Estimation for Wireless Communications – MIMO/OFDM Cellular and Sensor Networks

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Lecture Number 27

Introduction to Orthogonal Frequency DivisionMultiplexing(OFDM)

FFT at Receiver and Flat-Fading across Each Subcarrier

Hello! Welcome to this massive open online course on Estimation for Wireless Communication Systems. So currently we are looking at OFDM or Orthogonal Frequency Division Multiplexing and we are trying to understand the mechanism of OFDM and we also said OFDM is a modern wireless technology which when efficiently overcome inter-symbol interference, correct?

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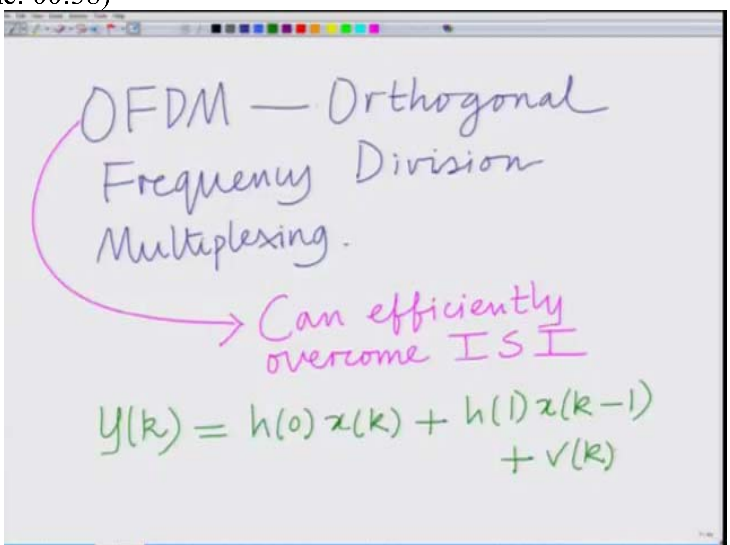

So what we are looking at currently is you are looking at estimation in the context of OFDM where OFDM stands for Orthogonal Frequency Division Multiplexing, correct? And we said, OFDM can be used to efficiently overcome, can efficiently overcome ISI or basically Inter Symbol Interference. For instance we have our, and we have seen this many times before, YK equals H zero, SK plus H one X K minus one plus VK.

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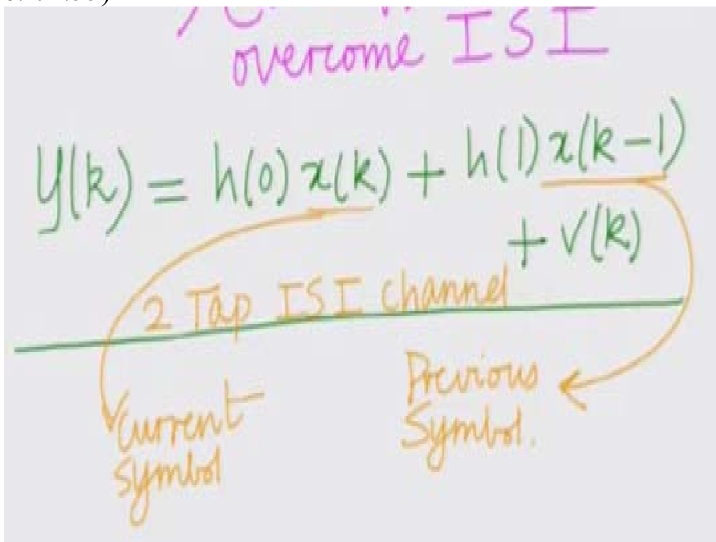

This is our two tap ISI channel so this is basically the model for your two tap ISI channel, but the tabs are zero and H one, XK is the current symbol, XK minus one is, XK minus one is the previous symbol, alright? So there is Inter Symbol Interference of the previous symbol XK minus one on the current symbol. And we said OFDM can be explained as follows.

So OFDM we have, let's consider a typical OFDM system with four subcarriers, that is, we have the symbol X zero, capital X zero, capital X one, capital X two, capital X three, these are the N symbols, that is, these are N equal to four symbols, these are loaded onto N equal to four subcarriers and the meaning of this is basically.

