Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks

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Lecture Number 35

Example - Frequency Domain Equalisation (FDE) for Inter Symbol Interference (ISI) Removal in Wireless Systems - Transmission Structure.

Hello, welcome to another module in this massive open online course on Estimation for Wireless Communication Systems. So currently we are looking at Frequency Domain Equalisation. What is abbreviated as FDE, alright? So in the previous model we have looked at the theory of FDE which is frequency which is Frequency Domain Equalisation FDE, right, this is FDE.

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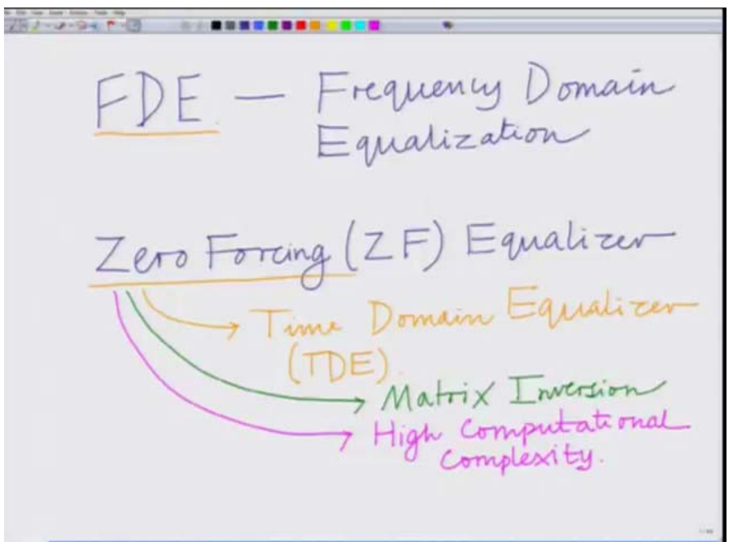

And this basically an alternative to what we have seen previously that is your 0 4 sync equaliser which is a time domain equalisation technique. So previously we have all seen the 0 4 sync equaliser. And this we can think of 0 4 sync equaliser as basically a time domain equaliser yeah. That is also what we said in the time domain equaliser or basically TDE.

And the disadvantage of Time Domain Equalisation is that we said it has matrix inversion, it involves remember 0 4 sync 0 4 sync equaliser involves matrix inversion therefore, it has a the time domain equaliser has a high computational complexity. It has a high computational complexity. On the other hand, when you look at Frequency Domain Equaliser Frequency Domain Equalisation is done in the frequency domain as the name applies.

And as we have seen in the previous model where we have described this scheme, it involves a simple computation in the frequency domain where basically you compute the estimate of the coefficient X hat L on each subcarrier L, alright. So basically it does not involve any matrix inversion and therefore it has a very low complexity which means it can be implemented efficiently.

So just to summarise FDE again all though you have seen it explicitly in the previous model where we have where we have derived or describe this technique FDE which is basically does not involve matrix inversion that is inverting that is computing the inverse of a matrix and hence Frequency Domain Equalisation is very efficient. Therefore, it is very efficient in the sense it has a low complexity.

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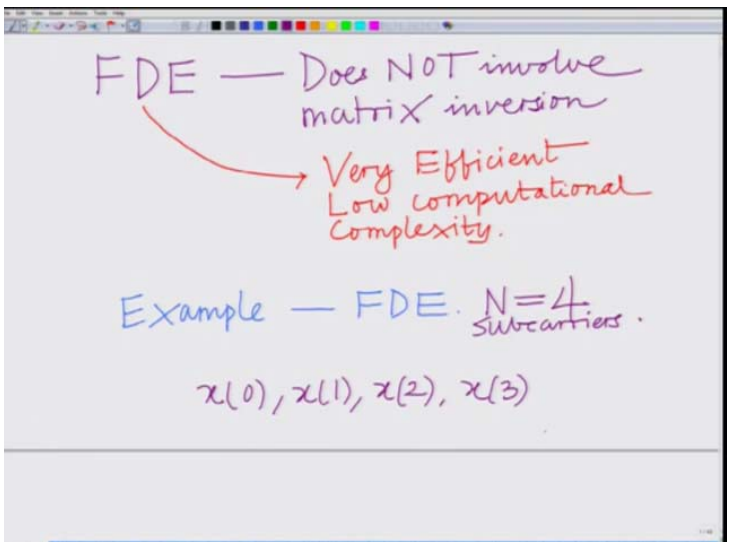

It has a low computational complexity, okay. For instant let us now in this module what we are going to do is basically let us look at an example to understand this Frequency Domain Equalisation better, alright. So in the previous model we have described the technique, in this module similar to past several times we have discussed and schemes.

Let us do an example to understand how this technique works, okay. So what we are going to do in this module is we are going to do an example for FDE or basically all Frequency Domain Equalisation, okay. So in Frequency Domain Equalisation what we said is X 0, X 1, X 2, X 3 again let us consider a system with N equal to 4 subcarriers, okay.

Just to consider a simple system, although it can be extended very easily to a system with a higher number of subcarriers, just to illustrate for the purpose of an example, let us consider a system with N equal to 4 subcarriers. We said that small x 0, small x 1, small x 2, small x 3, these are the time domain symbols, these are not the samples and that is something important to remember, it is an important difference in comparison to OFDM time domain.

