Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks
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Lecture Number- 30
Example-Orthogonal Frequency Division Multiplexing (OFDM) - FFT at Receiver and Channel Estimation Across Subcarriers

Hello, welcome to another module in this massive open online course on Estimation for Wireless Communication systems. So what we are seeing currently is we are seeing an example of an OFDM with N equal to 4 subcarriers. So we have N n equal to 4 example of OFDM.

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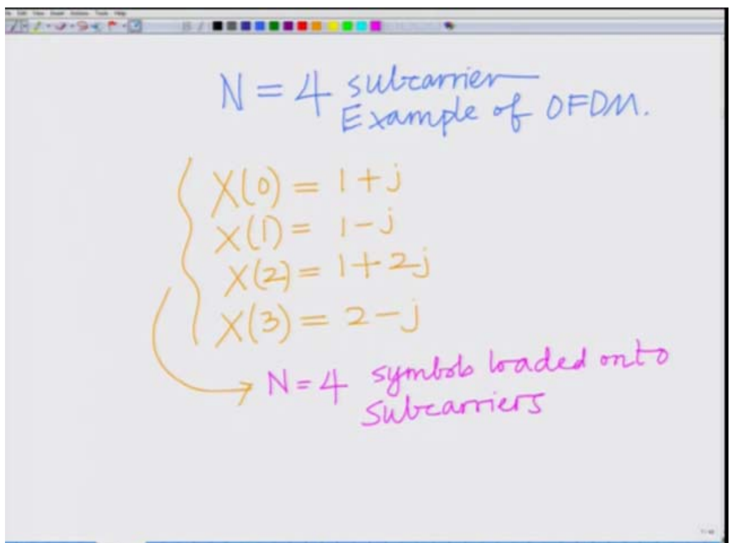

Alright, we consider the 4 symbols loaded onto the subcarriers to be 1 + J X 1, X 0 equals 1 + j, X 1 equals 1- j, X 2 equals 1 + 2 j and X 3 equals 2 - j.

These are the N equal to 4 symbols loaded on to the subcarriers, N equal to 4 symbols loaded onto the subcarriers, further what we do is to generate the samples from this, we generate the samples, how do we generate the samples by taking the… samples are generated by and equal to 4 point IFFT.

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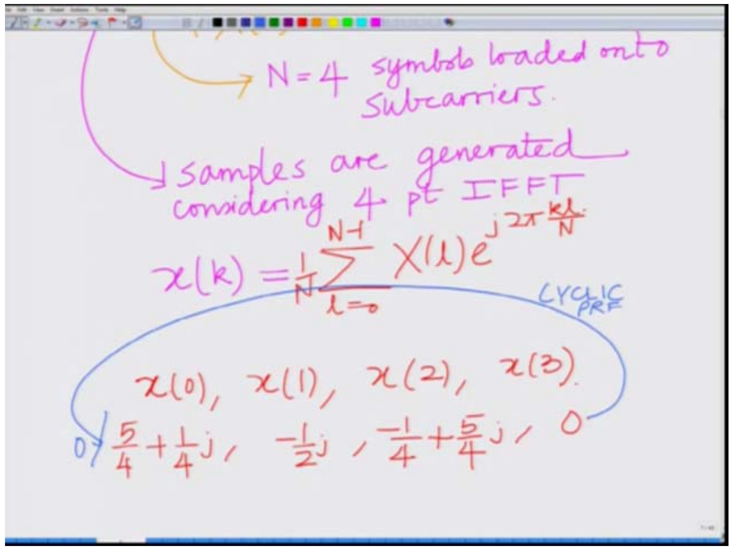

Samples are generated 4 point 4 point IFFT where the kth sample x k equals submission N equal to 0 to N - 1,1 over N X L e power j 2 pie k L divided by N. And we have seen that the samples are given as x 0, x 1, x 2, x 3 and the samples are basically your5 by 4 + 1 by 4 j, - half j, - 1 by 4 + 5 by 4 j, 0 and what we had did later in the next step was to take this and prefix it in before.

This is termed the cyclic prefix, this is your cyclic prefix, so this is the block of 5 samples with cyclic prefix, this is the block with cyclic prefix, this is transmitted across channel, across our 2 tap ISI channel L equal to 2 tap inter symbol interference channel. And after FFT at the receiver remember, at the receiver we perform the FFT.

After the FFT at the receiver, the received symbol pilot output across the Lth subcarrier is given as.

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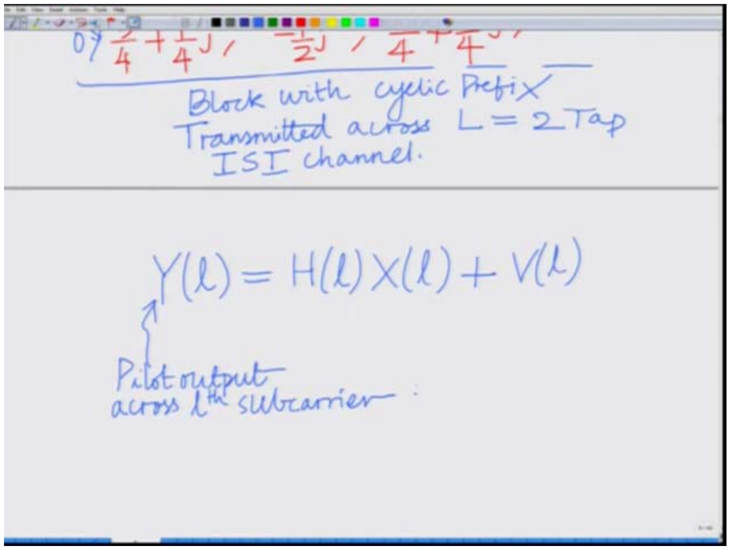

Y L which is the pilot output across the Lth subcarrier Y L equals H L times X L + V L. What is this? This is the pilot output across the Lth subcarrier. This is the pilot output across the Lth subcarrier. Now how to generate these pilot outputs that is let us say; we have considered the received sample.

