Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks

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Lecture Number - 31

Comb Type Pilot (CTP) Based Author Orthogonal Frequency Division Multiplexing (OFDM) Channel Estimation-Block Structure

Hello, welcome to another module in this massive open online course on Estimation Theory for Wireless Communication Systems. So previously we have looked at OFDM channel estimation that is; how to transmit the pilot symbols on the various subcarriers, find estimates of the channel coefficients in frequency domain and later from these estimates in the frequency domain, compute the estimates of the channel taps in the time domain alright.

So now we are going to look at a different scheme for OFDM channel estimation, this is known as Comb Type Pilot Based OFDM channel estimation, all right. So we are going to continue looking at OFDM channel estimation, where remember again OFDM, all of you should be familiar by now stands for Orthogonal Frequency Orthogonal Frequency Division Multiplexing.

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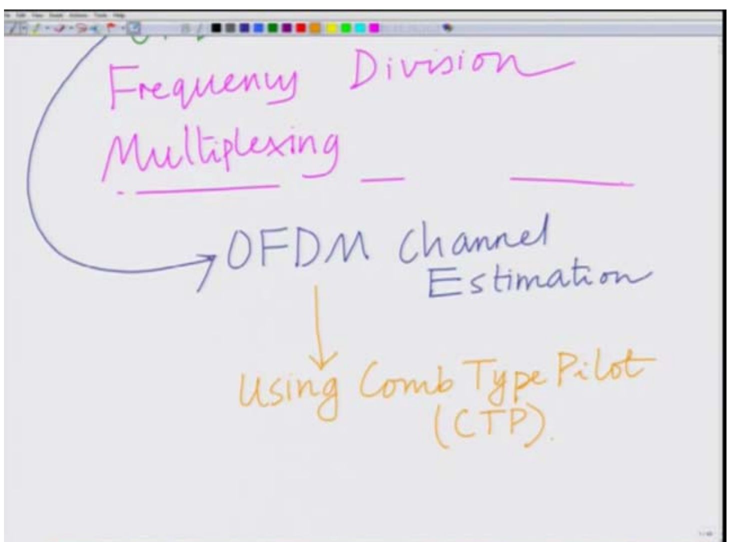

We are going to look at OFDM channel estimation and we are going to look at OFDM using Comb Type Pilot. So we are going to look at OFDM channel estimation using what are known as a comb type of pilot. So OFDM channel estimation using Comb Type Pilot symbols or we can also abbreviate this as CTP, where CTP represents a Comb Type Pilot transmission for an OFDM system.

So first let us understand what is this Comb Type Pilot transmission? And let us understand what is the motivation for Comb Type Pilot transmission; so let us go back to our L equal to 2 taps, so let us consider our N equal to 4 subcarriers OFDM system as we considered previously also. N equal to 4 subcarriers OFDM with L equal to 2 tap ISI channel, where ISI stands for Inter Symbol Interference.

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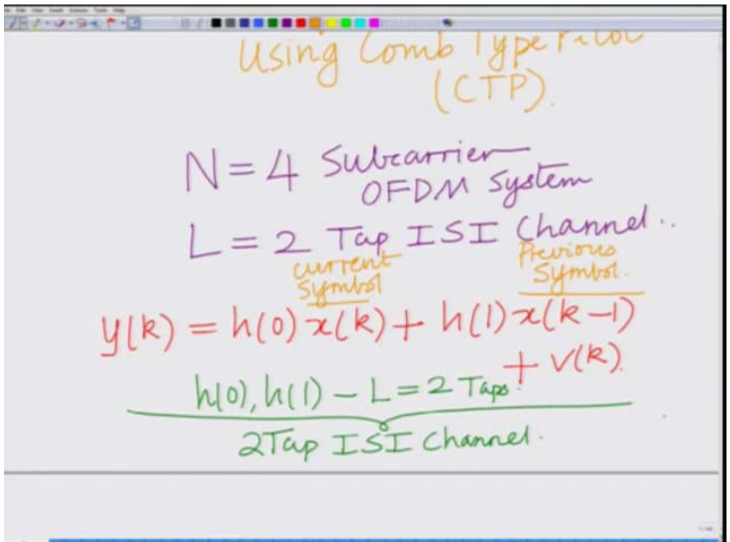

All of you should be familiar with that by now, so L equal to 2 tap ISI channel therefore, I have y k which is equal to h 0 times x k + h 1 times x k - 1 + v k. What is this? This is basically your this is basically your 2 tap ISI channel h 0, h 1, these are your L equal to 2 taps, these are L equal to 2 channel taps. X k is the again something we already seen, x k is the current symbol. This is the previous transmitted symbol.

And that is what we said; OFDM that is Orthogonal Frequency Division Multiplexing basically overcomes the Inter Symbol Interference, okay all right. So now once use OFDM, you can that the channel coefficients across the subcarrier in the time domain are given by the N point FFT in fact, 0 pared FFT of the channel taps in the time domain.

That is the channel coefficient in the frequency domain are given by the FFT of the channel tap, 0 pared FFT of the channel taps in the time domain. And this is something that we have already seen before, this is given as the channel coefficients, let me just write it down a bit more clearly. Channel coefficients H L in frequency domain are given by the 0 pared FFT of your channel taps in the time domain okay.

