Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks
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Lecture Number 33
Comb Type Pilot (CTP) Based Author Orthogonal Frequency Division Multiplexing (OFDM) Channel Estimation

Hello, so in the previous module we have looked at the Comb Type Pilot or CTP for OFDM channel estimation. Let us now look at an example for the same thing that is Comb Type Pilot based OFDM channel estimation, all rights. So what we want to do starting today's module is look at an example for our Comb Type Pilot that is CTP Comb Type Pilot or CTP based OFDM channel estimation, all right.

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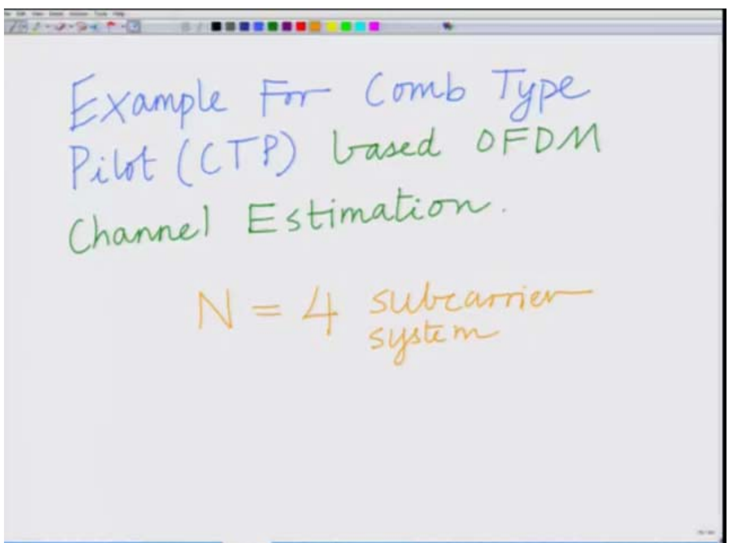

So let the symbols loaded onto the subcarrier, alright. So we have considered again we have start by considering an N equal to 4 an N equal to 4 subcarrier system, okay. And in this the information symbols, alright, let the symbols loaded onto the subcarriers, we are going to set them as X 0 equals 1 + j, X 1 equals 1 - j, X 2 equals 1 + 2 j, X 3 equals 2 - j that is same as before.

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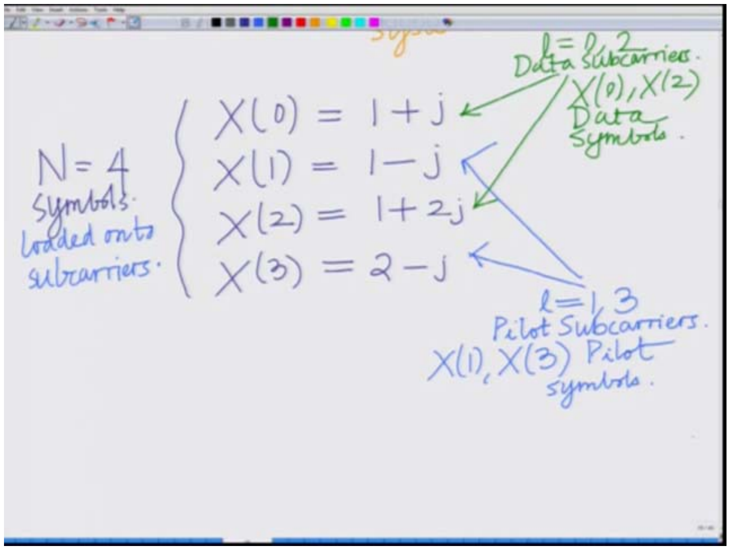

So these are N equal to 4 symbols loaded onto the subcarriers and these N equal to 4 symbols are loaded basically, that is the nomenclature that we use that these are loaded these are loaded onto the N equal to 4 symbols are loaded onto the subcarrier. However, as I said in Comb Type Pilot based channel estimation, not all the subcarriers are loaded with the pilot symbols, only a few of the subcarriers are loaded with pilot symbols.

So let us say again that X 1 again as in what we have done previously that X 1 and X 3 are pilot symbols that is L equal to 1 to 3 are basically your pilot subcarriers and X 1 and therefore X 1, X 3 are basically the pilot symbols loaded on the pilot subcarriers. And again X 0 and X 2 these, that is corresponding to L equal to 0, 2, these are my data subcarriers, alright.

X 0, X 2 are the data symbols, all right. X 0, X 2 are basically corresponding data symbols loaded onto the data carriers, okay. So now what we do is, we have this comb type pilot arrangement that is corresponding to the 4 symbols X0, X1, X2, X3. Now what we are going to do is similar to what we have done before. These are the symbols loaded onto the subcarriers.

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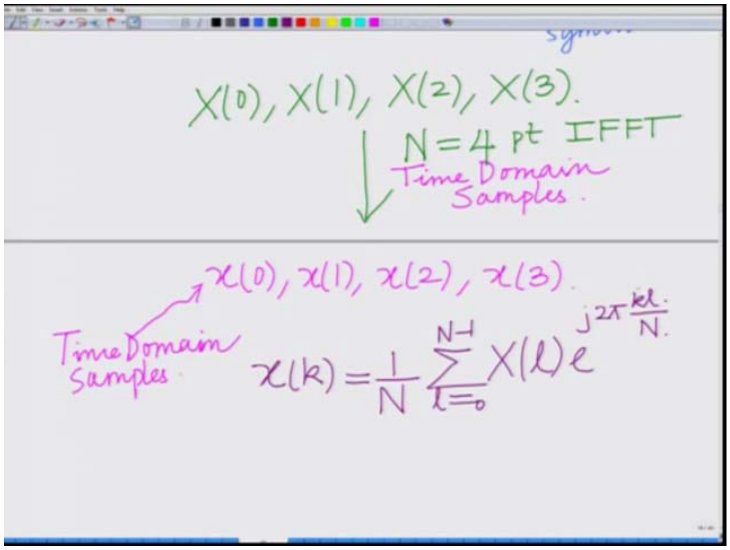

So we are going to take the N equal to 4 point IFFT Inverse Fast Fourier Transform to basically generate the time domain samples x 0, x 1, x 2, x 3, okay. So now we consider the N equal to 4 point IFFT to generate the time domain samples x 0, x 1, x 2, x 3, these are basically remember these are the time domain samples.

And these are generated as x k equals as we said these are the N point IFFT submission L equal to 0 to 1 e power j 2 pie k L by N. And we have already calculated this before previously in the context of the OFDM symbol. We have calculated the time domain samples already, so I am going to simply write that down over write those down over here.

