Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks
Professor A K Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur
Lecture Number - 28
Channel Estimation across Each Subcarrier in Orthogonal Frequency Division Multiplexing (OFDM)

Hello, welcome to another module in this massive open online course on Estimation Theory for Wireless Communication Systems. And we are looking at OFDM that is Orthogonal Frequency Division Multiplexing. And we said that OFDM very efficiently overcomes the Inter symbol interference in a wireless channel, all right.

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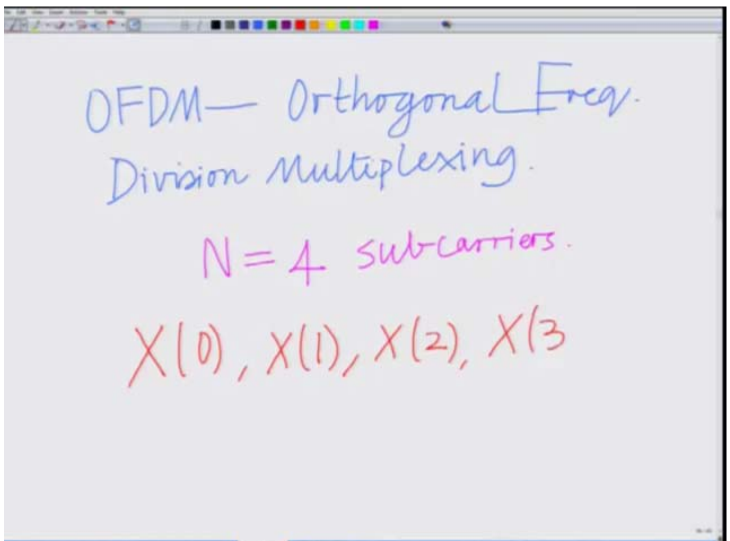

So we are looking at basically OFDM, which is your Orthogonal Frequency… Orthogonal Frequency Division Multiplexing and we are considering, as an example, we are considering a system with N equal to 4 subcarriers and we said that X 0, X 1,X 3 these are the N equal to 4 symbols loaded onto the subcarriers.

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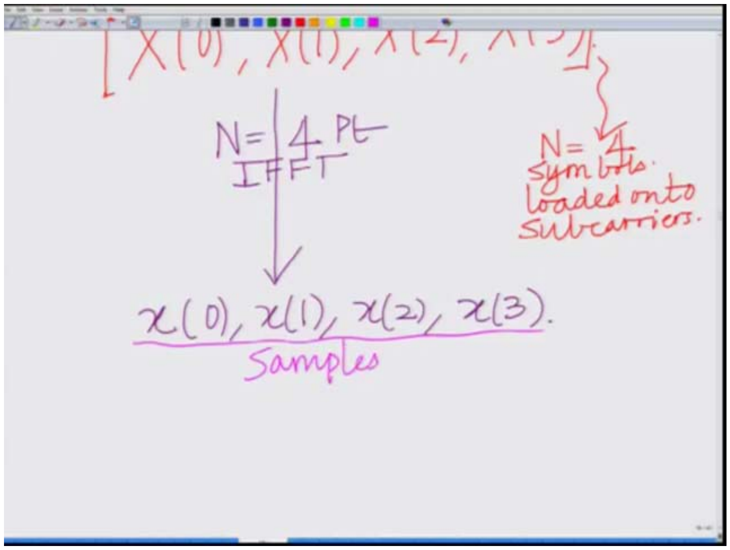

These are basically your N equal to 4 symbols loaded, which are basically loaded onto the, which are loaded onto the subcarriers.

Basically, which means that we are looking at the N equal to 4 point N equal to 4 point IFFT Inverse Fast Fourier Transform of this to generate the samples. So we loading capital X 0, capital X 1, capital X 2, X 3 onto the subcarriers which is, we are taking the N point or 4 point IFFT to generate the samples. And the samples are basically your X 0, X 1, X 2, X 3 where your kth sample.

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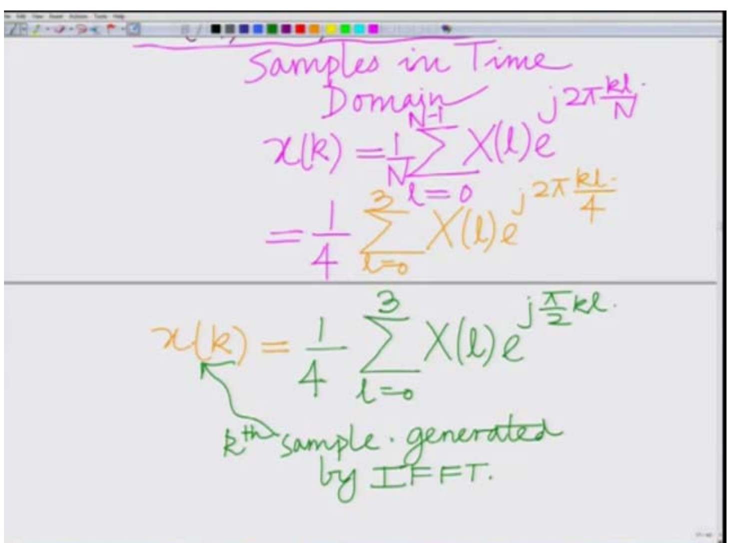

These are the samples in time domain which are actually transmitted over the channel and therefore, basically kth sample X k equals some L equal to 0 to N minus 1 x L e power 1 over N since this is an IFFT, e power j 2 pie k L divided by N. Now substituting N equal to 4, I have N equal to 0 to N minus that is 3 x L e power j 2 pie k L by 4 which means your sample x k equals 1 over 4 submission L equal 0 to 3 x L e power j pie by 2 k l.

So this is the kth sample which is generated by the IFFT. So the kth sample generated by the IFFT, correct. And now we have not simply transmitting the samples, we are adding a cyclic prefix that is we are taking some samples from the tail of the block and prefixing them at the head of the block and this is known as the cyclic prefix.

Precisely in this example we have taken one sample, but one can take more also, alright. Or that depends on the length or the number of taps in the channel response, okay.

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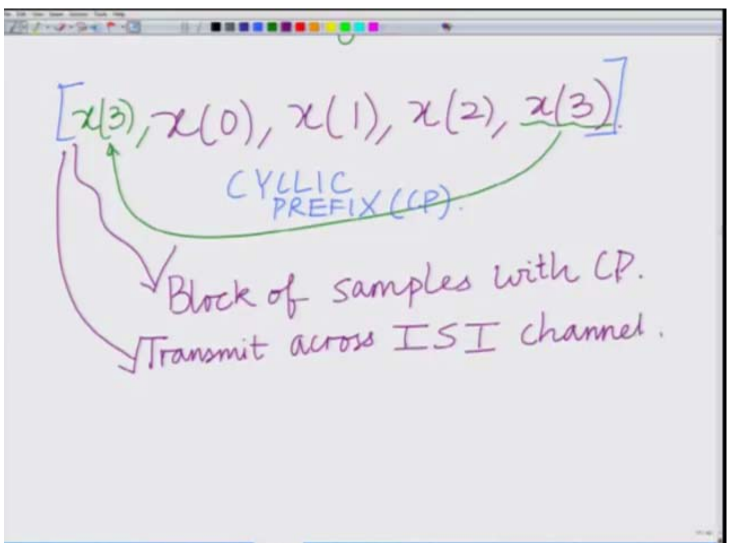

So basically, what we are doing is, we have X 0, X 1, X 2, X 3 and what I am doing is basically now I take X 3 and prefix and this is termed as your cyclic prefix or CP are basically the CP.