Where as in OFDM we are taking the symbols, loading them onto the subcarriers, performing IFFT and transmitting them in the time domain, transmitting the samples in the time domain. In the FDE, we are simply; we are not doing any IFFT at the transfer. We are simply taking the time domain symbols and adding a cyclic prefix and try transmitting them and that is the important aspect that one has to remember in FDE, okay.

So let us take a simple example, let this x 0 which are the time domain symbols these be given as.

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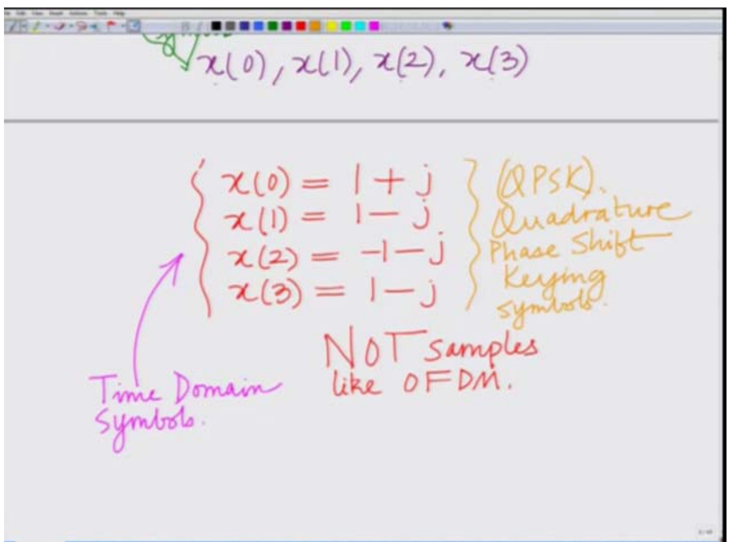

1 + j, x 1 equals 1 - J, x 2 equals - 1 - j and x 3 equals - j, these are your time domain symbols correct these are the time domain symbols. These are the, what are these, these are the time domain symbols, these are not again, these are not just to be explicit, these are not samples like OFDM, okay.

Please remember that these are not the samples and you can clearly see these are QPSK symbols that is Quadrature Phase Shift Keying that is, if you are familiar with communication, these are Quadrature Phase Shift QPSK that is Quadrature Phase Shift Keying what is Quadrature Phase Shift Keying? Quadrature Phase Shift Keying in a complex baseband is a constellation which has basically a real part and an imaginary part.

And the real part can take either of 2 voltage levels that is + 1 and - 1 and the imaginary part can also take either of 2 voltage levels + 1 and - 1 therefore, you have two times 2 that is 4 symbols. One is 1 + 1 j, 1 - j, - 1 + j, - 1 - j. So these are the 4 symbols in QPS, 4 possible symbols out of which you draw the various symbols.

So what we are saying is small x 0 is 1 + j, small x 1 is 1 - j, small x 2 is - 1 - j, small x 3 is 1 - j, these are the time domain symbols, alright. These are the time domain in fact, to be more explicit these are the time domain QPSK symbols all though one can choose any particular modulation, alright that is not a restriction in this case, okay.

And now what we are going to do is again as we have said we are going to take this time domain symbols, take these time domain symbols and add the cyclic prefix, so I have x 3, I take this x 3 and I add this x 3 over here.

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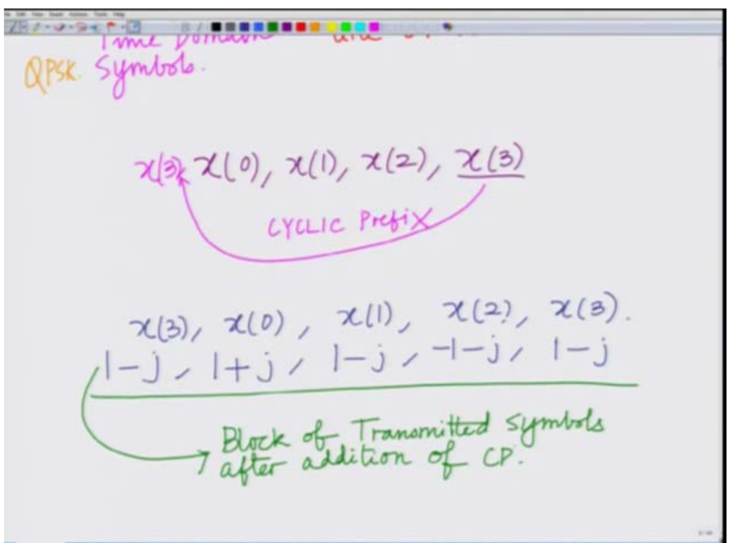

So this is the your basically and we have seen this also many times before, so this is the cyclic prefix. So in the sense that we are adding the cyclic prefix prior to transmission, it is similar to OFDM.

Remember, in OFDM also we add the cyclic prefix however, the important difference is while in OFDM we take the IFFT of the symbols and transmit the samples, in FDE we take the symbols and directly transmit them after adding the cyclic prefix, okay. So now therefore if I substitute this small x 0, what is your small x 0?

