Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks
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Lecture Number 34
Frequency Domain Equalisation (FDE) for Inter Symbol Interference (ISI) Removal in Wireless Systems

Hello, welcome to another module in this massive open online course on Estimation for Wireless Communications. So far we have looked at various techniques of estimation basically, various techniques of estimation to overcome Inter Symbol Interference. First we have seen Time Domain Equalisation that is based on 0 4 sync 0 4sync equaliser.

And we have also seen a very low complexity equaliser that is OFDM that is Orthogonal Frequency Division Multiplexing. It is not equalisation, but it is an interesting and very efficient transmission scheme. Not today, we are going to look at yet another scheme which is termed as Frequency Domain Equalisation for ISI that is removing Inter Symbol Interference.

So starting today we are going to look at what is known as FDE stands for Frequency Domain Equalisation.

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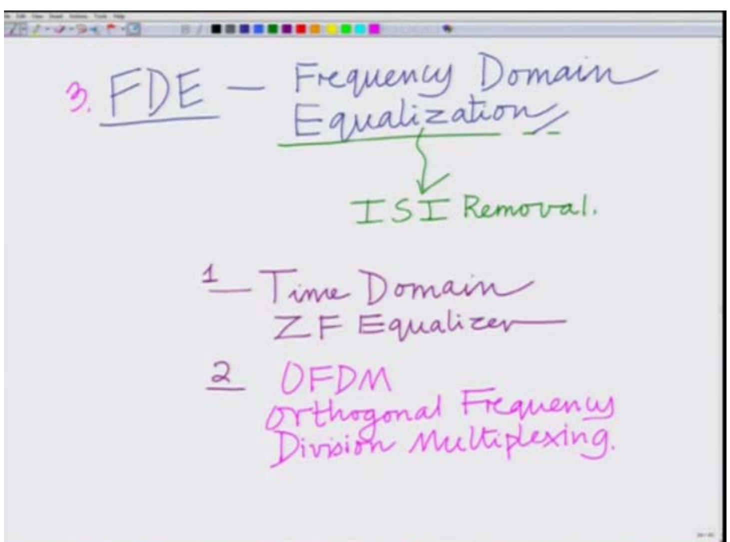

And naturally this equalisation, remember equalisation is where we remove this is for, equalisation is for ISI removal. That is, it is to remove the Inter Symbol Interference. And previously we have seen 2 different techniques that is FDE Frequency Domain Equalisation.

Previously we have seen Time domain that is, equalisation in the time domain that is your 0 4 sync equaliser. This is one of the techniques that we have seen; let us call it technical 1. Technique 2 is basically your OFDM or Orthogonal Frequency Division Multiplexing and this is basically the third technique that we are going to look at now.

That is termed as FDE or Frequency Domain Equalisation and as the name implies, this is where basically equalisation is done in a frequency domain that is, previously we have seen 0 4 sync equalisation valve basically equalisation is done in the time domain, all right. This is now frequency domain equalisation, where equalisation is done in the frequency domain, all right.

And it is very similar to OFDM, although there is a very important very subtle yet very important difference with respect to that of OFDM or Orthogonal Frequency Division Multiplexing. For let us see what is the mechanism for this FDE that is Frequency Domain Equalisation for removal of ISI, okay.

So in FDE that is Frequency Domain Equalisation, so let me just write this again because we are talking about FDE Frequency Domain Equalisation, what we have is basically instead of transmitting loading symbols on the subcarriers, we do not load the symbols on the subcarriers, but we transmit the symbols directly that is x 0, x 1, x 2, x 3 let us say these are the block of symbols, these are the N equal to 4 symbols.

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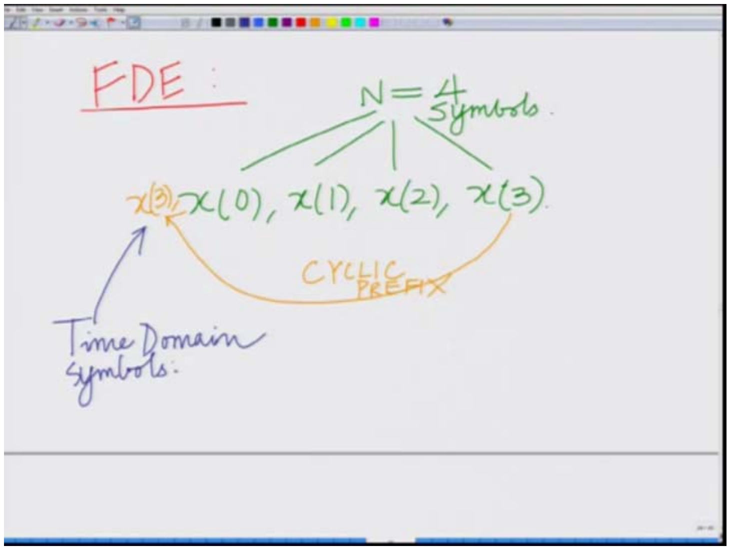

These are not the samples, but these are in OFDM, these are the small x in OFDM are the time domain samples which are generated by the IFFT of the symbols loaded onto the subcarriers. However, in FD we directly trough with the symbols, there is no there is no loading of the symbols on the subcarriers instead of that, the small x, small x 0, small x 1, small x 2, small x 3, these are the symbols.

