Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks

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Lecture Number- 29

Example-Orthogonal Frequency Division Multiplexing (OFDM) - Transmission of Samples with Cyclic Prefix (CP)

Hello, welcome to another module in this massive open online course on Estimation for Wireless Communication. So we are looking at estimation in the context of OFDM or Orthogonal Frequency Division Multiplexing based system and we have already seen the theory of basically how what is the system model for an OFDM system and basically how to carry out channel estimation.

That is, estimation of the various channels coefficients on the subcarriers as well as the channel taps in the time domain in the OFDM system. Now let us see a simple example to understand this better, so what we are going to do in this module or starting in this module is basically look at a step-by-step implementation of the OFDM system as well as channel estimation.

So let us look at an example, example of an OFDM system and in particular what we want to look at is channel estimation in OFDM system, all right.

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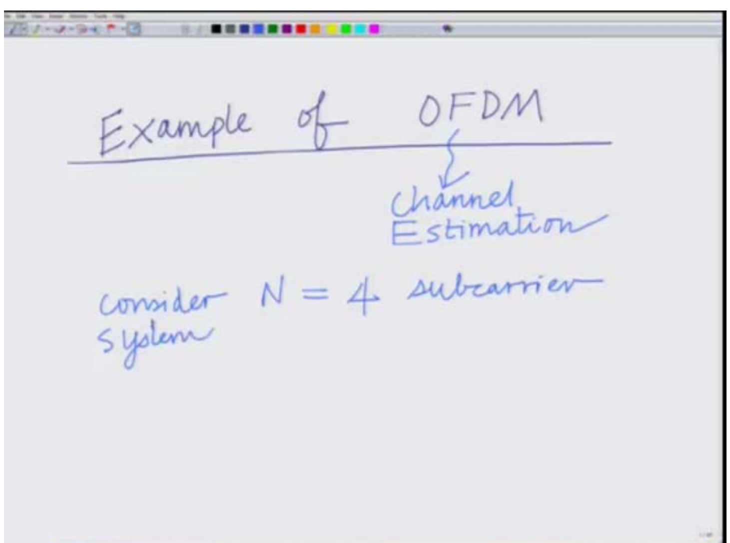

So as we have been seeing, let us consider N equal to 4 subcarrier systems, so consider, so similar to what we have seen in the previous modules, consider an N equal to 4 an N equal to 4 subcarrier system.

And therefore we have 4 symbols that are loaded onto the subcarriers, 4 pilot symbols, so these let these pilot symbols denoted by capital X 0 equals 1 plus j, capital X 1 equals 1 minus j, X 2 equals 2 plus j, capital X 3 equals 2 minus j, these are the 4 symbols, these are the N equal to 4 symbols onto the subcarriers. These are the N equal to 4 symbols which are loaded onto the 4 subcarriers, correct.

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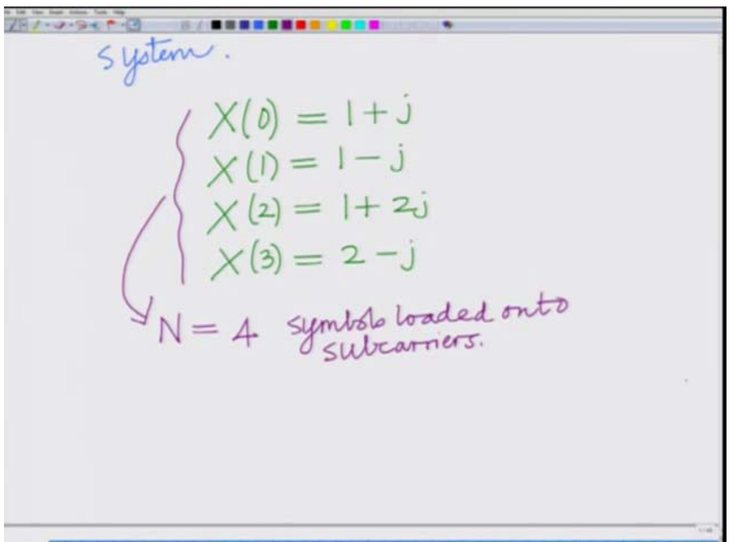

So we have capital x 0, x 1, x 2, x 3 which are the 4 symbols loaded onto the 4 subcarriers. Now we want to generate the samples in the time domain and remember that the samples in the time domain are generated by the N equal to 4 point IFFT. So the samples which are denoted by… so the time domain samples.

Let me write it clearly, this is the time domain samples are generated by N equal to 4 point IDFT.