So basically let me write this down, so what is the transmitted, your transmitted block is x 3, x 0, x 1, x 2, x 3 and now let us substitute the values, x 3 is 1 - j, you can see from above that basically your x 3 is 1 - j, x 0 is 1 + j, x 1 is 1- j, x 2 is - 1 - j and again x 3 is basically 1 - j and this is the block of transmitted symbols after addition of cyclic prefix. Block of transmitted symbols after addition of the cyclic prefix, okay.

And now let us again consider the 2 tap ISI channel that is, y k remember, this is also something that we have considered many time before. Again we are restricting ourselves to 2 taps because to keep the example simple.

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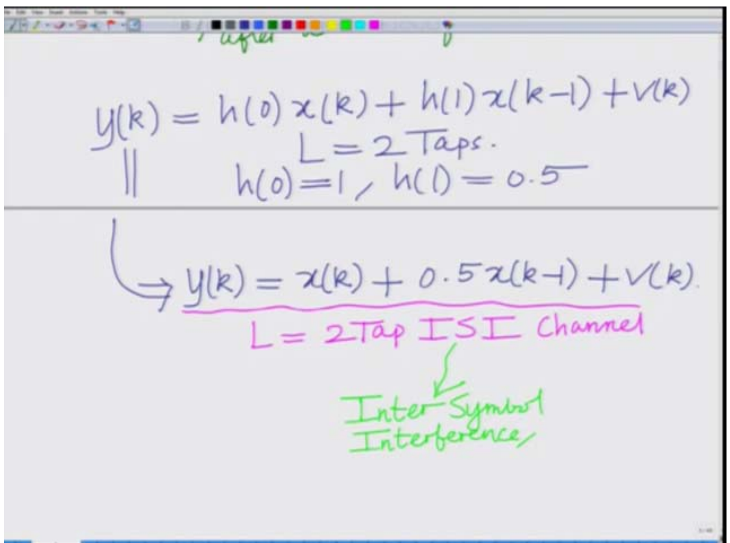

Although, it can be readily extended to a scenario with many more taps or in fact for any general number of taps L, okay. So we are considering again to repeat it with, we are L equal to 2 taps.

For the purpose of this example, let us specifically set h 0 equal to 1, h 1 equal to 0.5, which basically implies your y k equals, h 0 is 1, so it is simply x k + 0.5 times x k - 1 + v k, this is your 2 tap ISI channel, okay. So this is basically your y k equals x k + 0.5 x k - 1 + v k, this is basically your L equal to 2 tap ISI channel or ISI stands for Inter Symbol Interference.

L equal to 2 tap ISI channel where your ISI stands for Inter Symbol Interference. Just being a little explicit here, Inter Symbol Interference, okay. So we have y k equals x k + 0.5 x k - 1 plus v k where v k is of course the noise sample y k is the received kth output symbol, the small y k is received kth output symbol in the time domain, alright.

All the small quantities the small letters basically represent the time domain quantities; the capital letters represent the frequency domain quantities, alright. This is the notation that we have been following all throughout this course. In fact for the previous module also so that so as to keep the notation clear, alright. This is the same thing we did in OFDM and we are once again doing it in Frequency Domain Equalisation.

So small y is the time domain deceived output symbols, small x is the time domain transmitted symbols, small v is the time domain noise sample. And now when I transmit the cyclic prefix added symbols, we know that the channel operation basically the leaner convolution becomes a circular convolution because of the addition of the cyclic prefix.

Therefore, the output y can be represented as.

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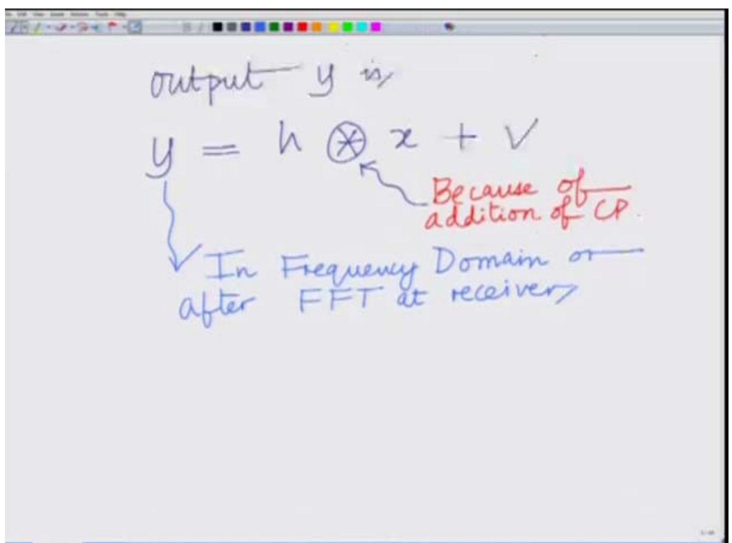

h circularly convolved with x + v, where circular convolution rather than linear it is circular convolution and that is important to keep in mind because of addition because of the addition of the cyclic prefix, this is circular convolution has this senior convolution has become a circular convolution, correct.

And therefore, in the frequency domain or basically after your FFT after the FFT at the receiver this is basically given as now you can see.