And now when you take this block with cyclic prefix, this is your block of samples. So the OFDM transmission takes place in blocks. So this is the block of samples with your cyclic prefix and you transmit these across the ISI channel. Transmit this across the Channel with inter symbol interference and what is the channel; we are considering the 2 tap inter symbol interference channel that is y k equals h 0 x k plus h 1 x k minus 1 plus V k.

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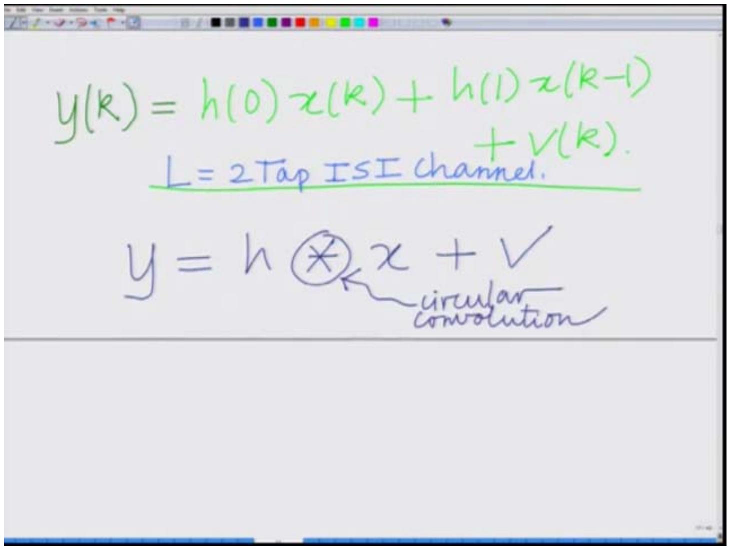

This is the 2 tap, this is your basically L equal to 2 tap ISI channel. This is your L equal to 2 tap ISI channel. And now when you transmit the samples with the cyclic prefix, that is a cyclic prefix block of samples across this ISI channel, what you get is that the channel action becomes a circular convolution.

So the output basically y is a circular convolution of the channel with the transmitted samples plus the noise and that is what we have also seen yesterday. That is also what we have seen in the previous modules that is, y equals h times x h and where this is basically your circular convolution in a time domain that is, in a domain of the circular convolution.

And it is important to remember that this is the circular convolution in the time domain for the samples.

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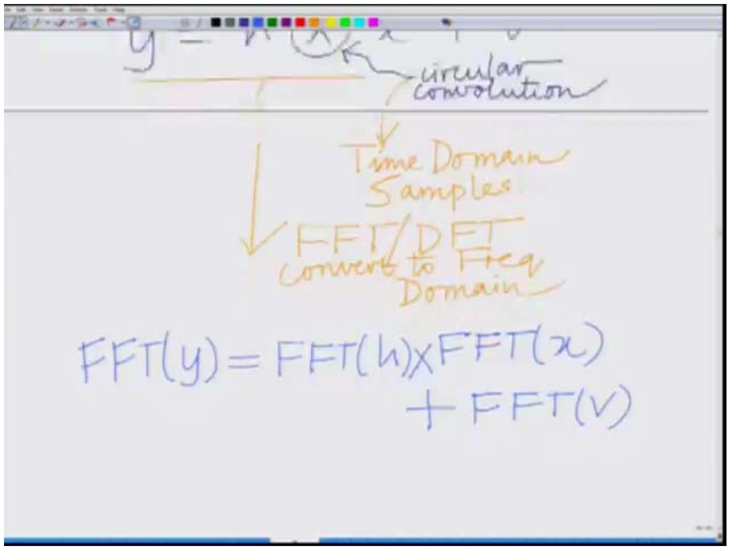

Okay, this is the circular convolution and the time domain that is the time domain for the sample. So there is still inter sample interference that is, the inter sample interference is still there, that has not been removed.

However, now once you take the FFT and convert it into a frequency domain. Now you take the whole system, you take the Fast Fourier Transform FFT or basically which is the same thing as your TFT and convert it into the frequency domain. And once you convert it into the frequency domain, what you have is that the circular convolution becomes a multiplication that is, FFT of h times FFT of x plus FFT of your noise.

Let me write this multiplication explicitly. So in the FFT domain, this becomes a multiplication. Multiplication in FFT domain or basically your frequency domain. Let us call this, rather than FFT, probably a better word for this is the frequency or subcarrier domain. Remember, in the frequency we are talking about the various subcarriers in the frequency domain.

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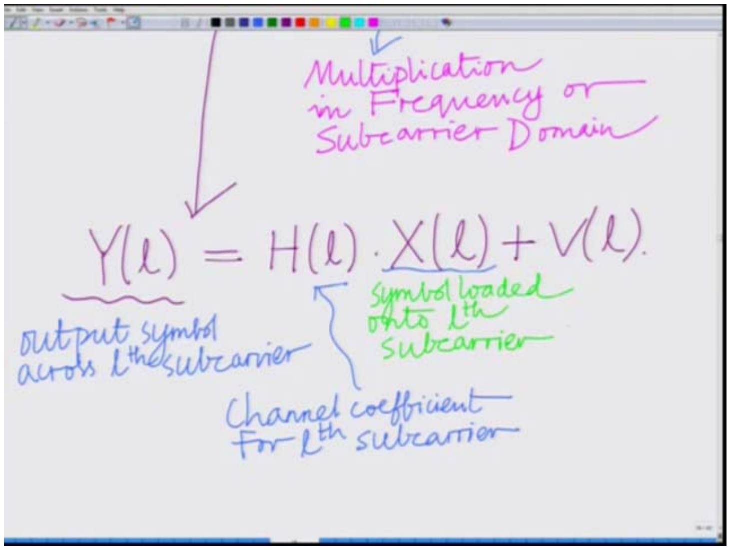

So in the frequency domain, it is a multiplication which means now basically if you look at the Lth subcarrier that is, I have y L that is in the frequency domain across Lth subcarrier is the product h L times x L which is the symbol transmitted across the Lth subcarrier plus V L where y L is the output symbol. Remember, this we said is the output; this is your output symbol across Lth subcarrier.

This is your channel coefficient for your Lth carrier, okay. This is the symbol loaded onto the Lth subcarrier. This is something that we have already seen the capital l. This is the symbol loaded onto the Lth, this is the symbol loaded onto the Lth subcarrier and V L is basically the on, this is your noise on the Lth subcarrier; this is the noise on the Lth subcarrier, so this is the model.