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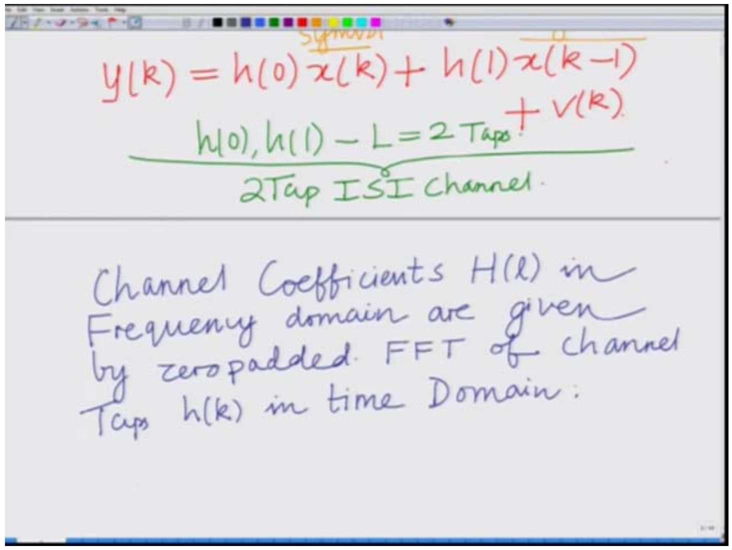

And that is basically can be demonstrated as follows, that is L channel taps I pad them, so these are my L equal to 2 taps, these are the L equal to 2 channel taps. These are the N - L equals basically 4 - 2, N equal to 4, L equal to 2 equal to 2, 0.

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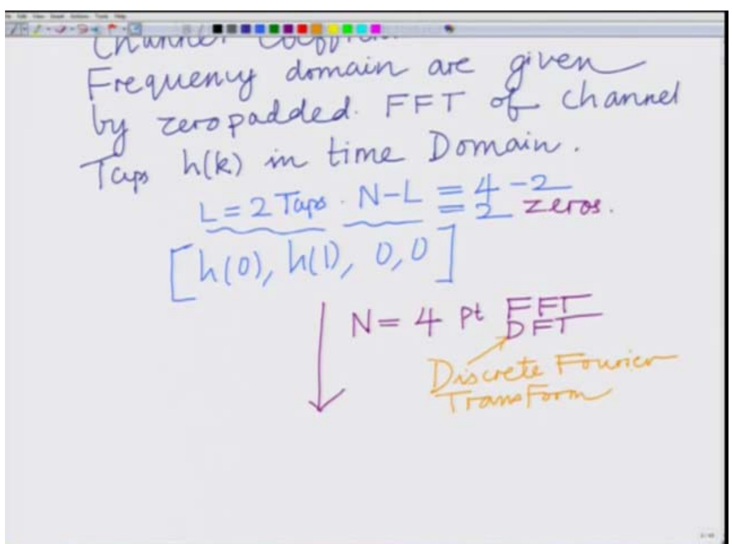

So you pad them with and this is also something that we have seen before. You pad them with N - L 0, N equal to 4 point now take your N equal to 4 point FFT, okay.

Or basically, your DFT your discreet where DFT basically stands for the Discrete Fourier Transform, remember DFT stands for the Discrete Fourier Transform. And this is the channel coefficients across the subcarrier that is your H 0, H 1, and H 2. H 3, so these are the channel taps and these are in the time domain.

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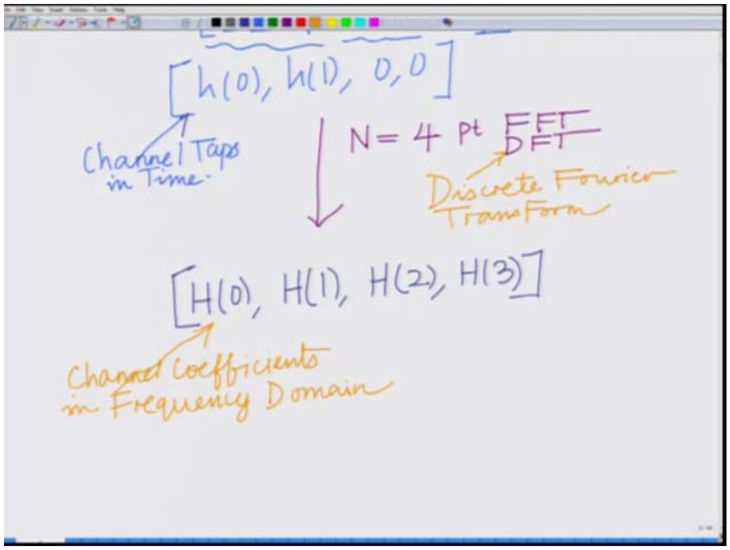

These are the channel taps channel taps in the time domain; these are channel coefficients or the subcarrier channel coefficients in the frequency domain. These are the subcarrier channel coefficients in the frequency domain therefore, we have the Lth subcarrier coefficient H L is basically given the FFT as a L equal to 0 in fact, there should be k equal to 0 FFT Lth FFT point of the channel taps h k e raise to - j 2 pie k L divided by N.