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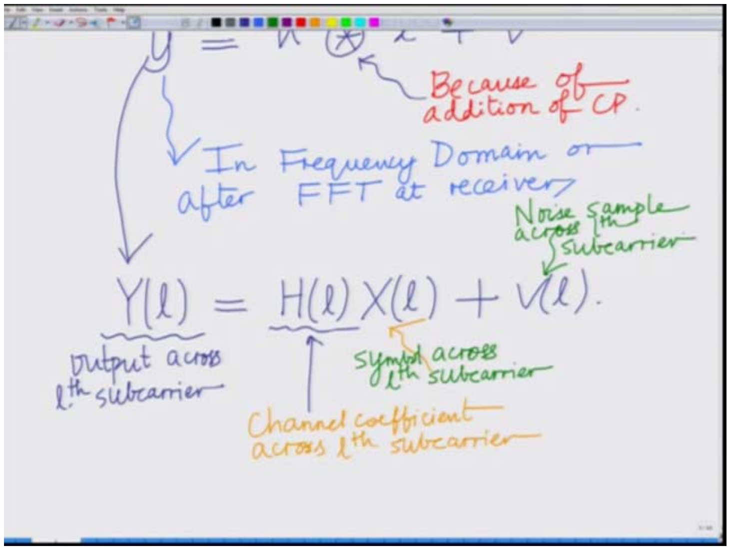

This is basically given as Y of L equals circular convolution becomes multiplication X L + V L and therefore this Y L is the output coefficient output across your Lth from carrier, H L is the channel coefficient across Lth subcarrier, this is the channel coefficient across Lth subcarrier.

Capital H L, this is the symbol across the Lth subcarrier and this is the noise sample across, this is the noise sample across the Lth subcarrier. Now let the output symbols, since we are considering the example, let the output symbol in time domain.

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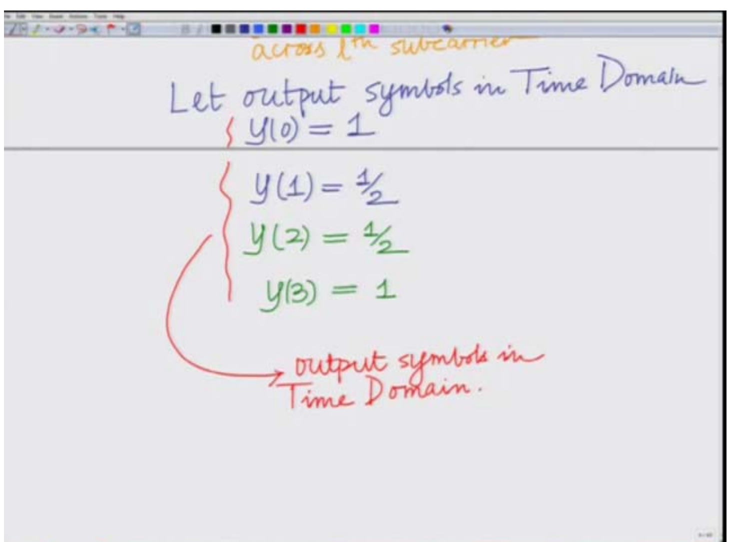

Let the output symbols in time domain be y 0 equals, let us say y 0 equals 1, y 1 equals half, y 2 equals half and y 3 equals 1, these are your output symbols in the time domain.

These are the output symbols in the time domain, okay. These are the small, alright. Now to correspond the um, calculate the corresponding output in the frequency domain remember, we have to take the FFT of these time domain symbols, alright. So the capital Ys are given by the FFT of the small ys. Which means I have to take the N equal to 4 0 point FFT, so the symbols across the subcarriers Y L.

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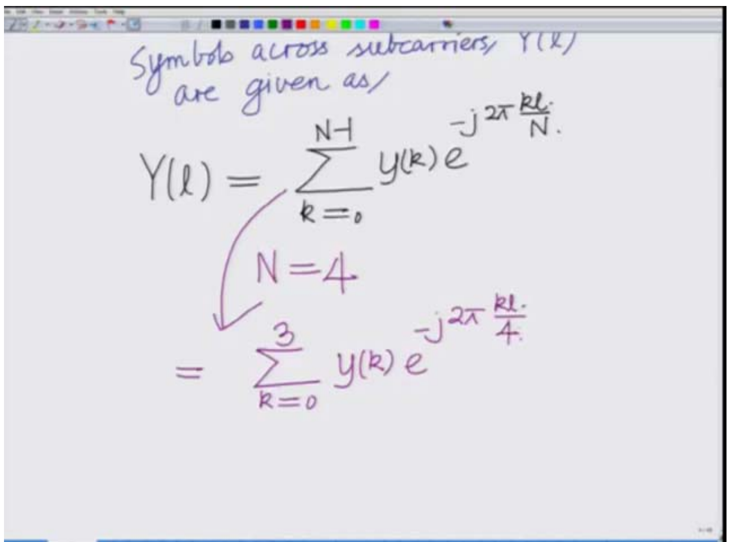

The symbols across the subcarriers Y L are given as; well we can write the expression for that Y L equals well this is similar FFT of the, this k equals 0 to N - 1, y k e power - j 2 pie k L by N and this is the expression for general FFT of length L. So now we have to set N equal to 4 which is the number of subcarriers of frequency points in our system which is equal to.

Submission k equal to 0 to N - 1 that is 3, y k e power - j 2 pie k L by 4 which is equal to submission k equal to 0 to 3 y k e power - j pie by 2 k L, okay. These are the symbols across the symbol across subcarrier L or symbols at Lth FFT 0 point or the sample.