These are the symbols which are directly transmitted with one modification that is, adding a cyclic prefix. So we add the cyclic prefix, so in that sense it is similar to OFDM that is, take x 3 which is from the end and we prefix it at the head of the block, this is termed as the cyclic prefix, this is something that we have seen several times. And now these cyclic prefix, so these are the time domain symbols.

So now these are not the time domain samples, but these are the symbol that is BPSK or QPSK or so on, basically moderated signals + the cyclic prefix. So the cyclic prefix added block of time domain symbols, now this is transmitted across the, this is transmitted across the ISI channel and remember, ISI stands for Inter Symbol Interference, yeah.

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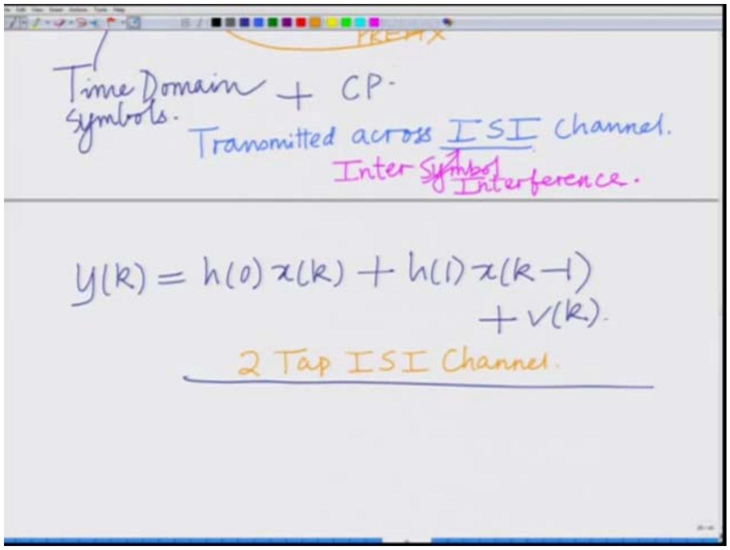

So this is transmitted across the ISI channel remember, our model for the ISI channel is the same, we are considering the 2 tap ISI channel, which is basically your y k current symbol y k current output symbol is h 0 x k + h 1 x k - 1 + v k. Remember, what is this? This is your 2 tap; this is your 2 tap ISI channel or 2 tap Inter Symbol Interference channel, all right. And now when we transmit these symbols, time domain symbols with cyclic prefix.

Of course, the effect of the channel is that of circular convolution therefore, I will have y which is h, the channel filter circularly convolved with x + v. Why does this arise?

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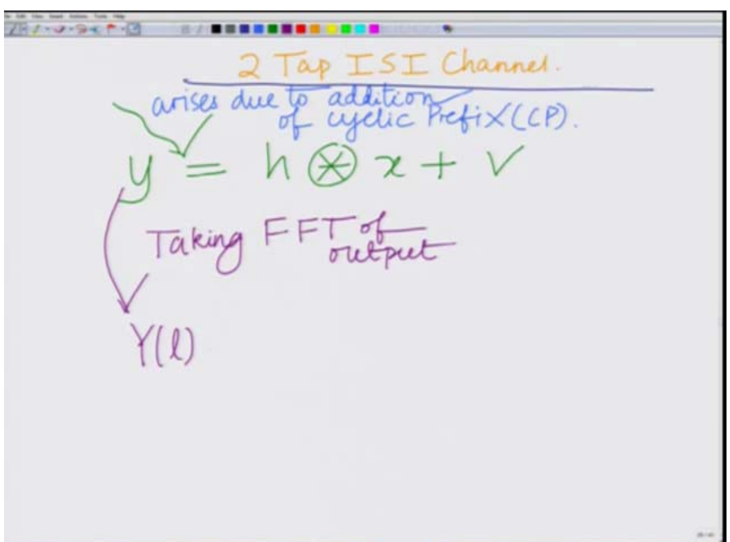

This arises because we are transmitting well arises due to basically addition of; this arises because of the addition of the cyclic prefix. And this is something that we have seen several times before, that is I take a block of sample.

Add a cyclic prefix and transmit it across an Inter Symbol Interference channel, the output y can be modelled as basically the channel h circularly convolved with this block of block of symbols, yeah or samples, whichever is transmitted + in the presence of additive noise, okay. So naturally, once you take the FFT in the IFFT domain across each sub carrier or across each kth frequency point.

It is going to be the multiplication of the channel FFT coefficient and basically the FFT coefficient of these symbols that are transmitted in the times domain. So now once we take the FFT, now once we take the FFT, taking the taking the FFT of the output we have across the Lth subcarrier Y L, that if I look at this across the Lth subcarrier, I have Y L equals H L times X L + V L, what is this?