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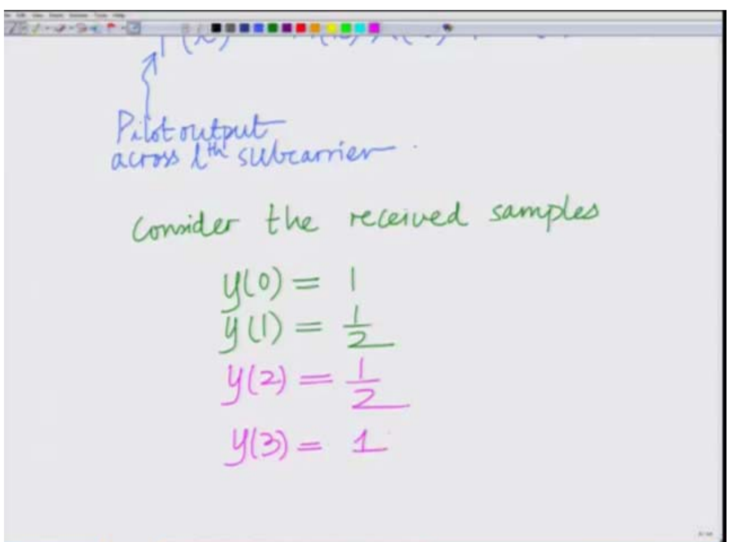

Remember, we have the received samples, so let us say the received samples are y 0 equals well 1, y 1 equals half, y 2 equals half and y 3 equals 1. Let us say we are considering this simple example of the received samples, and then what do we do? Basically we take the a 4 point FFT that is N equal to 4 point FFT to generate the output samples on the various subcarriers.

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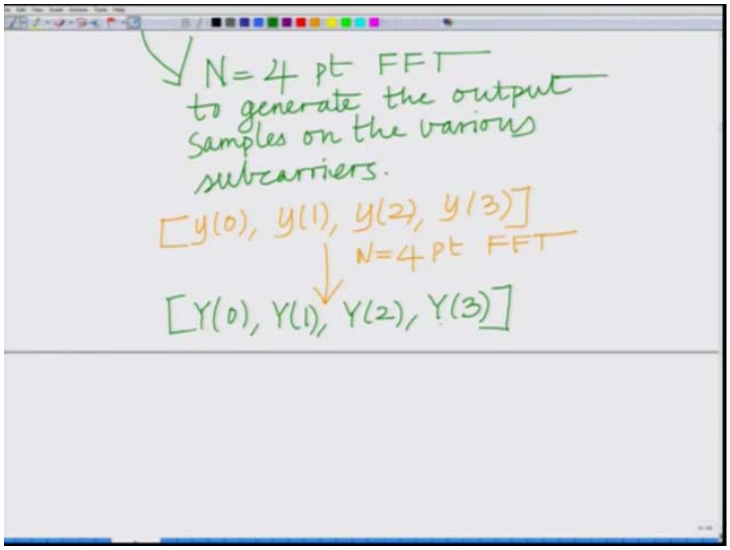

The output samples on the various subcarriers are basically generated so basically we have the time domain samples small y 0, small y 1, small y 2, and small y 3 alright. There is in the N output time domain samples, we take the N point FFT to generate the corresponding frequency domain samples or the output symbols corresponding to the various subcarriers, okay.

So we have y 0, y 1, y 2, y 3, you take N equal to 4 point FFT and generate across the subcarriers capital Y 0, capital Y 1, capital Y 2, capital Y 3 which are output symbols across the subcarriers. Therefore for the Lth subcarrier that is given basically by the Lth FFT output, that is equal to submission of the time domain samples k equal to 0 to N - 1 small x k, e raise to - j 2 pie k L divided by N.

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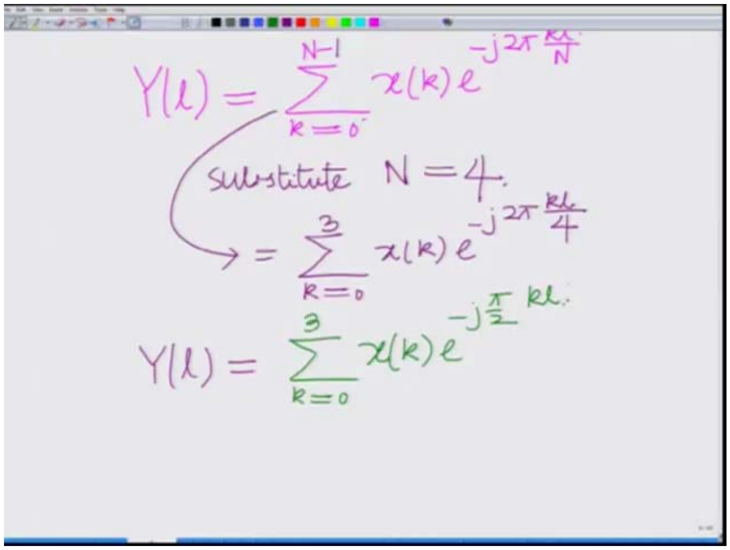

Now we substitute N equal to 4 sorry, this has to be N - 1. Substitute N equal to 4, so therefore this becomes this is equal to submission k equal to 0 to 3 x k e raise to - j 2 pie k L by 4, which means basically your Y L is equal to submission k equal to 0 to 3 x k e raise to - j pie by 2 k L, okay that is what we have. So basically the output symbols are generated by the corresponding N point FFT of the output samples, all right.

So now, let us compute output symbols y 0, y 0 is fairly straightforward, that is I substitute L equal to 0 above x k e power - j 5 by 2 k times 0, so basically this quantity is one and that is very simple.

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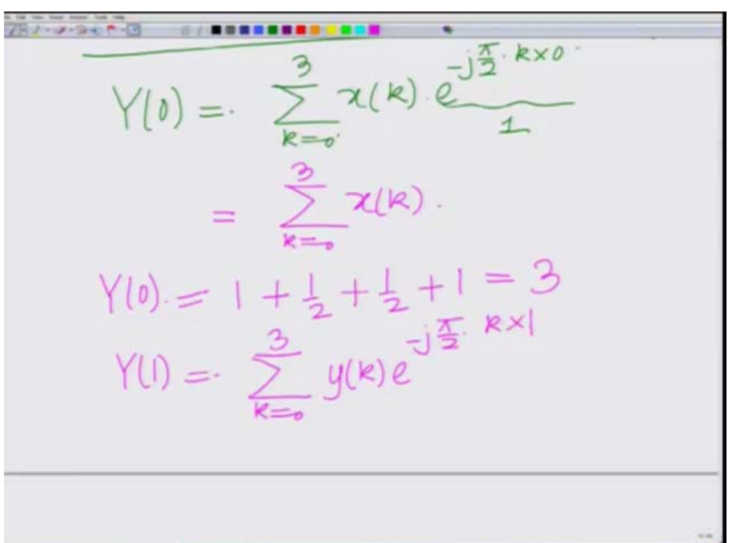

So basically k equal to, simply k equal to 0, the submission of the various time domain samples, which is equal to basically 1 + half + half + 1 which is equal to 3. So basically my y 0 is equal to 3.