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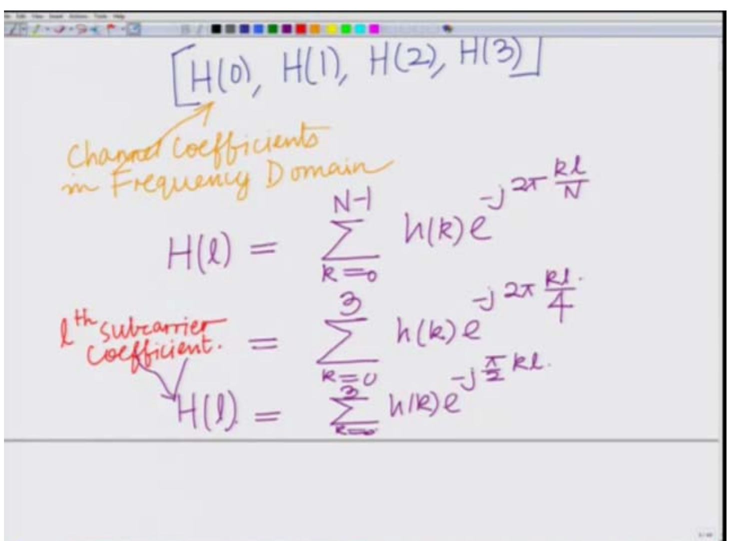

This something that we have seen before also and now substituting N equal to 4, this is basically k equal to 0 k equal to 0 to 3 h k e raise to - j 2 pie k L by 4, which is submission k equal to 0 to 3 h k e raise to - j pie by 2 k l. This is basically your H L which is the Lth subcarrier coefficient.

This is the Lth subcarrier coefficients, so H L is basically given by the Lth subcarrier coefficient is given by the Lth FFT point of the 0 pared channel taps, all right. So now let us right the explicit expressions for each H L and then we will see something interesting, so H 0 is basically your h of 0 + h of 1 e power - j pie by 2 k equal to 0 k equal to 0 and L equal to 0, yeah.

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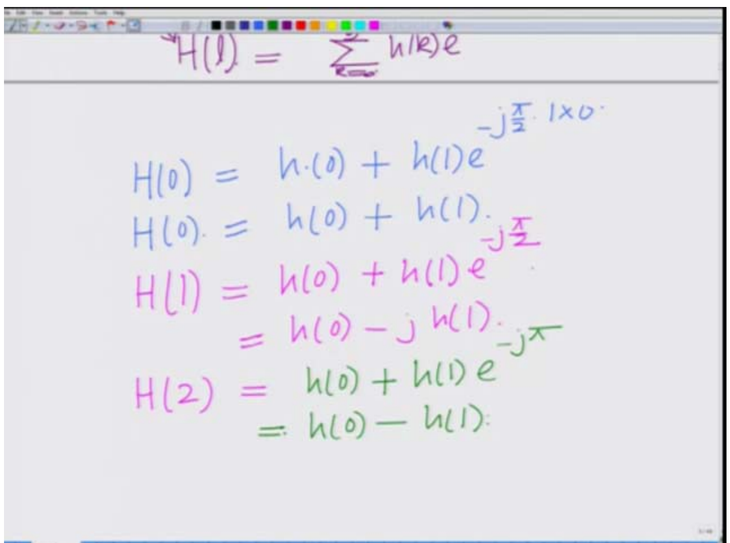

So that is in fact, 0 times 0 or basically k equal to 1 in fact, rather k equal to 1 L equal to 0, so basically this is h 0 + h 1. So you have H 0 equals h 0 + h 1. You have H 1 similarly is h 0 well h 0 + h 1 into e power to - j pie by 2, which is equal to h 0 + - j times h 1.

Similarly H 2, this is equal to h 0 + h 1 e power to - j pie, which is equal to h 0 - h 1 and H 3 which is equal to well you can check this, this is h 0 + h 1 e power to - j 3 pie by 2 equals h 0 + j h 1.

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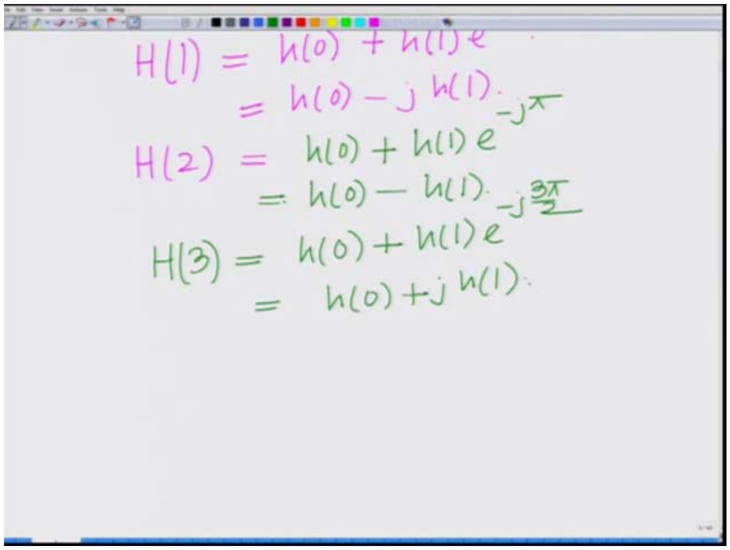

Now let us write this all together, write all this channel coefficients across the subcarriers and what you will find is H 0 equals h 0 + h 1, H 1 equals well h 0 - j h 1, H 2 equals h 0 - h 1, H 3 equals h 0 + j h 1.

