Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks
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Lecture Number 32
Comb Type Pilot (CTP) Based Author Orthogonal Frequency Division Multiplexing (OFDM) Channel - Estimation Scheme

Hello, welcome to another module in this massive open online course on Estimation for Wireless Communication systems. So we are looking at a different estimation scheme or different channel estimation scheme for Orthogonal Frequency Division Multiplexing that is OFDM system and we called this the Comb Type Pilot based estimate, right.

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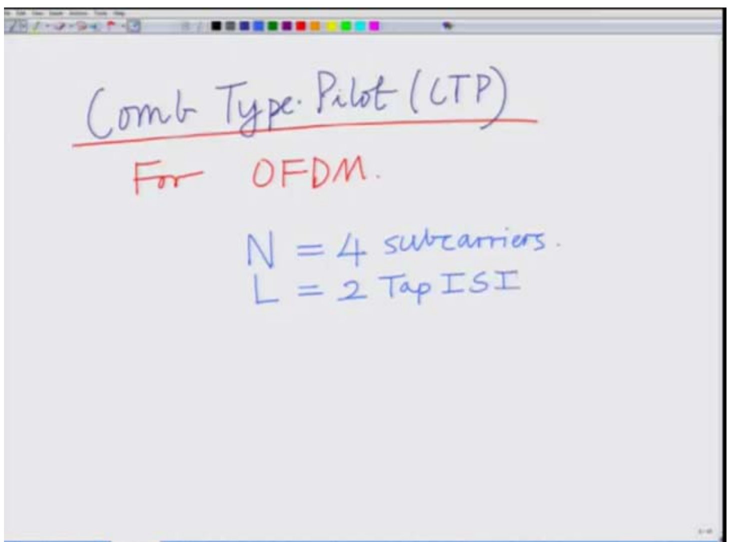

So we are looking at a Comb Type Pilot or basically CTP, this is your Comb Type Pilot for OFDM or Orthogonal Frequency Division Multiplexing, all right. And we have said again, just to refresh your memory, we are going to consider a system with N equal to 4 subcarriers and L equal to 2 tap ISI channel L equal to 2 tap ISI channel. And what we did in the previous module was, we looked at the 4 symbols loaded onto the N equal to 4 subcarriers.

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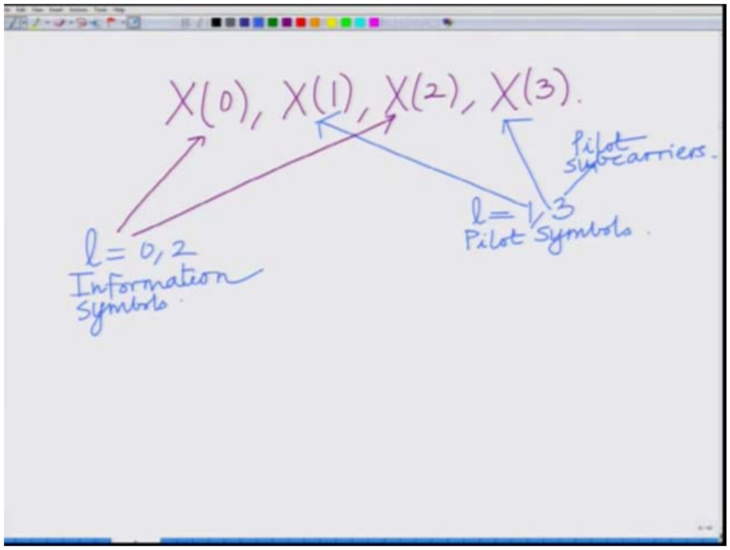

That is X 0, X 1, X 2, X 3, okay. And we said we are not going to load all the subcarriers with pilot symbols, rather we are going to load only to symbols that is corresponding to subcarriers L equal to 1, 3. These are pilot symbols and in fact, we said these are the corresponding L equal to 1 to 3; these are your pilot subcarriers, okay.

And similarly, we have X 0 and X 2, the rest of the subcarriers, that is subcarriers L equal to 0, 2 these are your information subcarriers, these are loaded, loaded with the regular information symbols, that is the transmitted information over the channels. So these are loaded with your regular information symbols. Now what we do is now we have basically what we have is we have 2 pilot subcarriers.

That is X 1, X 3; these are the 2 pilot symbols and the information symbols. That is X 0, X 2. And now we have basically what we have is basically the pilot symbols are sort of meshed with the information symbols therefore, this is known as the Comb Type Pilot arrangement that is what we saw yesterday.

And now once we have this block of symbols, that is pilot symbols and information symbols that are loaded onto the subcarriers, we take the N point, that is N equal to 4 point IFFT to generate the time domain samples, okay. That is similar to their regular OFDM. So now what we do is we do your N equal to 4 point IFFT to generate the samples, and these are your x 0, x 1, x 2, x 3, okay which other time domain samples.

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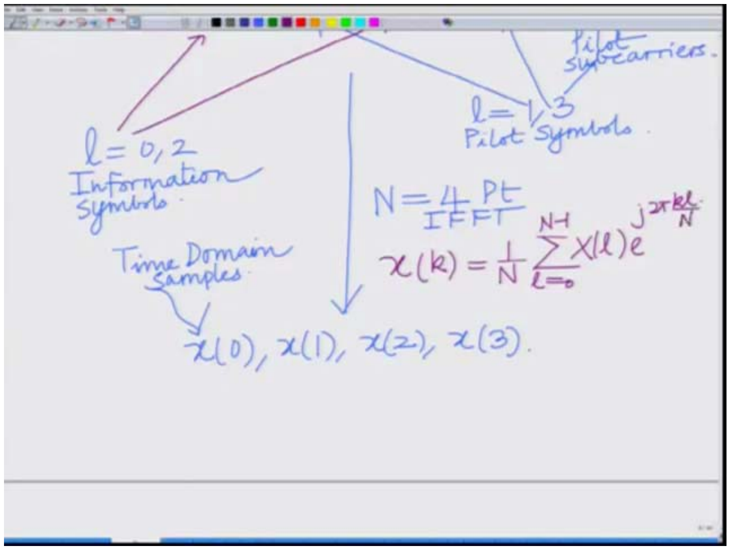

These are the time domain samples and in fact it is these are given as the kth sample is x k equals 1 over N submission L equal to 0 to N - 1 X L e power j 2 pie j k L divided by N. Now once you generate the time domain samples, we add the cyclic prefix that is basically where you have x 3, that is something symbols from the tail prefixed in the beginning, so you have now x 3, x 0, x 1, x 2, x 3.