Let us compute y 1, y 1 equals submission k equal to 0 to 3 y k e raise to - j pie by 2 k times 1 which is basically k itself, so this is submission k equal to 0 to 3 y k e raise to - j pie by 2 k.

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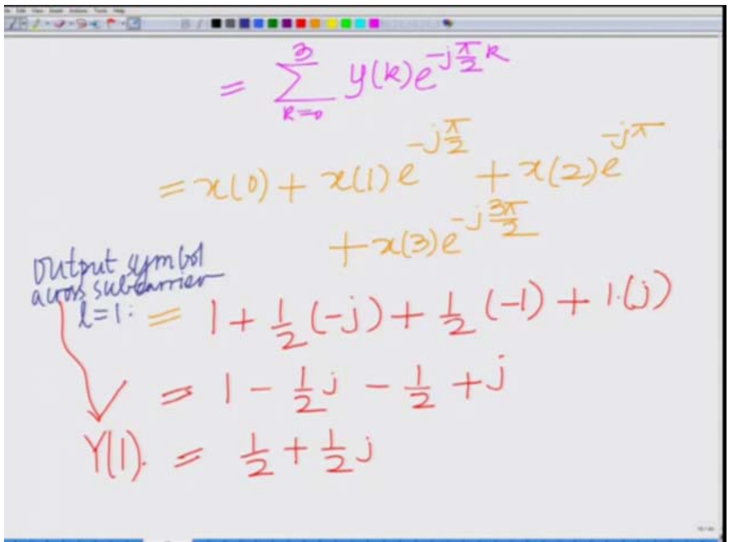

And now expanding this submission that is basically small x 0, I have small x 0 + small x 1 e raise to - j pie by 2 plus small x 2 e raise to - j pie + small x 3 e raise to - j 3 pie over 2. Now substituting the values of this time domain samples I have.

1 + half times - j + half times - 1 + 1 times j and you can check this, this is basically 1 - half j - half + j equals basically half + half j. So this is, this is basically what is this? This is y 1 that is the symbol received across the output symbol across the first subcarrier. Remember, y 1 is output symbol across subcarrier L equal to 1. Output symbol across the subcarrier L is equal to 1, okay.

Similarly, one can compute the output symbol across the subcarrier L is equal to 2 that is.

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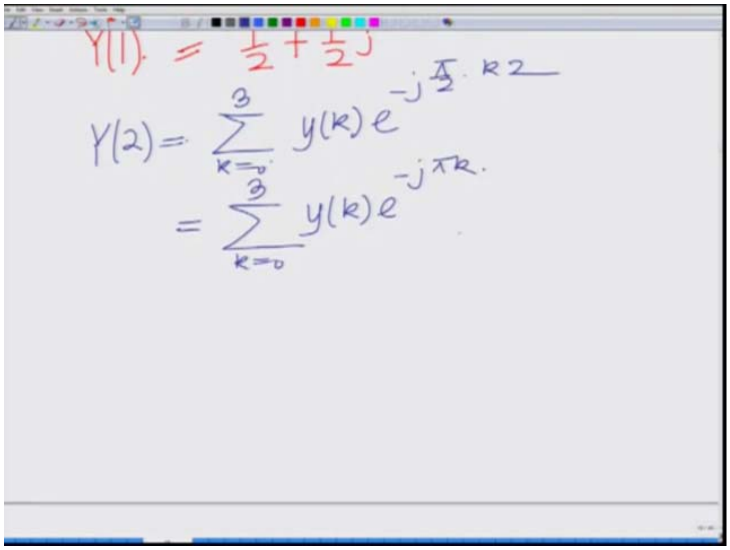

submission k equal to 0 to 3 y k e raise to - j pie by 2, substitute L equal to 2, so this is submission k equal to 0 to 3 y k e raise to - j pie k or basically k pie, which is basically I am sorry I think I have been making made a small mistake here.

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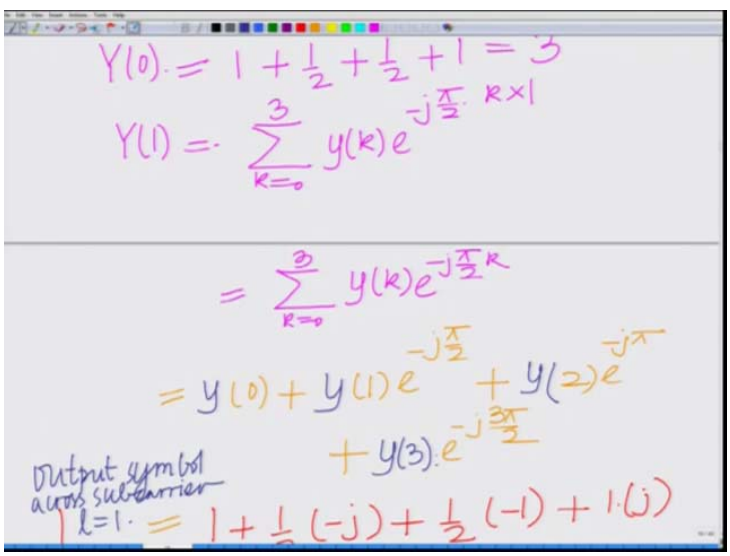

I think this has to be the samples y 0, y 2, y 1 and this is of course y 3 that is the samples across small y. I think I have made all of these small x should be replaced by the small y.