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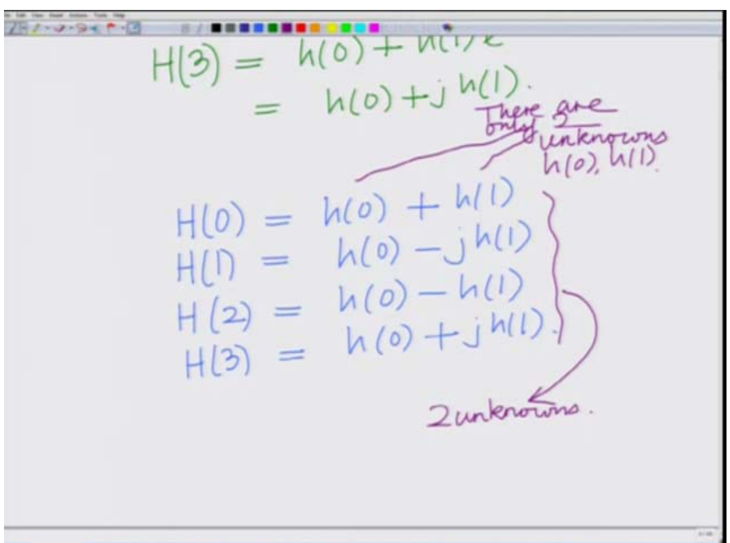

And what you realise from this is basically, there are only 2 unknowns. These are your channel taps h 0, h 1, so out of all these you can realise that there are only 2 basically only 2 unknowns. If you view this as a set of equations, basically if you view this as a set of 4 equations in the h 0 and h 1, you realise that basically there are only 2 unknowns, correct.

That is, h 0, h 1, yeah basically there are as many unknowns as the channel taps so this is L, L equal to e equals to 2 in this scenario, so there are 2 unknowns. So therefore you only need 2 equations to estimate the channel taps, which means you need only estimates of 2 subcarrier channel coefficients to estimate the channel taps h 0, h 1, that is the point.

The point is, there are 4 equations and 2 unknowns. There are N equations basically, there are here there is redundancy, because there are N equal to 4 equations, only L equal to 2 unknowns.

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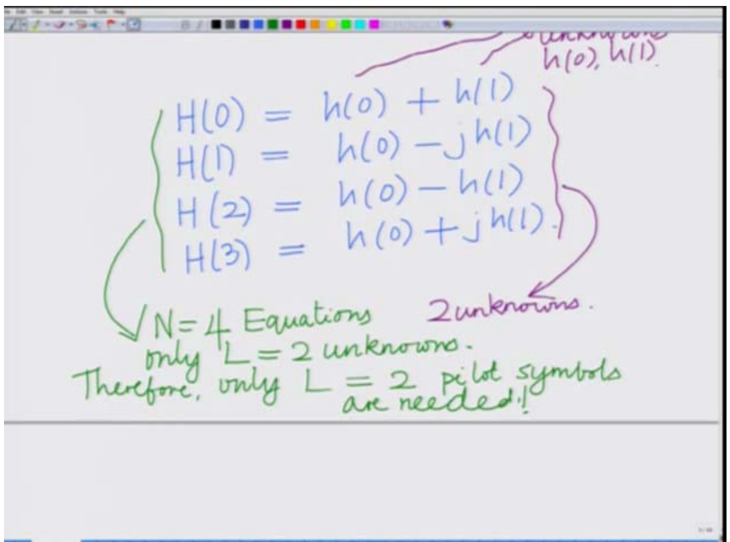

Therefore, only L equal to 2 pilot symbols are needed, this is in fact a very interesting observation, what we are saying is basically there are only 2 unknowns. We need to transmit only 2 pilot symbols.

So basically there are L unknowns, L is the number of channel taps, so you need only L pilot symbols and the rest of the symbols that are loaded onto the subcarriers, remember the N - L can be loaded with N - L subcarriers can be loaded with information symbols and this is basically interesting aspect that we are going to explore now, yeah.

So what I am going to say is basically we need L pilot symbols, so basically remember we have started with X 0, X 1, X 2, X 3, what are these?