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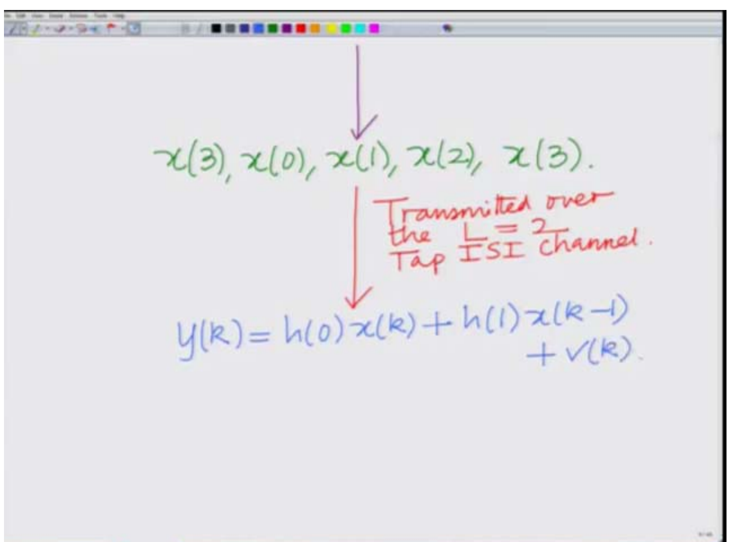

And these are transmitted basically over the transmitted over the L equal to 2 tap ISI channel, all right. These are transmitted over the L equal to 2 tap ISI channel which is your y k equals h 0 x k + h 1 x k - 1 + V k, all right. So once you transmit the cyclic prefix basically once you transmit the time domain samples with the cyclic prefix over this Inter Symbol Interference channel.

Now what happens basically the channel, the action of the channel over the samples is that of circular convolution therefore, what you have is the received output y is h which is circularly convolved with x + v, okay.

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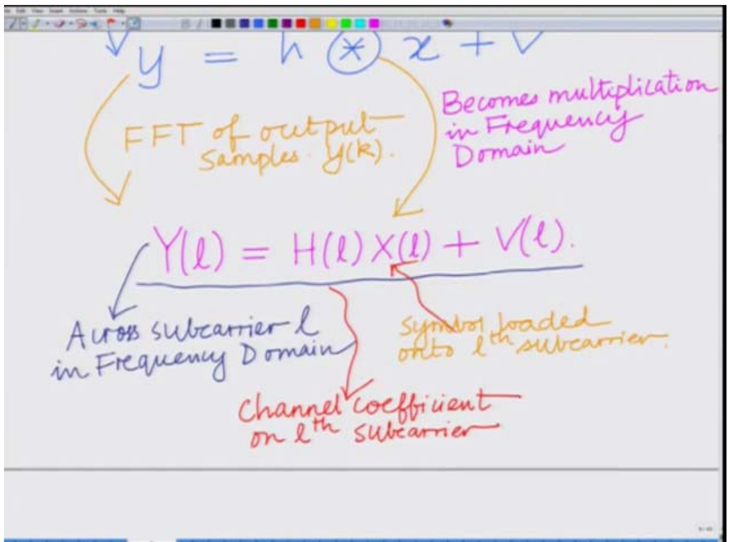

Now once you take the FFT, you take the FFT of the output samples y, okay. This circular convolution becomes multiplication in the FFT or the frequency domain.

Yeah, because in the frequency domain because multiplication in the in the frequency domain therefore, across subcarrier L you have Y L equals H L times X L + V L, yeah. So this is basically across subcarrier L in the frequency domain, all right. So this is something that you have to remember, that is this is in the FFT or in the frequency domain.

That is you take the FFT of the received output samples, all right, because the channel is circularly convolved with the time domain samples. Therefore, in the frequency domain the corresponding channel coefficients are basically multiplied by the corresponding FFT coefficients of the samples, but the FFT coefficients but the samples remember are generated by the IFFT of the symbols loaded onto the subcarriers.

Therefore, the FFT of the samples is nothing but the symbols loaded onto the subcarriers. Therefore, the output L of the Lth subcarrier is the channel coefficient H L times X L which is the symbol loaded on the Lth subcarriers + noise, which is the noise on the Lth subcarrier that is given by Lth FFT point of the time domain noise samples, okay.

So we can look at this, this is the channel coefficient on the Lth subcarrier and this is the symbol loaded onto the Lth subcarrier, okay. Now specifically for channel estimation remember; we are interested in the pilot symbol or the subcarriers therefore, we have to extract the output symbols corresponding to these pilot subcarriers.

And remember that these pilot subcarriers correspond to L equal to 1 and L equal to 3, these are our pilot subcarriers, okay. So our pilot subcarriers are, these are basically our pilot subcarriers. Therefore, now what we are going to do, we are going to extract the output symbols corresponding to the pilot subcarriers.