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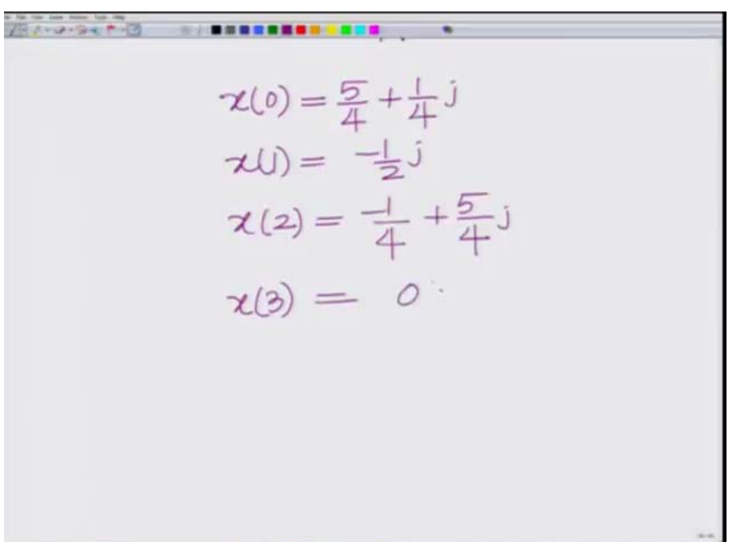

So the time domain samples are given as x 0 equals 5 by 4 + 1 by 4 j, x 1 equals - half j, x 2 equals - 1 by 4 + 5 by 4 j and x 3 equal to 0. So the block of time domain samples therefore is given as is therefore given as.

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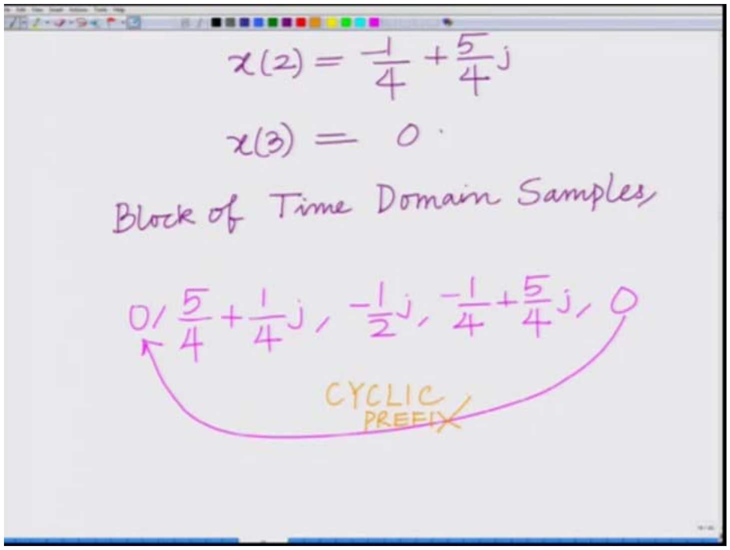

5 by 4 + 1 by 4 j, - half j, 0. And you take the 0 and place it over here remember, this is basically adding the cyclic prefix and this is the block that is transmitted.

This is the cyclic prefix added block, so this is basically your block with cyclic prefix added and this is transmitted over the disease transmitted over the ISI or basically the Inter Symbol Interference channel.

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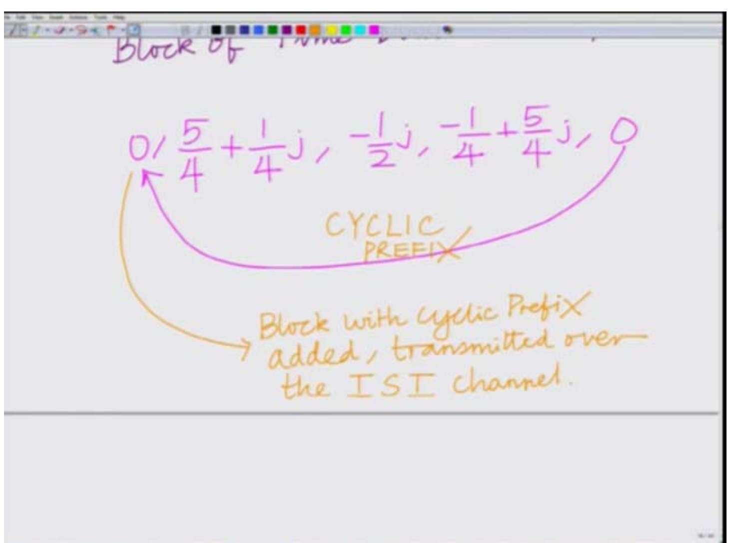

So taking the time domain samples of this Comb Type of subcarriers on which the information symbols and the pilot symbols are arranged in a Comb Type fashion, alright.

You take the N point IFFT to generate the time domain samples, add the cyclic prefix, now the cyclic prefix added block is what is actually transmitted over the ISI channel, alright. And similarly, similar to several times before, we are considering the 2 tap ISI channel, that is y k equals h 0 x k + h 1 x k - 1.

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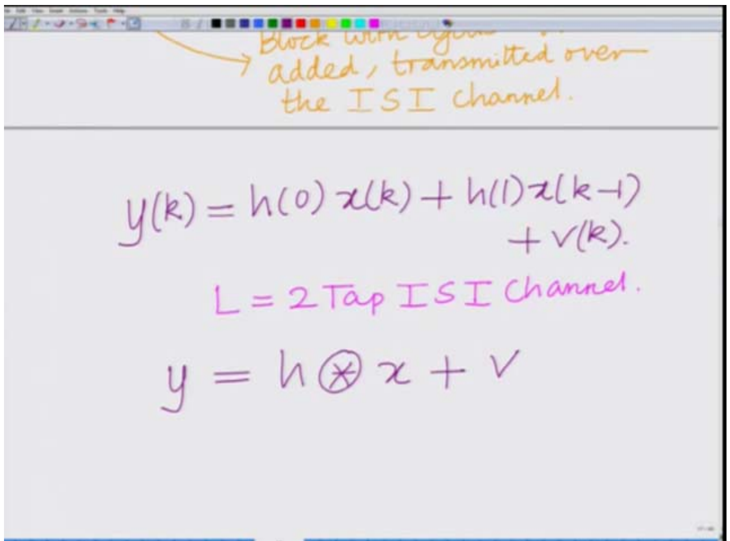

Basically, what is this? This is your L equal to 2 tap ISI channel. This is the L equal to 2 tap ISI channel, okay.

And therefore the action of the channel tap is now y is equal to the action of the channel because you are transmitting cyclic prefix added block of samples, the action of the channel is basically circular convolution in the presence of additive noise, alright. And therefore now once you take the FFT of the output samples, FFT of the received output samples.