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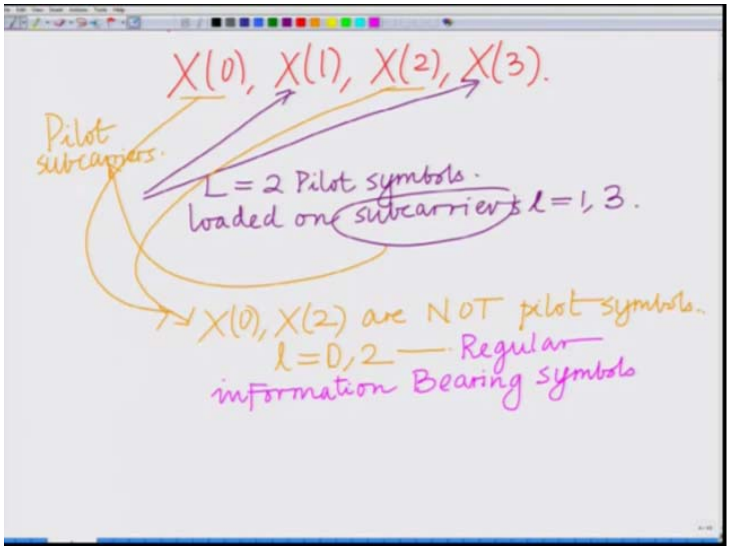

These are the N equal to 4 symbols loaded onto your loaded onto the subcarriers. Previously, all these N equal to 4 symbols were pilot symbols however, now we will only need 2 pilot symbols. So let us say basically, this X 1, X 3 are the L equal to 2 pilot symbols.

These are loaded onto subcarriers basically loaded onto subcarriers, so 1 and 3. That is L equal to that is, subcarrier 1, subcarrier 3, in fact the subcarriers carrying pilot symbols, these are known as the pilot subcarriers, these are known as the. These subcarriers which are loaded with pilot symbols, these are known as the pilot subcarriers, okay. So the rest X 0, now these X 0, X 2, these are not pilot symbols, so these can be.

So the rest can be, these are X 1, X 2 these are not your pilot symbols. Well, if these are not pilot symbols that is, corresponding to L equal to m I am sorry, these are X equal to 0, 2, so corresponding to subcarrier 0, 2, these are not pilot symbols, so if these are not pilot symbols, then what are these what are symbols that are loaded onto the subcarriers 0 and 2?

These are your regular information symbols; these are the normal information symbols; that is the information bearing symbols. So these are basically what these are? These are your regular or your normal, these are your normal information bearing singles, so what you are doing is basically instead of transmitting the pilot symbols on all the subcarriers, you are transmitting pilot symbols on only L subcarriers.

In this case, L equals to on the rest N - L that is which is 2 again, in this case you are transmitting the regular information bearing symbols. So you if you look at the symbols loaded onto the subcarriers that has the following structure, you have X 0, X 1, X 2, X 3.

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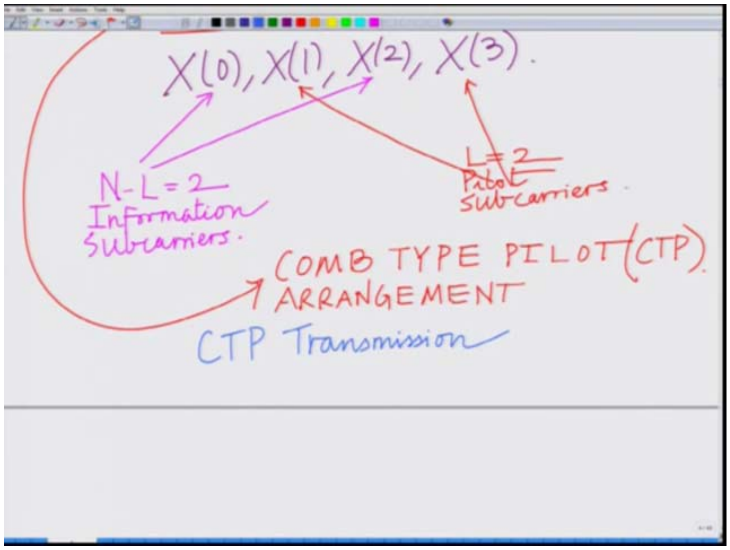

Out of these your X 0, X 1, these are your L equal to 2 pilot subcarriers, correct. These are your L equal to 2 pilot subcarriers.

And interestingly, these X 0, X 1, these are the information symbols, these are your L equal to 2 in fact, N - L equal to 2, these are your N - L information subcarriers. So what we have is basically we have pilot symbols on the few subcarriers, data symbols, sorry information symbols on the few subcarriers and these subcarriers are basically meshed, they arranged like a comb, this is known as a, Comb Type Pilot arrangement.

Basically because not all the subcarriers are carrying pilot symbols, few of them are carrying pilot symbols, that is X 1, X 3 inters place or spaced in between them, arranged between them are basically the information subcarriers, that is X 0 and X 2 and these are basically meshed, arranged like a meshed, this is known as a Comb Type Pilot arrangement, this is like a mesh or like a comb basically, all right.

So this is known as a comb type pilot arrangement, so such a transmission, this is basically known as your Comb Type Pilot arrangement or basically you can say Comb Type Pilot transmission. We are going to denote this by this acronym CTP, so this is basically your CTP pilot transmission or CTP based transmission. Comb Type Pilot transmission, okay.