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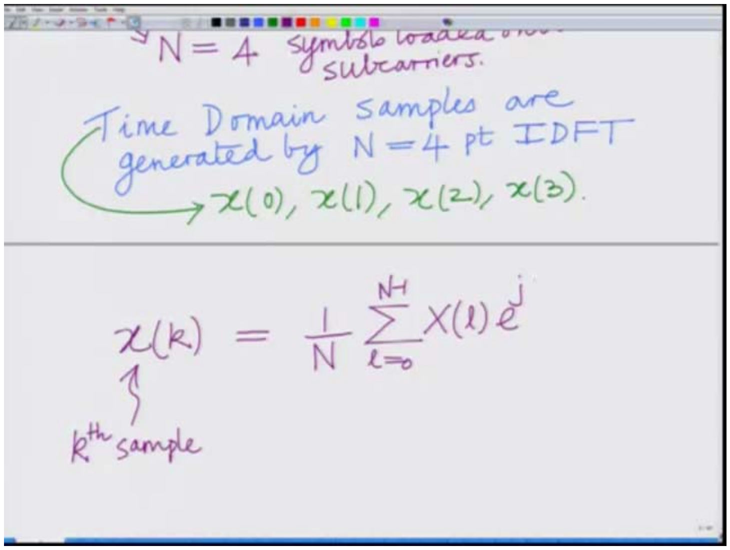

That is, basically these samples are denoted by X0, X1, X2, X3 and these are basically your time domain samples which are generated by the 4 point IFFT.

And therefore, xk which is the kth sample, what is x k, x k is the kth sample and this is equal to 1 over N submission L equal to 0 to N minus 1, x L e raise to j 2 pie k L divided by N which is equal to which is equal to 1 over 4, L is equal to 0 to 3 x L, e raise to j, 2 pie k L divided by 4 which is in turn, thus simplifying this as far as possible, this is L equal to 0 to 3 x L, e raise to j pie by 2 k L, okay.

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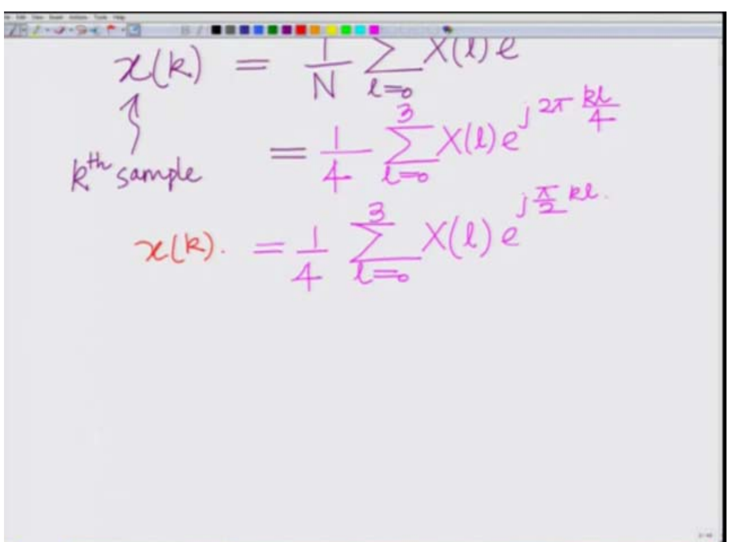

So this is basically your x k. This is the kth subcarrier, this is of course x capital X L is the symbol that is loaded onto the Lth subcarrier and the time domain samples are basically given by the IDFT of the symbols loaded onto the subcarriers. So we have the symbols loaded onto the subcarriers, so let us now find what the corresponding time domain transmitted samples are, okay.

So x of 0, let us start with sample x of 0, that is sample corresponding to time index 0, that is 1 over 4 submission L equal to 0 to 3, x L e raise to j pie by 2 k, for k I have to substitute 0 because I am considering the sample at k equal to 0.

