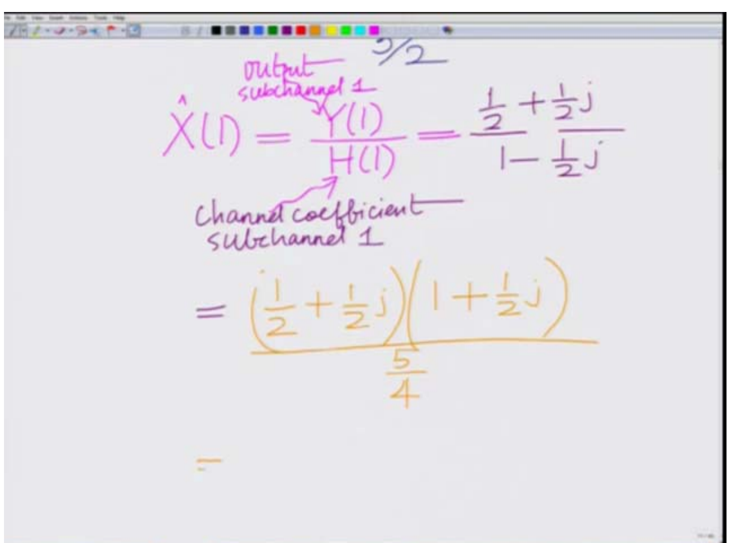

Capital X hat of 1 equals basically your again the very same thing, capital Y 1 divided by capital H 1 this is the Y 1, again to remind you, this is the output across the sub channel L sub channel 1 that is in the frequency domain, this is the channel coefficient for sub channel 1, which is equal to basically half + half j divided by 1 - half j.

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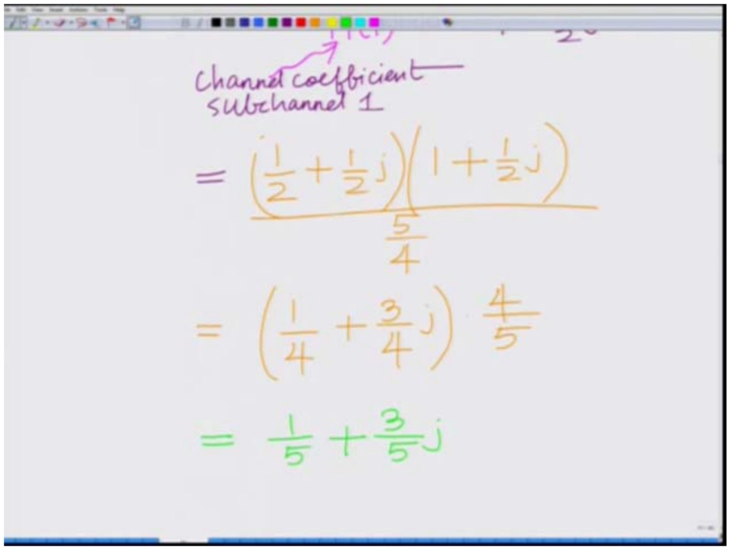

And therefore, this can now be simplified as half + half j times 1 + half j divided by well, divided by 5 by 4 which is equal to 1 by 4 + 3 by 4 j into 4 by 5 and this is therefore equal to equal to 1 by 5 +3 by 5 j, okay. And capital X hat of 2 capital X hat of 2 equals Y 2 divided by capital H 2, but Y 2 remember the output across sub channel 2 is 0 divided by H 2 is half, so this is 0.

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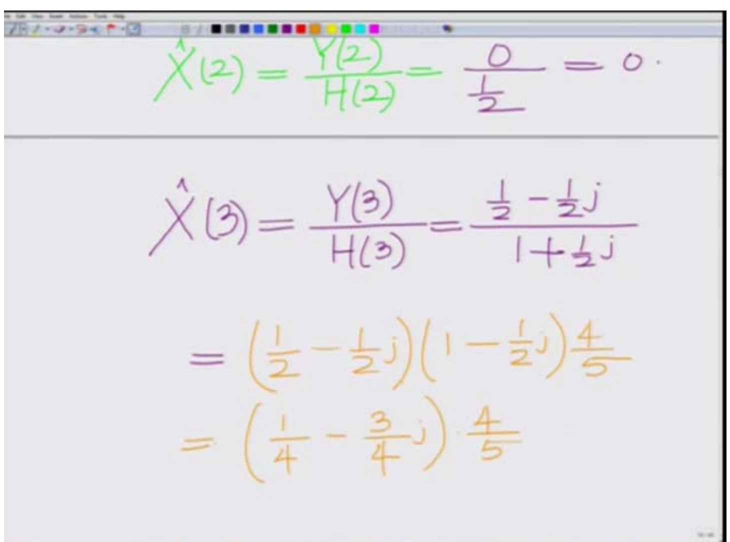