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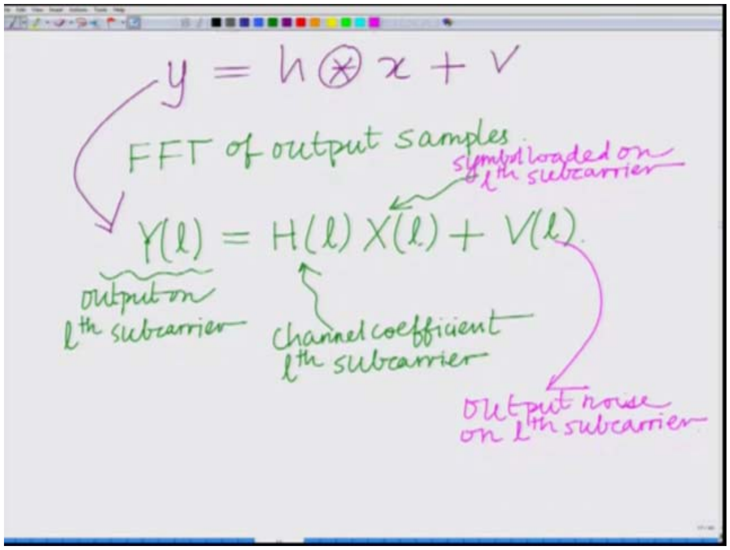

FFT of the received output samples; I have Y L equals H L X L + V L, where Y L is the output on the Lth subcarrier. This is the output on Lth subcarrier; this is the channel coefficient on the Lth subcarrier. This is the symbol transmitted or symbols loaded and this is the, the output noise on the Lth subcarrier, all right.

So this is the model that we have that is the Y L equals H L times X L + V L this is the symbol that we have, this is the system model that we have in the frequency domain that is the input output model across each subcarrier that is across the Lth subcarrier, alright. And now we said in the Comb Type Pilot in the Comb Type Pilot based channel estimation scheme.

We have to extract the output symbols corresponding to the pilot subcarriers, alright. So that is what we are going to do next, so let us now let say, the output samples, let us first consider the output samples.

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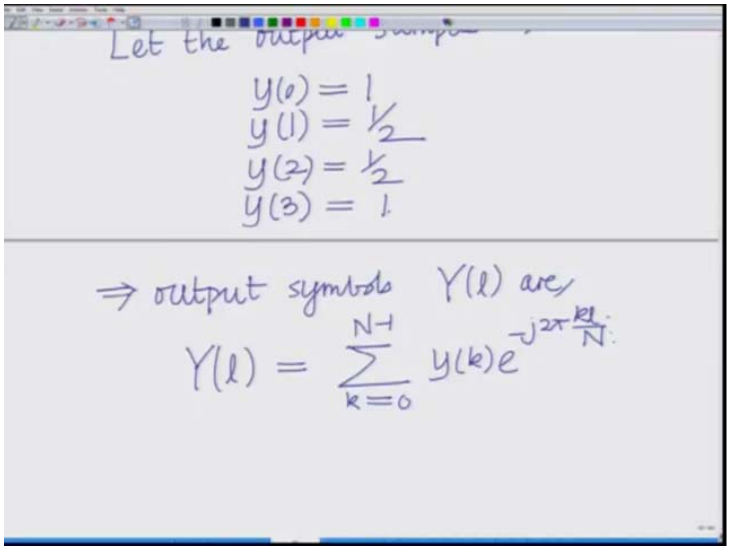

Let the output samples be y 0 equals 1, y 1 equals half, y 2 equals half, y 3 equals 1. Therefore, the output symbols Y L implies symbol Y L are basically remember the Y L are given by the FFT of the time domain samples y L.

So they are basically you have k equal to 0 to N - 1 y k that is the FFT e power - j 2 pie k L divided by N. Therefore, Y L these are basically given as FFT of time domain FFT of the time domain output samples y k and that is what we have written over here.

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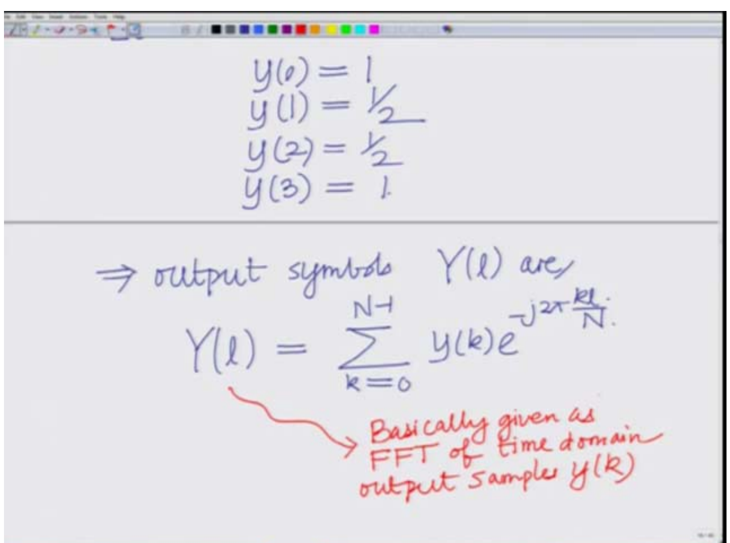

And let and we have already said y 0 equals 1, y 1 equals half, y 2 equals half, y 3 equals 1.

So now once we take the FFT and this is again something that we have calculated before that is this is again considering these output samples, we have calculated the FFT of these output samples in the context of the OFDM example and therefore, these the output symbols across the various carriers Y 0 are given as 3, that is once you compute the FFT, these are the values that you will get.

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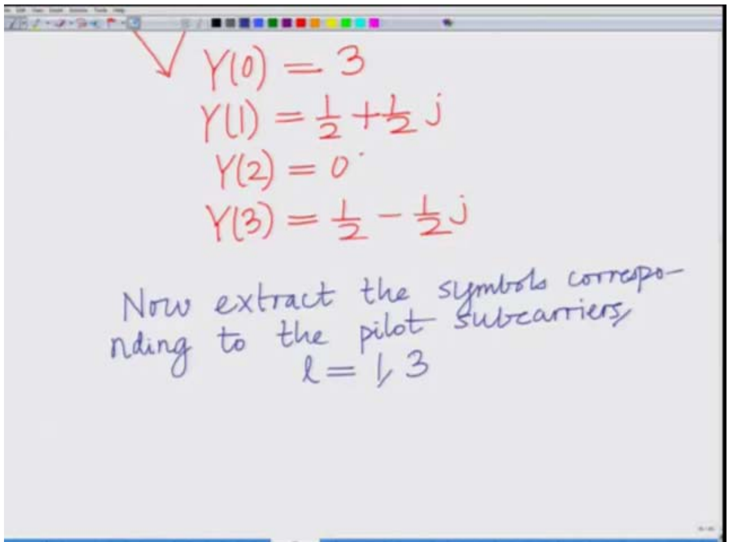

Y 1 equals half + half j, Y 2 equals 0 and Y 3 equals half - half j. Now extract the symbols corresponding to the pilot subcarriers, now we extract the symbols corresponding, now we expand the symbols corresponding to the pilot subcarriers that is, pilot subcarriers are L equal to 1, 3. Therefore, we extract the symbols the symbols corresponding to the pilot subcarriers.