And what we are going to do is, we are going to take these symbols with this Comb Type Pilot transmission that is the information and pilot symbols. And now we are going to do, so this is your comb type basically, block of symbols, right.

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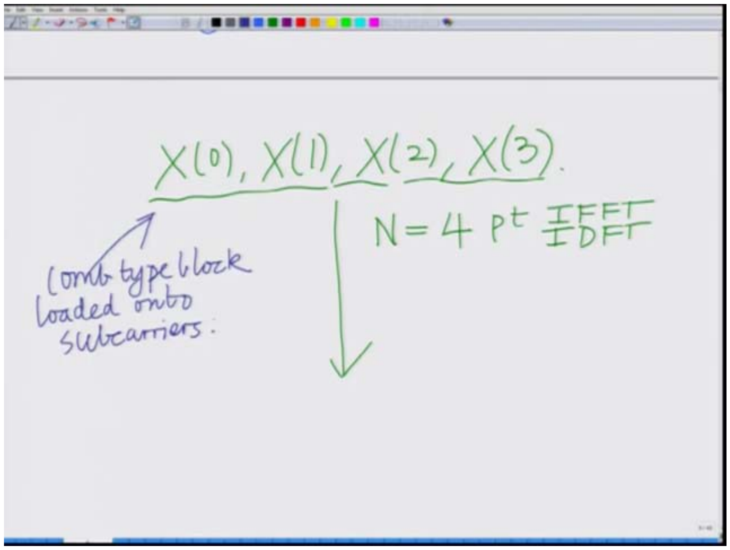

And now we are going to do N equal to that is the same as before. Now we are going to do the N 4 point IFFT which is the basically the efficient version of the IDFT Inverse Discrete Fourier Transform, all right.

So this is your Comb Type Pilot symbol that is Comb Type block this is a comb type block loaded onto the subcarriers. You generate the samples, now you take the N equal to 4 point IFFT and you generate the samples x 0, x 1, x 2, x 3.

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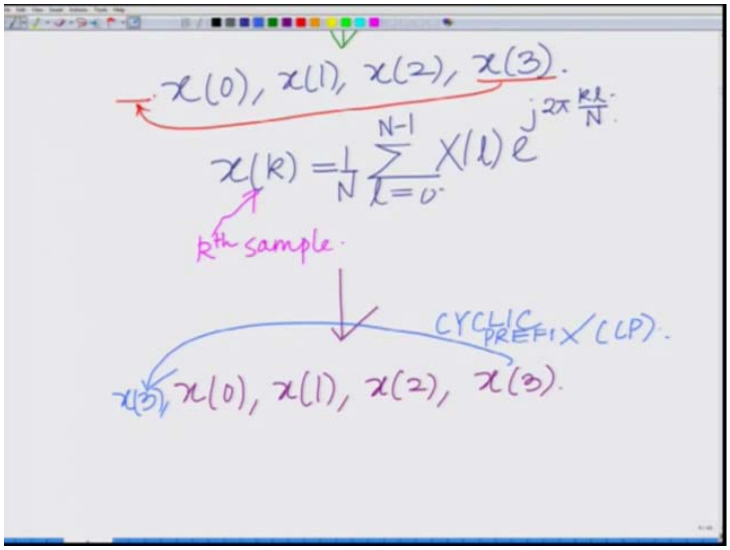

These are generated by the N point IFFT as basically x k equals L equal to 0 to N - 1, 1 over N x L e power j 2 pie k L by N, this is the N point IFFT and this is your kth sample.

And this is something that you have seen before, this does not change. The only thing that is changed is, now instead of transmitting, the only thing that has changed in this scenario is instead of loading the pilot symbols onto all of the carriers, now basically you are loading pilot symbols onto the few onto a few subcarriers, data symbols or information symbols on to the rest of the rest of the subcarriers.

And together with this with this block, which is comb type block, you take the N equal to 4 point IFFT or the Inverse Fast Fourier Transform or Basically the IDFT Inverse Discrete Fourier Transform to generate the time domain samples, okay. And once you generate the time domain samples.

Again the next operation is also similar, is also same in fact, identical where you basically take x 3 from the tail of the block, prefix it at the head. So basically your next step is again the same thing, that will be basically you take x 3, so you have x 0, x 1, x 2, x 3 and you take x 3 from the tail of the block, you prefix this, of course we have seen this many times before now, this is known as the cyclic prefix, that is CP.

And this thing also remains the same and this is the block of transmitted symbols with cyclic prefix.