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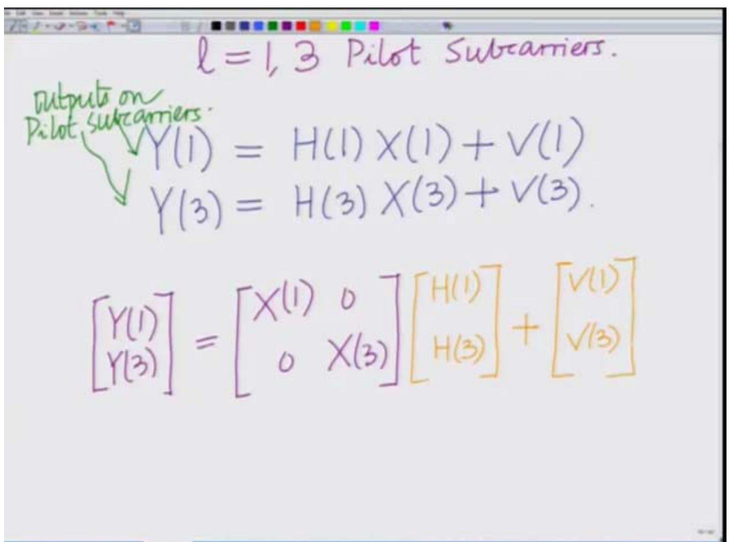

That is Y 1 equals H 1 X 1 + V 1, Y 3 equals H 3 X 3 + V 3, all right. Since L equal to 1 and L equal to 3 are basically the pilot subcarriers therefore, we are extracting output signals Y 1 and Y 3 corresponding to these pilot subcarriers, okay. So these are outputs corresponding to the pilot subcarriers. These can be used for channel estimation, right. These are outputs corresponding to your okay, now writing it in matrix form I have.

Again I can write it, again similar to what we did previously for OFDM, I can write it in matrix form, this Y 1 Y 3 equals X 1 X 3 times H 1 H 3 + your noise factor V 1 V 3. And now therefore what you have is basically you have.

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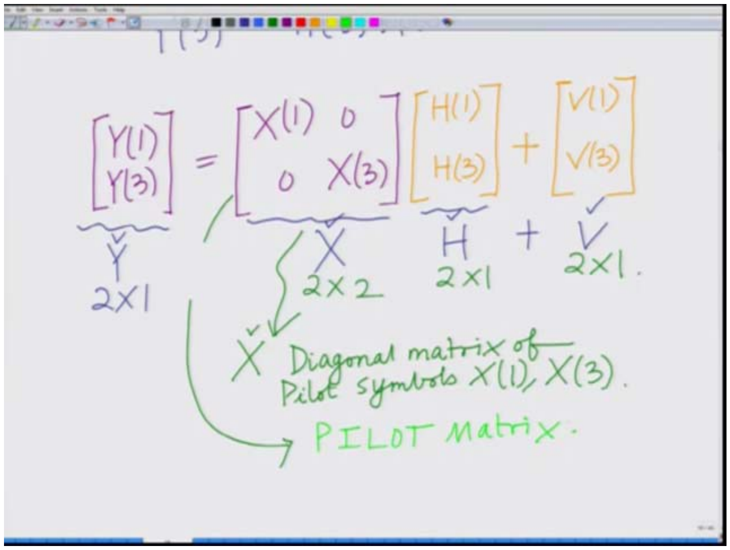

Let us call this output vector Y check, let us call this diagonal matrix of pilot symbols as X check, this is in fact the pilot matrix. Let us call this vector as your H check + this is V check.

So basically Y check, what is this? This is your 2 cross vector, this is also a 2 cross 1 vector basically, an L cross 1 vector this is I am sorry this is a 2 cross 2 matrix, this is a 2 cross 1 vector and this is a 2 cross 1 vector. And observe again that this X check, this is the diagonal matrix of pilot symbols, this is the diagonal matrix of pilot samples X 1 and X 3. That is the diagonal elements are the transmitted pilot symbols X1 and X 3, all right.

So therefore, it is a diagonal matrix and in fact this is also the pilot matrix for this comb type pilot based OFDM channel estimation, okay. So this is basically our pilot matrix, okay. So this is our this is our 2 cross 2 pilot matrix. And now obviously a system model, what is our equivalent system model or equivalent system model, is Y check, we have this Y check equals X check times H check + V check.

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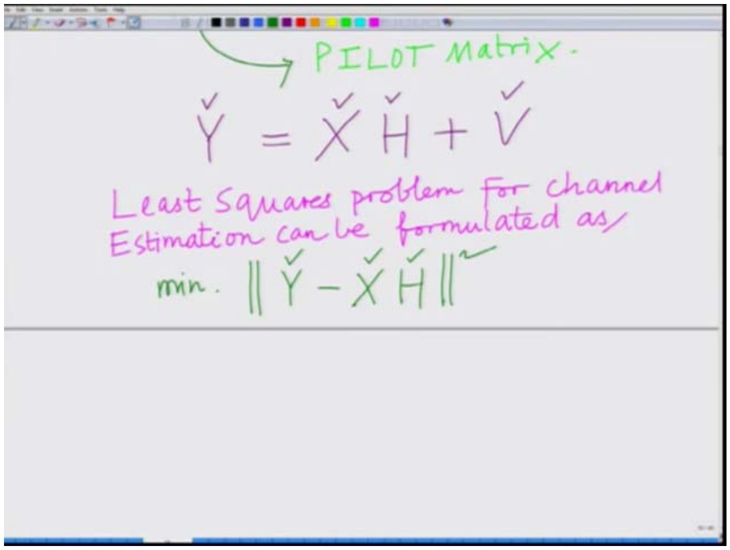

Therefore, the Least squares problem for channel estimation can be formulated as; the least squares problem for channel estimation can now be formulated as norm Y - X or norm Y check - X check whole square. This is the same, this is your and one has to find the H check which minimises this Least Squares cost function all right.

So we have to find this lector H check, right. And that is found from the Least Squares minimisation problem that is, the H check which minimises this Least Squares cost function, all right. So we have to find this vector H check, right. And that is found as from the Least Squares minimisation, that is the H check which minimises this Least Squares cost function that is, norm of Y check - X check times X check norm whole square, okay.

And we know the solution for this least squares cost function, again similar to what we have done previously in the case of OFDM.