Therefore it goes without saying these are Y 1, Y 3, alright.

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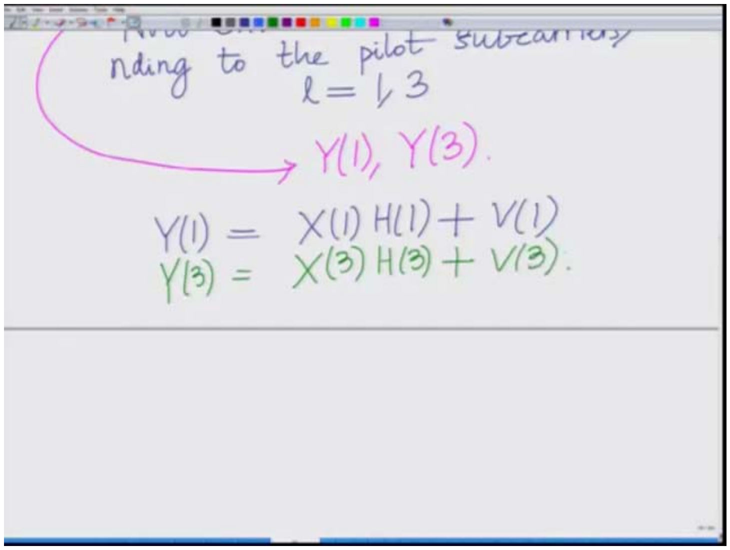

Since the pilot symbols are transmitted on subcarriers 1 and 3; that is L equal to 1 and 3. The symbols corresponding to these subcarriers are Y 1 and Y 3. And now we extract them, that is we use these and these only for the purpose of channel estimation because these are the pilot symbols or these are the pilot subcarriers, okay.

So let us do that and what we have is now if we write this model. Remember, that is what we have seen. If we write this model across the pilot subcarriers, I have Y 1 equals X 1 H 1 + V 1. And we also have Y 3 equals X 3 H 3 + V 3. Now writing this in matrix notation what I have is

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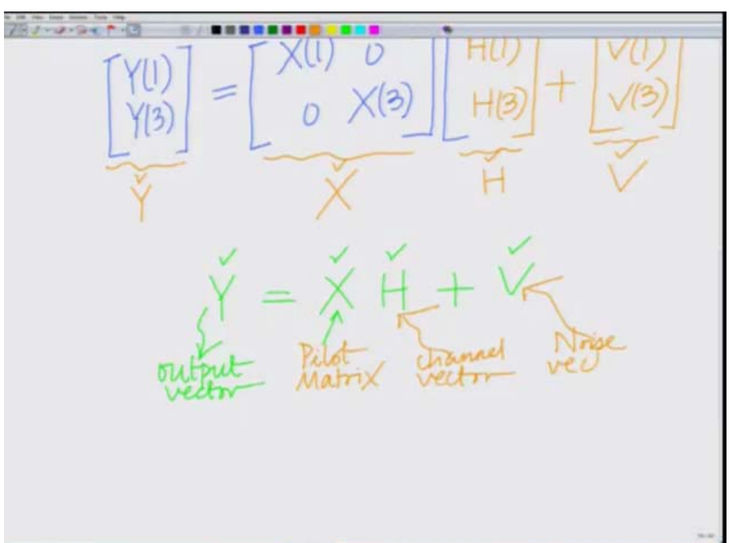

Y 1 Y 3 equals X 1 X 3, the diagonal matrix times this vector H 1 H 3 of channel coefficient + V 1 V 3.

And remember this we called Y check, this we called X check, this we called the vector H check, this we called the vector V check. And now the least squares therefore we have the system model which is basically your Y check equals X check X check plus V check, this is the output vector, this is your pilot matrix, this is your channel vector containing coefficient X 1 X 3, this is your noise vector.

Therefore the least squares function now at this point again this is something you should be familiar with. Least squares function for channel estimation.

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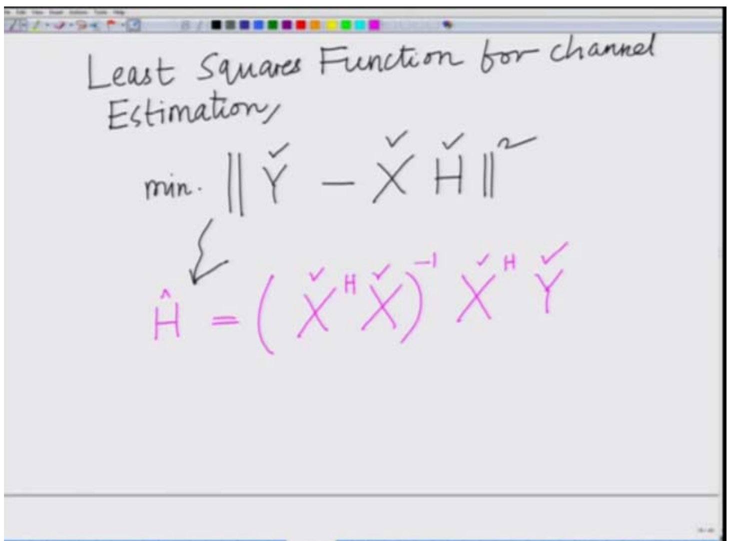

That is your Y check that is your Y check - X check H check whole square that is minimise the Least squares cost function Y check equals X check, minimise norm Y check - X check H check whole square.

And the Least squares solution therefore the channel estimate H hat remember is basically your X check her X check inverse X check her Y check, this is the least squares solution and remember we said X is this X check matrix is a square matrix, it is invertible and it is diagonal, it is diagonal and it is invertible. Basically, it is an invertible matrix therefore, this can be simplified as.