And finally capital X hat of 3 that is the estimate of the FFT coefficient of the estimate of the third FFT coefficient of the transmitted symbols is capital Y 3 divided by capital H 3 which is equal to half - half j divided by 1 + half j equals right and this can again be simplified as using the property of complex numbers as half - half j times 1 - half j into 4 by 5, which is equal to 1 by 4 - 3 by 4 j into 4 by 5, which is equal to 1 by 5 - 3 by 5 j.

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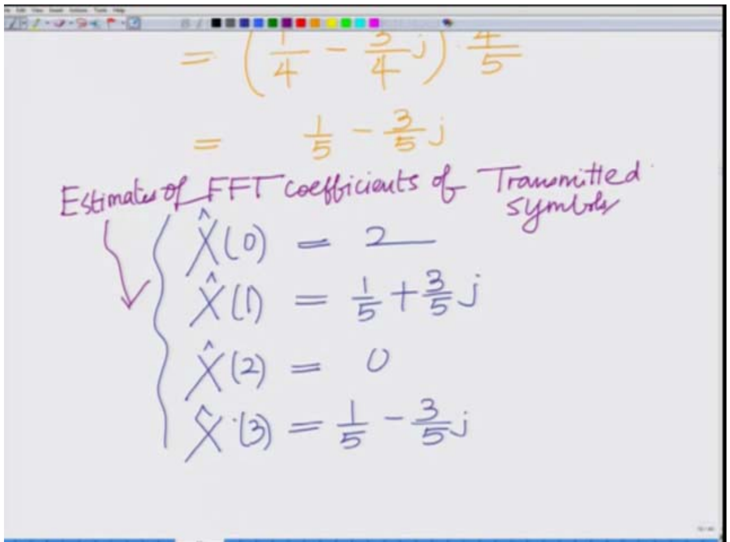

So we have basically completed the estimates of the FFT coefficients of the transmit symbols and these are now to summarise you have capital X hat of 0 equals 2, capital X hat of 1 equals 1 by 5 + 3 by 5 j, capital X hat of 2 equals 0, capital X hat of 3 equals 1 by 5 - 3 by 5 j and what are these?

These are the estimates of the FFT coefficients of the transmitted symbols. These are estimates of FFT coefficients, these are the estimates of FFT coefficients of the we have computed the estimates of the capital Xs, the capital Xs are the FFT coefficients of the transmitted symbols, we have computed the respective estimates that is the capital X hats, okay.

So now we have the estimates of the FFT coefficients of the transmitted symbols, now how to get the transmitted symbols themselves that is estimates of the transmitted symbols that is basically, take the inverse FFT of this estimated FFT coefficients, right. So the capital Xs are given by the FFT of the small x, so if you take the IFFT of the capital Xs, you will get the small Xs, right.

So take the capital X hats, right, the estimates of the FFT coefficients, take the inverse FFT and you get the estimates that a small x hats, that is the estimates of the time domain symbols, okay. And that is basically as simple as that, so basically you have capital X hat of 0, capital X hat of 1, capital X hat of 2, capital X hat of 3, these are estimates of FFT coefficients of your transmitted symbols.