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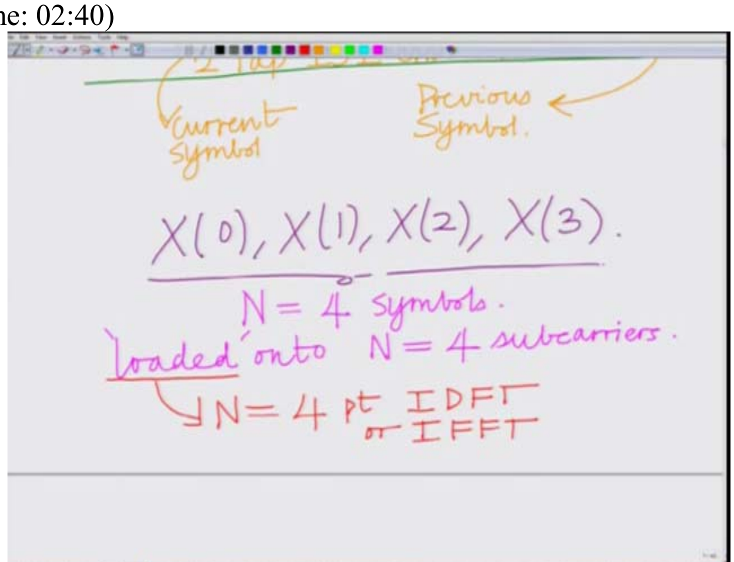

The meaning of this term loaded is basicallywe perform N equal to four point, N equal to four point, IDFT or IFFT, the Inverse Fast Fourier transform which is basically the Inverse Fast Fourier Transform one can perform IDFT or equivalently IFFT, IFFT is nothing but the same as the IDFT, but it's an efficient, it is a fast algorithm to perform IDFT, that is, Inverse Fast Fourier Transform, okay?

So now, therefore, the samples from this, what you have is from this capital X zero, X one, X two, X three we generate what are known as the samples, that is, by IFT that is the samples X zero, X one, X two, X three, and these are generated by the, these are the N equal to four samples and these are generated by the IDFT as and all of you must be familiar with the IDFT.

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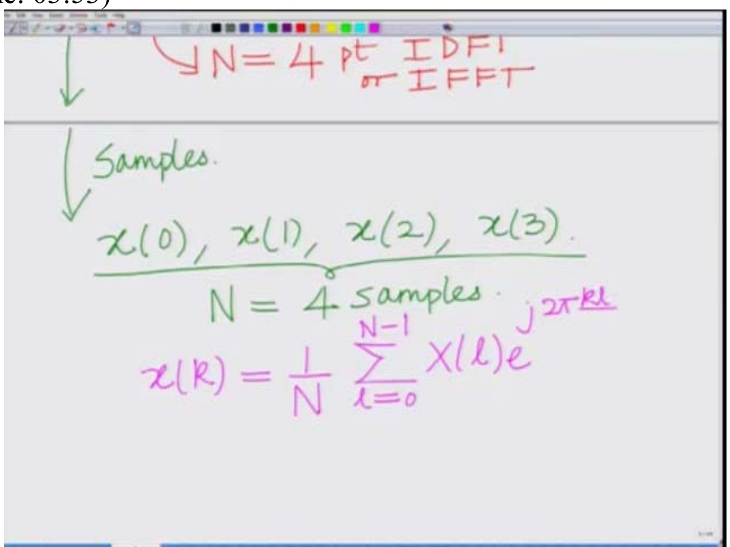

That is basically we have X of K equals one over N summation L equal to zero to N minus one, capital X of L that is the symbols loaded on to the subcarriers, E to the power of J two pi KL by N. And now I am going to substitute N equal to four so this gives me basically one over four summationL equal to zero to three capital XL E raised to J two pi KL by four, which is equal to.

Therefore, your XL is basically equal to one over four summation L equal to zero to three, XL E raised to J, pi by two KL, alright? So this is basically the sample, so these are the symbols loaded on to the subcarrier that is your BPSK QPSK symbols etc. and these are the, these are the samples and now we said we are not going to, we want to transmit the samples.