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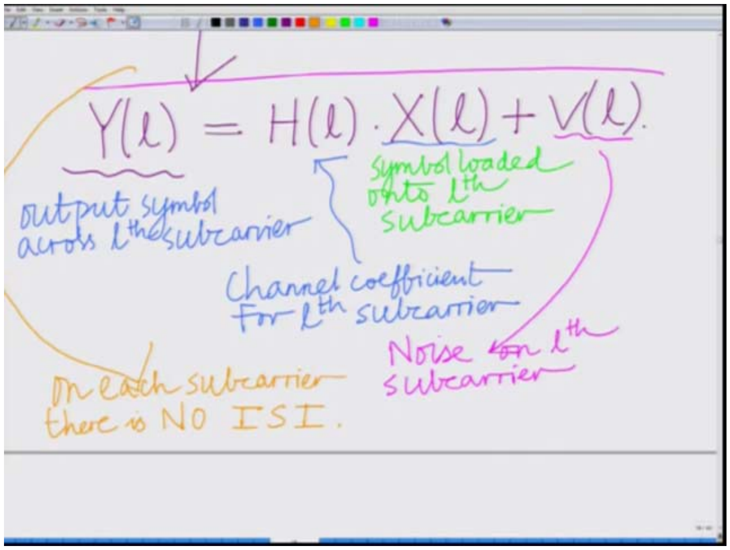

And therefore what you are saying on the subcarrier, on each subcarrier there is no… And this is important to observe that on each subcarrier there is no ISI. That is there is no Inter Symbol Interference on each subcarrier because y L equal the coefficient h L times the symbol x L on the subcarrier plus V l. So basically there is no interference Inter Symbol Interference on the symbol x L capital x L which is basically the symbol loaded onto the subcarrier.

Therefore, it very efficiently removes ISI using simply the IFFT at the transmitter and the FFT operation at the receiver. And now since we have N equal to 4 subcarriers, we may simply expressively write it down so that we can formulate the system model, then see how the estimation of this of this channel coefficient can be done. And that is indeed going to be very simple. So if I write the system for the N equal to 4 subcarriers.

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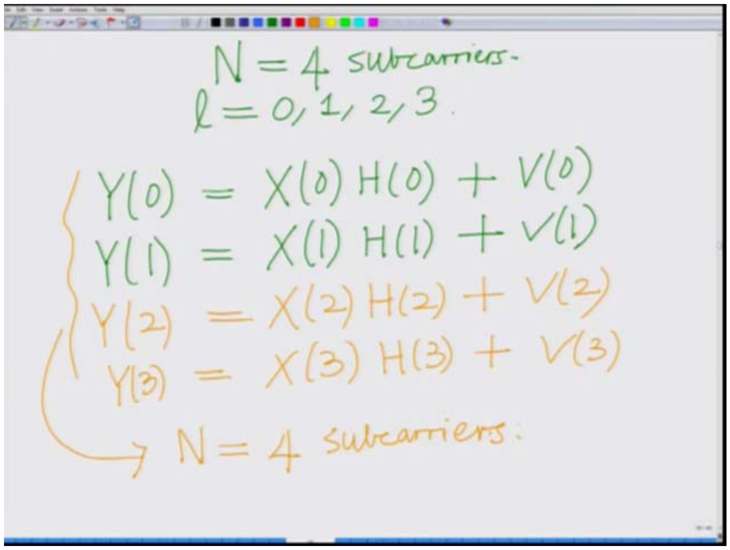

So remember we said we have N equal to 4 subcarriers are responding to L equal to 0, 1, 2, 3 which means across subcarrier 0 I have y 0 equals x 0 h 0 just writing h x as x h because this is convenient. y 1 equals x 1 h 1 plus V 1 by the same token, again this is very simple, something that you can already guess y 2 equals x 2 h 2 plus V 2 and y 3 equals x 3 h 3 plus V 3, this is across your…

N equal to 4, this is across your N equal to 4 this is across your N equal to 4 subcarriers. And now I can write this as a matrix, so basically what I can do now is I can basically make a vector out of this, I can use it, I can model it, I can represent it using vector notation. That is what we have done many times before and it is going to be very simple.Now if I write it as a vector, now we can clearly see, write it is as a vector.

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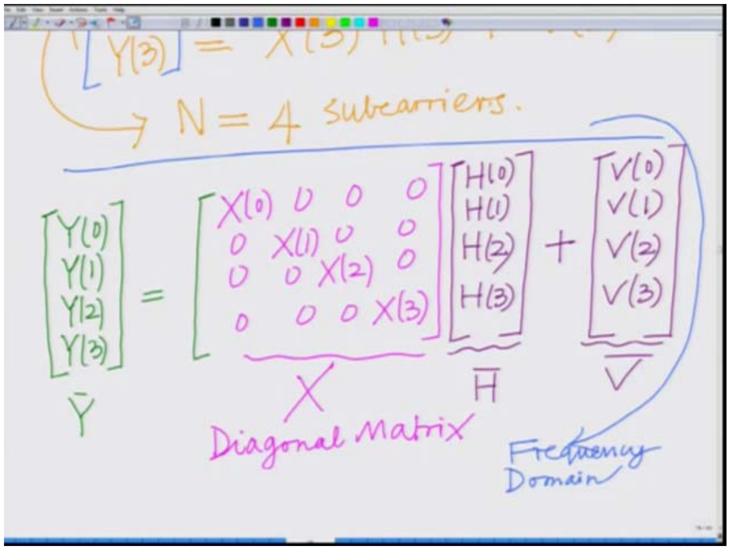

That is, I have y 0, y 1, y 2, y 3, let me call this vector y bar. You can see this is basically this diagonal matrix of x 0, x 1, x 2, x 3, this is basically.

Let us call this matrix x which is your diagonal matrix, times the matrix of channel coefficients h 0 or the vector of channel coefficient across each subcarrier rather h 2, h 3 plus again the noise vector it is the noise across each sub carrier capital V 0, capital V 1, capital V 3, so this is your channel vector, this is your noise vector.

And remember, this whole model is basically in the frequency domain because we are writing it across each subcarrier. This whole model represents the frequency domain representation of this OFDM system. That is y L equals x L the symbol loaded onto the subcarrier times h l, the channel coefficient across subcarrier plus capital V l, which is the noise sample on subcarrier l.