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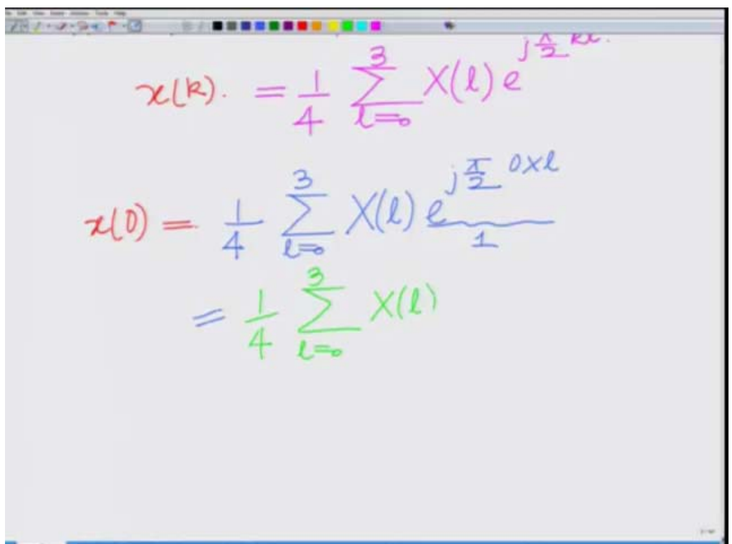

0 times L which is basically now this quantity is basically 1. So this is 1 over 4, simply submission L equal to 0 to 3 of x L. Now therefore x 0 your x 0 is 1 over is basically 1 over 4.

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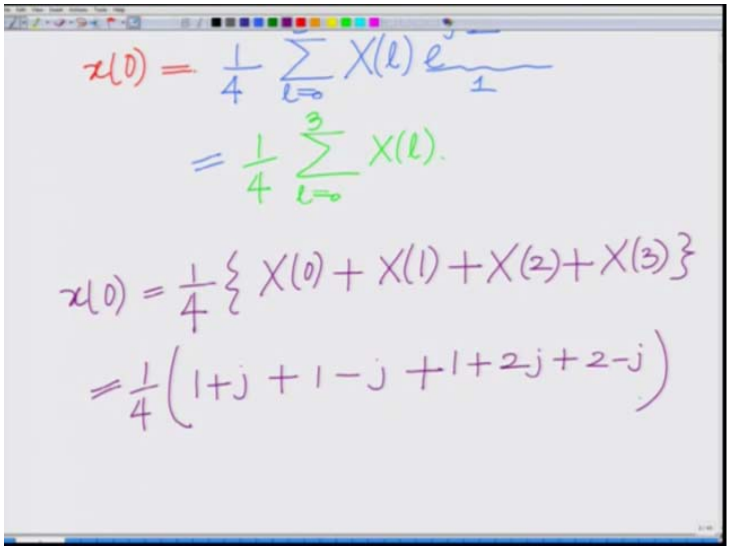

Capital X 0 plus X 1 plus X 2 plus X 3, which is equal to, what is this equal to, 1 over 4. Now substitute for capital X 0 which is 1 plus J plus capital X 1 which we have said is 1 minus J plus capital X 2 plus 1 plus 2j plus capital X 3, which is 2 minus j. Remember, we have already given the values of capital X that is the symbols.

These are the symbols that are loaded onto the subcarriers.

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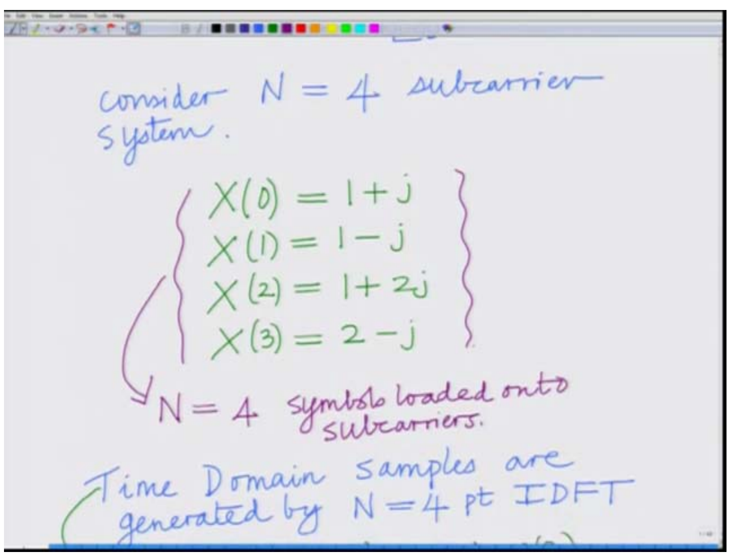

So I am simply substituting them over here. So I have 1 over 4, 1 plus j plus 1 minus j plus 1 plus 2 j plus 2 minus j which is equal to 1 plus 4 into 5 plus J which is equal to 5 by 4 plus 1 over 4 plus times j. This is the complex symbol; this is the x small x 0 that is the sample transmitted at time 0.