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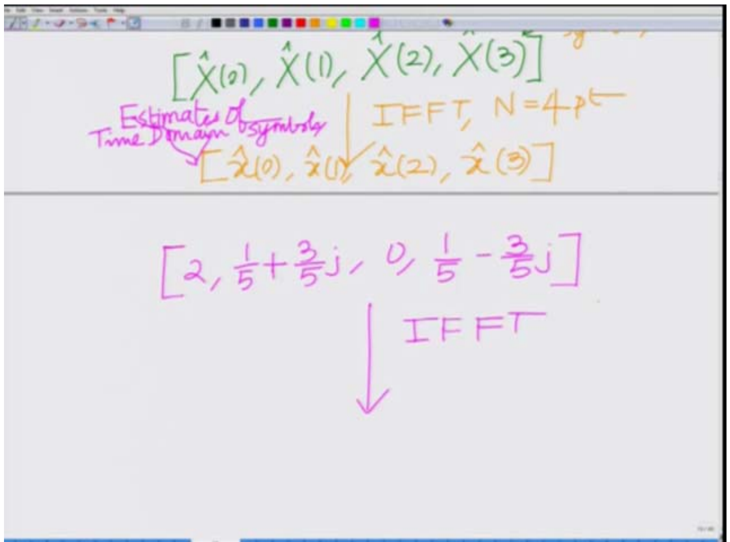

Naturally are the estimates of the FFT coefficients, you take the IFFT that is your N equal to 4 point IFFT and what you will get are the estimates of the time domain transmitted symbols. You get the estimates of the time domain symbols and therefore, let us do that now, so we have basically your X capital X hat 0 is 2, capital X hat 1 is 1 by 5 + 3 by 5 j, capital X hat 2 is 0, capital X hat 3 is 1 by 5 - 3 by 5 j.

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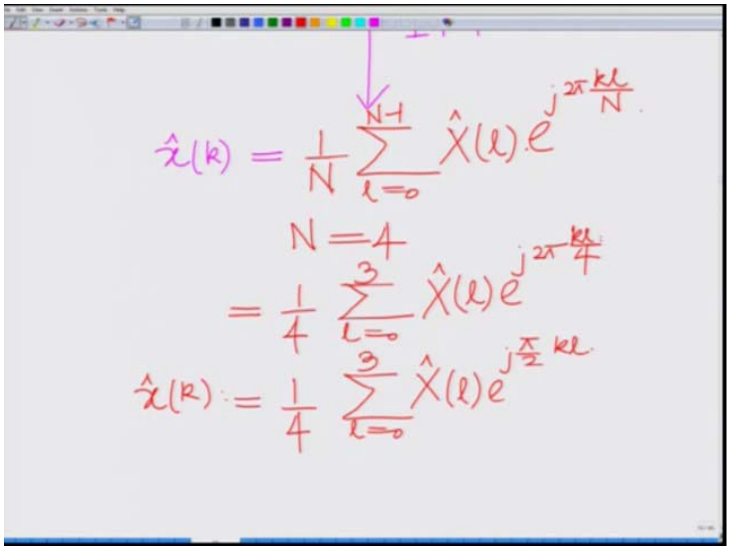

And basically now you take the IFFT of this, that is basically what we are saying is small x hat of k is given the by the IFFT of the capital X hats of L, L equal to 0 to N - 1, capital X hat L e to the power of j 2 pie k L divided by N, substitute N equal to 4 and what we have is basically small x hat of k equals 1 over 4 small L equals 0 to 3 capital X hat of L e power j 2 pie k L divided by 4.

Which is equal to 1 over 4 L equal to 0 to 3 capital X hat of L e power j pie by 2 small x hat of k, okay. Small x hat of k equals 1 over 4 submission L equal to 0 to 3 capital X hat of L e power j pie by 2 times k L, okay. And therefore, now let us compute the estimate of the symbol 0 capital X hat of 0.

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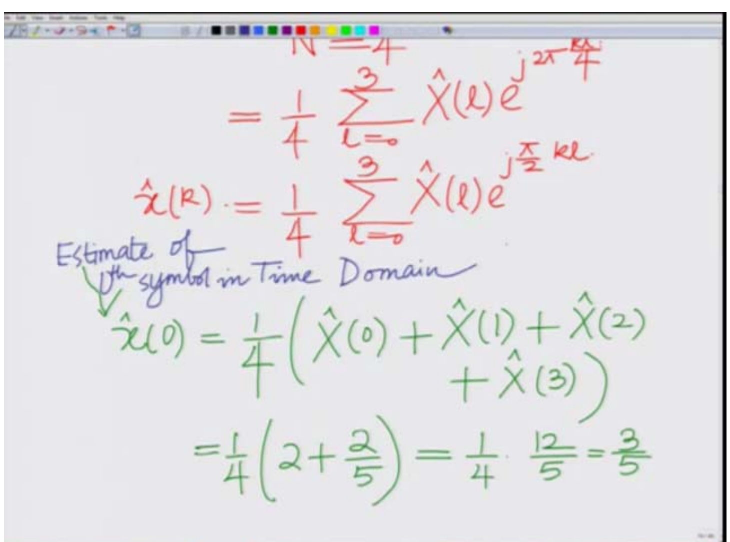