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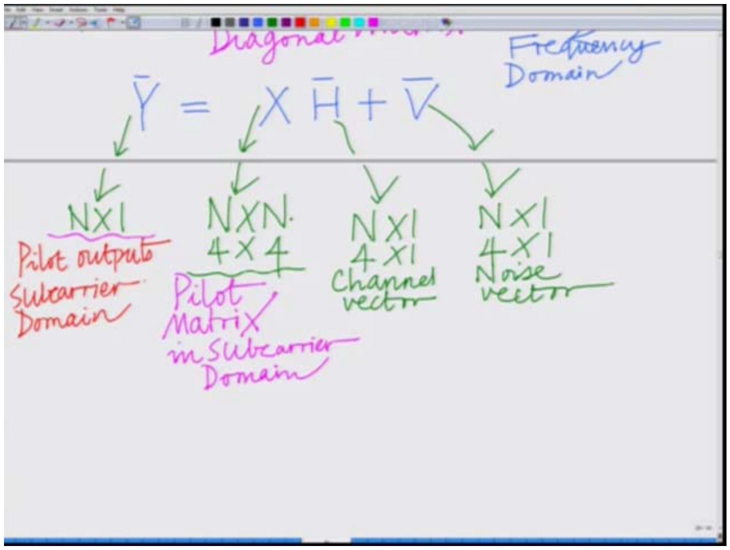

And now we have converted into matrix model where y bar equals x times h bar plus V bar where x equals, you can see this is an N cross N that is in, this case 4 cross 4 diagonal matrix. Y bar this is N cross 1 vector, h bar is your N cross 1 that is again 4 cross 1 coefficient vector. And V bar is basically your N cross 1 or basically your 4 cross 1 noise vector.

This is an equivalent channel vector, you can think of this as basically your channel vector in the frequency domain and you can think of this as the pilot matrix in the subcarrier domain. That is, the diagonal matrix x if we are transmitting pilot symbols on to the [sum] subcarrier, so this is the pilot matrix in the subcarrier domain. That is where, and this is the receive pilot outputs in the subcarrier domain.

For channel estimation, these are the pilot outputs in the subcarrier domain. Basically what we are saying is, we are considering capital X 0, capital X 1, capital X 2, capital X 3 to be the N pilots loading onto the N subcarrier. So basically what we are saying is we have 4 purposes of channel estimation, if we load the pilots onto the subcarriers, so these are your pilots loaded onto the subcarriers.

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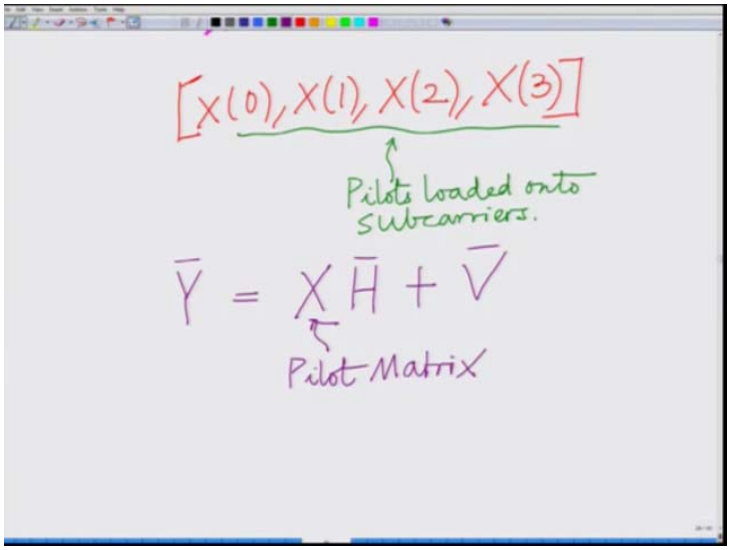

These are the pilot which are loaded onto the subcarriers and therefore what I have is, now I have a familiar y equals x h bar plus V bar where this is the pilot matrix. This is something what we have seen many times before, even in the downlink channel estimation where we have the receive vector basically y equals pilot matrix x times the channel vector h plus the noise vector.

And therefore now, one can formulate the channel estimation problem that is, the estimation of the coefficient vector capital H bar as a Least Squares problem. So naturally I can now formulate this as a Least squares problem and it is going to be a Least simply Squares problem. So channel estimation, I'm going to formulate this as a Least squares problem. This is going to be your Least Squares that is LS problem for channel estimation.

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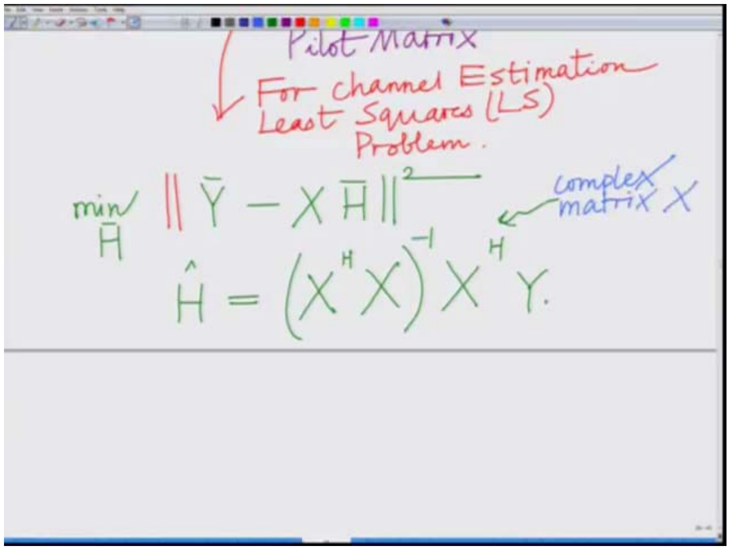

Which is simply now like many times before, y minus h x bar square and we would like to minimise it with respect to h bar that is, we would like to find the h bar which minimises this Least Squares function and that gave the Maximum Likelihood Estimate and remember that is also something that we had derived and the least squares solution again is very simple.

That is, h hat equal x hermitian considering complex symbols. X hermitian that is simply replacing the transposed by hermitian x hermitian x inverse x hermitian y that is basically… Remember, previously we had x transpose x inverse x transpose. Why I am simply replacing the transpose by hermitian, because of to allow complex matrix x, yeah.And therefore what we have and now therefore now let me write this again a little bit clearly.

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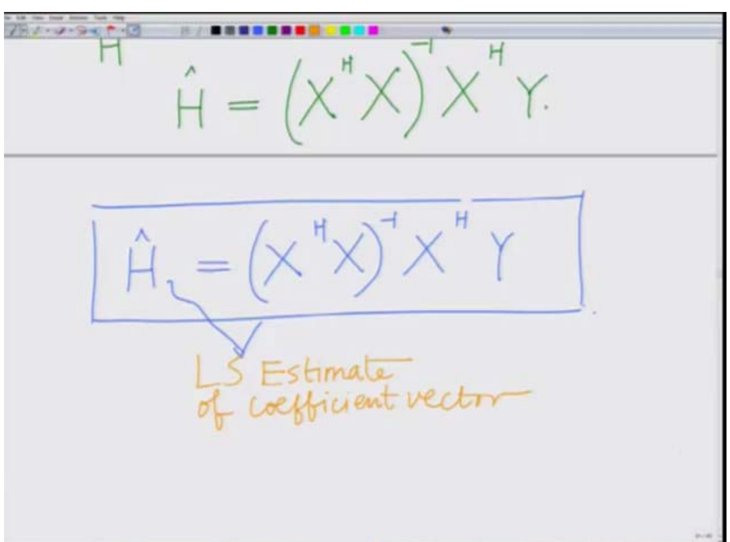

x hermitian x inverse x hermitian y, this is the Least Square estimate of the coefficient vector. This is the Least Square or the LS estimate of the coefficient vector, right. x hermitian x inverse x hermitian y right, where y basically vector y is the FFT of the outputs received of the sample, output samples received in the time domain.