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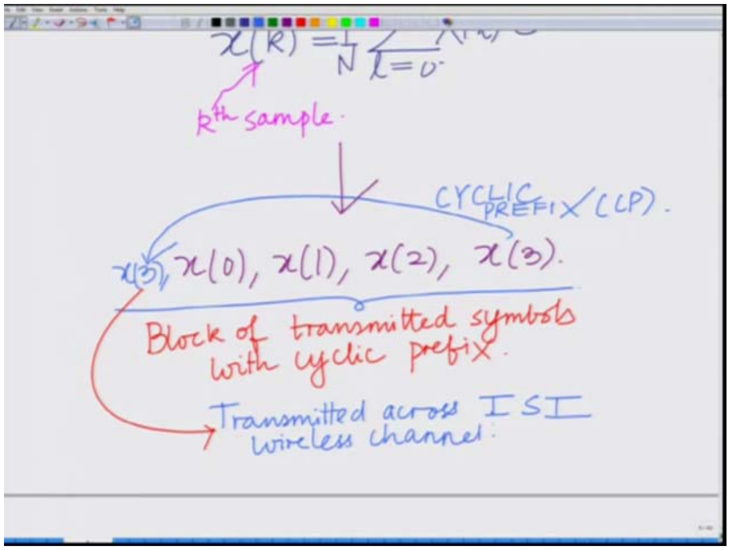

This is the block of transmitted symbols with cyclic prefix, something that we have already seen before. This is this block is transmitted across the ISI that is the ISI channel, this is the transmitted across the ISI, the block of the samples transmitted across your ISI channel basically, which is your y k equals, that is equal to well.

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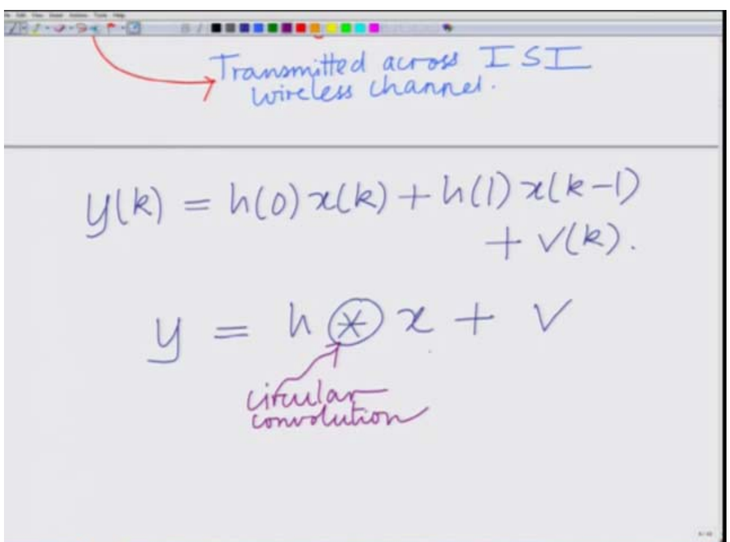

Or let me just write it little bit more clearly, that is y k equals h 0 x k + h 1 x k – 1 + V k and then we have seen many times before that when you transmit this block of samples with the cyclic prefix, the action of the channel in the time domain is basically that of a circular convolution.

Therefore, what you get at the output is basically the y which is equal to y the received output y is the channel filter h circularly convolved with h in the presence of additive noise V and this is the key aspect, this is your circular, this is the circular convolution, okay.

Now therefore, what this change is basically in the frequency domain, it simply becomes a multiplication.

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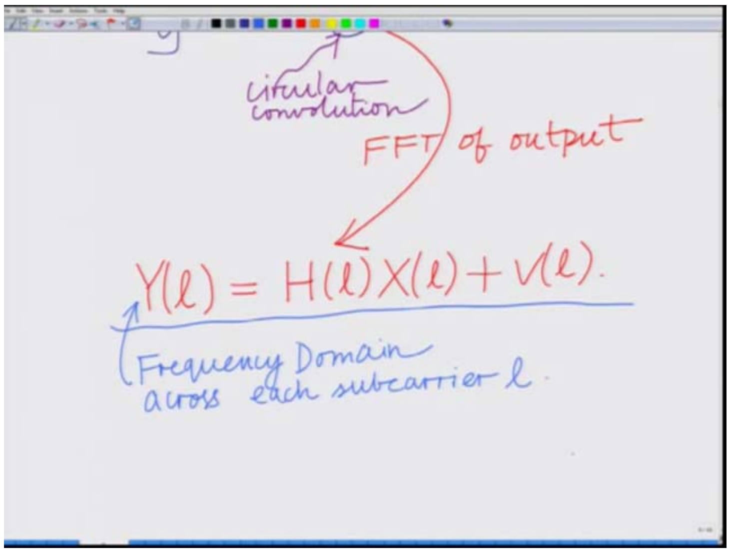

That is once you take the FFT of output that is, FFT of the output symbol that is y 0, that is the received symbols y, in the frequency domain I have Y L equals H L times X L + V L. So basically, remember this is in the frequency domain, this is in the frequency domain across each subcarriers L.

Now remember for channel and here comes the difference between the previous channel estimation scheme and this Comb Type that is CTP based channel estimation. Previously, we had looked at all the subcarriers because the pilot symbols were transmitted on all of subcarriers. However, in this scene, remember in CTP pilot symbols are only transmitted on the pilot subcarriers.

Therefore, we extract the pilot subcarrier that is, these are the only subcarriers which we are about which I mean with which we are concerned with regards to channel estimation, so we extract the pilot subcarriers. Remember the pilot subcarriers corresponding to L equal to 1 and L equal to 3. That is, subcarriers 1 and 3 are the pilot subcarriers, so therefore you extract the pilot subcarriers.