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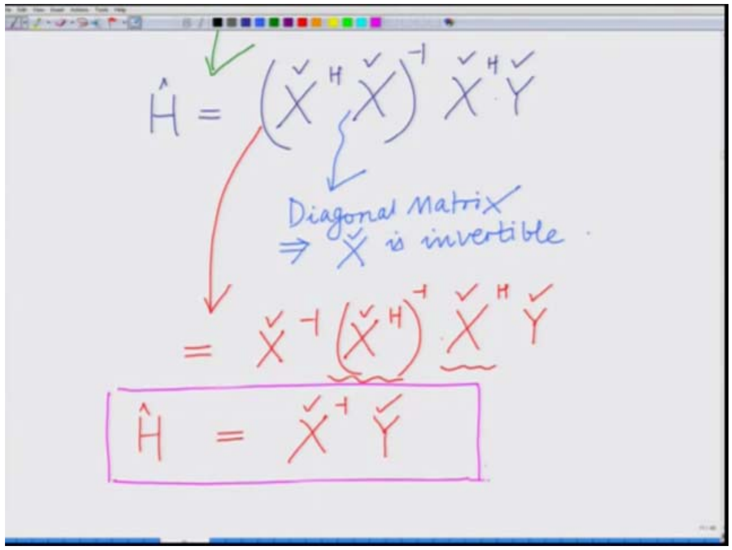

I have the estimate H hat equals X check hermitian X check that is the pseudo-inverse of X check times in fact, X check hermitian times Y. However, now this is a diagonal matrix and invertible. X check is a diagonal matrix implies X check is invertible. Therefore, specifically for this scenario, I mean although it is not generally valid.

I can write this as is X check is an invertible matrix, X check her, so this becomes X check inverse X check hermitian inverse times X check hermitian times Y check, X check hermitian times inverse times X check her, this is identity, so this basically becomes your X check her. So your H hat is basically is your in fact, this X check inverse times Y. So that is the estimate of the channel coefficient in fact, now this can be implemented rather very easily, because X check is a diagonal matrix.

So the inverse of the diagonal matrix is nothing but another diagonal matrix with each with the reciprocals of the diagonal elements. Therefore, what we have is basically in fact, H hat 1 that is the estimates of the subcarriers.

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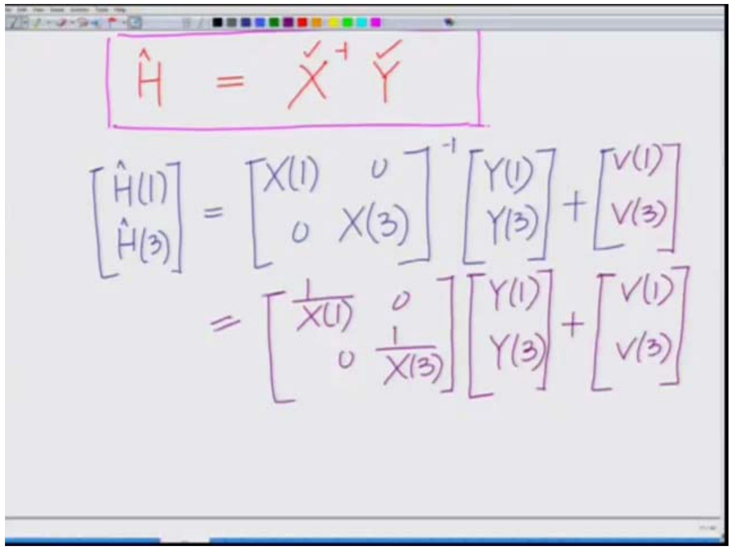

Estimates of the channel coefficients of the pilot subcarriers are basically your inverse of this diagonal matrix times Y 1 Y 3 outputs on the pilot subcarriers + the noise V1, V 3 on the pilot subcarriers.

And therefore, now you can see this is nothing but inverse of the elements, a reciprocal of the elements on the diagonal, because it is a diagonal as we said X check is a diagonal matrix, so its inverse is another diagonal matrix with the reciprocals of the diagonal elements, which implies finally if you look at this, if you look at this element by element. Therefore, the estimates are H 1 again you can see this is 1 X 1 over X 1 times Y 1, H hat 3 is 1 over X 3 times Y 3.

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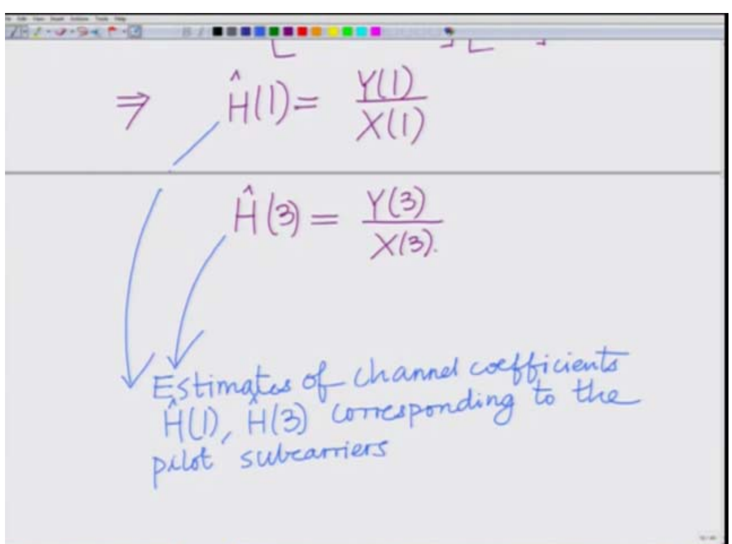

I am sorry, there should not be, there is no noise in the estimate, so this is basically this part and therefore what we have over here is H hat 1 equals Y 1 divided by H 1, H hat 3 equals Y 3 divided by X 3. So these are basically, what are these? These are the estimates of the channel coefficients on the pilot subcarriers. These are the estimates of channel coefficient H hat 1, H hat 3 corresponding to the corresponding to the pilot subcarriers.