You can simply see is 1 over 4 capital X hat of 0 + capital X hat of 1 + capital X hat of 2 + capital X hat of 3, which is equal to 1 over 4 times 2 + 2 by 5, correct. And therefore, this is equal to 1 over 4 into 12 by 5 which is equal to 3 by 5. That is small x hat 0 which is the estimate of the transmitted symbol 0 or the 0th transmitted symbol in the time domain, right is basically 3 over 5 that is what you have just calculated, right.

This is the estimate of the 0th symbol in the time domain. This is the estimate of this is the estimate of the 0th symbol in the time domain, okay. Small x hat of 1 equals 1 over 4 capital X 0 + x 1 e power j pie by 2 + capital X 2 e power j pie + capital X 3 e power j 3 pie by 2, which is equal to 1 over 4, 2 + 1 by 5 + 3 by 5 + j, 1 over 5 + 3 by 5 j times j + 0 + your 1 over 5 - 3 over 5 j times - j which is basically equal to.

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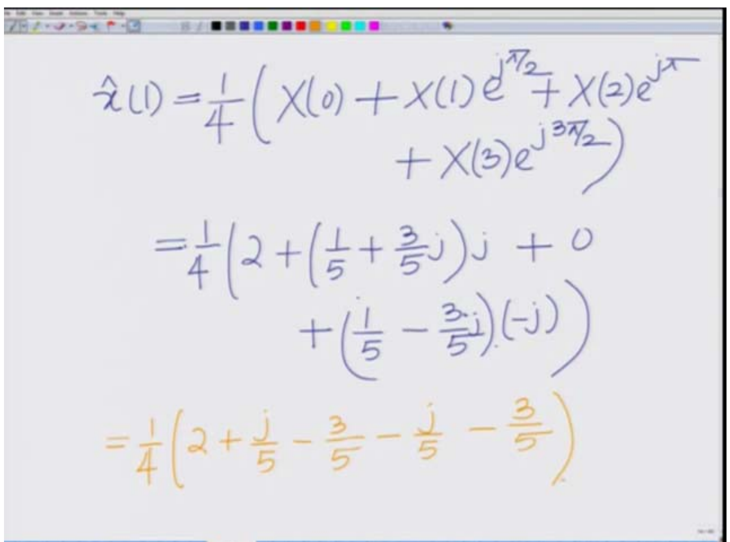

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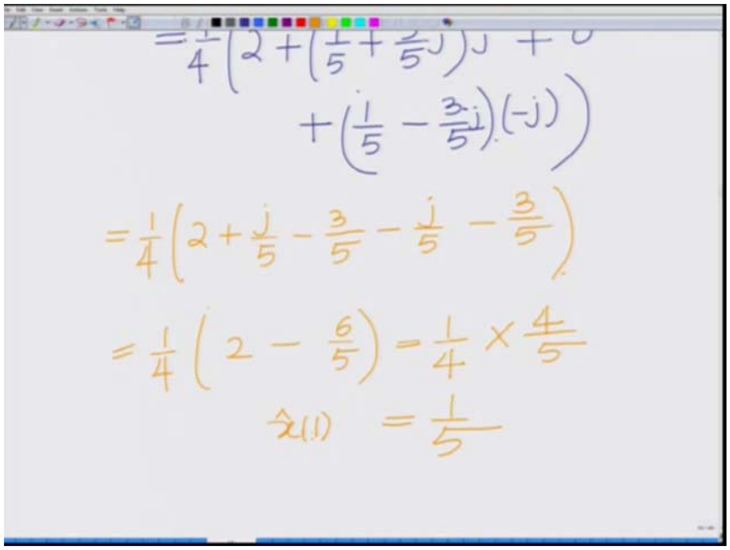

1 over 4, 2 + j by 5 - 3 by 5 - j by 5 - 3 by 5 which is equal to 1 over 4, 2 - 6 by 5 which is equal to 1 over 4 times 4 divided by 5 which is equal to 1 over 5 and this is small x hat of 1, okay that is the estimate of you have computed small x hat of 1, here we have computed small x hat of 0, let me just mark this clearly.