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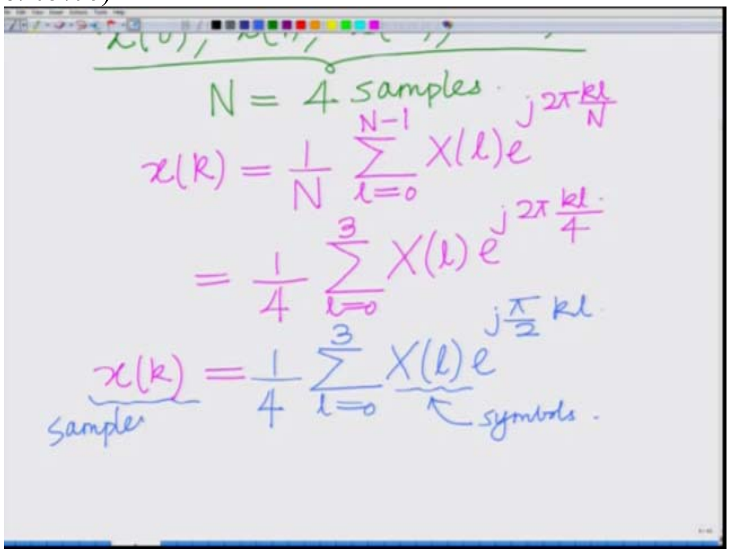

But we don't submit the samples as it is, we have small modification before transmission of these samples over the channel we add the cyclic prefix. That is we take a few symbols from the tail of the block and prefix them at the head of the block. Since we are cycling symbols towards the end towards the head this is known as a cyclic, this is the cyclic operation and also since we are prefixing them it is known as cyclic prefix.

So in the second step what we do is you have your samples X zero, X one, X two, X three and what I have is now again I am basically, cycling the samples from the end towards the beginning. This is known as a cyclic prefix. This is denoted by the term CP, okay? And now this block with CP added, this is transmitted over the channel.

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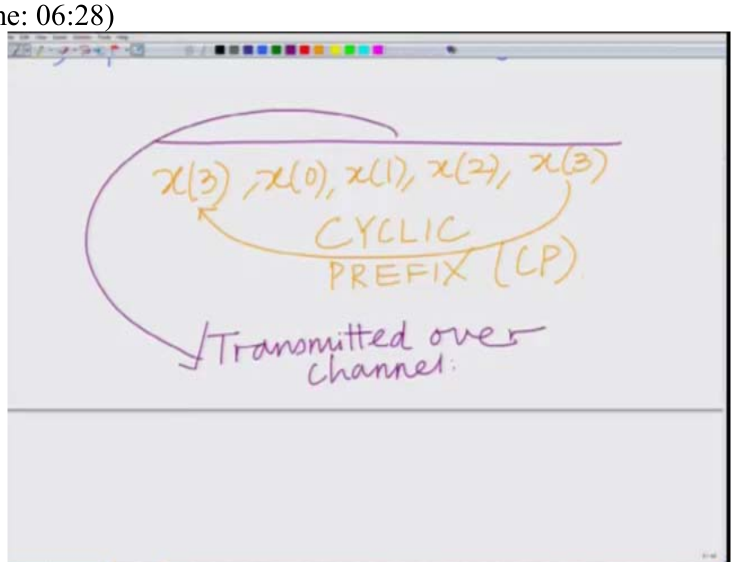

Remember our channel is our frequency selective channel, that is, YK equals H zero, XK plus H one XK minus one plus VK. Now when I transmit the samples your cyclic prefix CP and its samples is X three, X zero, X one, X two, X three, now observe the output corresponding to X zero, output corresponding to X zero is Y zero equals H zero times X zero plus H one times the previous symbol, but the previous symbol to X zero is X three.

This is the previous symbol because the addition of cyclic prefix, this is your previous symbol to X zero because of the addition of the cyclic prefix, therefore this is X zero plus, I'm sorry, this is X three, this is a previous symbol to X zero plus V zero, alright? So what we have done is basically because we've added a cyclic prefix, alright?

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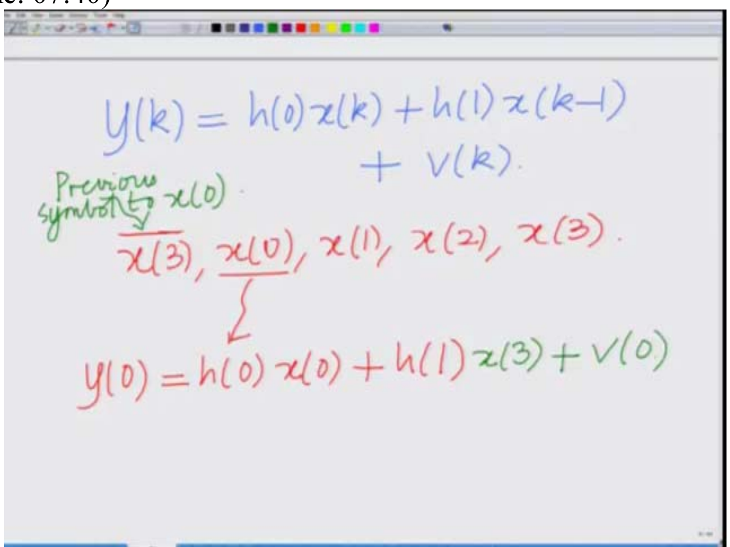

And (X) the sample X zero has interference from the previous sample, and the previous sample X three. Therefore, we have H zero times X zero plus H one times X three plus V zero, okay, so that is basically what we have at the output that is corresponding to Y zero, okay?