And that is something important to realise that is where as previously when we load pilot symbols onto all the subcarriers, we have the estimates of all the channel coefficients corresponding to all the subcarriers. However, now we do not have the estimates of the channel coefficients corresponding to all the subcarriers, we can estimate only those channel coefficients corresponding to the pilot subcarriers.

And from these estimates of the channel coefficients on the pilot subcarriers, we are going to see that interestingly we can recover the estimates of the channel coefficients on the rest of the subcarriers. And that and the trick to do that lie in realising the following. Now if you look at, if you go back and look at these channel coefficients in the previous module.

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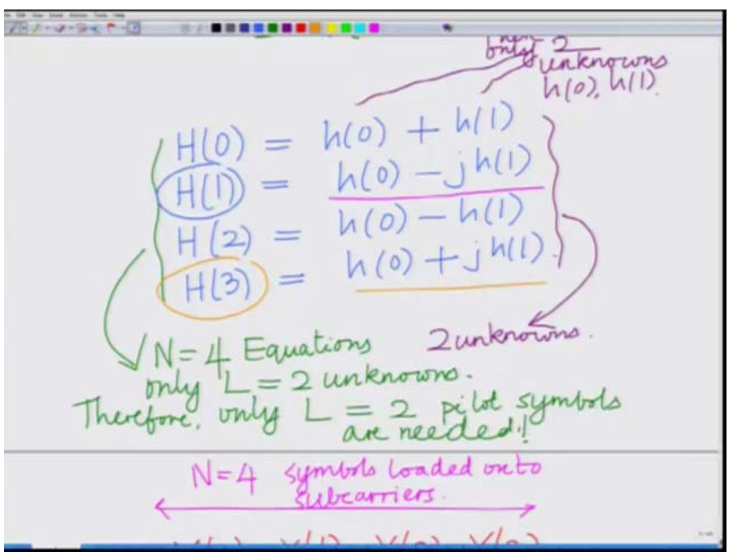

Now what you can see is basically, the channel coefficients on the subcarriers can be expressed as basically the channel coefficients on the coefficients, the channel taps, all right. So I have H 1 which can basically be expressed as h 0 - j h 1 and I have H 3 which can be expressed as h 0 plus j h 1. Therefore, now what you have is if you go back so therefore what you can see is the following thing.

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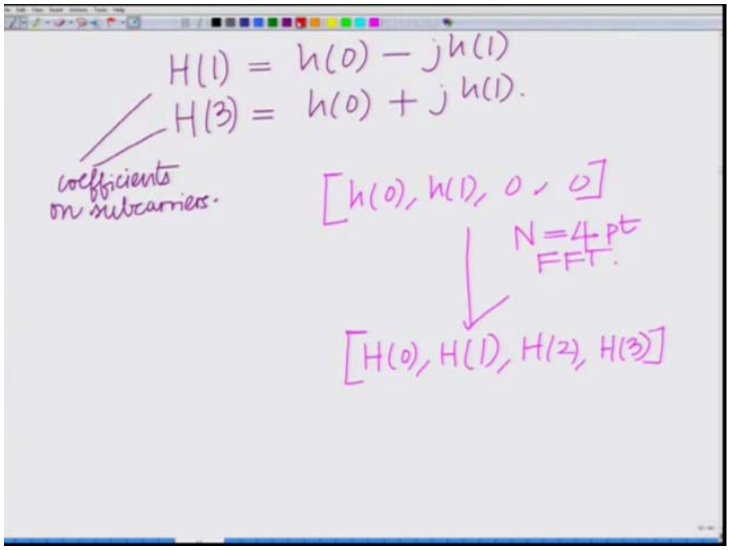

Therefore, I have H 1 equals h 0 - let me write it correctly, h 0 - j h 1 and H 3 equals h 0 + j h 1, yeah. So these are the coefficients on subcarriers and h 0 and h 1 remember, these are the channel taps in the time domain. These are the channel taps in the time domain in fact, the channel coefficients are basically, if you remember the channel coefficients are basically derived by the 0 pared FFT of the channel taps.

So I have H 0 H 1, I looked at the N equal to 4 point FFT or the Fast Fourier transform or H 2 and H 3 and this is basically your N equal to 4 point FFT. So the channel coefficients the channel coefficients in the across the subcarriers the channel coefficients in the frequency domain are basically given by the FFT of the channel taps. In fact, the 0 pared FFT of the channel taps in the time domain, all right.

Now if you look at this, I have H 1 H 1 that is the channel coefficient across carrier 1 equals h 0 - j h 1 and H 3 that is the coefficient across subcarrier 3 equals h 0 + j h 1. Now I can write this in a matrix form and that will become interestingly that will become H 1, H 3 equals we have 1, - j, 1, j into the channel tap vector h 0, h 1.

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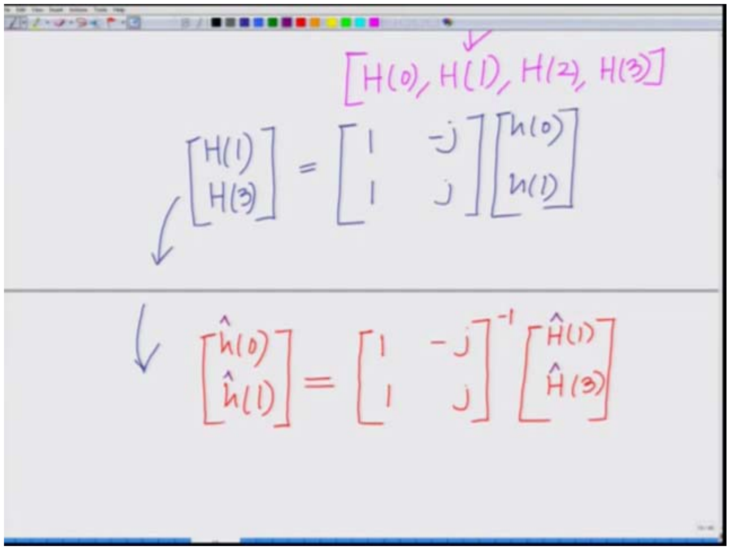

And now if you look at this therefore, now if you look at this it is easy to see that the channel taps in the time domain that is h hat 0 h 0 h 1 is the inverse of this matrix this transformation matrix 1, - j, 1, j, the inverse of this times H 1, H 3. And in fact the only difference is basically we have the estimates of the external coefficients H 1 H 3 therefore, I have the estimates that correspondingly therefore, if you have estimates on the right, you will have estimates on the left.