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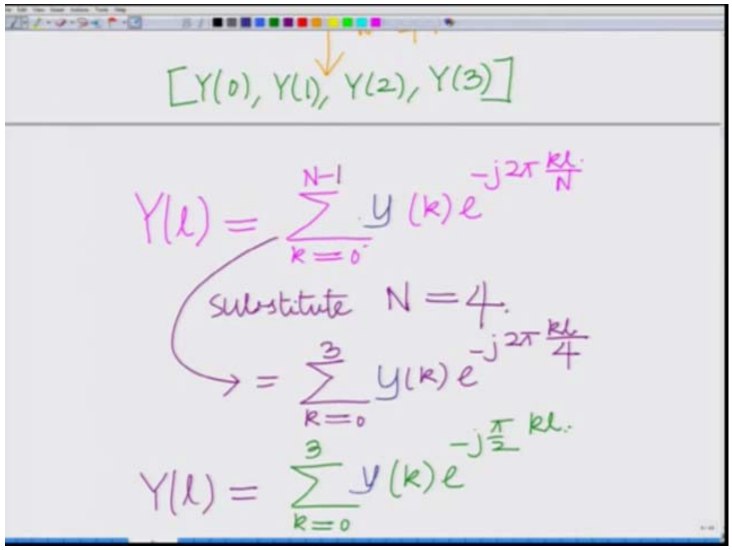

These are the corresponding samples, so this is also has to be replaced by small y because the output sample y of L corresponding to the carrier L is given basically by FFT of the output samples, time domain samples which are the small ys okay alright.

So basically just to correct these things; that is, replacing the small xs by the small ys, what I have is.

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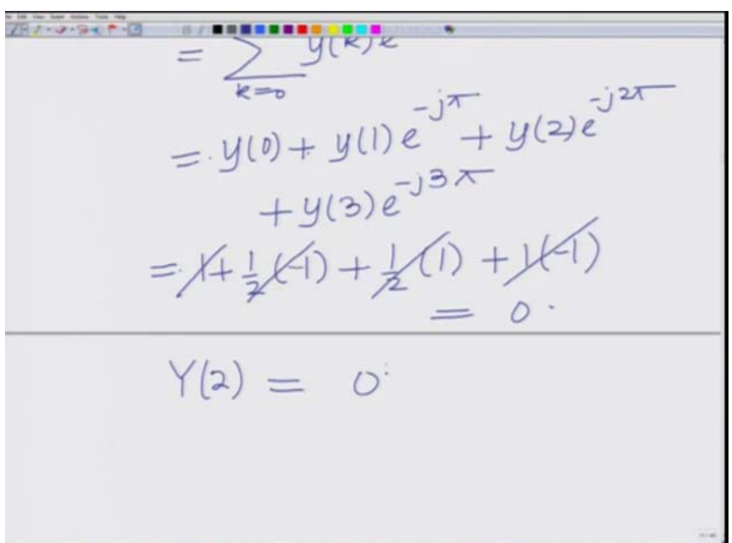

I think this is y k, this is y 0 + y 1 e raise to - j pie + y 2 e raise to - j 2 pie + y 3 e raise to - j 3 pie which is equal to 1 + half into - 1 + half into 1+ 1 into - 1 and an all can see all this cancels, I have a 1, I have a - 1, I have - half, I have a half, so this basically equal to 0.

So what I have is y 2 is equal to 0. Further y 3 that is output symbol corresponding to subcarrier 3 is equal to submission k equal to 0 to 3 y of k e raise to - j pie by 2 times 3 k, which is basically k equal to 0 to 3 y k e raise to - 3 pie by 2 k, which is basically if you look at this.

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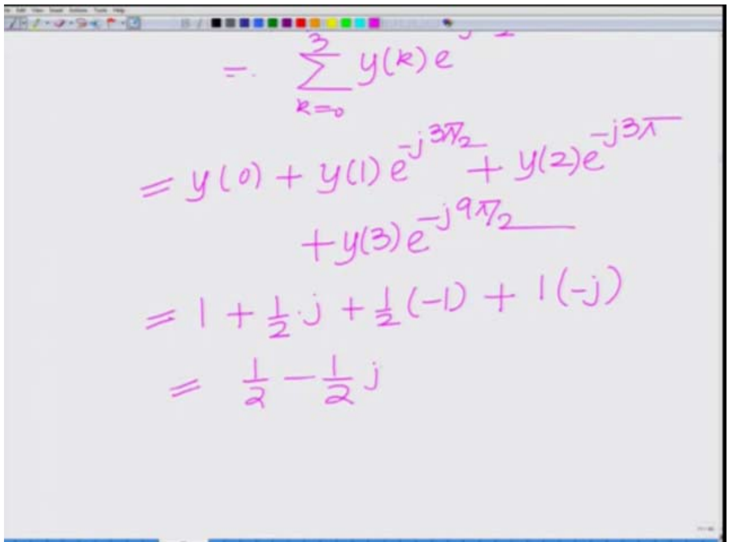

This is equal to y 0 + y 1 e raise to - j 3 pie by 2 plus y 2 e raise to - j 3 pie + y 3 e raise to - j 9 pie by 2 which is equal to 1 + half times j + half times - 1 + 1 times - j, which is basically equal to equal to half - half j and just to check if we have any type of graphical errors about, just to carefully check once more.

That is basically replacing all the small xs small xs by the small ys which are the time domain output samples, okay. So these are basically the small ys, just to clarify it once again because there seems to be some confusion. These are the time domain output samples.

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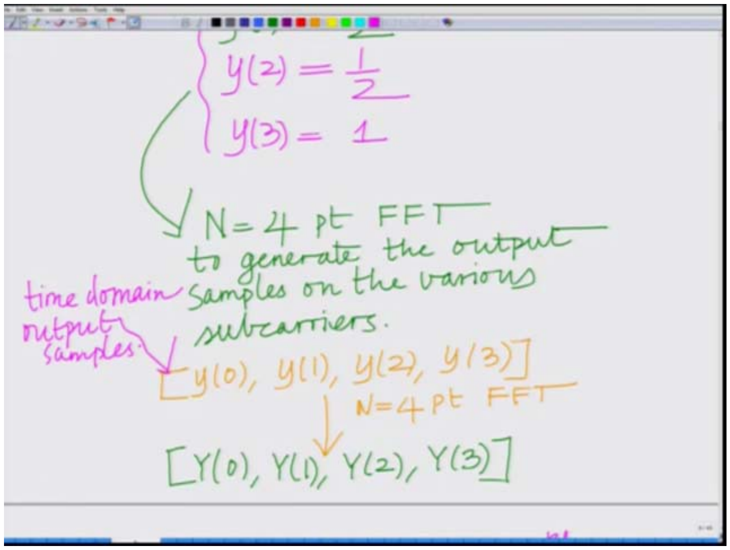

and the small xs are the time domain input samples. These small xs, these are the input samples, so I hope that aspect is clear.