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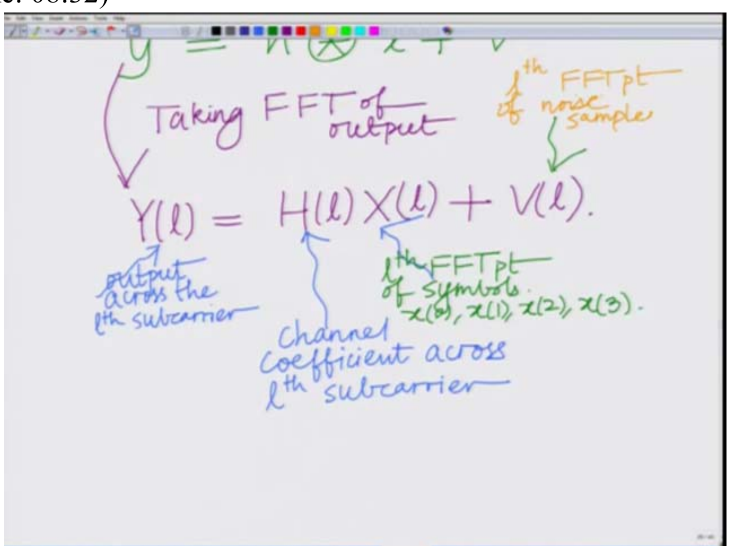

This is across the subcarriers yeah, taking the FFT across the Lth subcarrier in fact, Y L is the output symbol across the Lth subcarrier, H L is the channel coefficient across the Lth subcarrier, X L is basically your symbol transmitted on the that is basically, now this is basically not the symbol on the Lth subcarrier, but this X L is the Lth FFT point of the symbols.

This is simply now the Lth FFT points of symbols remember, because we are transmitting the symbols in the time domain. The Lth FFT point of the symbols x 0, x 1, x 2, x 3 and V L is basically your Lth FFT point of the noise samples, okay. So now let us explore this quantity capital X L again. X L as we said is the Lth FFT point, this is the Lth FFT point of your transmitted symbols.

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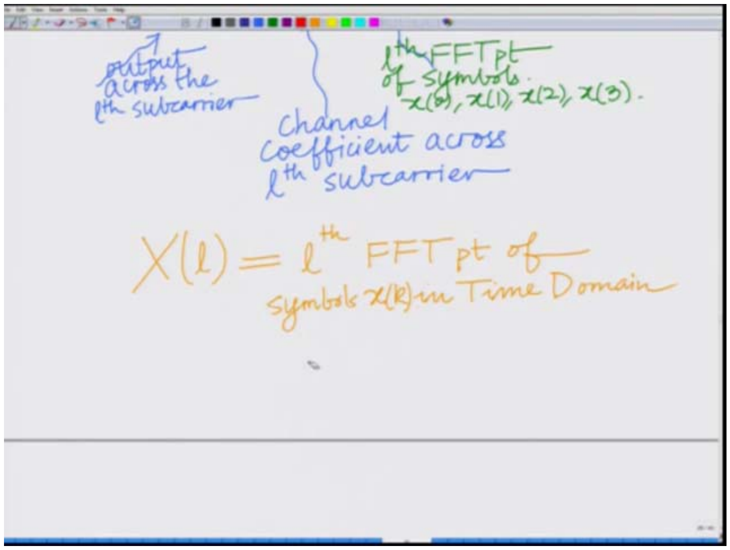

That is the symbols in the symbols that is basically symbols are the x ks yeah, the small x ks, which are the small x ks in the time domain. Which means basically your X L is the N point FFT of that is, how do we generate these X Ls that is basically you have your symbols in the time domain x 0, x 1, x 2, x 3 and you take, now what are these in the context of FDE? These are the time these are the time domain symbols in FDE.

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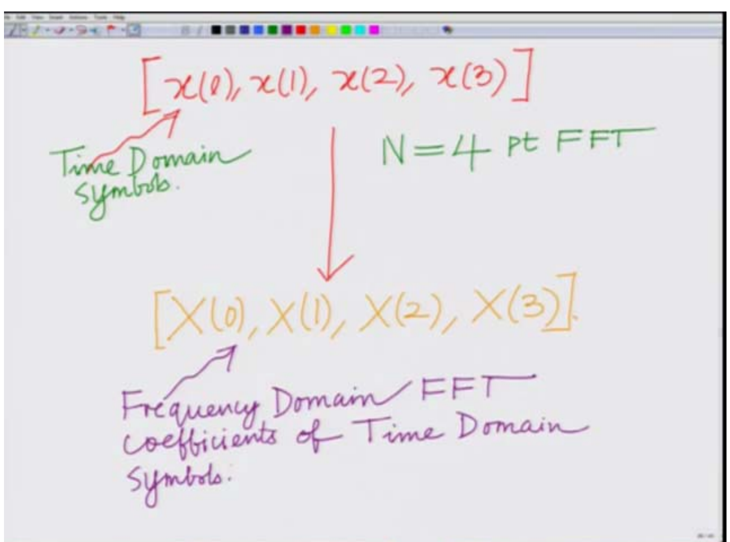

Remember, previously in OFDM, these were the time domain time domain samples and these are generated by the IFFT of the symbols loaded onto the subcarriers. However in FDE, the important differences these are not the time domain samples, but these are the time domain symbols, because we are directly transmitting the symbols in the time domain after adding the secret prefix rather than generating samples, okay.

So once we take the FFT of this, that is the N equal to 4 point FFT, what we have is simply the capital X Ls which are simply the FFT coefficients of the transmitted symbols. So what we have is, now we are looking at, we take the N equal to 4 point FFT of this and we what we get are basically we get the capital Xs which are X 0, X 1, X 2, X 3 and these are simply the these are simply your frequency domain FFT coefficients of the time domain symbols.

These are not, these are simply the frequency domain and that is the important difference with respect to OFDM. These are simply the frequency domain FFT coefficients of your, of the time domain symbols which are the small xs x k. In fact, your capital X L equals that is the Lth FFT point submission k equal to 0 to N - 1 small x k e raise to - j 2 pie k L divided by N.