So basically, I have the estimates of the channel coefficients H hat 1 H hat 3 on subcarriers 1 and 3. And from this I can from these estimates, I can basically recover the estimates of the channel taps, the h hat 0 and the h hat 1 in fact that is, that holds the key to this entire process because remember when we started, we said that there are only 2 unknowns in this system and these 2 unknowns are in fact the channel um, the channel taps.

The time domain channel taps that is your h 0 and h 1. So basically, I need to 2 equations to determine this time domain channel taps and those 2 equations are given by my H 1 that is the subcarrier coefficient or estimate of the subcarrier coefficient on subcarrier 1 and H 3 that is H hat 3 that is the estimate of the channel coefficient on subcarrier 3. And from this now therefore I can find the estimates of the time domain channel taps.

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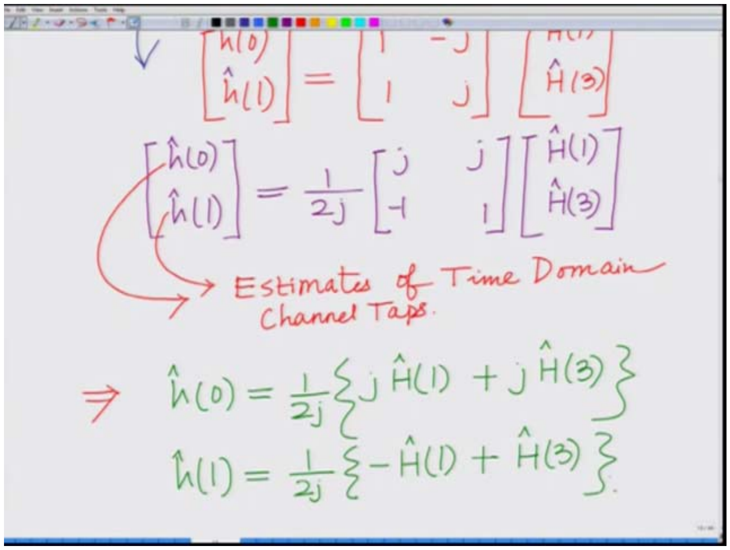

And of course there is 2 cross 2 matrix that is easy to invert that is 1 over the determinant that is 1 over 2 j times interchange the diagonal entries that is, j and 1 and well then the negative of the diagonal entries, okay. So interchange the diagonal entries and the negative of the diagonal, so it is j, - 1 and that is basically equal to now H hat 1, H hat 3 and that basically gives me my h at 0, h hat 1.

These are the estimates of basically, what are these; these are the estimates of the time domain channel taps. And that is interesting, so you can use this linear transformation to get estimates of the time domain. These are the estimates of the time domain channel taps and that is what we have. And therefore, now you can see that h hat 0, this basically implies that, now writing it out I have h hat 0 equals 1 over 2 j times well j times H hat 1 + j times j hat 3.

And similarly, h hat h hat I am sorry similarly h hat 1 is equal to 1 over 2 j - H hat 1 + H hat 3, okay. So these are the estimates of the, and remember we already said, these are basically your estimates of the channel taps in the time domain. Now from the estimates of the channel time in the time domain.

So basically these are given as h hat 0 equals 1 over 2 j times j times H hat 1 + j times H hat 3; h hat 1 equals 1 over 2 j times - H hat 1 + H hat 3, so these are obviously H hat 1 and H hat 3 are the estimates of the channel coefficients across the pilot subcarriers in the frequency domain.

And now basically from these, the rest of the subcarrier coefficients can now be calculated, so now I can use the estimates of the channel taps. h hat 0 and h hat 1 in the time domain and compute the estimates of the rest of the subcarriers coefficients and that can be done also as follows. Now again we go back to where we expressed the subcarrier channel coefficients in terms of the channel types and see how we can use that once again.

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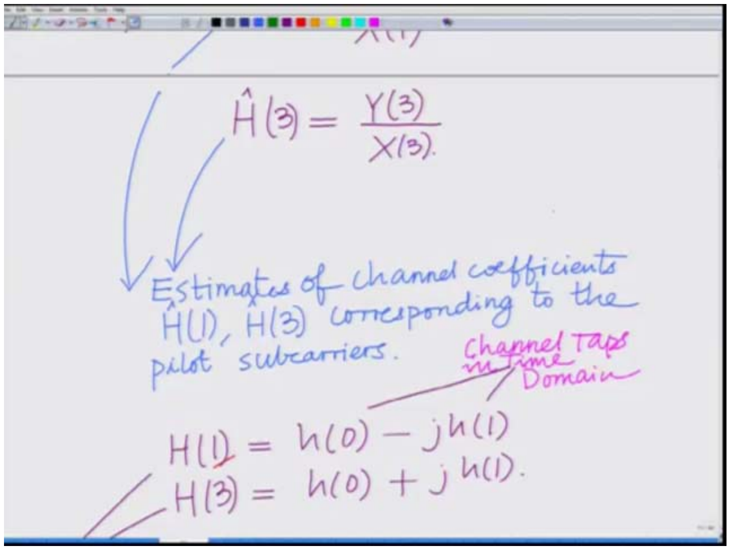

And that is basically done as follows, now if we look at this again I have H 0 basically, you can see equals h 0 + h 1, all right.