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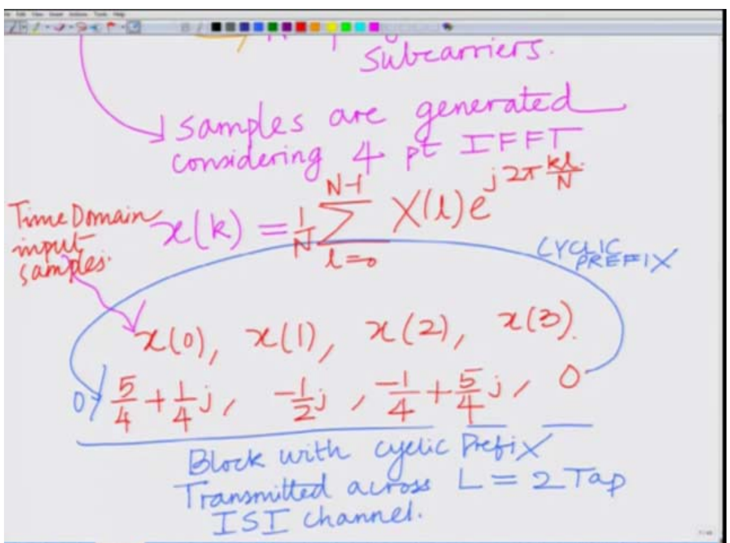

So these are the time domain input samples, that is when we said the time domain input samples, these are the samples which are transmitted over the channel and the time domain output samples or corresponding output samples which are basically received over the channel, all right. So this is the difference between the time domain input samples and the time domain output samples, all right.

So now basically we have found that the corresponding frequency domain output symbols, the capital Y 0, capital Y 1, capital Y 2, capital Y 3.

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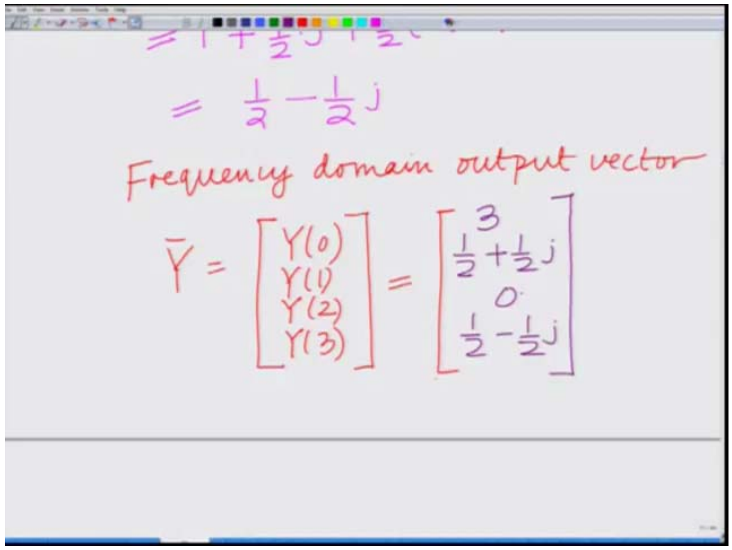

And therefore the frequency domain output vectors comprising of symbols received across the subcarriers as basically y bar equals Y 0, Y 1, Y 2, Y 3 and this is basically equal to this is basically equal to 3. Let me just write it down clearly, this is basically equal to 3 half plus half j 0 half - half j.

Now let us the pilot matrix or pilot matrix x, this is our pilot matrix, remember this is a diagonal in the case of OFDM, this is equal to remember, this is equal to x, this is your diagonal matrix.

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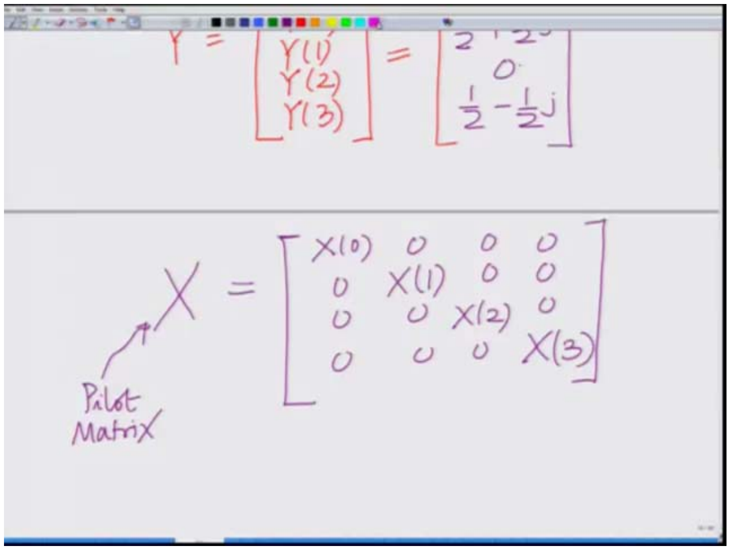

X 0, X 1, X 2, X 3 0 0 0 0 0 0. Remember this what is this? This is a diagonal matrix, now I am going to substitute the symbols loaded onto the subcarriers.