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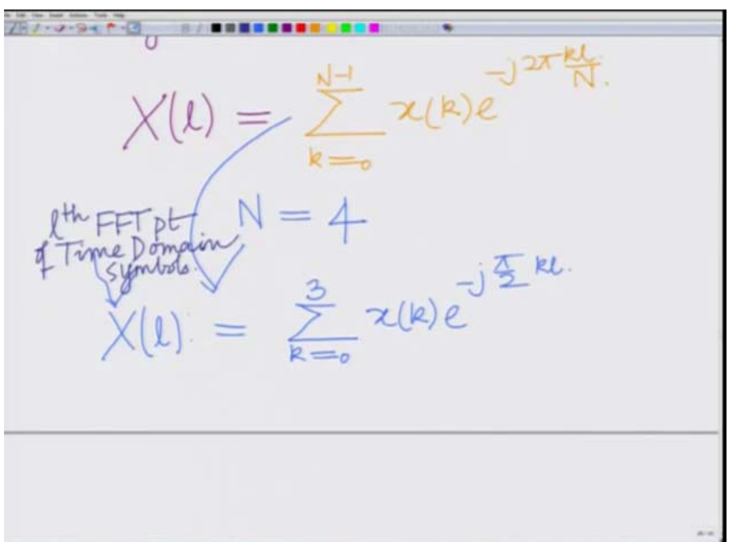

Substituting N equal to 4 since we are considering a system with N equal to basically 4 therefore what we have, this is basically equal to submission k equal to 0to 3 x k e raise to - j pie k L, okay. So X L is the Lth FFT point. So what is the X L? X L is the Lth FFT point of the time domain transmitted symbols. What is this? This is the Lth FFT point, Lth FFT point of the time domain symbols, okay that is X L.

And the rest are again same. That is if you look at capital Ys the capital Ys, which are basically generated by the FFT of the time domain output symbols that is basically you have the small ys, the small y k, y 0, y 1, y 2, y 3 and you take the N equal to 4 point IFFT, you get the capital Y 0, capital Y 1, capital Y 2, capital Y 3, these are the N point N point IFFT of the time domain output samples, the time domain output symbols that are received.

These will give the FFT of the time domain output symbols give the frequency domain output symbols across the subcarriers, okay across each FFT point, okay. So what we have is you have y 0, y 1, y 2, y 3, you take the N equal to 4 point FFT.

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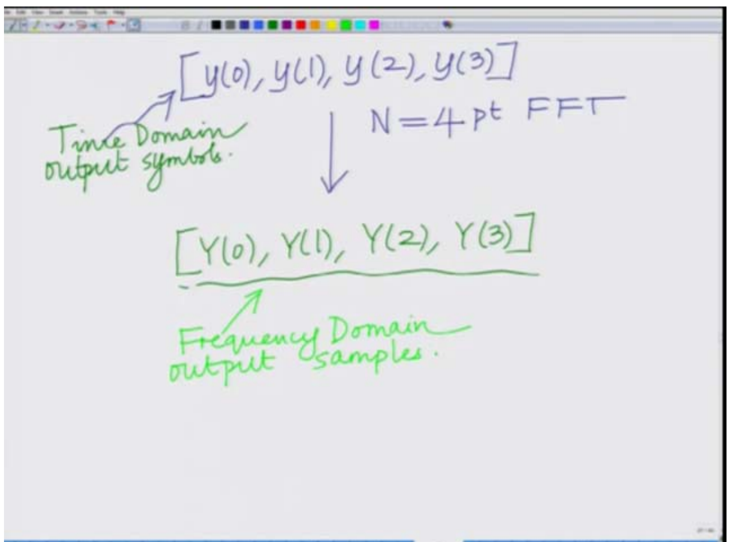

These are time doing output symbols, these are the time when output symbols and what you get is Y 0, Y 1, Y 2, Y 3 and these are basically, these are your frequency domain output samples, yeah.

The frequency domain FFT coefficients of the received time domain symbols, yeah. You take the FFT of the time domain symbols that is the small ys, you get the capital Ys, which are the frequency domain output samples. That is output Y L is the output sample across each subcarrier L or across each career frequency point or each Lth frequency point in the FFT domain, okay. Similarly, now the capital Hs are basically the 0 pared FFT of the channel taps.

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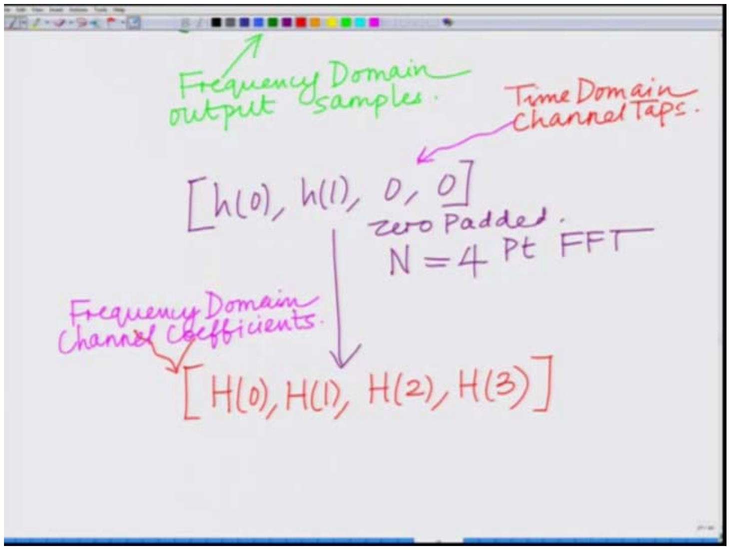

You have L equal to 2 channel taps, you pad N - L 0, you take the N equal to 4 point FFT in fact, this is the 0 pared, this is the 0 pared FFT to generate the capital Hs, H 0, H 1, H 2, H 3, what are these? These are basically your frequency domain channel coefficients. These are your frequency domain channel coefficients, yeah; these are the time domain channel taps that is something that we have already seen.