And similarly I have Y one, now the rest can be written in a straight forward manner Y one equal to X zero times X one plus H one times the previous sample that is X zero plus V of one your Y two equals H zero times X of two plus H one times the previous symbol sample X one plus V of two and Y three equals H zero, X three, plus H one times the previous sample that is X three plus V of three.

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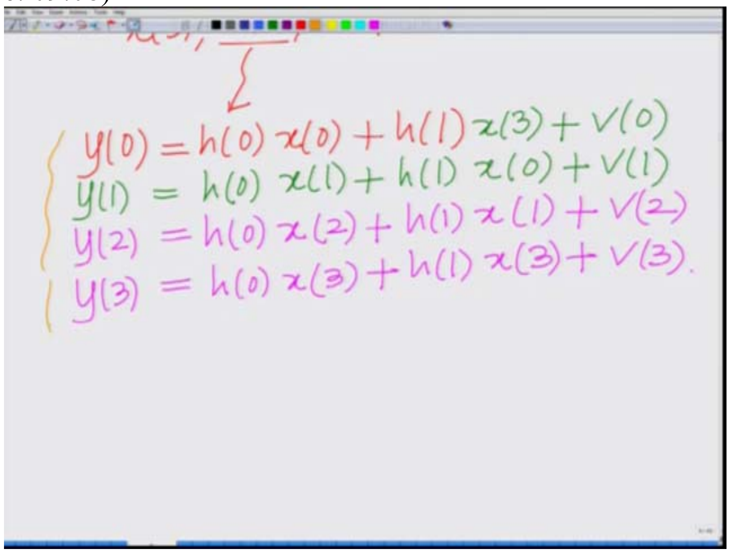

And now these are the samples, and now we said yesterday that is if you look at Y zero that is, if you look at Y zero, Y one, Y two, Y three, that can be basically represented as the circular convolution of the channel filter H with the transmitted samples X plus the noise and that is the key operation, that is the advantage that this (basical) this addition of cyclic prefix is giving as a.

Therefore we have which something that we have seen in in an elaborate manner yesterday in great detail is basically the observation that now because of the addition of the cyclic prefix what I have is Y becomes the circular convolution of H with X in the presence of added (()) (10.45).

So this is basically nothing but your, this is your circular convolution and as a result now if you take the FFT we know in the frequency domain, circular convolution in the time domain becomes in the frequency domain it becomes the multiplication. Therefore that is the FFT of H times the FFT of X plus the FFT of V which is basically your (()) (11.25)

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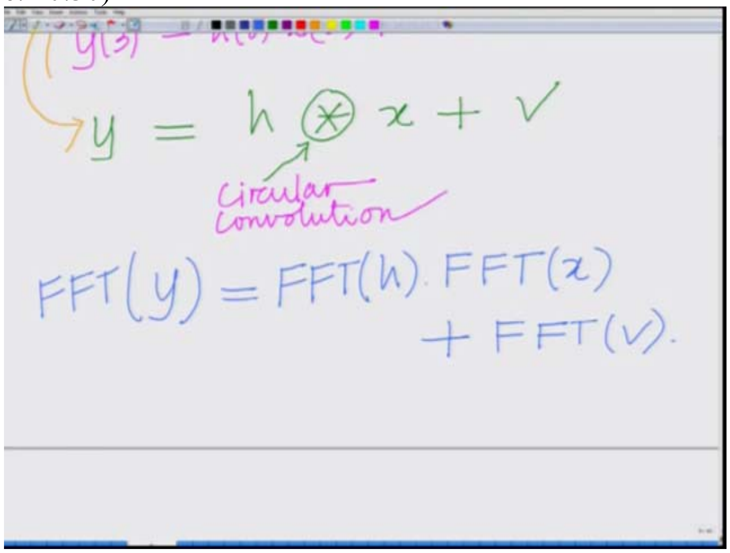

Therefore we have this interesting property because circular convolution and the time is basically multiplication in the frequency where FFT domain. So once you take the FFT of the samples at the output,basically the action of the channel in the frequency domain, basically now can be represented as a simple multiplication.