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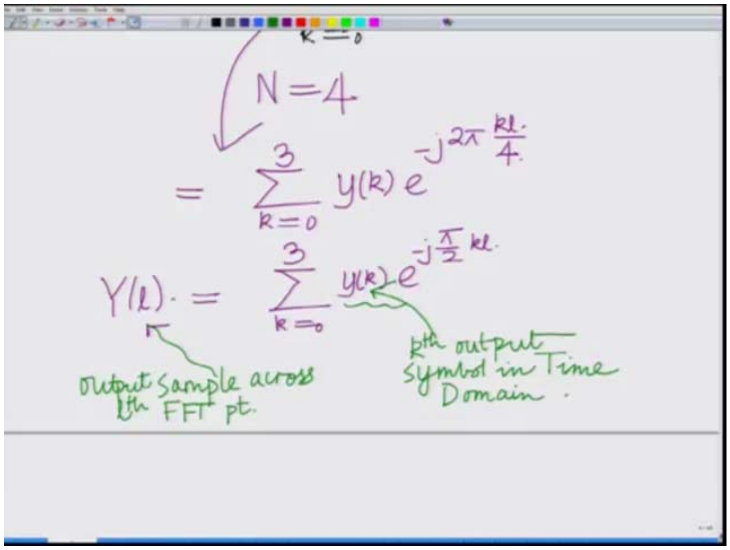

Let us put it the sample, this is in the FFT domain, so let us say this is the output sample, output sample across the Lth FFT point.

These are the for instance, y k is the kth output symbol in the time domain. Small y k is the kth output symbol in the time domain. So small ys are the output symbols in the time domain, capital Ys are the output samples in the frequency domain, you take the FFT of the small Ys and what you will get is the output. That is you will get the capital Ys which are the output samples in the frequency domain across the various FFT points, okay.

And now what we have is, we have already calculated this FFT before, so I am not going to repeat that, y 0, y 1, y 2, y 3, okay which are basically 1, half, half, 1 and we take the N equal to 4 0 point FFT.

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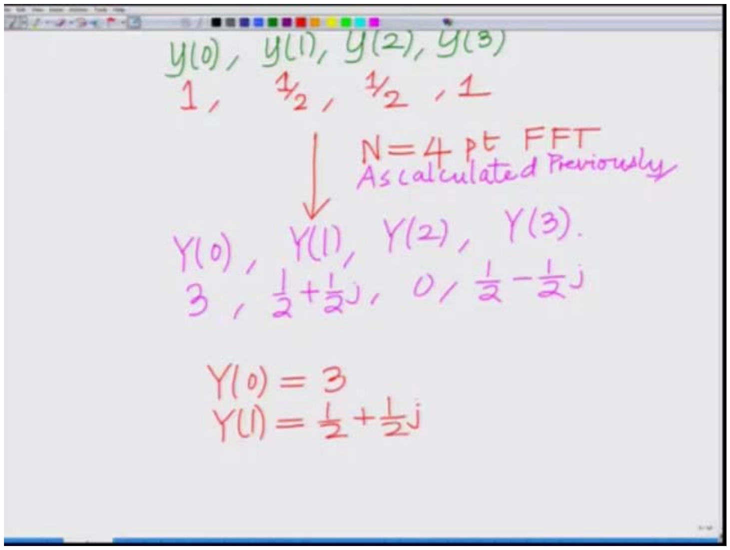

This is something that we have already seen in the context of OFDM, N equal to 4 0 point FFT I am sorry not IFFT, N equal to 4 0 point FFT of the received output samples as I am going to write "as calculated previously".

We have calculated this in the previous module and this you can see is given as your capital Y 0, your capital Y 1, your capital Y 2, capital Y 3 which are basically 3, half + half j, 0 and half - half j. Just to be a little bit more explicit, let me write down Y 0 equals 3, Y 1 equals half that is capital Y 1 which is the output sample across the subcarriers or the first FFT 0 point is half + half j.

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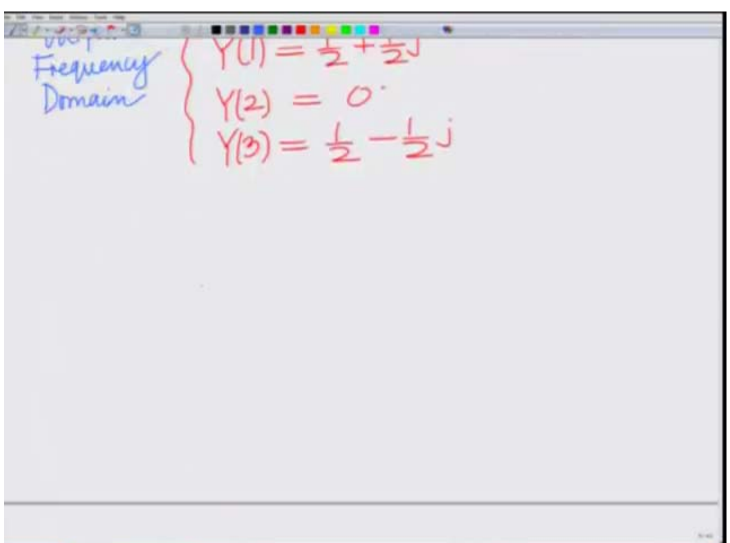

Y 2 equals 0, Y 3 equals half - half j; these are the outputs across the as we have already written over here, these are the output samples across the various FFT points or the output samples across the subcarriers. These are in the frequency domain that is important to remember, the capital Ys are in the outputs in the, these are the outputs in the frequency domain that is something important to remember, okay.