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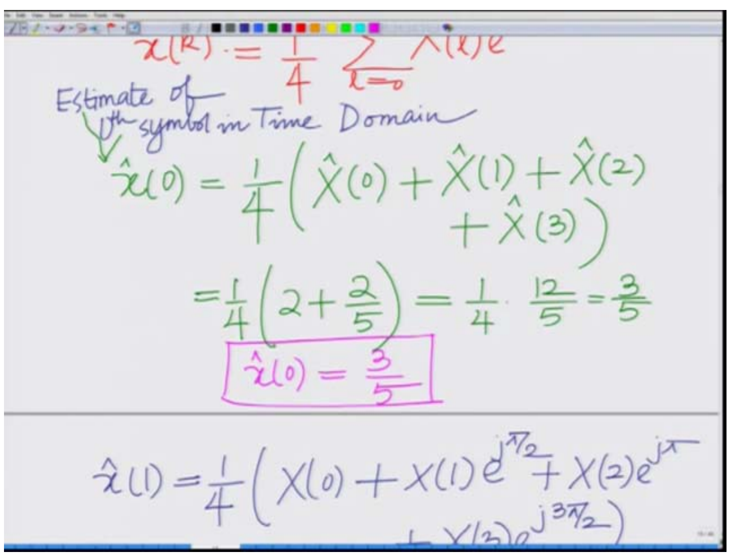

Small x hat of 0 equals 3 by 5 and small x hat that is the symbol that is the small x hat of 1 which is the estimate of the first time domain that is small x hat of 1 that is the time domain symbol x of 1 small x of 1, okay. Let us similarly compute small x hat of 2 that is the estimate of the time domain symbol small x of 2, so small x hat of 2 equals 1 over 4, again simply substituting in that IFFT relation that we have.

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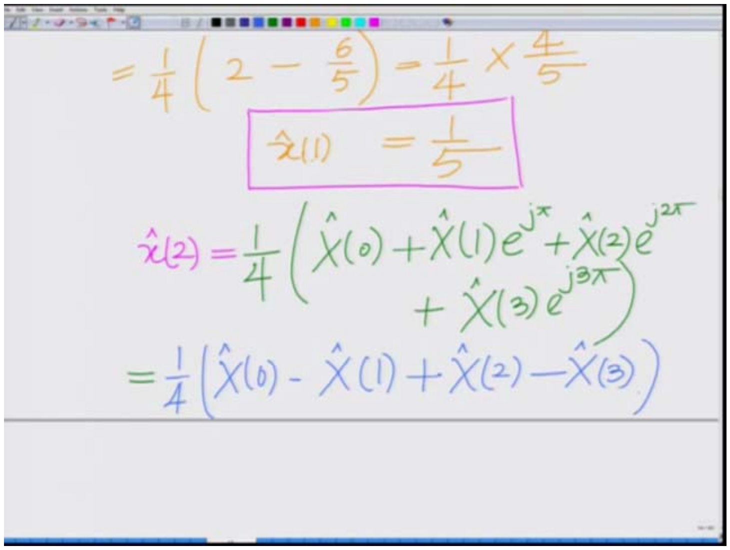

Capital X hat of 0 + capital X hat of 1 just making sure that we have written the estimates, these should be estimates because what we are working with are really the estimates capital X hat of 3, so this is X hat of 1 into e power j pie + X hat of 2 e power j 2 pie + capital X hat of 3 into e power j 3 pie, which is equal to your 1 over 4 capital X hat of 0 - capital X hat of 1 + X hat of 2 - capital X hat of 3.