Therefore the FFT of the output samples Y is basically the FFT equals the FFT of the channel filter H times the FFT of the transmitted samples X plus the FFT of the (()) (11.54). And now let us see how you get the (FF) how you get the FFT of course the FFT of Y is basically you have samples Y zero, Y one, Y two, Y three, so you take the FFT of this sample.

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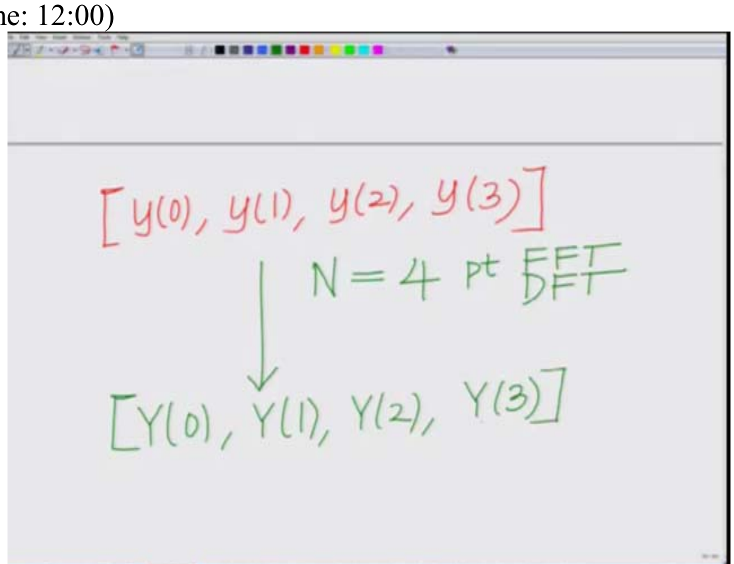

Obviously we are talking about the N equal to four point FFT where N is the number of subcarriers, so the size of the FFT is always fixed that is N equal to four point FFT that is

Fast Fourier Transform or basically also the DFT, I don't need to mention this the FFT is simply a fast algorithm for DFT that is Discrete Time Fourier Transform.

And that gives you the symbols across the subcarriersY zero, Y one, Y two, Y three where each Y L is basically generated by the FFT of the small Y, that is basically summation K equal to zero and I'm writing the expression for the N point of FFT, XK E to the power of minus J twopi KL divided by N and substitute N equal to four.

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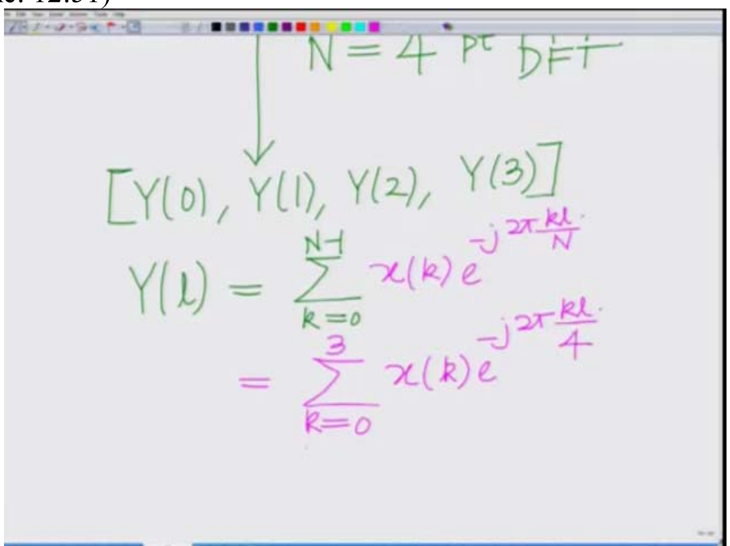

This is basically your K equal to zero to N minus one which is three, that is XK E to the power of minus J two pi KL divided by four, which is basically equal to so your YL equals summation K equal to zero to three. XK E raised to minus J pi by two KL. So basically, this what we are doing here is we are substituting N equal to, we are substituting N is equal to four.