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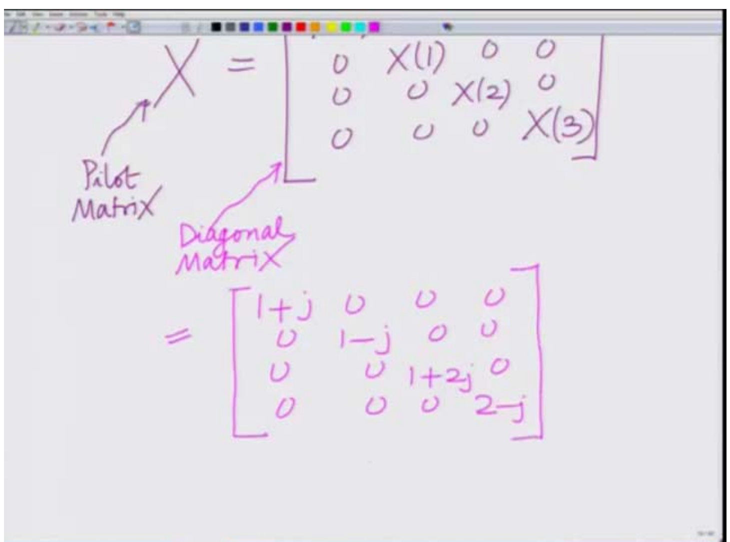

These are 1 + j, 1 - j, 1 + 2 j, 2 - j, okay. And therefore, now the least square estimate of the channel coefficient across the subcarriers, that is the capital H S are given by basically the pilot matrix X inverse times Y, right. We said that, we derived that formally that because X is a diagonal matrix which is invertible, this is simply the inverse of X inverse times Y.

So therefore H hat equals X inverse times Y and x is a diagonal matrix, so the universe is simply the inverse of each of the diagonal elements that is that is the reciprocal of each of the diagonal elements.

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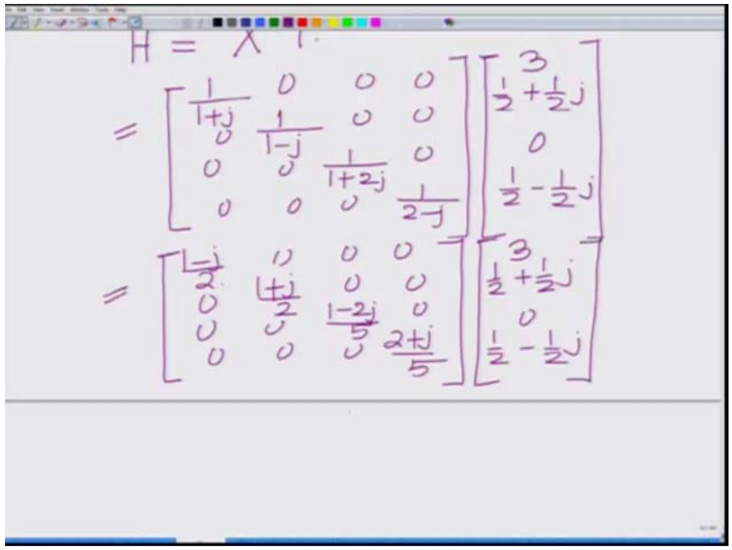

1 over 1 + j, 1 over 1 - j, 1 over 1 + 2 j, 1 over 2 - j, which is basically multiplied with your output vector containing the samples received across the subcarriers, that is Y bar.

So this is X bar, this is diagonal matrix X, this has to be capital Y bar, this is 3 half + half j, 0 half - half j, which is basically now again is fairly simple to simplify this at this point, this is 1 - j divided by 2, 1 + j divided by 2, these are the diagonal elements I am writing, the rest of the elements are 0 N , 1 - 2 j divided by 5, 2+ j divided by 5, 0 0 0 0 0 0 0 0 0 0 0 times 3 half + how j, 0, half - half j.

And you can multiply this out now, and you can clearly see that basically your H hat which are basically your vector, which are of vector of channel coefficients across the subcarriers, right.

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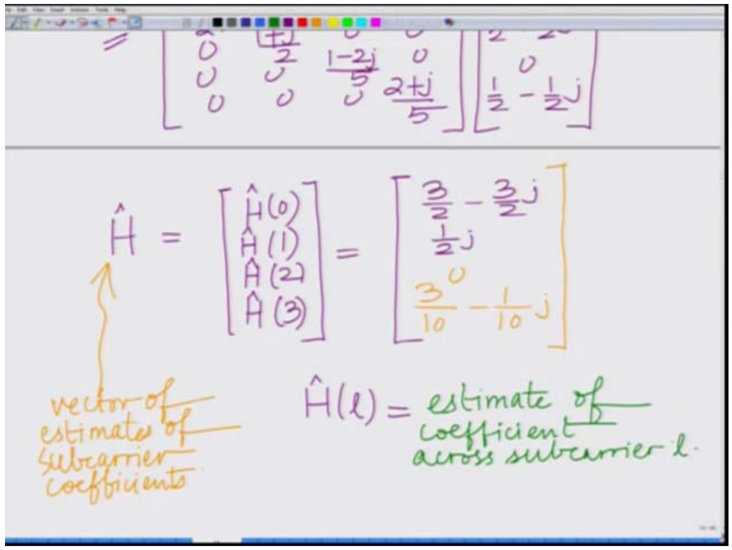

Estimates of the channel coefficients across the subcarriers rather. This is equal to well, multiplying 3 by 2 - 3 by 2 j and half j and this is 0 and this is 3 by 10 - 1 by 10 times j, all right.

So H hat denotes the vector or basically estimates of the channel coefficients across the various subcarriers, all right. H hat 0 is the estimate of the channel coefficient, capital H hat 0 across subcarriers 0, H hat 1 across a carrier 1, so on and so far up to H hat 3, all right. So basically we have H hat vector estimates vector of estimates of subcarriers coefficients for instance, H hat of L equals basically your estimate of the coefficient across subcarrier L.

So now we have found the estimates of the coefficients across the various subcarriers that is the H hat 0, H hat 1, H hat 2, H hat 3, that is the capital H s, right. From this we can also compute the estimates of the channel taps, remember the small H s, H 0 and H 1 and remember to do that what we said is basically if you remember again.

What we said is basically H 0, H 1 if you look at this.