These are your time domain these are the time domain channel taps, okay. And finally of course you take the small Vs which are the time domain noise samples, yeah. And you take the FFT of these, you generate the capital Vs, which are the frequency domain noise samples, that is the noise across each subcarrier L. Okay, so you take the small Vs which are v 0, v 1, v 2, v 3, you take the N equal to 4 point FFT.

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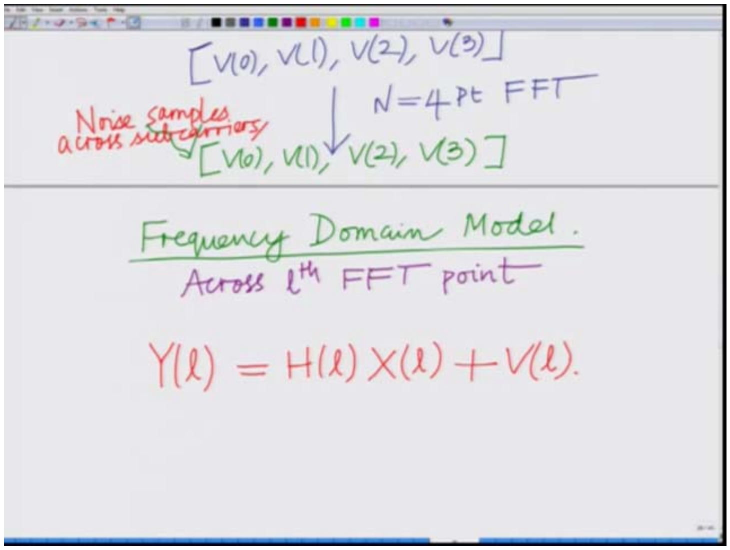

You get the capital V 0, capital V 1, capital V 2, capital V 3; these are basically what are these? These are basically the frequency domain noise samples across the subcarrier. These are basically the noise samples across the subcarriers and therefore now what we have is the frequency domain model across Lth sub carrier or the Lth FFT, the frequency domain model across the Lth FFT point and that is given as.

Y L equals H L X L + V L, Y L equals H L times X L + V L, okay. Now we can perform frequency domain equalisation that is one can estimate this Lth FFT coefficient what is this? Remember this is the Lth FFT coefficient, now we are doing estimation in the frequency domain. We estimate this Lth FFT coefficient, now we can see this Lth FFT coefficient can be estimated as X hat L equals Y of L divided by H of L that is what you can see.

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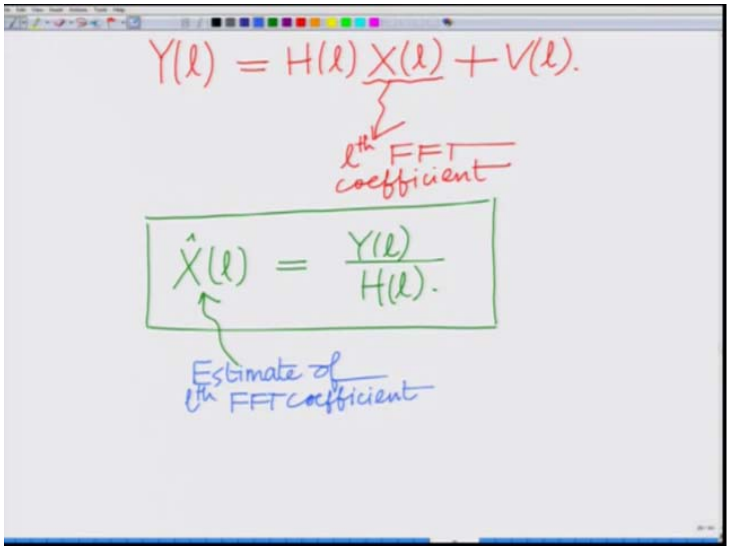

Yeah, Y L equals H L times X L +V L. And therefore, X hat L X hat of L, this is the estimate of the Lth FFT coefficient. This is the estimate of the Lth FFT coefficient, this is equal to Y L which is the output which is output sample across the Lth subcarrier and capital H L which is the channel coefficient across the Lth subcarrier therefore, if you do Y L divided by capital H L, you get the estimate capital X hat L.

That is the estimate of the Lth FFT coefficient of the transmitted symbol sequence, which is the small xs, okay. So therefore, what we are saying is basically capital X hat 0 equal capital Y of 0 divided by capital H of 0.