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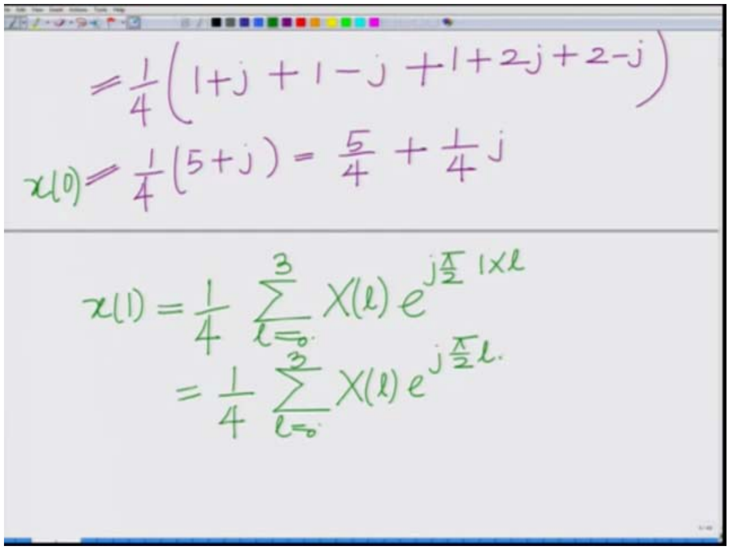

Similarly, let us compute small x 1 which is the sample transmitted at time, okay. Again it goes without saying that small x 1 corresponds to k equal to 1. That is the sample corresponding to your time instant 1, L equal to 0 to 3 x L e power j pie by 2, instead of k I have substituted 1, 1 times L which is basically 1 over 4 submission L equal to 0 to 3 x L e power j pie by 2 L which is now, if I substitute the symbols loaded onto the subcarrier.

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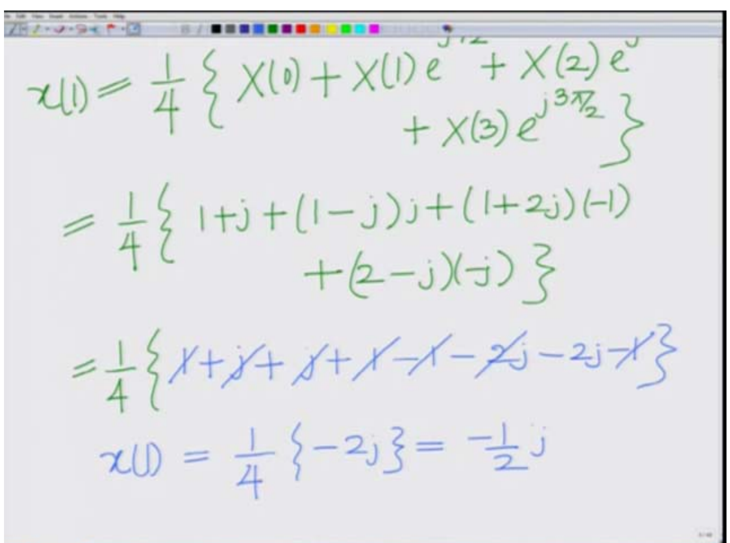

I have x 1 equals 1 over 4, times x 0 plus x 1 into e power j pie by 2 plus x 2 e power j pie plus x 3 e power J 3 pie by 2 which is equal to basically your 1 over 4 times 1 plus j plus 1 minus j into j plus 1 plus 2 j into minus 1 plus 2 minus j into minus j. And what is this equal to, this is equal to your 1 over 4 plus 1 plus j plus j plus 1 minus 1 minus 2 j minus 1 and this is equal to basically, let us write this down.

This is x of 1 which is equal to 1 over 4, and can see there is basically a 1 minus 1 which goes 1 minus 1 which goes, so I am left with is j j minus 2 j, so this is basically minus 2 j divided by 4 equals minus half times j, okay. Again you check all these calculations, please do them yourself and try to check these calculations, so we have calculated the sample x 1 small x 1, corresponding to k equal to 1 as minus half j, okay. Now let us compute small x 2.