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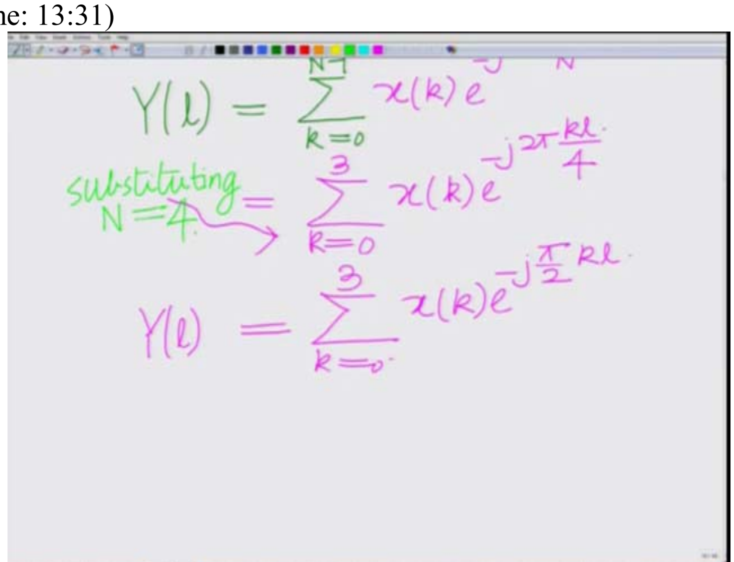

And that is what you are saying is the capital Ys that capital YL that is the received symbol across subcarrier L and FFT domain is now given by the four pointer, N equal to four point FFT or Fast Fourier Transform, DFT Discreet Fourier Transform of the received output samples Y zero, Y one, Y two, Y three, so this is basically what is this YL? This is the received symbol in frequency domain across; this is the received symbol across subcarrier L.

Similarly one can do the same thing for, now let us look at, of course we have to take the FFT of the channel filter. Now look at that, the channel filter has two channel tabs, H zero, H one and I have to take the four point FFT therefore I have to obviously pad with zeroes. So these are basically L channel tabs, where L equal to two.

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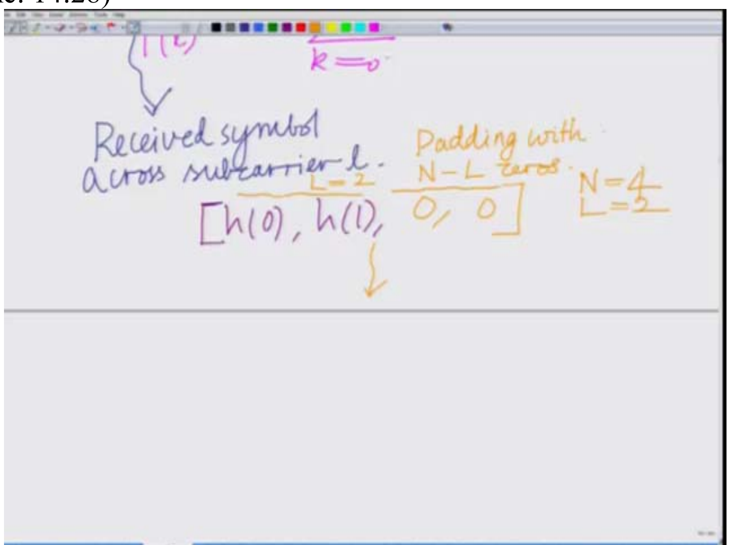

I have to pad it with N minus L zero, so N equal to four, L equal to two, because basically I have L channel tabs and I have to take the N point FFT so naturally I have to pad with N minus L zero. So this is basically the zero padded FFT. So padding with N minus L zeroes, and now you take the FFT of this to give you capital H zero, capital H one, capital H two, capital H three and these are the coefficients across the subcarrier, these are the coefficients or you can also say channel coefficients.

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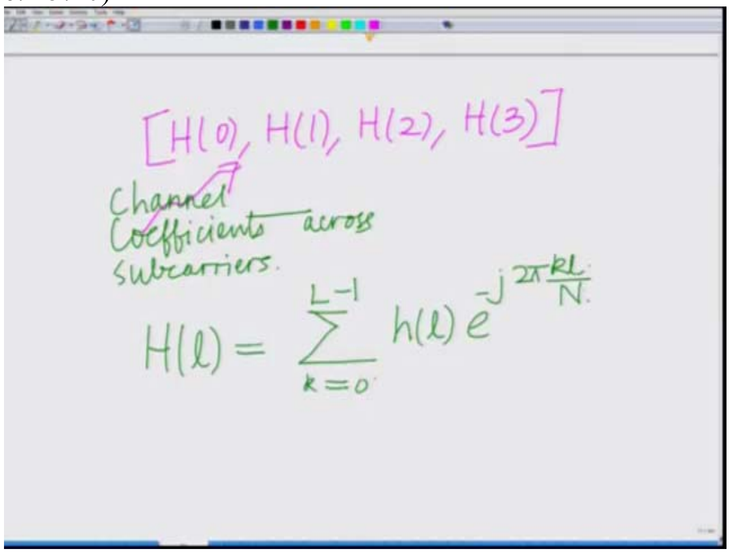