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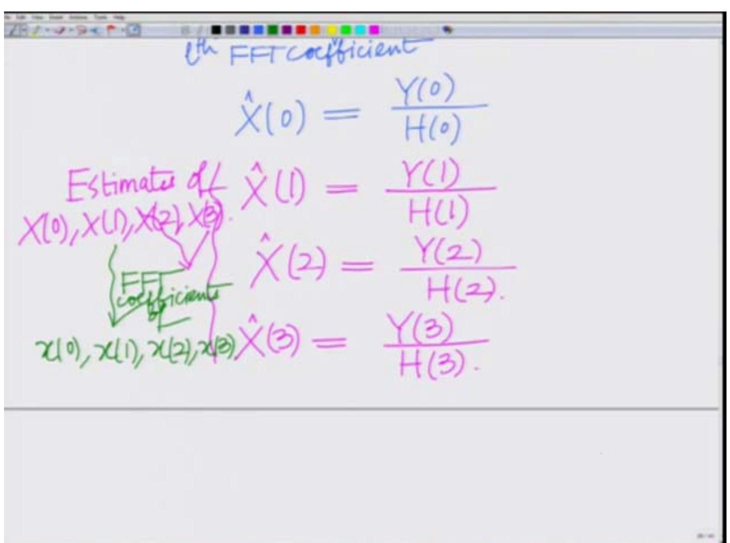

Capital X hat of 1 equals Y of 1 divided by H of 1. Capital X hat of 2 equals Y of 2 divided by H of 2, and Capital X hat of 3 equals Y of 3 divided by H of 3, okay. So these are basically what are these; um

These are basically estimates of the these are estimates of basically your capital X 0, X 1 capital X 0, capital X 1, capital X 2, capital X 3 and these in turn what are these; These are the FFT coefficient of the symbol sequence that is, small x 0, small x small x 1, small x 3 and therefore what we see is basically you have the small xs, which are the symbols in the time domain.

You take the FFT of this, you generate the samples that is the capital X 0, capital X 1, capital X 2, capital X 3 therefore, from the capital Xs, if you perform the IFFT, you have you can generate the small xs, all right. So naturally because the capital Xs are the FFT of the small xs, so the small xs are the IFFT, that is you take capital X you can take capital X as X 0 capital X 1, capital X 2, capital X 3.

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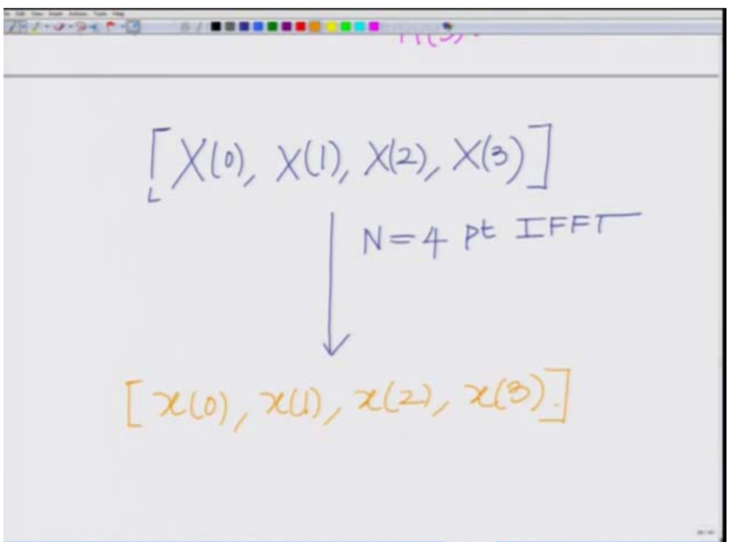

And if you perform the N equal to 4 point, now if you perform IFFT, you generate the symbols in the time domain and the symbols in the time domain are basically your small x 0, small x 1, small x 2, small x 3, so these are the symbols in the time domain, right. These are the symbols in the, these are the symbols in the time domain. Now, all we have basically if you can see, we do not have the capital Xs, but we have something close.

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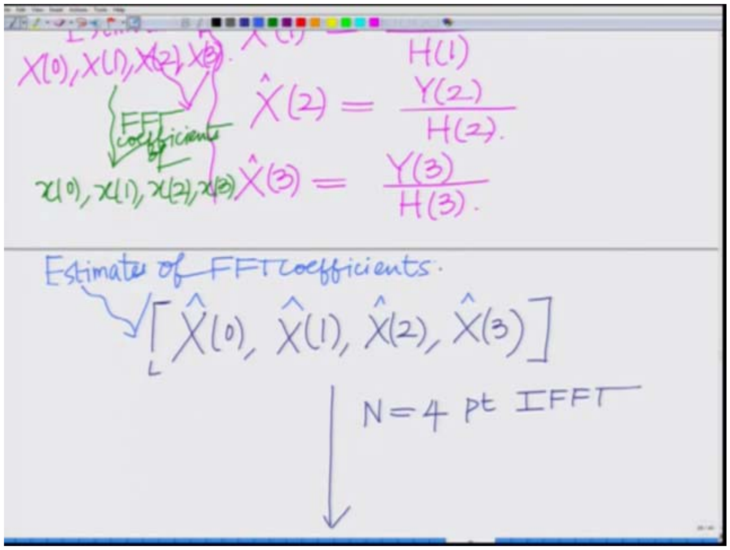

We have the estimates of the capital Xs, we have the estimates of the FFT coefficients, we have the estimates of your FFT coefficients of the symbols, we have estimates of the FFT coefficients of the symbols therefore, if we take the IFFT what we will get is we will not get the symbols, but rather the estimates of the symbols.