Basically, y bar is the vector of outputs across the various subcarriers, correct. And x is basically the diagonal matrix consisting of the pilot symbols which are loaded onto the subcarriers and now if you observe something interesting, this matrix x is a diagonal matrix which is basically invertible, right.

So if you look at this is slightly different from the Least Square that we have seen before, your x is a x basically equals. In fact, let me just write it expressively.

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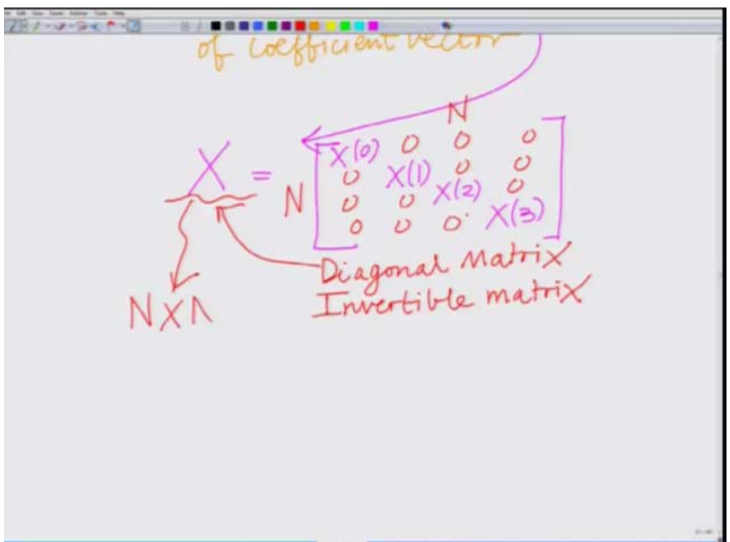

This is your x 0, x 1, x 2, x 3 which is an only diagonal entries are nonzero, rest of the entries are 0. So basically your x is a diagonal matrix.

X is a diagonal matrix which is invertible and therefore what you have, so this matrix is basically this is an invertible matrix. This is basically an invertible matrix which means now what you have is normally, see remember previously we said, so this is an N cross N matrix, so X is basically a square.

In fact, diagonal matrix and hence x is invertible. Previously, when we considered the pilot matrix, remember the pilot matrix we said is N cross M where N is a number of pilot vectors and [a] M is basically the number of antennas at the base station. That was in the case of multi-antenna downlink channel estimation.

Here we have something very interesting because of the nature of OFDM channel estimation. We have, if the pilot matrix x is N cross N, it is a square matrix. So therefore in fact it is invertible and hence in this scenario particularly, x hermitian x, since x is a invertible matrix, this is not always true, so this becomes x inverse times x hermitian inverse.

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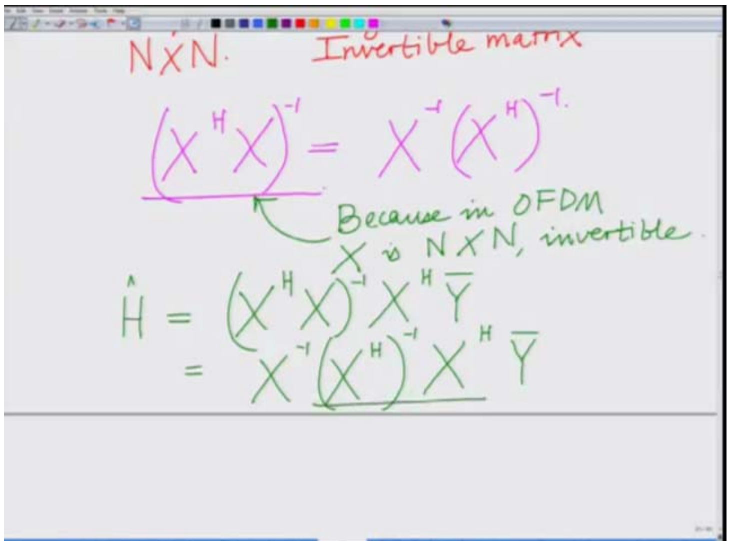

This is only for this scenario because x is invertible. This is not because x X is invertible, this is not generally true, and this is only true in this scenario because in OFDM and this OFDM scenario X is N cross N. It is an N cross N invertible matrix. And therefore now if I substitute this, I have h hat equals x hermitian x inverse x hermitian y bar which is basically now x inverse into x hermitian inverse into x hermitian into y bar.

Of course you can see x hermitian inverse into x hermitian, this is identity, so this is simply now very interestingly simply x inverse y bar where x is basically your, where x is basically, this x is basically your pilot matrix pilot matrix.

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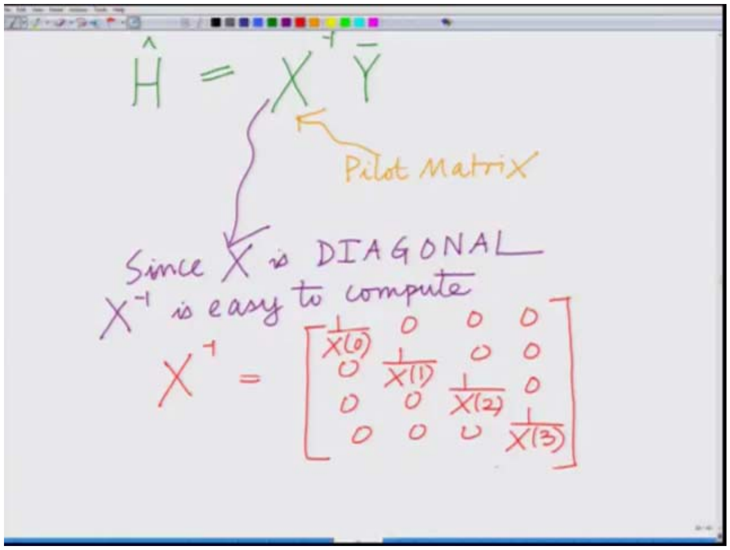

And also since the pilot matrix is diagonal, since it is a diagonal matrix, x inverse is also very easy to compute.

If basically the inverse of each of the diagonal elements that is the reciprocal of each of the diagonal elements. Therefore, x inverse simply, since x is a diagonal matrix, this is also important to note. Since x is diagonal, x inverse is easy to compute. When I say easy, it means it is sufficient, x inverse is simply basically your 1 over x 0, 1 over x 1, 1 over x 2, the rest are 0s.

This is your diagonal matrix and therefore now if I look at h hat.