These are the channel coefficients across the subcarrier. How do we generate them naturally what we do is we have H of L, the coefficient across subcarrier L equals summation K equal to zero to L minus one, K equal to (z). Remember we only have channel coefficients L channel coefficients from K equal to zero L minus one so that is H of L, E raised to minus J two pi KL divided by N, I'm sorry, this is K which is equal to summation K equal to zero.

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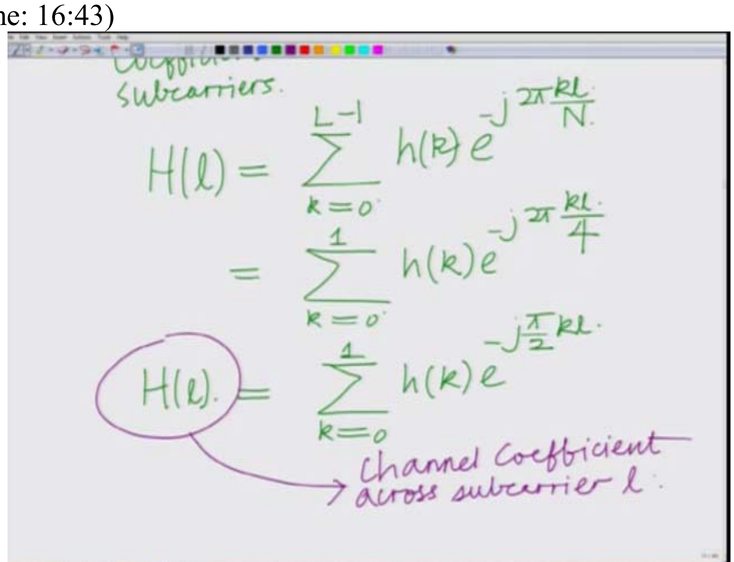

In this case L equal to one, so K equal to zero, to basically, L one, that is H of K E raised to minus J two pi KL divided by four which is basically equal to your summation K equal to zero to one, HK E raised to minus J, pi by two KL, this is your HL as we said this HL is basically the channel coefficient or the effective channel coefficient across, across the L^{th} subcarrier, alright?

So the capital HL, which is the channel coefficient across subcarrier L in the frequency domain is given by the FFT of the the (ta) channel tabs in the time domain. Of course these are only capital L channel tabs, and we have to take N point FFT, right? We have to consider the N point FFT therefore naturally you have to pad it with N minus L zeroes, Okay?

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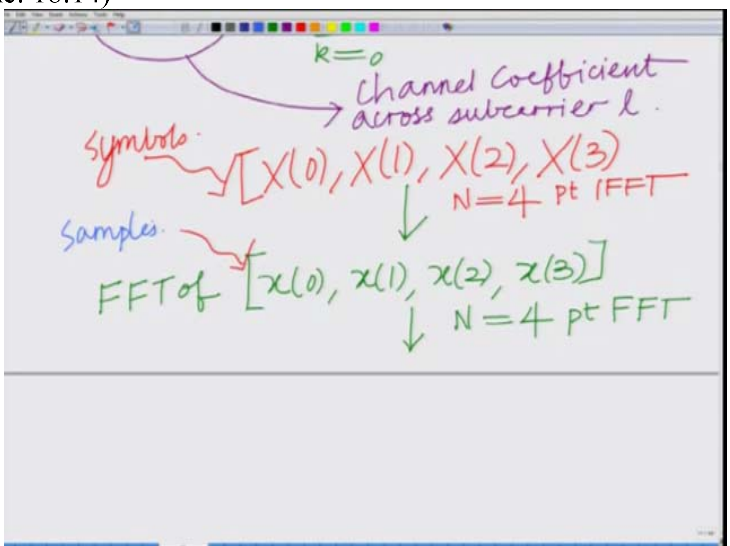

And now let us look at the FFT of the samples X, FFT of the samples X,now we want to take the FFT that is obviously the N point FFT. N equal to four point, N equal to four point FFT but observe that these are basically given by the IFFT of the symbols, remember where we started from, we started with X zero, X one, X two, X three, and we did the N equal to four point IFFT to get the sample.