Now what do we need, we also need remember that is the other thing that we need to compute the estimates of across the subcarriers, we also need these capitals H Ls, let me go all the back here and circle this.

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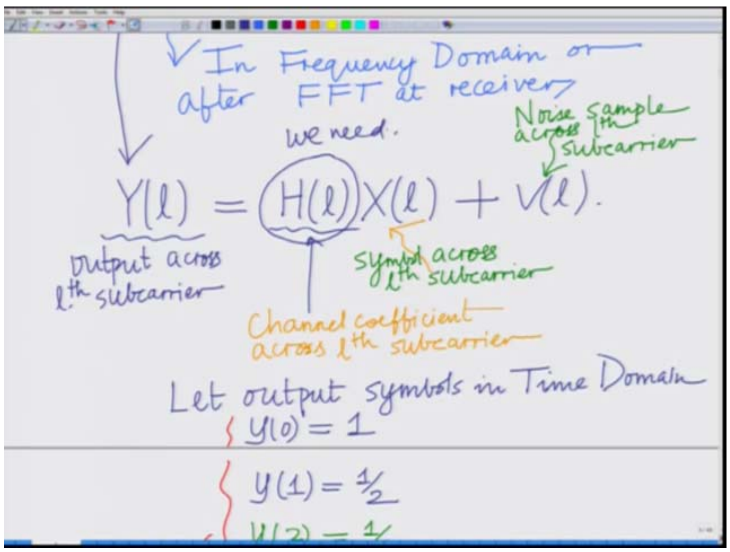

So we need these H Ls basically, what are these capital H Ls, if you remember the capital H Ls are basically the coefficients, the channel coefficient across the various carriers.

And these are calculated by the 0 pared FFT of the channel taps in the time domain, alright I would like to remind you of that.

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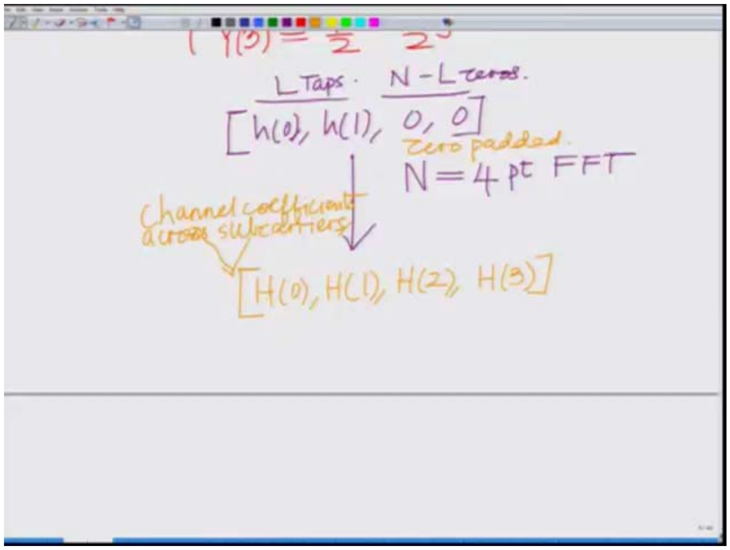

So you take your 0 parrot channel taps that is, h 0, h 1, pat them with 0 that is N - L 0s, you have L taps, which means you are going to have N - L 0s for a general scenario. In this case N equal to 2, L equal to 2, so you have 2 taps and N - L, N equal to 4, so you have two 0s, alright.

And you take the 0 pared FFT, N equal to 4 0 point FFT and just to keep in mind that this is the 0 pared FFT and what you get are the channel coefficients across the, what are these; these are the channel coefficients, these are the channel coefficients across the subcarriers. Now how do we get this? Therefore, if I write the expression for H L

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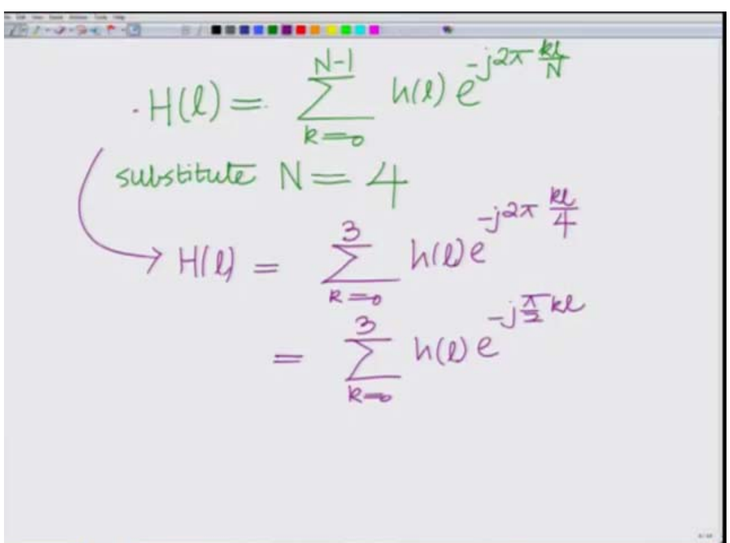

I can write H L equals submission k equal to 0 to N - 1, the time domain channel taps times e power - j 2 pie k L divided by N, substitute N is equal to 4, let me write this explicitly, substitute N equal to 4 and therefore we have H of L equals k equal to 0 to 3 j 2 pie k L divided by 4, which is equal to k equal to 0 to 3 H L e power - j pie by 2 k L.