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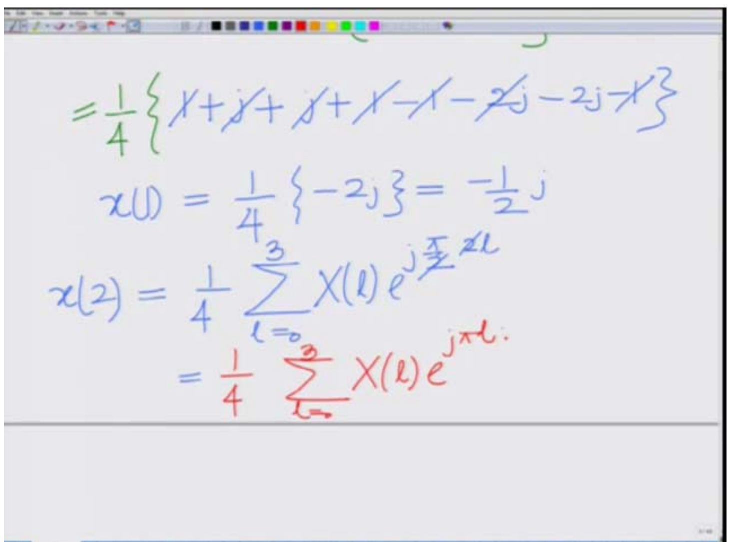

Small x 2 equals with 1 over 4, L equal to 0 to 3 capital X L e power j pie by 2, substitute k equal to 2, 2 times L of course the 2 go away and what we have is 1 over 4, L equal 0 to 3 capital x L e power j pie L that is small x 2. Now substituting of course, the capital X L which is the symbols loaded onto the subcarriers.

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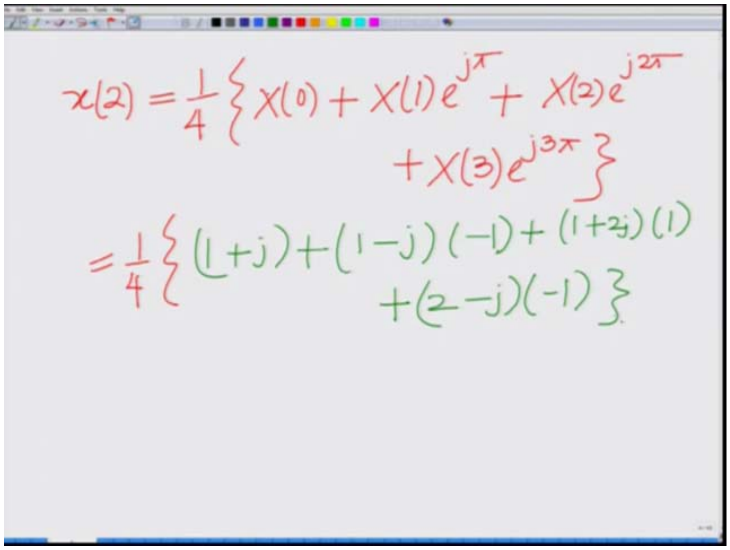

I have 1 over 4 x 0 plus x 1 e power j pie plus x 2 e power j 2 pie plus x 3 e power j 3 pie which is equal to 1 over 4 again hoping that I am making no mistakes and this is 1 plus J plus 1 minus j times minus 1, e power j pie is minus 1, plus 1 plus 2 j into e power j 2 pie is 1, 2 minus j into minus 1 and what is this, this is therefore equal to 1 over 4, 1 plus j minus 1 plus j plus 1 plus 2 j minus 2 plus j.

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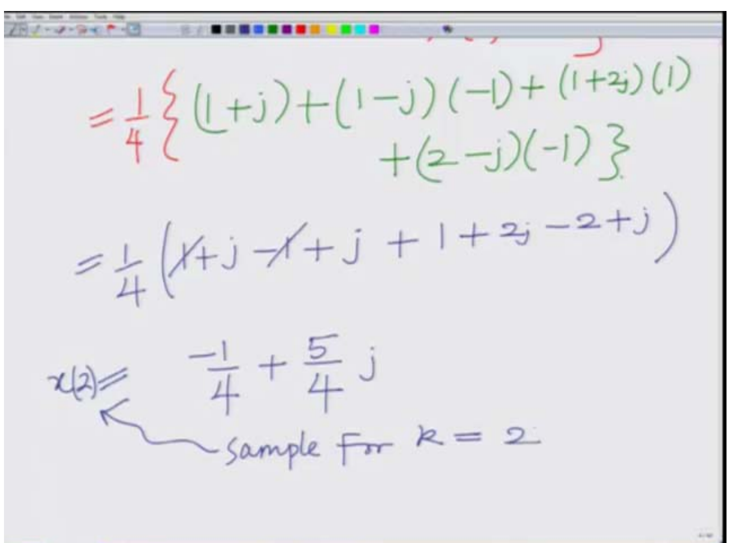

And this is basically, if I am correct again, we have 1 minus 1, that is 0. So basically I have basically I have 1, what is this, this is basically equal to well I have over here. I have a 1 minus 2, so this is minus 1 over 4 and I have well basically I have 2 j plus 2 j plus j that is 5 j.