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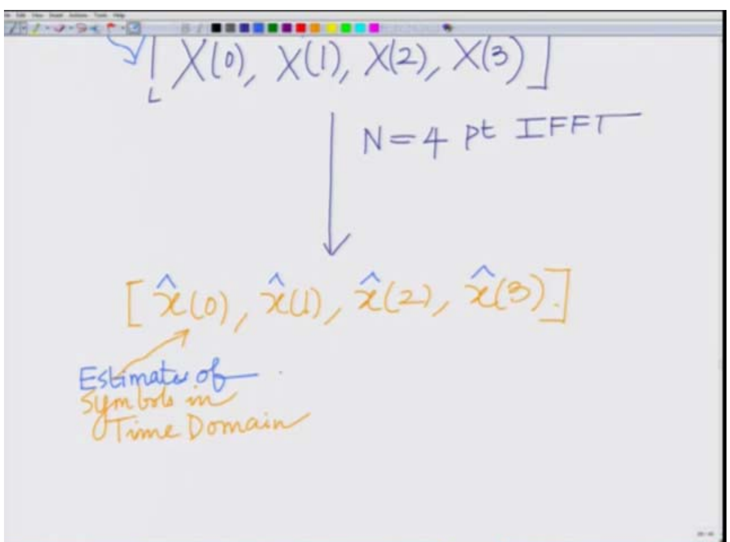

So we will get, so what we are saying is the samples that is the capital Xs are generated by the FFT of the symbols, right.

So therefore, if I take the capital Xs and perform the IFFT, naturally I can generate the symbols in the time domain. However, what I have is from the Frequency Domain

Equalisation basically what I have is; I have estimates I have estimates of the FFT coefficients the capital Xs. Therefore, if I take the IFFT naturally I can generate the estimates of the symbols that is the small xs in the time domain.

And you can see, this equalisation is a very simple process. All have to do is take Y 0 divided by H 0, there is no matrix inversion, so this is also known as a 1 tap, single tap unlike the time domain equalisation.

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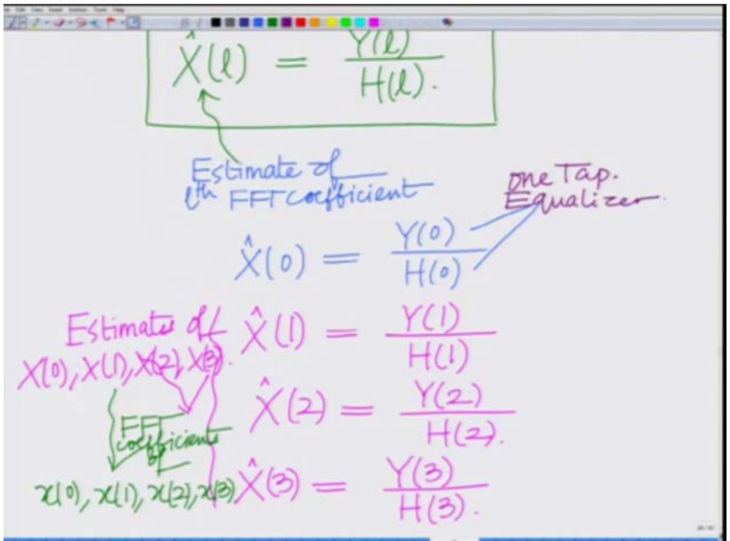

That is if you remember the 0 4 sync equaliser which was done in the time domain. In the time domain equalisation, there was a matrix inversion, yeah depending on what is the order of the equaliser if you remember.

So in time domain equalisation where basically we are computing the equaliser, there is a matrix inversion that is required. However, in frequency domain equalisation there is no matrix inversion; there is simply a division by the channel coefficient capital H L that is we are take simply taking the Y L which is the output Lth output sample in the frequency domain across the Lth FFT point dividing it by capital H L.

Which is the channel coefficient corresponding to the Lth subcarrier or basically the Lth FFT channel or the Lth FFT point capital Y L divided by capital H L gives us capital X hat of L, which is the estimate of the Lth FFT coefficient of the transmitted symbol sequence in the time domain. And therefore now you take these estimates capital X hat of L and you perform the IFFT and what that gives is basically, that gives you the symbol estimate.

So this is a slightly roundabout fashion that is, first what we are doing is, we are taking a time domain system, moving it into frequency domain using the FFT, doing the equalisation in the frequency domain and from the equalised from the equalisation done in the frequency domain, we are reconstructing the time domain symbols.

Now we are doing this in a roundabout fashion, because we have seen the equalisation in the frequency domain is much less complicated, it is simply a 1 tap equaliser that is, you have to divide by basically the corresponding channel coefficient on each subcarrier, all right. So this low complexity motivates us to use equalisation in the frequency domain rather in the rather than the time domain.

And therefore, now naturally what we have is the symbol estimates in the time domain.