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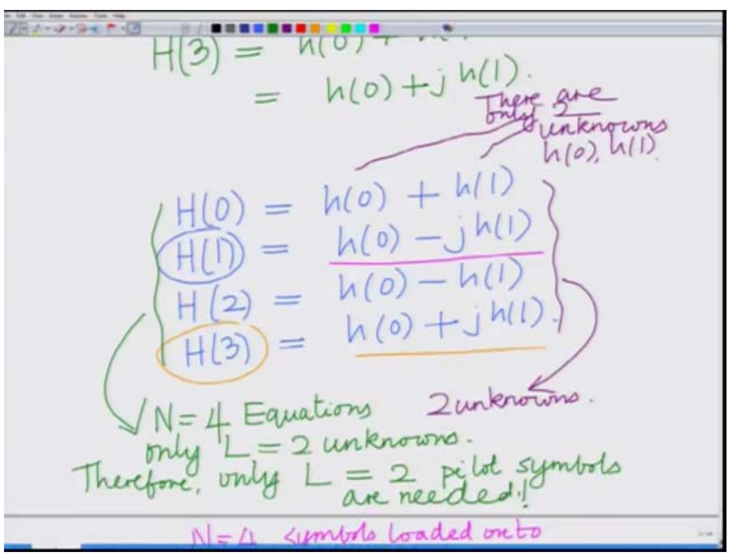

And the coefficient across subcarrier 2 is h 0 - h 1, okay. So now I can go back again and I can use this property.

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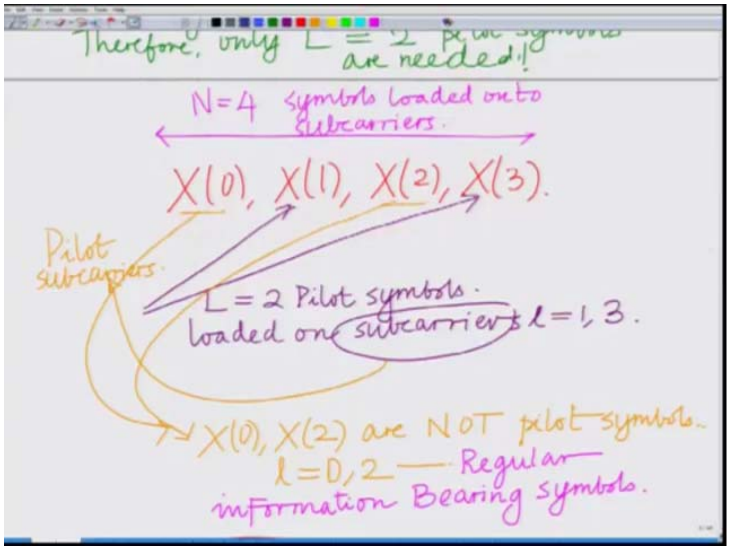

So basically what I have is interestingly what I have is.

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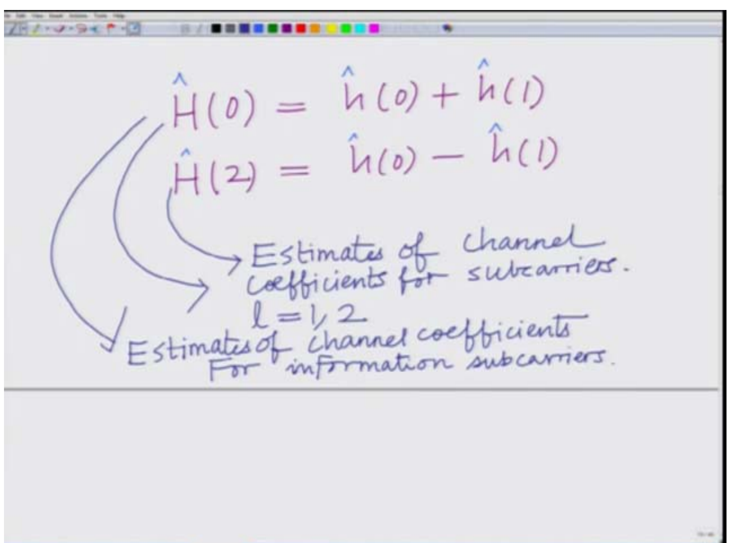

I have H 0 equals h 0 + h 1; H 2 equals h 0 - h 1 and now I can basically do these estimates as H 2 H hat 2 equals.

So basically now you can see the estimates H hat 0, H hat 2, these are estimates of your channel coefficients for subcarriers L equal to 1, 2. Or which are basically the estimates of the channel coefficients for the non-pilot subcarriers or estimates of channel coefficients for the information subcarriers. These are basically your, these are the estimates of the channel coefficients basically for the information subcarriers and that is what we have over here.

So if you look at this, we have something very interesting. So now that completes it basically, now if you look at this that basically completes the problem. Now we have the estimates of the channel coefficients on all the subcarriers. Basically, H hat 1 and H hat 3 we estimated from the pilot subcarriers, all right from the pilot symbols transmitted on the pilot subcarriers from these we estimated the channel taps.

And from the channel taps we estimated the rest of the channel coefficients corresponding to the non pilot subcarriers, all right. So that is very interesting, so even though you are transmitting only 2 pilot symbols on 2 subcarriers that is you are not transmitting pilot symbols on all the subcarriers, you are only transmitting a few pilot symbols on a few pilot subcarriers, you are able to basically extract the estimates of the channel coefficient on all the subcarriers.

So basically let us note that down that is, although one is transmitting L equal to 2 pilot symbols, one is octane the estimates of channel coefficients on all the subcarriers.