And in fact, you can write explicitly because although we are taking the submission from k equal to 0 to 3, it is a 0 pared FFT there are only 2 channel taps, so I can write k equal to 0 to 1 in fact yeah.

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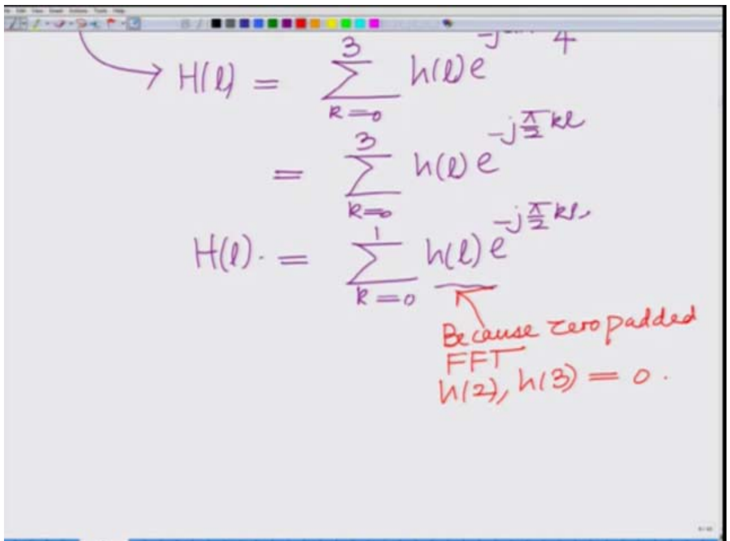

And that is something that you can remember because this is a 0 pared FFT, why? Because this is 0 pared FFT and h 2, so we have h 0 and h 1 and h 2, h 3 are really speaking 0 point.

So let this channel taps, one way to think about this 0 pared FFT is to basically say that only h 0, small h 0 and small h 1 are nonzero, while small h 2 and h 3 that is the second and third channel taps are 0 okay, that is the same thing okay. Now let us compute the FFT, now we know and we have already done this also before, but let us just do this for the sake of completeness, we know that the channel tap h 0 equal to 1, h 1 equals 0.5.

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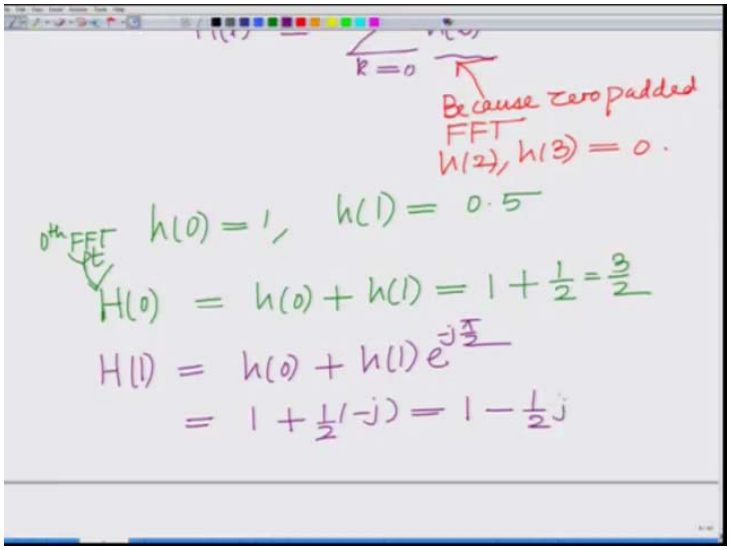

Therefore, capital H 0 that is the 0th FFT point, this is equal to h 0 + h 1 equals 1 + half equals 3 by 2. The capital H 1 equals small h 0 + small h 1 e power - j pie by 2 which is 1 + half into - j equals 1 - half j. Capital H 2 that is the channel coefficient across the second FFT 0 point or the second subcarrier equals H 0 + H 1 e power - j pie equals basically 1 - half equals half.

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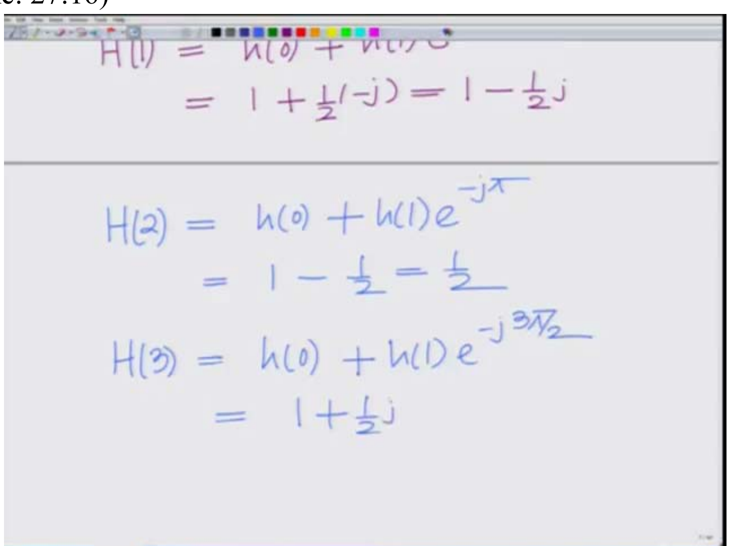

And capital H 3 is basically your capital H 3 is the channel coefficient across the third FFT 0 point or third subcarrier, this is small h 0 + small h 1 into e power - j 3 pie by 2 equals 1 + half j and therefore now just writing all these things explicitly.