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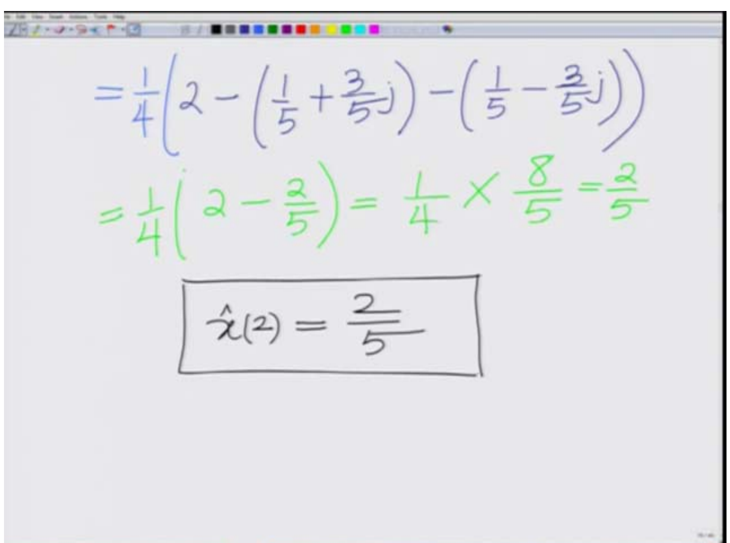

Which is equal to basically; 1 over 4, 2 - 1 by 5 + 3 by 5 j - 1 by 5 - 3 by 5 j which is equal to 1 over 4, 2 - 2 by 5 which is 1 over 4 times 8 divided by 5, which is equal to 2 by 5. Therefore, small x hat of 2 estimate of symbol small x of 2 equals basically 2 by 5 so far, we have computed small x hat of 0, small x hat of 1 and small x hat of 2 and now all that remains is to compute small x hat of 3.

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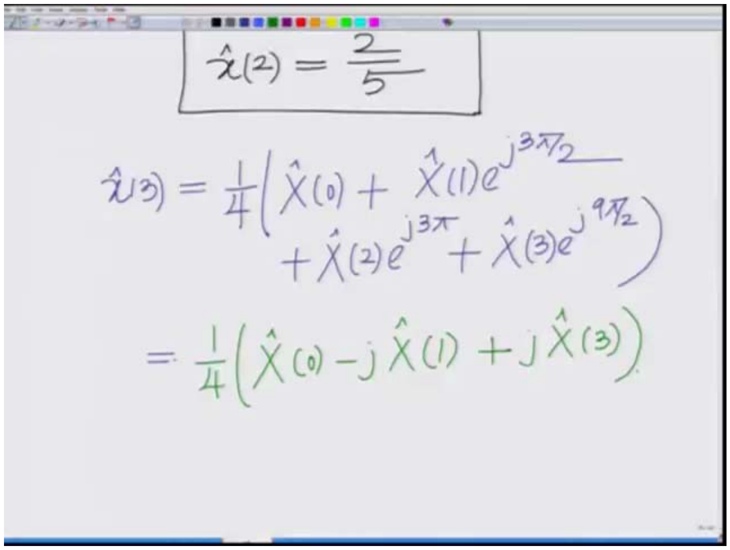

And that can also be computed similarly, let us just complete that small x hat of 3, which is the estimate of time domain symbol small x of 3 is 1 over 4 well X capital X hat of 0 + X hat of 1 e power j 3 pie by 2 + capital X hat of 2 e power j 3 pie + capital X hat of 3 e power j 9 pie by 2, which is equal to your 1 over 4 capital X hat 0 - j capital X hat of 1 plus j capital X hat of 3 you can see.

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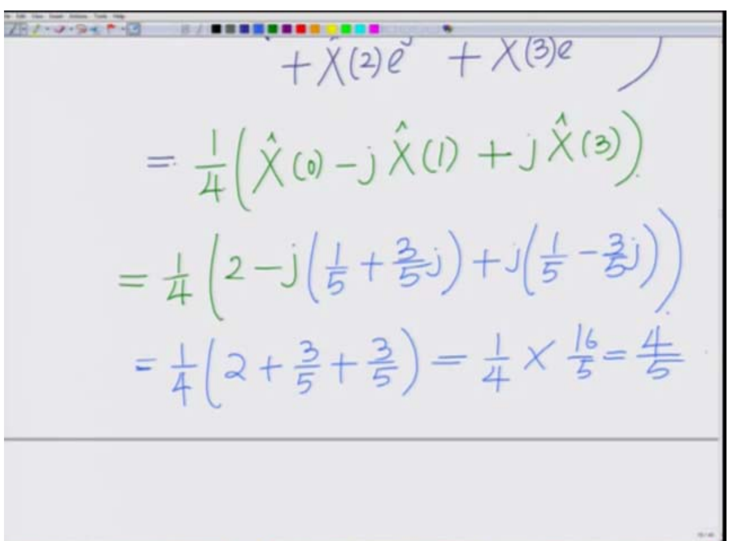