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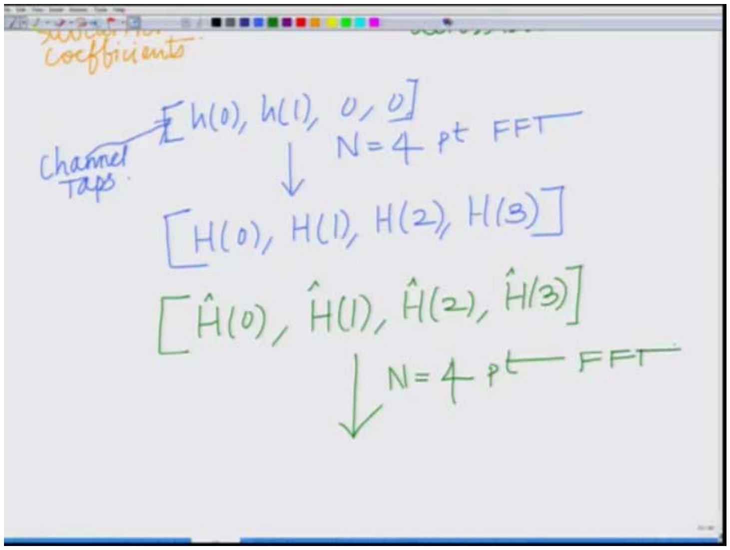

0 pared and if you take the N equal to 4 point FFT, you generate the coefficient across the subcarriers, is not it? So your H 0, H 1, H 2, H 3 and these are the channel taps. Now if you take the IFFT, now what we have a something slightly different, instead of this what we have is, we have the estimate of this.

Like these are the, we have the estimates, so naturally if you take the IFFT of the channel coefficients, you get back the channel taps. However, what we have are the estimates of the channel coefficients H 0, H 1, H 2, H 3 and now once you take the FFT that is N equal to 4 point FFT, what you get here are basically estimates of the channel taps or estimates of channel taps in time domain.

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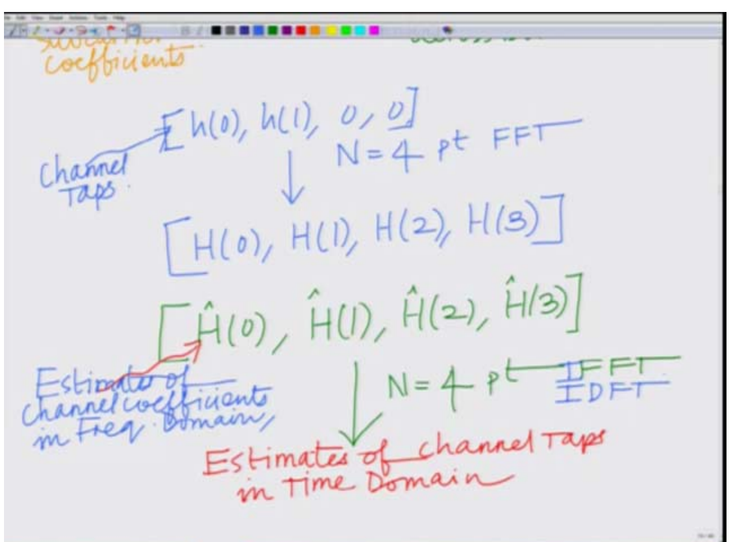

These are the estimates of the channel coefficients in frequency domain, yeah that is important to keep in mind. Estimates of channel coefficients these are the estimates of the channel coefficients in the frequency domain and that is something that is important to keep in mind.

From that you take the FFT, you get the estimate of the channel coefficients sorry, you take the IFFT or the IDFT, because from the frequency you have to go to time, so you take the IFFT or IDFT and you get the estimates of the channel taps in the time domain. So basically what you have is.

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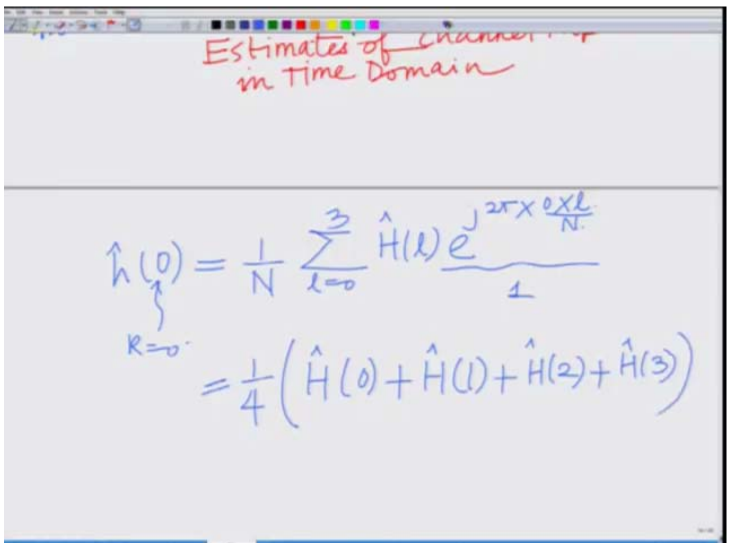

H hat 0 equals 1 over N 1 over N submission L equal to 0 to 3 H of L e power j 2 pie now k, this corresponds to k equal to 0.

So substituting 0 into 0 times L divided by N, of course this is 1. So again substituting N equal to 4, what this is? Simply this is the sum of the of course I have to have the estimate some of the estimates of the channel taps H hat 1 + H hat 2 + H hat 3, which is basically equal to now you substitute the estimates of the channel coefficients across the subcarriers computed above.

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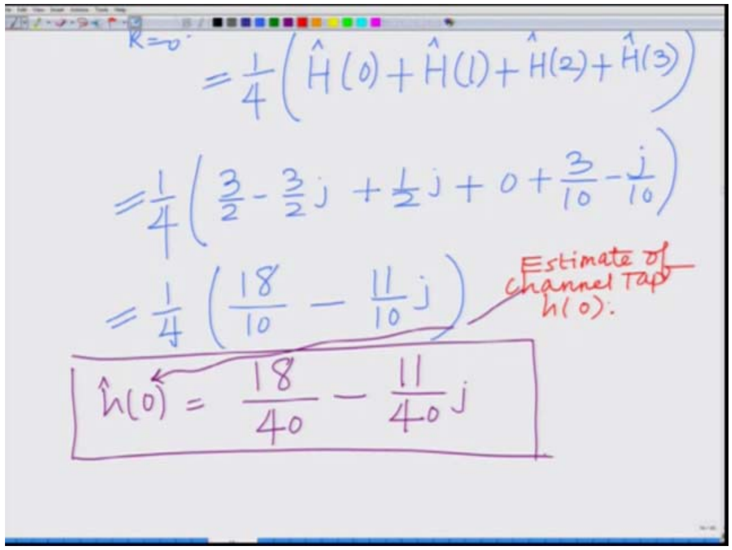

So this is 3 by 2 - 3 by 2 j + half J + 0 + half j + 0 + 3 by 10 - j by 10, which is equal to, if you look at this, 1 over 4, this is equal to well, this is equal to 1 over 4 18 by 10 - 11 by 10 j, which is basically H hat 0, therefore you can say H hat 0 equals 18 by 40 - 11 by 40, I think this is the estimate of channel tap H 0, this is what is H hat 0, this is estimate of channel tap H 0.