So plus 5 by 4 divided by j and this is in fact your x 2 that is sample corresponding to k equal to… Sample for k equal to, this is the sample corresponding to k equal to 2.

And similarly I thing we can do the sample small x 3 corresponding to k equal to 3. Just to be a little bit more explicit, to illustrate how to compute these things, although I am sure that all of you could do this simple 4 point IIFT yourself, just expressively illustrate the process.

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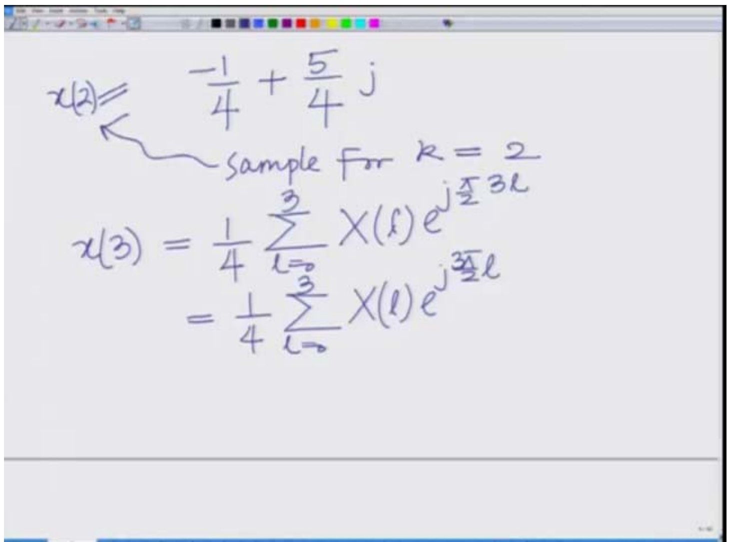

So this is x of 3 equal 1 over 4, again submission L equal to 0 to 3 x L e power j pie by 2, 3 times L which is 1 over 4 submission L equal to 0 to 3 x L e power j with 3 pie by 2 times L. And now again when I substitute the symbols of the subcarriers that is, x 3 equals 1 over 4.

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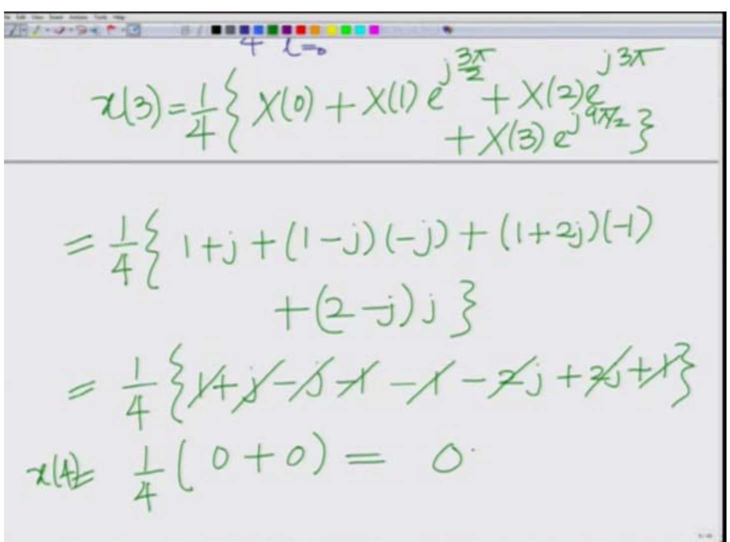

What do we have is x 0 plus x 1 into e power j 3 pie by 2, plus x 2 into e power j well L equal to 2, so this is 3 pie plus x 3 into e power j, 9 pie by 2 because L equal to 3, this is equal to 1 over 4, x 0 is 1 plus j plus 1 minus j times minus j, that is e power j pie by 2, plus well 1 plus 2 j into minus 1 plus2 minus j into e power j 9 pie by 2 that is j.

And when you simplify this, what do you have when you simplify this, so that is 1 over 4 and once again you can check this, 1 over 4, 1 plus J minus j minus 1 minus 1 minus 2 j plus 2 j plus 1 and what do you have over here. In fact, you have 1 by 4 times 0 plus 0. You can see, this is j minus j, 1 minus 1 minus 1 plus 1, 2 j minus 2 j, so this goes, so what you have is 0 and this is basically your x of 3.