We are avoiding capital X hat of 2 because that is 0 anyway okay. And therefore this is equal to 1 over 4 2 - j times 1 over 5 + 3 over 5 j + j times 1 over 5 - 3 over 5 j, which is equal to 1 over 4 times 2 + 3 by 5 + 3 by 5, which is 1 over 4 times 16 divided by 5 equals 4 divided by 5 therefore, small x hat of 3 equals 4 divided 5, we have computed the estimates of the various symbols in the time domain.

Small x hat of 0, small x hat 1, small x hat 2, small x hat 3, which was the aim, right? So basically this is Frequency Domain Equalisation, right. We have to ultimately compute the estimates of the time domain symbols in the time domain, okay. So now let us write these things down again, let us again writes these things down.

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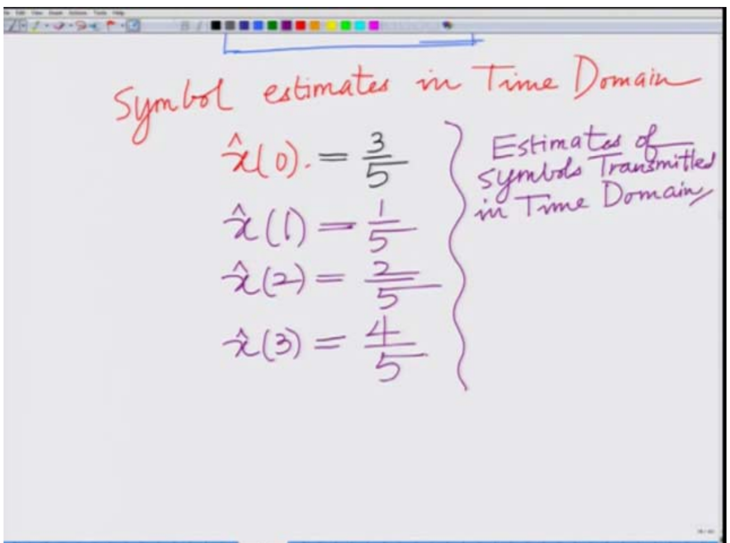

So the symbol estimates finally one can write the symbol estimates in the time domain, so the symbol estimates in the time domain are given as small x hat of 0 equals 3 over 5, small x hat of 1 equals 1 over 5, small x hat of 2 equals to over 5 and small x hat of 3 equals 4 over 5, so these are the estimates of symbols transmitted in the time domain, estimates of the symbols transmitted in the time domain.

And that completes your FDE which is basically the frequency that completes this example of new FDE, which is Frequency Domain Equalisation. And as you have seen, which is the same point that we are stressing since the beginning that is similar to OFDM, at the same time it is different. Similar in the sense, we add the cyclic prefix prior to transmission.

However, it is different from OFDM because we are not transmitting the IFFT samples of the symbols rather we are transmitting directly the cyclic prefix added symbols. So the previous models, we have looked at the theory of FDE which is Frequency Domain Equalisation and in this, we have completed a simple example which illustrates how is the transmission and how is the equalisation done in the frequency domain.

Followed by basically how do you reconstruct the estimates or how do you find the estimates of the transmitted symbols in the time domain, all right. So this comprehensively difficulty explains or elaborates on this paradigm of Frequency Domain Equalisation through an example, all right. So please go through this again to understand it completely and thoroughly all right.

And so we will stop this module here on Frequency Domain Equalisation and I would like to also remind you that Frequency Domain Equalisation is basically much more its preferred compact to Time Domain Equalisation because of its low complexity. Basically, it involves only IFFT similar to OFDM it involves only IFFT and FFT operation.

But there is no matrix inversion unlike the time domain equaliser or basically the 0 4 sync equaliser which actually involves the computation of a matrix inverse okay. So we will stop this module here and we will explore other aspects in the subsequent modules, thank you very much.