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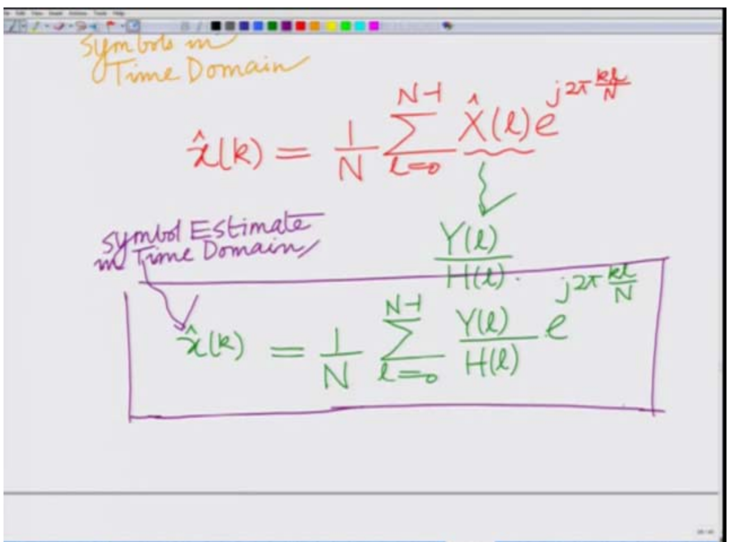

The x hat x is the IFFT of the capital X hat of L e to the power of j 2 pie k L by N. And now we know this X hat of L is basically equal to Y of L divided by H of L. So which I can substitute now, so this is basically 1 over N your submission L equal to 0 to N - 1 Y of L divided by H of L times he to the power e to the power of j 2 pie k L by N.

This is your x hat of k and that basically gives you, this is the symbol estimate in the capital x hat k, which is the this is the symbol estimate in the time domain okay, so what we have is the symbol estimate in the time domain. And in fact, one can also at this point you can also substitute N equal to 3, N equal to 4. Let us also do that for the sake of completeness.

If you substitute N equal to 4 what you will have equals x hat of k equals 1 over 4 that is for this particular example.

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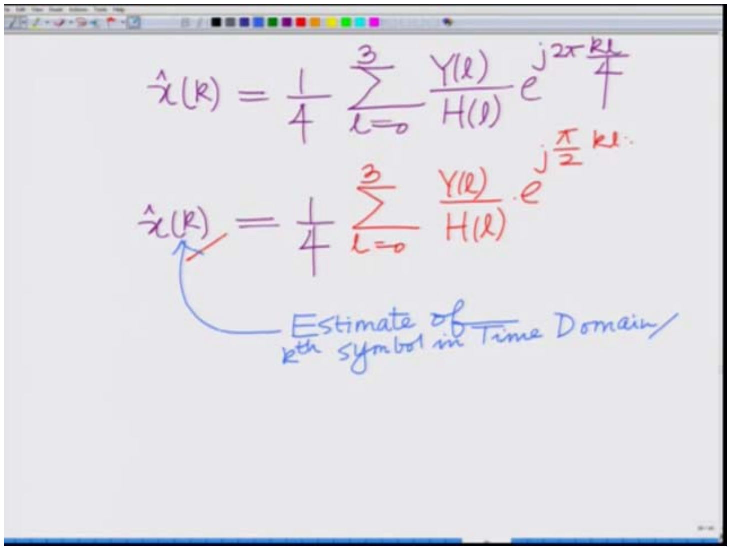

L equal to 0 to 3 Y L divided by H L e to the power of j 2 pie k L divided by 4, which is basically, you can write this as your x hat of k equals 1 over 4 submission L equal to 0 to 3 Y L divided by H L e to the power j 2 pie e to the power of j in fact, I can directly write this as pie over 2 k L, okay.

So x hat k is basically your estimate of the kth symbol in the time domain. In fact, this is the symbol estimate; this is the estimate of the kth symbol. Let me also write this x hat k is the estimate of the kth symbol in the time domain. So basically what we have done in this model is we have started with frequency division Frequency Domain Equalisation, we have said that Frequency Domain Equalisation is alternative to time domain equalisation.

Which is something that we have already seen before where we develop the 0 4 sync in the time domain. The frequency domain equaliser is basically motivated by its efficiency and its low complexity, so what we do is we transmit unlike OFDM and we transmit the samples in the time domain, here we directly transmit the symbols that is, we take a block,

Add the cyclic prefix and this cyclic prefix added block of symbols is transmitted in the time domain across the ISI channel that is Inter Symbol Interference. At the output basically, you take the FFT of the received symbols. Received symbols are small ys in the time domain; you perform now once you perform the FFT of the received symbols in the time domain, look at the output channel output samples; that is the capital Ys in the frequency domain.

Using this you perform Frequency Domain Equalisation that is compute the estimates of the capital X hat estimate capital X hats, which are the estimates of the FFT coefficient of the symbols. That is, each capital X hat L equals Y L that is the Lth FM the point of the received output in this divided by capital H L, which is the Lth FFT coefficient of the channel taps the 0 pared the Lth 0 pared FFT coefficient of the channel taps.

So capital Y L divided by capital H L gives you capital X hat L, which is the estimate of the Lth FFT point of the transmitted symbol. You take the IFFT now of this capital X hat of L, all right and you take once you take the IFFT. That gives you the small x hat of k that is the estimate of the symbols transmitted in the time domain, so this is basically Frequency Domain Equalisation.

Which is basically equalisation by removing the Inter Symbol Interference in the frequency domain, at the same time it has parallels to OFDM because again it involves both IFFT and FFT operations. However, yet it is very different from OFDM and you can go through this module again in detail to understand it better, all right.

Okay, so we will stop this module here and we will look at an example of FDE Frequency Domain Equalisation in the subsequent module, all right thank you very much.