So what you have is basically, let me summarize what you have, you have the transmitted samples in the time domain which are given as x 0, x 1, x 2, x 3 and therefore these samples, now let me write down the values that we have computed.

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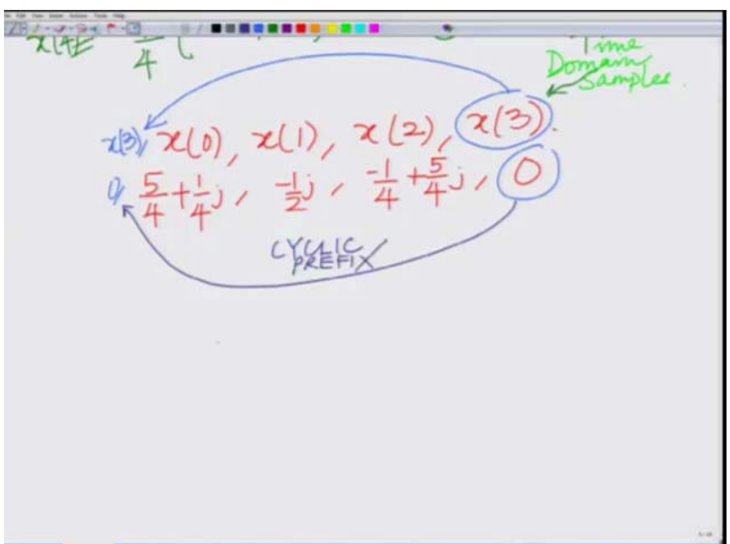

5 by 4 plus 1 by 4 j, minus half j, minus 1 by 4 plus 5 by 4 J, 0. This is x 3, so what I have done is beneath each time domain sample, I have written down the, so these are your time domain samples.

These are basically your time, beneath each time domain sample I have written the corresponding value of the sample and now what we do as they said. So we have generated the samples small x 0, small x 1, small x 2, small x 3 as the IFFT of the symbols capital X 0, X 1, capital X 2, capital X 3 which are loaded onto the subcarriers. Now the next step is of course adding the cyclic prefix that is, I take one sample, I mean typically it is not one sample, but for this example one sample is enough.

We will take the sample from the tale of this block of samples and again repeat it or prefix it at the head, this is known as the cyclic prefix. For instance here, I am going to take this x 3, repeat this x 3 over here or in other words, take this 0 and repeat the 0 over here, so this is basically what we have said and I think we should be similar with this by now.

This is basically what we said is the cyclic prefix. So the block of transmitted samples over the channel with cyclic prefix, so the block. This is important to remember what it is. This is block with the cyclic prefix.

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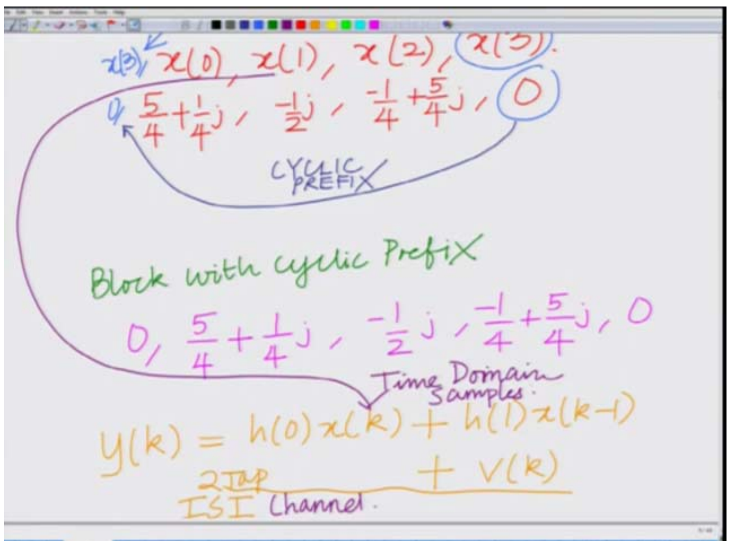

The block with cyclic prefix is basically your 0, let me just write it down very carefully. This is 0 5 by 4plus 1 by 4 j minus j minus 1 by 4 plus 5 by 4 j, 0.

So now what we are doing is, we have added the cyclic prefix to the time domain samples, transmit them over the channel. And remember that the channel that we are considering in particular, that is the channel that we had considered in the example that we had seen for this illustration of this OFDM is the simple 2 tap channel, which are the taps, h 0 and h 1. So the channel that we are considering, let me remind you, that is y of k, this is a 2 tap.

h 0 x k plus h 1x k minus 1 plus v k. Remember, this is our two tap channel. 2 tap in fact, ISI channel into symbol interference channel, okay. This is your 2 tap ISI channel, okay. And now once you transmit, these are the samples, remember these x ks, these samples are nothing but just let me illustrate it one more time.