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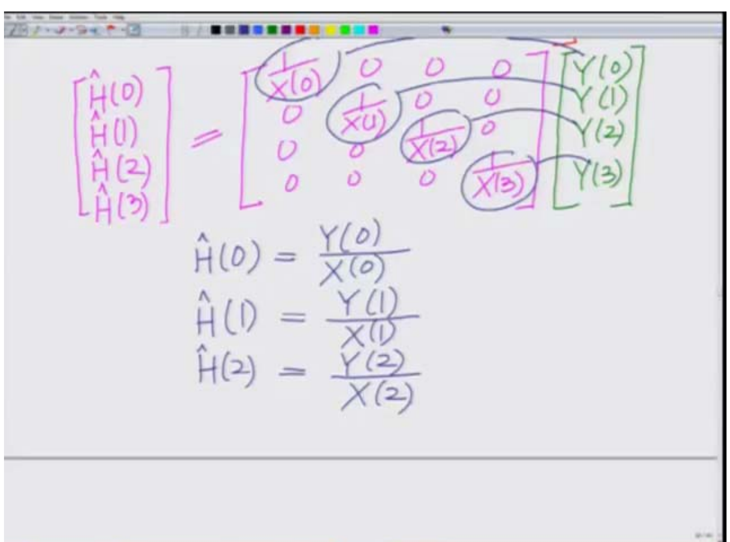

Estimate of the vector h hat that is estimate of h hat 0, h hat 1, ha hat 2, h hat 3, this is simply x inverse that is 1 over x 0, 1 over x 1,1 over x 2, 1 over x 3 0 times the symbol vector received across the subcarriers, that is y bar which is y 0, y 1, y 2, y 3 and now therefore now since this is diagonal matrix.

Now if you look at this, you have 1 over x 0 multiplying by y 0, you have 1 over x 1 multiplying y 1, you have 1 over x 2 multiplying y 2, x 3 multiplying y 3. Now therefore the channel estimates are very simple, it is simply h hat 0 equals y 0 divided by x 0, h hat 1 is basically your y 1 divided by x 1, h hat 2 equals y 2 divided by x 2 and finally h hat 3 equals y 3 divided by x 3.

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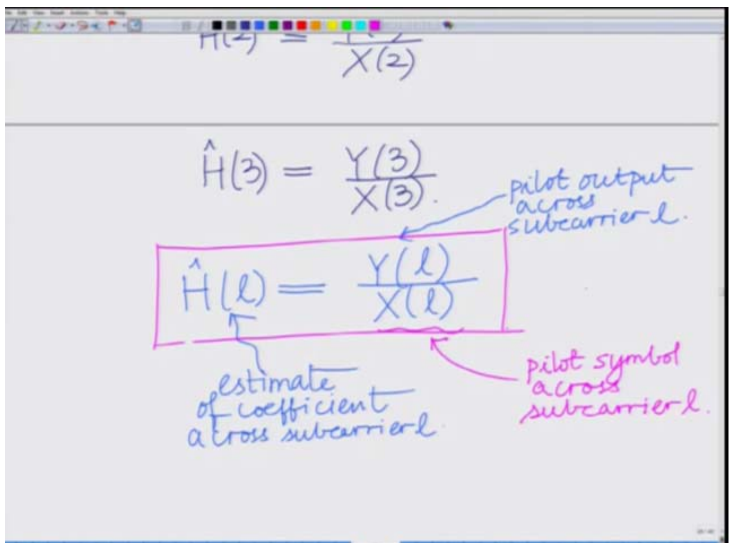

That is basically the estimate of the channel coefficient across each subcarriers N is simply the pilot output across the subcarrier divided by the pilot input across the subcarrier and that is very simple to see. So basically summarising this channel estimates, h hat k equals y k divided by x k, this is what, this is the estimate of coefficient or let us write it in terms of l, since that is the notation that we are using.

h hat L equals y L divided estimate of coefficient across subcarrier l, y L symbol across subcarrier l. The pilot output symbol, pilot output across subcarrier L and x L is basically your pilot symbol. This is basically the pilot symbol across subcarrier L and therefore, if we look at this across each subcarrier, we have h hat l.

It is very simple, the estimate of coefficient across subcarrier N is simply y L divided by x L where x L is the pilot symbol loaded onto the Lth subcarrier, all right. So the estimate h hat 0 is equal to, you can clearly see.

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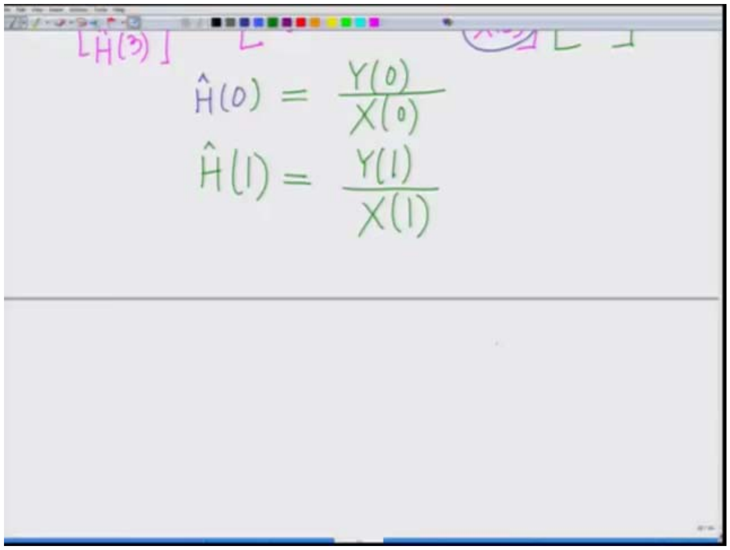

This is y 0 divided by x 0, h hat 1 equals y 1 divided by x 1.

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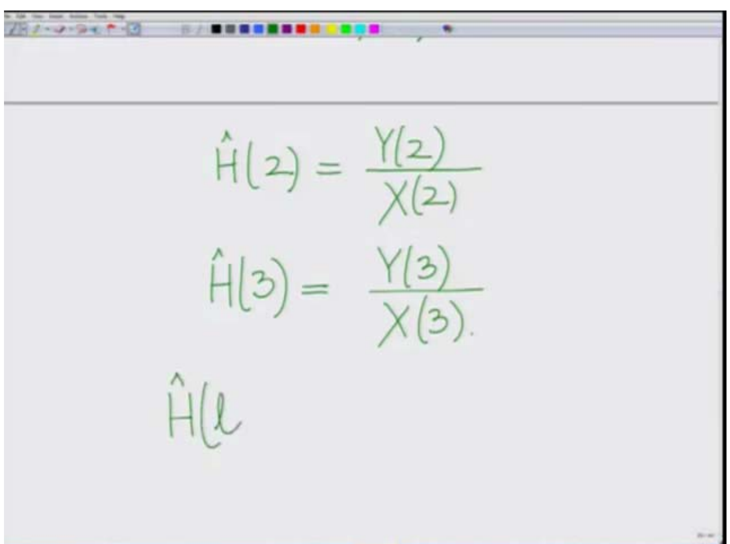

h hat 2 similarly equals y 2 divided by x 2 and h hat 3 this is equal to y 3 divided by x 3.And in general therefore one can say the estimate of the coefficient on the Lth subcarrier h hat L equals y L divided by x l.