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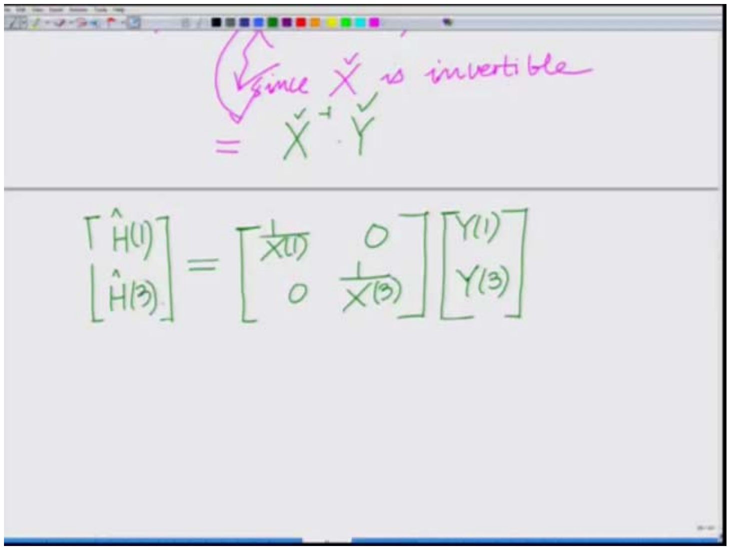

Since X check is invertible this is basically equal to your X check inverse times Y check and since X check inverse is diagonal, the inverse of X check is simply the inverse of the diagonal elements that is 1 over X 1, 1 over X 3 times Y 1 Y 3 and this is basically your estimate. This is your estimate of the channel coefficients. Therefore, now I can calculate the estimate H hat 1 as simply that is your Y 1 divided by X 1.

And we had we know both of this for instance, Y 1 is something that we know from here, this is your value of Y 1.

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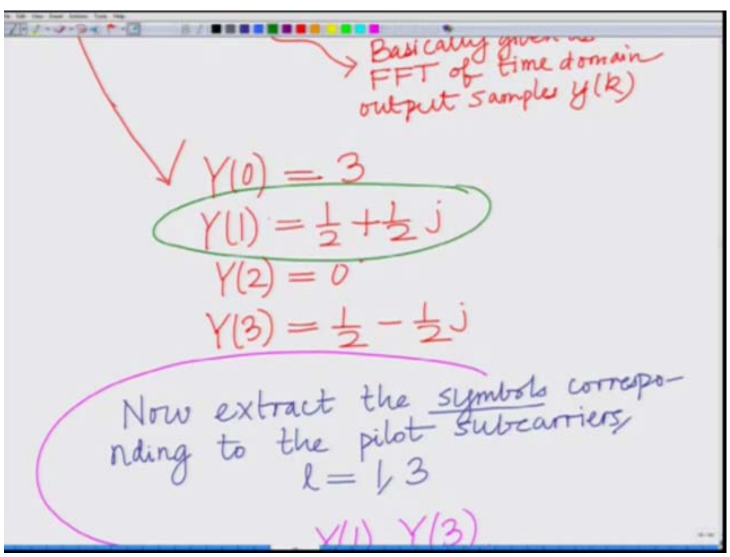

And X 1 is something that you know from here that is the symbol loaded that is the pilot loaded onto the subcarrier 1 that is 1 – j.

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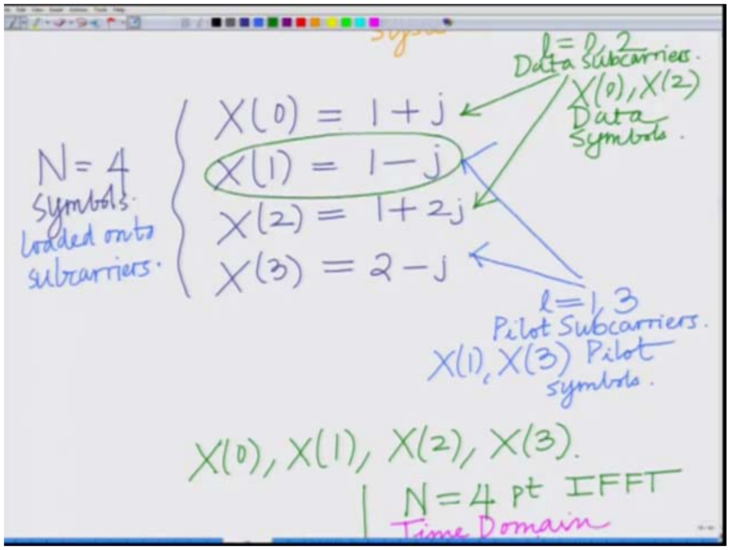

And therefore what we have is now I can compute H hat 1 alright. H hat 1 H hat 1 which is the basically remember H hat estimate of the channel coefficient across subcarrier 1.

What is this? This is estimate of channel coefficient across subcarrier 1.

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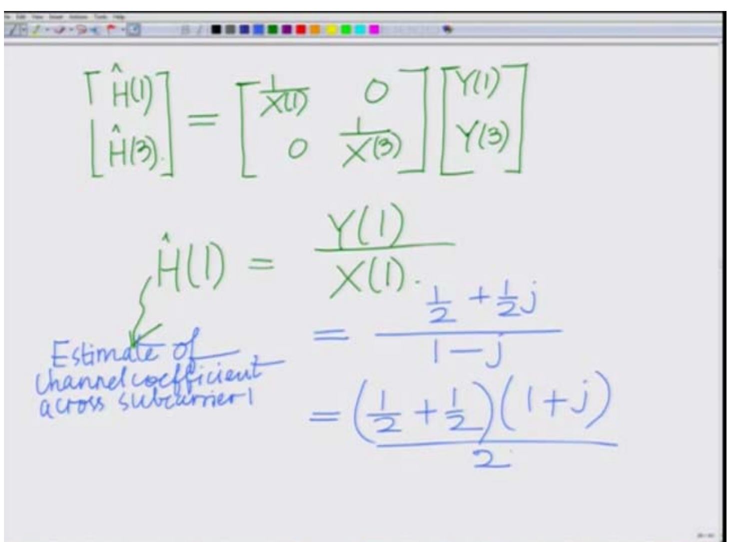

And this is equal to basically, this is equal to your, what is this? This is basically your half + half j divided by 1 - j which is basically equal to again just rewriting it using the properties of complex numbers that is 1 + j divided by 2, which is equal to which is equal to 1 over 4 times 1 + j times 1 + j which is 1 + j square.

And you can check, this is basically this is equal to half j, alright.