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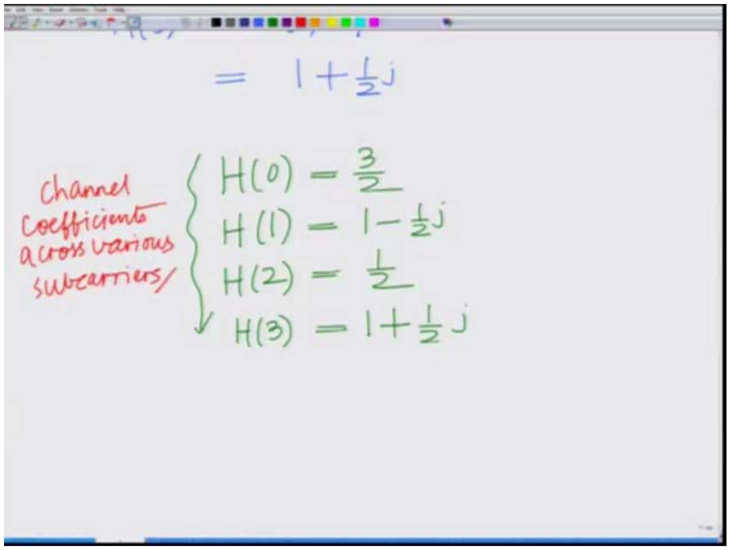

Once again you have capital H 0 equals, capital H 1 equals 1 - half j, capital H 2 equals half, capital H 3 equals 1 + half j, okay.

So these are the channel taps, these are your channel coefficients, so these are the channel coefficients across the various subcarriers, okay so we have the capital Hs okay. And now we have to do is basically now if you realise what we have in the frequency domain across the Lth subcarrier.

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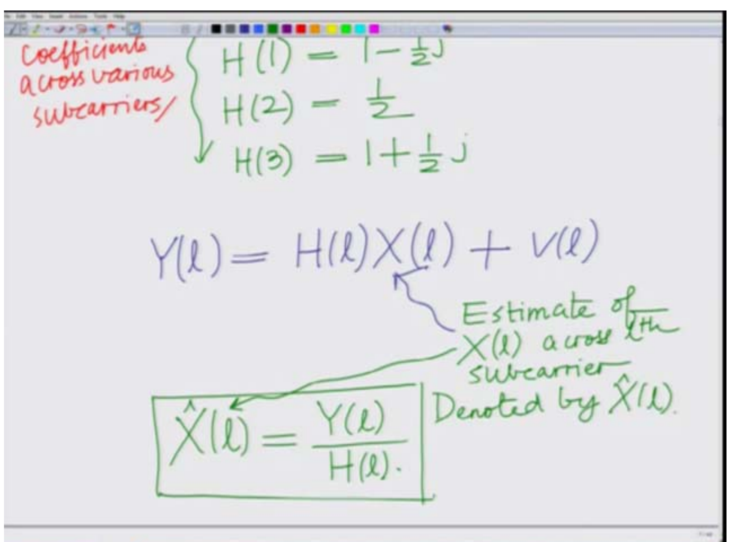

We have Y L equals H L times X L + V L. And we have to compute the estimate of the coefficients across the Lth subcarrier, right.

That is what we do in FDE that is estimate of X L across Lth subcarrier and that is denoted by your capital X hat of that is denoted by capital X hat of L. And remember, capital X hat of L equals Y of L divided by H of L; this is how we compute the estimate of X of L across the Lth subcarrier.

That is X L, which is the FFT coefficient, X of L which is the Lth FFT coefficient of the transmitted time domain symbols the small xs. X small x 0, small x 1, small x 2, small x 3, okay. So this we will compute in the next module and also compute the subsequent so these capital Xs are estimated in the frequency domain.

Then you have to perform remember the IFFT to compute the um corresponding estimates of the transmitted symbols in the time domain, okay. So these 2 steps we will do in the subsequent module, so what we have done in this thing is basically we are considering FDE which is Frequency Domain Equalisation, we are demonstrating a simple example for Frequency Domain Equalisation.

We are considering a block of transmitted QPSK symbols, we are considering an N equal to 4 subcarrier system, we are considering the QPSK symbols small x 0, small x 1, small x 2, small x 3, we have demonstrated what the cyclic prefix added block of transmitted symbols is, okay corresponding received output samples, the FFT of the samples, FFT of the 0 pared channel taps, okay.

Now in the next module we have to talk about what are the estimates of the capital Xs that is the FFT coefficients in the frequency domain and finally perform the IFFT to get the estimates of the symbols in the time domain, okay. This we will do in the subsequent module, thank you very much.