These x ks are basically, is nothing but these are the time domain samples that we have got after this is what we have got after the IFFT of the symbols loaded onto the subcarriers.

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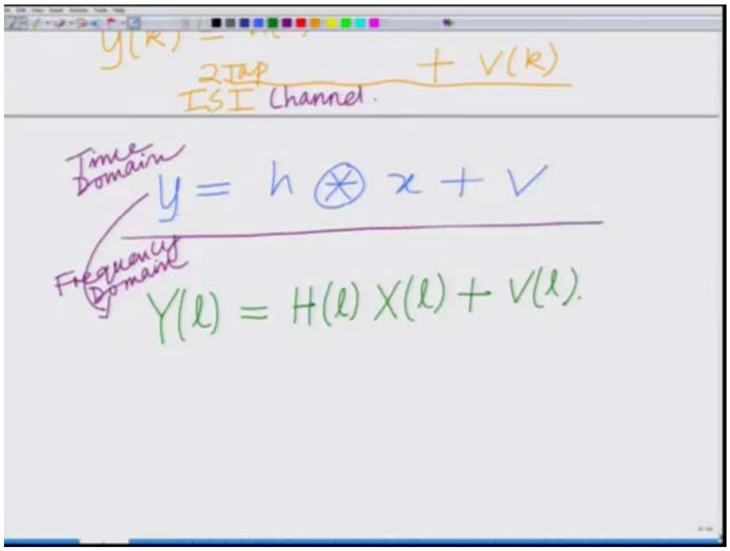

And therefore now what we are saying is, the action of the channel is one of circular convolution because of the cyclic prefix, you have y equals h circularly convolved with x plus the noise, which means now you take the FFT at the receiver, so basically this is time domain, on the top is the time domain.

On the bottom, is the frequency domain and in the frequency domain across each subcarrier remember we has that is the fundamentally question of OFDM Y L equals H L into X L plus V L, this is across of subcarrier L, across subcarrier L across subcarrier L. What are these Y Ls, these Y Ls are the symbol pilot output received across, the pilot outputs that are received across the subcarrier across the subcarrier L. So we have capital Y L which is the pilot output received across subcarrier L okay.

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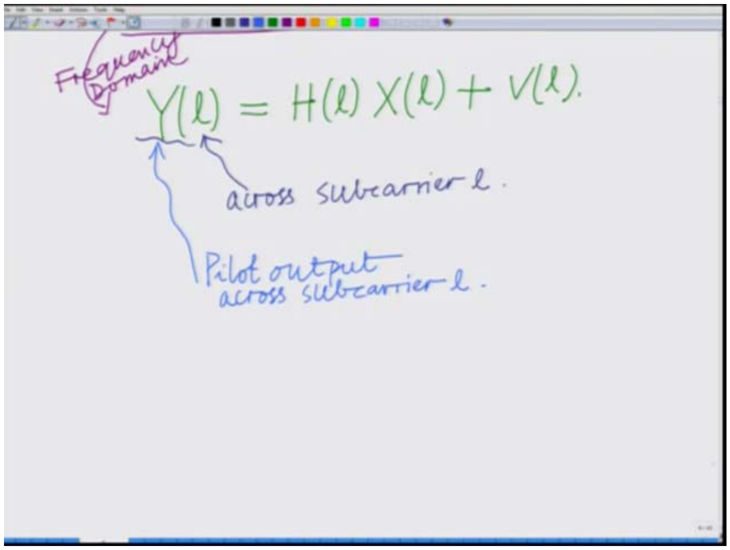

So what we have done so far is, we have considered the simple n equal to 4 subcarrier OFDM system okay. We have loaded the symbols capital X 0, capital X 1, capital X 2, capital X 3 to basically of the subcarrier, that is basically performed the IFFT or the IDFT to generate the time domain samples at the cyclic prefix. And we look that what happens at the receiver (()) (21:31) the FFT.

Now in the next module or in the secured module, we will look at what is the operation, what are the step-by-step operations at to be performed at the receiver in the simple OFDM example, alright. Starting with basically computing the pilot outputs received across each subcarrier and subsequently estimating the channel taps or the channel coefficient in the frequency domain as well as the channel coefficient or the channel taps in the time domain, okay.

So let us stop this module here and we will continue this in the subsequent module, thank you.