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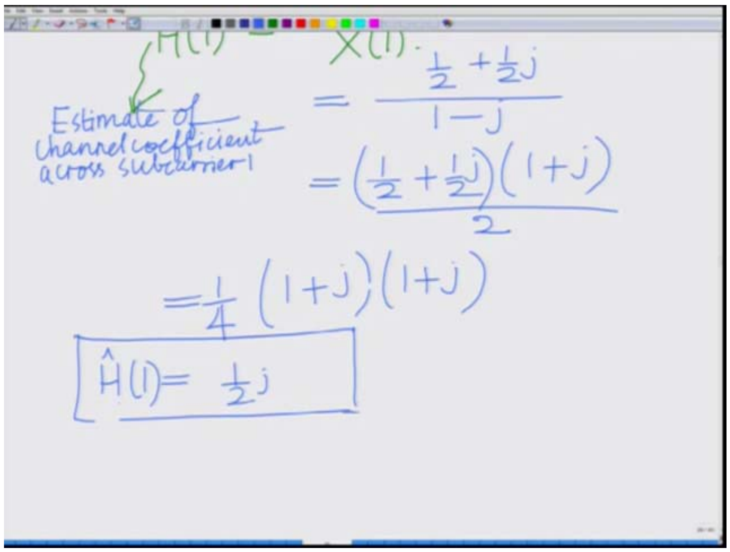

So H hat 1 that is the estimate of the channel coefficient, so what we have calculated so far is the estimate of the channel coefficient H hat 1 across subcarrier 1. This is Y 1 divided by X 1, so this is half j. Similarly, we can calculate the estimate H hat 3 across subcarrier 3 and this is equal to the estimate of channel coefficient across subcarrier 3.

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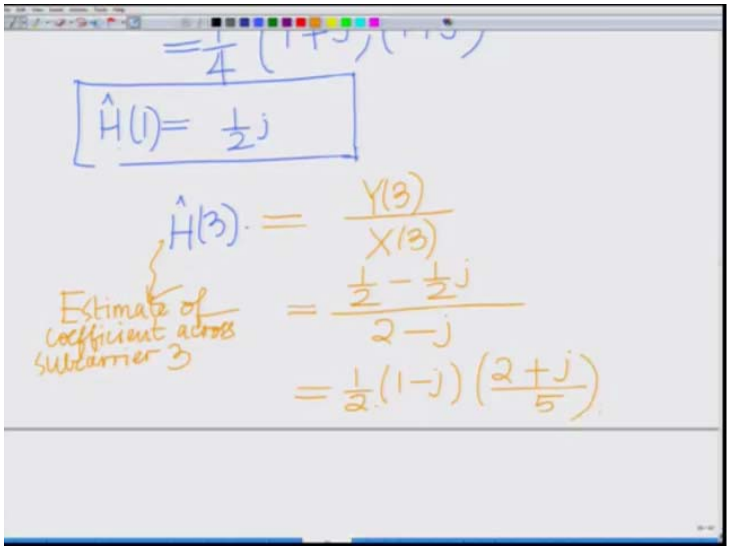

That is, estimate and this is equal to Y 3 divided by X 3 and once again from above we know both Y 3 Y 3 and X 3, that is Y 3 equals half - half j divided by X 3 that is, 2 - that is basically equal to 2 - j, which is half into 1 - j into again 2 + j divided by 5 and you can also verify this.

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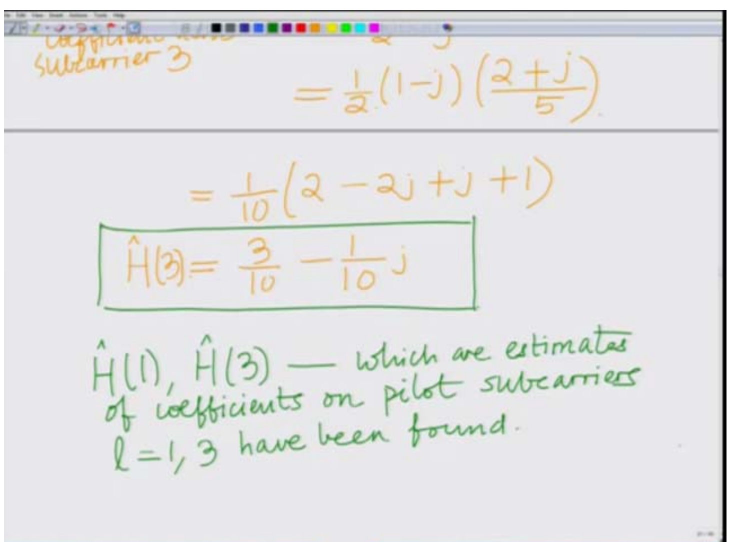

This is basically you 1 by 10 time 2 - 2 j + j + 1 and this is equal to 3 by 10 - 1 by 10 j, okay.

H hat 3 equals 3 by 10 - 1 by 10. So therefore now what we have found is we have found H hat 1, H hat 3, which are the estimates of the channel coefficients on the pilot subcarriers that is, L equal to 1 and 3. Therefore, H hat 1, H hat 3 which are estimates of coefficients on pilot subcarriers L equal to 1, 3 have been found, okay.

And now what is the next step remember, we have looked at Comb Type Pilot estimation from the estimates of the channel coefficients on the pilot sub carriers. We have computed the time domain estimates of the channel taps. Remember, that is the next step and that can be illustrated as follows. So remember we have H 1 remember equals h 0 - j h 1 and H 3 equals h 0 + j h 1, which implies writing this as vectors.

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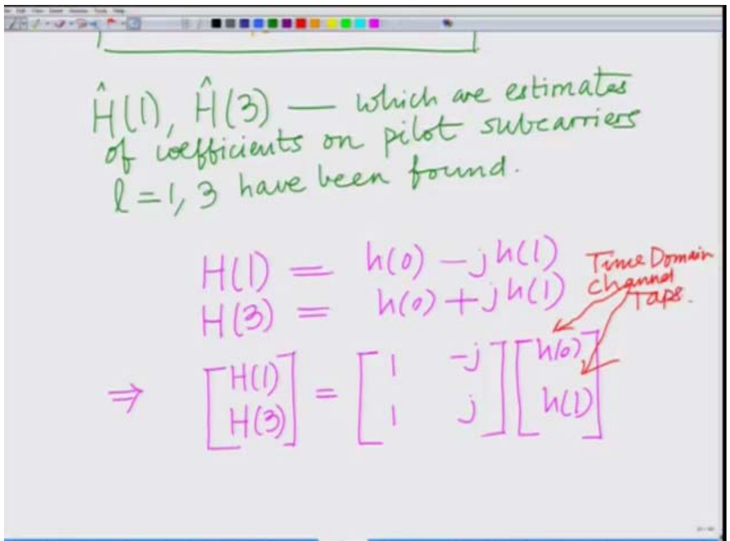

Writing this using matrix notation, I have H 1 H 3 equals 1 - j, 1 + j times h 0 h 1. Now what are these? These are basically your time domain channel taps, right. These are the time domain channel taps of the ISI channel. These are the time domain channel taps of the ISI channel and these are of course the coefficients on the pilot subcarriers. These are the coefficients on the pilot subcarriers.