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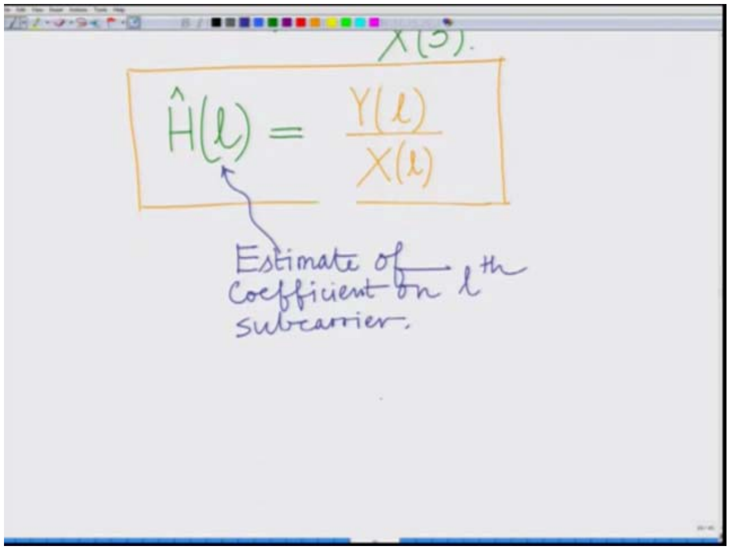

This is what is L h hat, L is basically, this is the estimate of the coefficient on the Lth subcarrier. This is the estimate, this if the estimate of the coefficient on the Lth subcarrier, all right.

So we are saying h hat L equals y L, that is the received pilot symbol y L on the Lth subcarrier divided by x L which is the transmitted pilot symbol on the Lth subcarrier. And therefore now if I look at this, now if I look at the coefficient h hat 0, h hat 1, now remember what we said earlier, remember.

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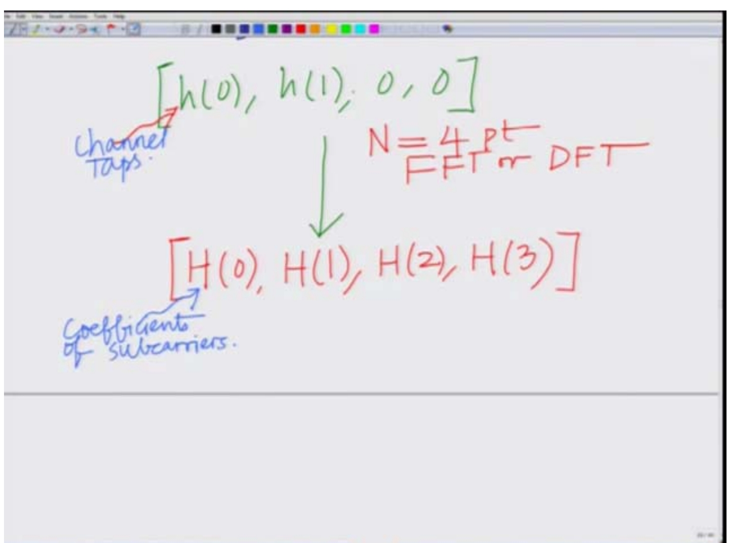

The h hat, the capital H, what are they, they are the given by the FFT of the channel taps, remember we had [h 1], h 0, h 1, 0 pared and you look at the N equal to 4 point IFFT or N equal to 4 point FFT or basically your DFT, you get the capital H S which are the basically the channel coefficients corresponding to the subcarriers, right.

These are the coefficients for the subcarriers, so what are these, these are your channel taps and these are the coefficients corresponding to the various coefficients of the subcarrier. So the coefficients of the subcarrier, channel coefficients of the subcarriers are given by the 0 pared FFT of the channel taps. But therefore, from the estimates of the coefficients of the subcarriers, if you want to construct the channel taps, we have to look at the IFFT.

Right, because the channel taps FFT use the channel coefficients of the subcarriers, the coefficients on the subcarriers, if you take the IFFT or the IDFT, you get back the taps. So now we have the estimates of the channel coefficient on the subcarrier. How do we find the channel taps, estimates of the channel taps, we take the IFFT or IDFT.

And therefore naturally what we want to do is…

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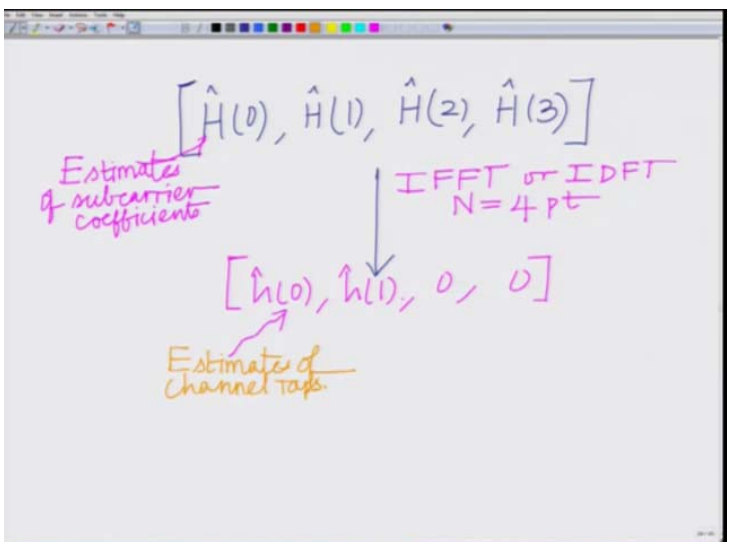

I have h hat of 0, h hat of 1, h hat of 2, h hat of 3 and what do I do, I basically take the IFFT or basically IDF. N equal to 4 point… To get back your h hat 0, to get back the estimates of the, what are these, these are the basically your estimates of the channel taps. So these are the estimates of the so these are estimates of the subcarrier coefficients.These are the estimates of your channel taps.