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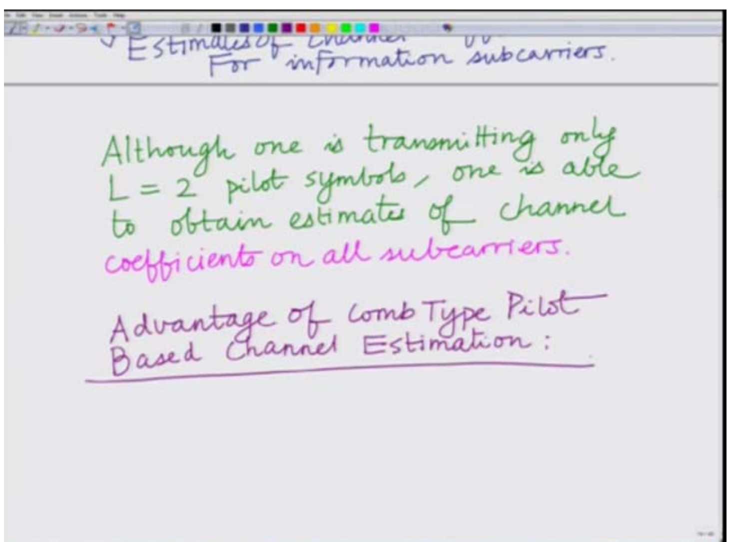

And therefore what is the advantage of this scheme, what is the advantage of this, Comb Type Pilot scheme again that is obvious, something that we have already said at the beginning that is, we are only transmitting pilot symbols on a few subcarriers.

Therefore, the rest of the subcarriers can be used to transmit the information or the data symbols and that significantly increases the bandwidth efficiency, all right. So let us note that down also, that is, what is the advantage of your Comb Type Pilot based channel estimation scheme? What is the advantage of Comb Type Pilot based channel estimation scheme?

And that is something that we have already seen, just to repeat it one last time that is I have X 0, X 1, X 2, X 3.

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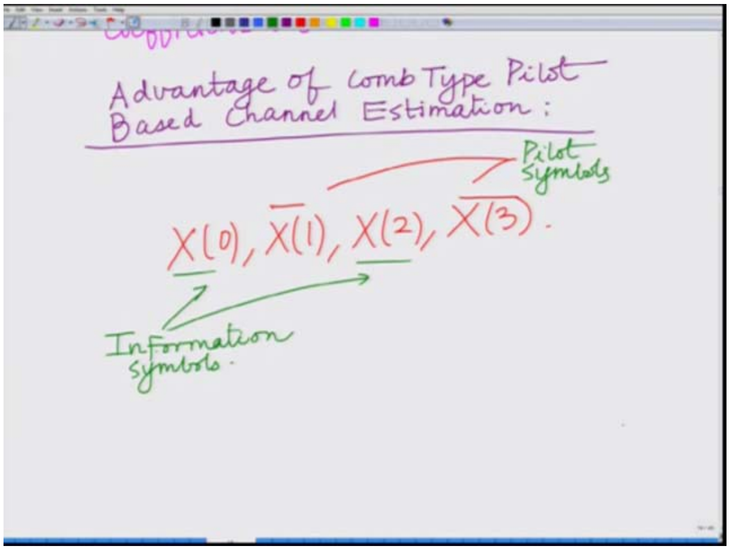

X 1 and X 3, these are your pilots or your pilot symbols. X 0, X 2 these are your information symbols, these are the information symbols. So since you are transmitting pilot symbols and information symbols, that if you are not entirely transmitting pilot symbols, so since the pilots are loaded fewer subcarriers in the Comb Type Pilot transmission.

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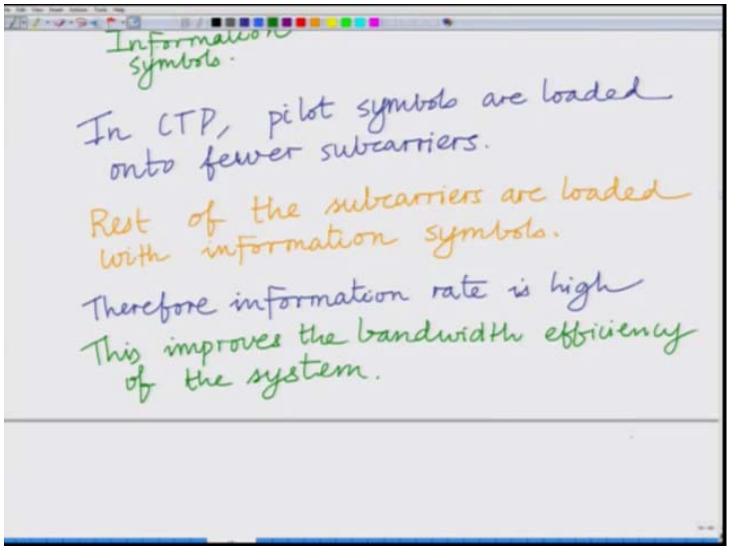

Pilot symbols are loaded pilot symbols are loaded onto fewer subcarriers. And rest of the subcarriers are loaded with information, rest of the carriers are loaded with information symbol therefore, this improves the bandwidth efficiency. Therefore, information rate and this improves the bandwidth efficiency that is also something interesting to note. This improves the bandwidth efficiency.

This significantly improves the bandwidth efficiency of the system, all right so that is what we have seen. So that is basically in summary the Comb Type Pilot based channel estimation scheme for OFDM what is what is interesting about this in comparison to the previous conventional pilot based OFDM channel estimation scheme is basically, you are loading pilot symbols only onto fewer subcarriers.

Therefore, the rest of the subcarriers can be used to transmit information symbols, all right. And the channel estimation proceeds by first estimating the channel coefficients in the frequency domain across the pilot subcarriers, convert them back to the estimates of the channel taps in the time domain and from the estimates of the channel taps in the time domain, estimate the rest of the channel estimate the channel coefficient corresponding to the rest of the sub carriers.

That is the non-pilot subcarriers or the information symbol subcarriers and therefore, now even though you are transmitting a fewer pilot symbols, you are basically able to establish the channel coefficients corresponding to all the sub carriers. That in summary basically captures the spirit and also basically the method of this Comb Type or the CTP based channel estimation for an Orthogonal Frequency Division Multiplexing OFMD System, all Right.

So we will stop this module here and subsequent module, we will look at an example to illustrate this further, thank you. Thanks very much.