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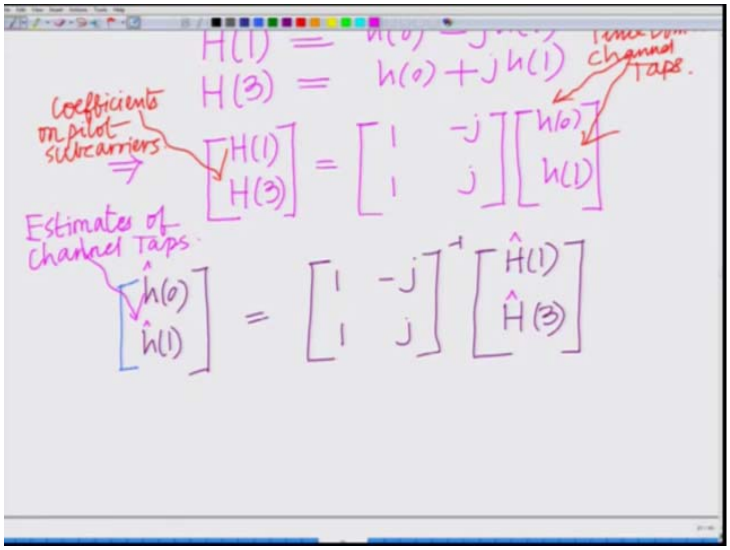

Now therefore, I can find the channel taps as the channel taps can be found simply as h 0 h 1 equals this 1 - j, this matrix inverse times H 1 H 3 and that is basically what we have. And the last step is basically to realize that we have the estimates of these channel coefficients on the pilot subcarriers, so if we use the estimates here on the left hand side also, you will get estimates.

So these are what these are? These are the estimates of the channel taps. Estimates of time domain channel taps or basically your ISI channel. So what we have is basically we can express h 0 h 1, the time domain channel taps in terms of H 1 H 3. But if you only have the estimate that is H hat 1 H hat 3.

What you are going to get is naturally basically the estimates of the time domain channel taps that is h 0 and h 1, alright. And that is how we calculated the estimates of the time domain channel taps, okay. And therefore now I am going to calculate this, I am going to substitute H hat 1 H hat 3, so this is 1 - j, 1 + J inverse of H hat 1. Now H hat 1 we have already calculated that is half j.

This H hat 3 we have calculated that is 3 by 10 - 1 by 10 j and therefore this is equal to now we will calculate the inverse of this matrix. This inverse of this matrix 1 - j, 1 j that is simple to compute because it is 2 cross 2 matrixes.

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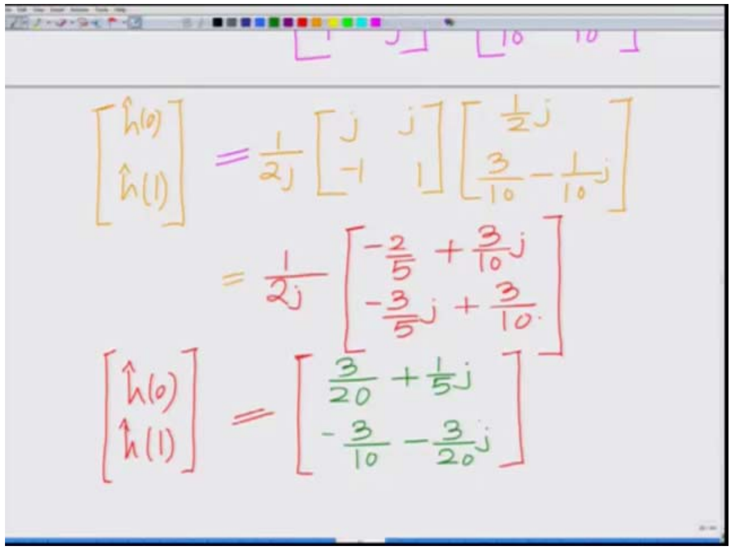

That is, 1 over 2 j times (())(28:02) the diagonal elements negative of the of diagonal elements. So this is the inverse of the matrix, this times half j 3 by 10 - 1 by 10 J.

These are the estimates of the time domain channel taps h hat 0 h hat 1. And now multiplying, I can find this is basically 1 over 2 j 1 over 2 j, 3 by 20 + 1 by j - 3 by 10 - 3 by 20 j. And now finally you can compute these estimates of the time domain channel taps as basically I think this is not completely correct I think. Let me just rewrite this, this is 1 over 2 j - 2 over 5 + 3 over 10 j - 3 over 5 j plus 3 over 10, okay.

And therefore what I am going to have here basically what I have over here is the estimates of the channel coefficients or estimates of the channel taps in the time domain which is basically 3 over 20 + 1 over 5 j - 3 over 10 - 3 over 20 J and these are the estimates of, so basically what we have is, these are the estimates of the channel taps in the time domain. These are the estimates of the channel taps these are the.

So basically these are your estimates of the channel taps in the time domain, okay. So what we have computed are the estimates of h hat 0 and h hat 1.

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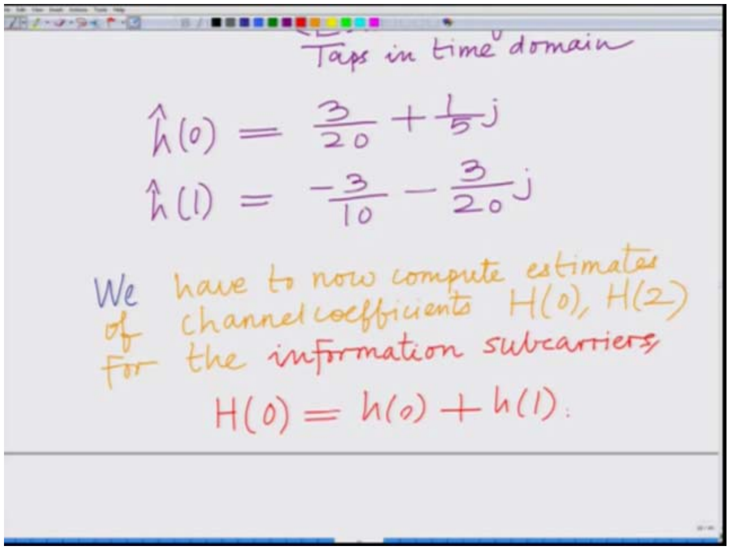

And these are given as basically h hat 0 equals now naturally 3 by 20 + 1 by 5 j and h hat 1 equals - 3 by 10 - 3 by 20 j. So now we have the estimates of the channel taps in the time domain. Now we have to compute remember, estimates of the rest of the channel coefficients in the frequency domain.