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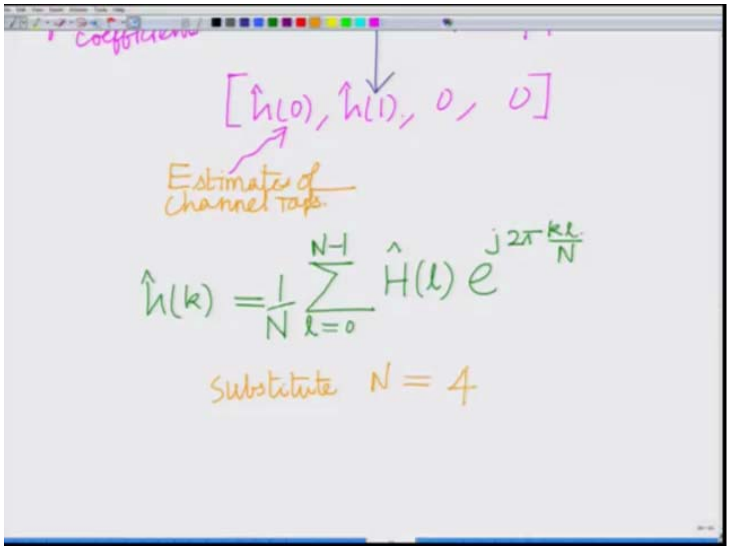

In other words, basically what we are saying is this h hat k which is the estimate of the channel tap of the subcarrier k on or the estimate of the kth channel tap is given by the N point IFFT, that is 1 over N, L equal to 0 to N minus 1, h Hat of L, e raise to j 2 pie kL by N and now here, I am going to substitute an equal to 4.Now substitute N equal to 4.

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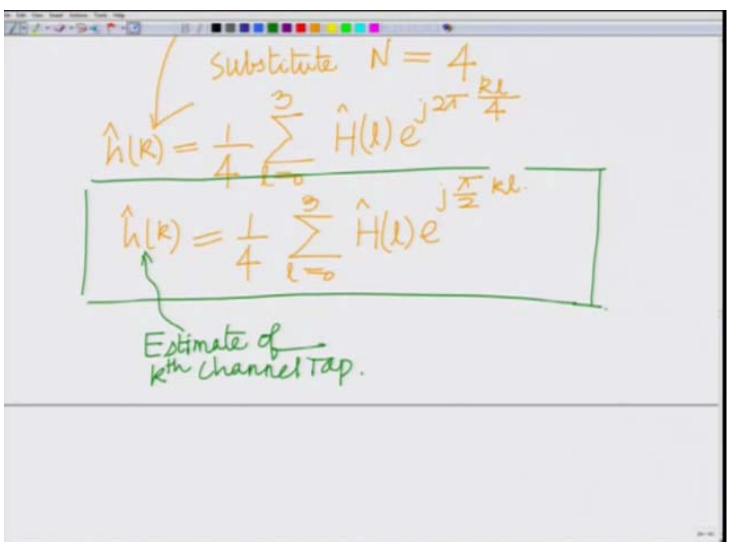

Which means what I have is h hat of k equals 1 over 4 L equal to 0 to 3, h hat of L e power j 2 pie k L [k L] divided by 4 which is basically 1 over 4 L equal to 0 to 3 h hat of L e power j pie to k times L. This is the expression for the estimate of the kth; this is estimate for the expression of the kth channel tap. So this is the estimate of the kth channel tap.

Expression for the estimate of the kth channel tap which is given by the IFFT of the estimates of the subcarrier coefficients that is, estimates of the channel coefficients of the various subcarriers. And the [IF] size of the IFFT or the IDFT is N equal to 4 point. Okay, for example, h hat 0, if you look at h hat of 0, I now have to do is, I have to substitute basically your k equal to 0 corresponds to k equal to 0.

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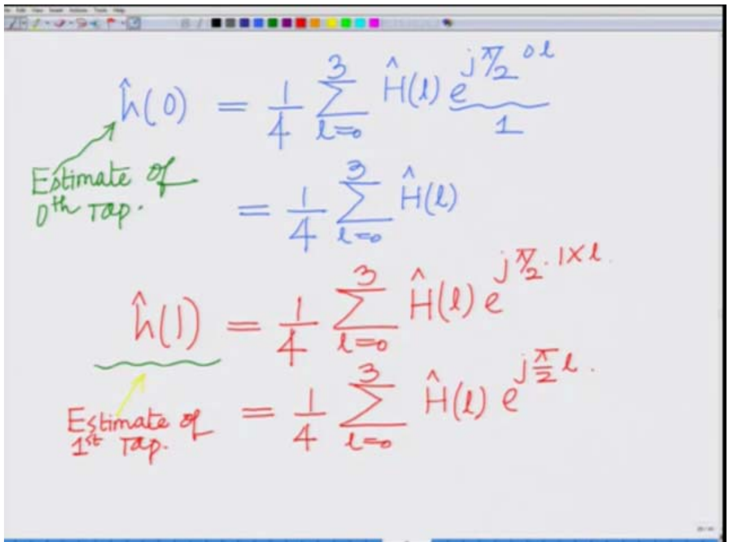

So this is 1 over 4, submission L equal to 0 to 3, h hat L , e power j pie by 2, k equal to 0 0 tads L, so this quantity is 1, so this is basically 1 over 4 submission L equal to 0 to 3, h hat of L, e power, well this thing is one, so it is simply submission 1 over 4 L equal to 0 to 3 h hat of L and h hat of 1 which is the estimate of the first channel tap is 1 over 4, basically that corresponds to k equal to 1, L equal to 0 to 3, h hat of L, e power j pie by 2, k equal to 1 into L.

So this is basically 1 over 4, L equal to 0, h hat of L, e power j pie by 2 into L. So this is estimate of channel tap 0 or estimate of 0 (()) (37:48) and this is basically your estimate of the first tap. This is basically the estimate of the first tap.

This is estimate of the channel tap 1, okay. So basically what we have over here, now what we have done, we have comprehensively demonstrated how to do channel estimation for an OFDM that is Orthogonal Frequency Division Multiplexing System.

So in this module and the past modules, what we have seen is first we have formulated, we have developed the model for OFDM or Orthogonal Frequency Division Multiplexing based transmission where we said we have symbols which are loaded into the subcarriers that is, basically perform the IDFT followed by the cyclic prefix, transmit them over the frequency select or the Internet symbol interference channel, right.

At the receiver perform the FFT, that converts, that basically because of the cyclic prefix action of the channel is that of circular convolution, so basically in the FFT domain the action of the channel is that of multiplication, the channel coefficient is multiplied across each subcarrier with the transmitted symbol plus of course you have additive noise.

And then we formulated, considering these symbols loaded onto the subcarrier to be pilot symbols, we formulated the least squares channel estimation problem, we estimated the channel coefficients corresponding to the various subcarriers.

In fact, we said that it is very simple, what we have to do is we have to take the received pilot symbol capital Y L across subcarrier L divided by the transmitted pilot symbol capital X L on the Lth subcarrier, that is capital Y L divided by X L that gives capital h hat L. The estimates of the channel coefficient across subcarrier L remember, this is the channel coefficient in the frequency domain.

Now to get the time domain channel coefficients, what paved we have to do is we have to take the IFFT similarly, the N equal to 4 point IFFT and then you get the corresponding estimates of the channel taps in the time domain. So that is how, that is how enquiry or in summary how OFDM channel estimation works, alright.

So we will stop this module here, in the subsequent module we will look at a simple example, because examples illustrate things clearly, so we will look at element example to see how this whole mechanism of OFDM channel estimation works, alright so we will stop this module. Thank you very much.