And by the same token how will be compute H hat 1 which is the estimate of channel tap. This corresponds to estimate of channel tap H 1 corresponding to k equal to 1 that is basically your submission L equal to 0 to 3 H hat of L e raise to well the IFFT.

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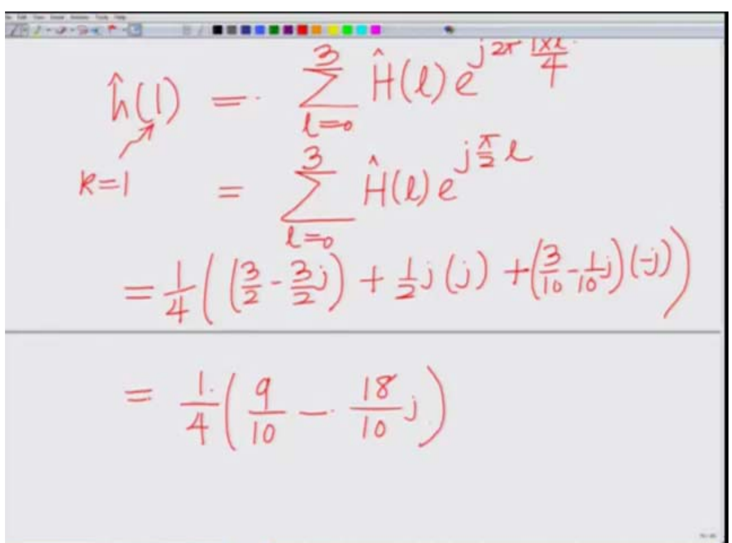

So e raise to j 2 pie, you have to substitute k equal to 1 times L divided by 4, which is submission L equal to 0 to 3 H hat L e raise to j pie by 2 L, which is equal to well…

This is equal to, substitute the various quantities what you have is basically 1 over 43 by 2 - 3 by 2 j times 1 + half j times j, of course there is 0, + 3 by 10 - 1 by 10 j times - j and you go with the submission and what you will find is basically you have here 1 by 4, 9 by 10 - 18 by 10 j, which is equal to 9 by 40 - 18 by 40 j and this is H hat 1, this is estimate of the channel coefficient across the… What is this?

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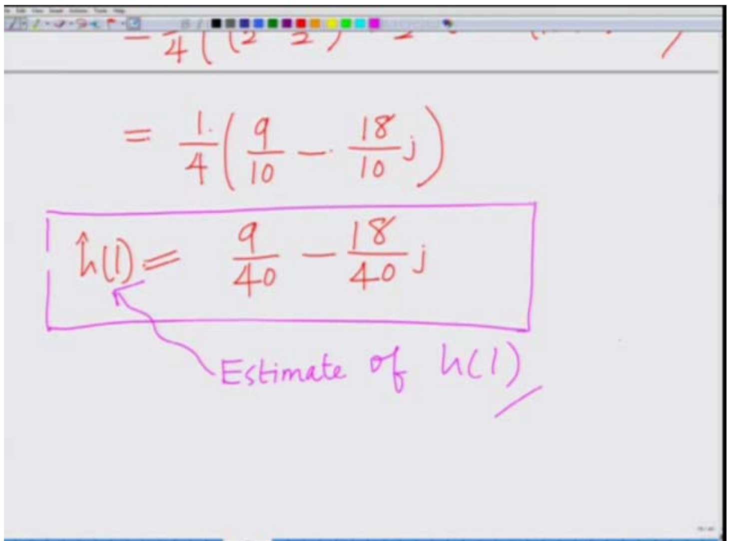

This is basically your estimate of the channel coefficient across the… Estimate of channel coefficient in fact it is not a channel coefficient, it is the time domain channel tap, it is the times domain channel tap, estimate of H 1 or channel tap H 1. So basically this is the estimate of channel tap H 1, this is the estimate of channel tap H 0, so we have H hat 0, we have H hat 1 and that basically gives us the estimate of the various channel taps.

So that is what we have done inside if you look at this example, what we have done over this module and the previous module, we have comprehensively Looked that this OFDM estimation scenario in a very thorough fashion and it is basically a sequence of a large number of steps and one should remember the logical ordering of these steps.

Basically you have the symbols that are loaded onto the subcarriers in the frequency domain, you convert it into the time domain b y locating the IFFT of the symbols loaded onto the subcarriers at the cyclic prefix, transmit them, you receive the output samples in the time domain which other small ys, You take the FFT of these output samples; you generate the capital Ys which are the frequency domain symbols that are received across the various subcarriers, right. You have the frequency domain symbol outputs; you have frequency domain symbols loaded onto the subcarriers, all rights. Now you get the channel coefficients, estimates of the channel coefficients in the frequency domain that is the FFT domain.

You perform IFFT of this and then you get the estimates of the channel tap in the time domain, all right. So that is basically once you understand OFDM, I think that sequence of steps is fairlY Logical and also to a certain extent in tutor, all right. And this example again clarifies in a step-by-step fashion.

How to go about channel estimation in an OFDM system (()) (30:52) in a simple OFDM system with 2 tap ISI channel that is an Inter symbol Interference channel with 2 channel taps and number of subcarriers N equal to 4, because of the limitation of the scope of this class, we cannot do an elaborate system with a large number of subcarriers and a large number of channel tap.

Although, from this example I think it is fairly clear to everyone who has followed it that this can be very readily generalize to a system with a large number of subcarriers and of course a large number of channel taps. And that can be implemented very efficiently using the software that is available or by coding it appropriately to simulate it, all right.

So basically this concludes this simple OFDM example and I argue to go through again this example to understand the working of the OFDM system as well as the channel estimation process better, all right. We have concluded this module here, thank you.