Remember, corresponding to the information subcarriers that is, L equal to 1 and 2 and these can be computed as follows. Now we have the estimates of the channel taps in the time domain. All that is left is to compute, we have to now compute, so we have to compute the estimates of the channel coefficients H 0, H 2 for the information subcarriers.

And therefore, these can now be determined as, now we have seen that basically well H 0 equals simply h 0 + h 1 that is what we have seen yesterday, that is H 0 is the 0^{th} point FFT of the 0^{th} 0 pared time domain channel taps h 0 h 1.

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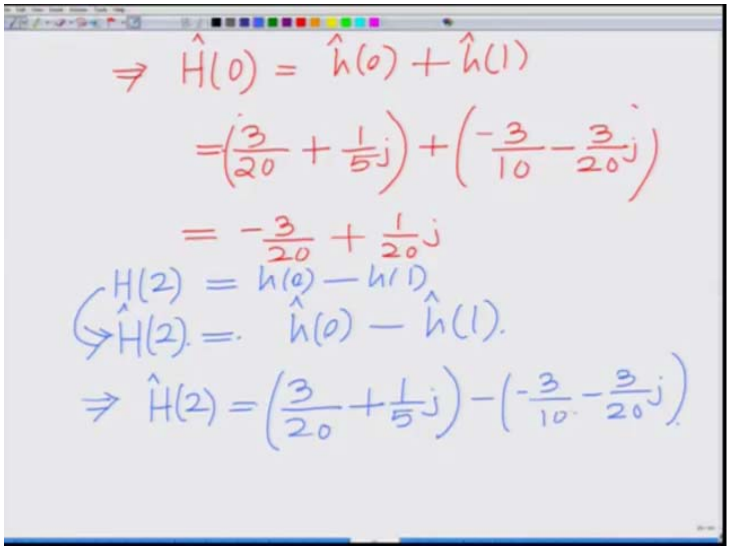

So this is implies H hat 0 equals h hat 0 + h hat 1, which is equal to 3 by 20 + 1 by 5 j + - 3 by 10 - 3 by 20 j equal - 3 by 20 + 1 by 20 j and similarly, H hat 2 H hat 2 corresponding to subcarrier L equal to 2.

This can be calculated as h hat 0 - h hat 1 and this follows remember, because h of 2 equals h 0 - h 1 h 0 - h 1 therefore, that implies H hat 2 equals h hat 0 - h hat 1 implies H hat 2 equals 3 by 20 + 1 by 5 j - - 3 by 20 - 3 by 10 I am sorry - 3 by 10 - 3 by 20 j equals 9 by 20 + 7 by 20 j that is equal to your H hat 2.

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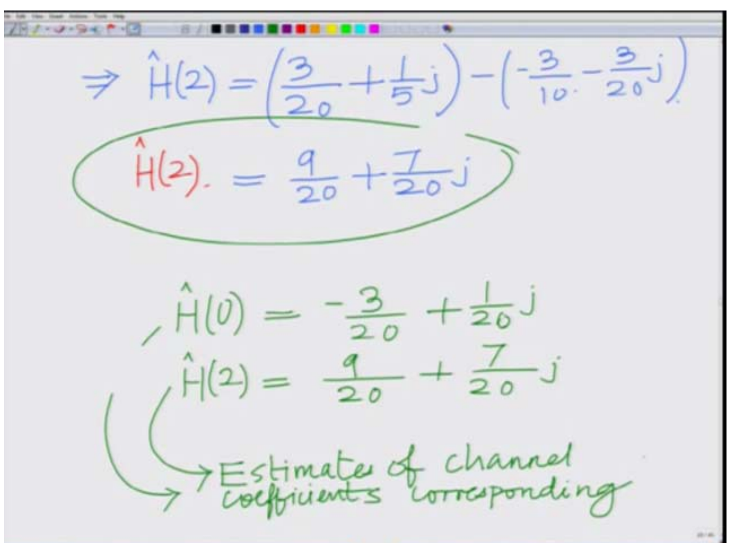

So now basically we have calculated the value of H hat 1, this is the value of your H hat 1 and this is the value of H hat 2. Let us write this down again, so basically your H hat 1 equals - 3 by 20 + 1 by 20 j and H hat 3 I am sorry, this is H hat 0 rather, this is not H hat 1, this is H hat 0. So now we have calculated the value of H hat 0, so I have H hat 0 equals - 3 by 20 + 1 by 20 j and H hat 2 equals 9 by 20 + 7 by 20 j.

And these are the estimates of the channel coefficients corresponding to the, these are the estimates of the channel coefficients corresponding to the data subcarriers or the information subcarriers or the non-pilot information or basically the non-pilot subcarriers. So basically, what we have seen is basically we have seen an example of this Comb Type Pilot based channel estimation scheme.

Where information symbols, where pilot symbols are loaded only on a only on much fewer number of subcarriers. So we have seen an N equal to 4 subcarrier example, where pilot symbol symbols are loaded onto 2 subcarriers that is L equal to 1 and L equal to 3 and the data symbols are loaded onto the remaining subcarriers, that is L equal to 0 and L equal to 2.

And therefore what you do now at the receiver is basically first estimate the channel coefficients in the frequency domain on the pilot subcarriers. These are the H 1 H 3, from these estimates the time domain channel taps in the time domain that is the h 0 h 1 and from these estimates of the time domain channel taps, re-estimate or recompute basically the estimates of the channel coefficients of the rest of the subcarriers.

That is the non-pilot subcarriers or basically the data or the information subcarriers that is, these are H 0 and the H hat 0 and H hat 1, all right. So this basically sort of comprehensively illustrates a simple example but comprehensively illustrates the principle behind the Comb Type Pilot based channel estimation for OFDM and as we have seen, this is preferable compared to conventional channel estimation.

Where the pilot symbols are loaded on all the subcarriers. In Comb Type Pilot, remember the pilot symbols are sort of meshed with the data symbols therefore, you are transmitting only a few pilot symbols therefore, and this is highly efficient because you are transmitting pilot as well as information, so this improves the data rate of the system.

So that in a sort of nut shell basically summarises the motivation and the procedure for this Comb Type Pilot OFDM channel estimation. So we will stop this module here and look at other aspects in subsequent module, thank you very much.