So in CTP since pilots are transmitted in CTP Comb Type Pilot transmission CTP only on L equal to 1, 3 subcarriers, that is basically the pilot subcarriers.

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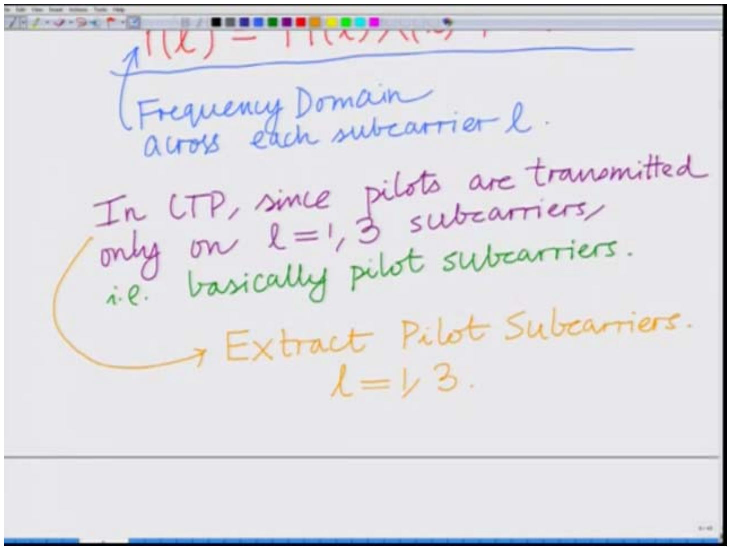

So what we are going to do is we are going to extract the pilot subcarriers that is, corresponding to L equal to 1, 3. And now once we extract the pilot subcarriers, what you see is basically the corresponding model for the pilot subcarriers is basically Y 1 equals h 1 times X 1 + V 1, Y 3 equals X 3 + V 3.

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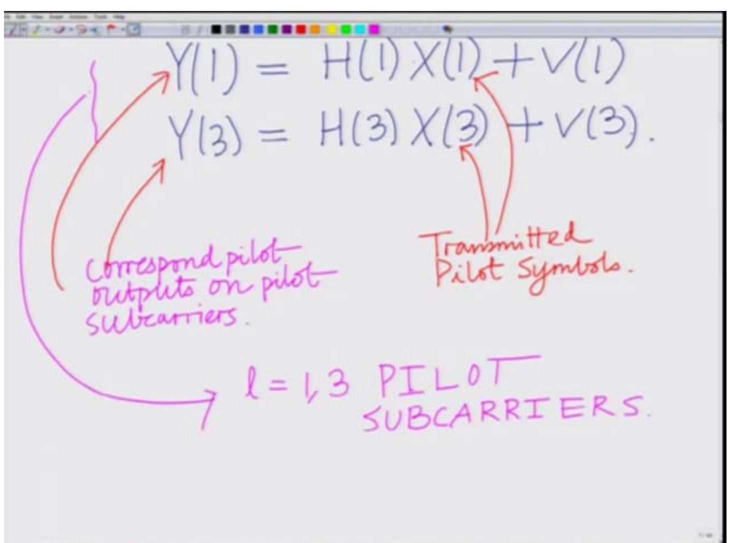

And remember these X 1 and X 3 are your actual transmitted pilot symbols. These X 1 and X 3, these are the transmitted these are the transmitted pilot symbols, so these can now be used for channel estimation and Y 1, Y 3, these are the corresponding pilot outputs on the pilot subcarriers, that is important to keep in mind.

These are the outputs, but on the pilot subcarriers, these are the corresponding, these are the corresponding pilot outputs on the pilot subcarriers, okay. Okay so what we have and therefore, these are basically L equal to 1, 3, these are your pilot subcarriers. And that is basically what we can see at this point, all right.

So let us stop this module here, so what we have seen in this module is basically that we have seen a different kind of pilot transmission strategy, that is the Comb Type Pilot transmission for an OFDM system, where what we are doing is we are not choosing to transmit pilot symbols on all the carriers, rather we have rather we have pilot symbols which are loaded onto L, that is 2 subcarriers in this scenario.

And the rest, that is N - L subcarriers are loaded with the regular information, so now we have a Comb Type Pilot arrangement, you take the FFT of this the IFFT of this to generate the time domain samples at the cyclic prefix. Now transmit the samples with the cyclic prefix added across the Channel. At the receiver, you basically have the received samples once you take the FFT at the receiver.

You basically in the, that is in the frequency domain, the output across each subcarrier Y L that is the symbol received across each subcarrier Y L equals H L, the channel coefficient across subcarrier L times X L which is the symbol loaded onto the subcarrier + V L which is the noise on the subcarrier L. Now out of all the carriers, remember for channel estimation, we are only interested in the pilot subcarriers.

Therefore, we extract the received pilot outputs corresponding to the pilot subcarriers. In this case, the pilot subcarriers are denoted by L equal to 1 and 3, so you extract the pilot subcarriers, all right. So we will stop here and we will continue with this further to look at how the channel is subsequently estimated by extracting these pilot symbols corresponding to the pilot location or the pilot subcarriers, thank you very much.