So these are basically your symbols, and these are basically your samples. So remember the when we started with in this OFDM, we considered the symbols, the capital Xs and we performed the IFFT to get the samples that is small Xs. So naturally once you do the FFT of the samples you get back the symbols that are loaded on to the subcarriers and that is the other important point to keep in mind.

So once you take the FFT, so basically we had the symbols in the frequency domain, which were loaded on to the subcarriers and you consider the IFFT to get the samples. Now once you take the FFT of these samples you are back in the frequency domain and you get the symbols that are loaded on to the various subcarriers and that is basically the important point to keep in mind.

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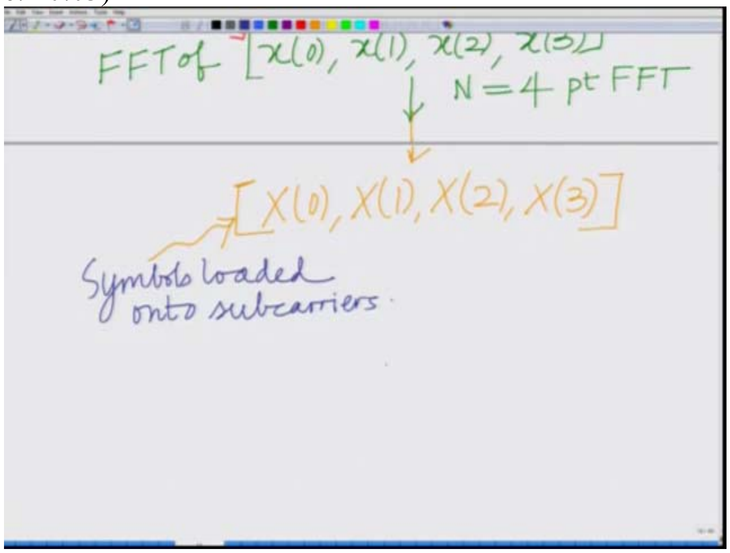

So this basically N equal to four point FFT so this basically this gives back your X zero, X one, X two, X three. These are basically your, these are basically the symbols or the modulated symbols loaded on to the various subcarriers and that is what you are getting back after you perform the FFT and naturally the only remaining thing is the noise, that is also very simple.

You have the small Vs, which are the noise samples (()) (20.45) noise samples in the time domain you do, you do the N equal to four point, let me write it a little bit more clearly, you do the N equal to four point FFT of this and what you get back is, is what you get, is the noise that capital Vs, the noise across each subcarrier, the noise samples across the subcarriers.

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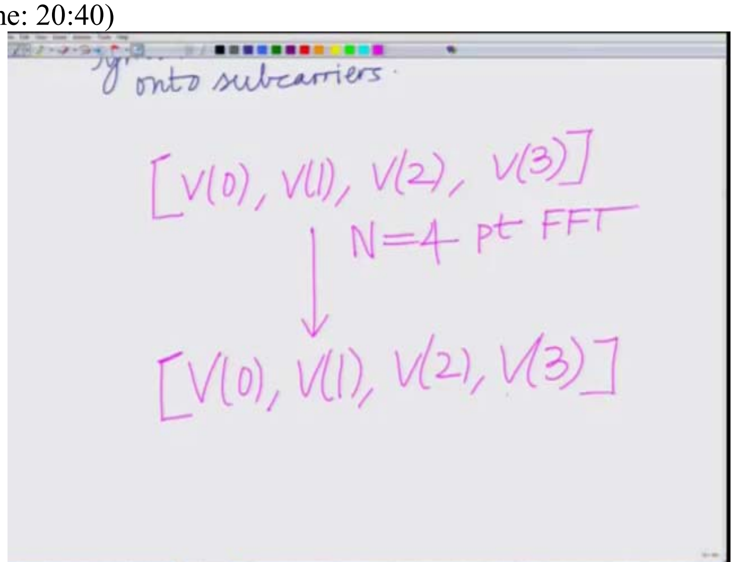

What are these? These are your noise samples across subcarriers, these are the noise samples across the subcarrier and of course we have V of L which is given by the FFT that is summation K equal to Zero to N minus one small VK which is the noise sample, E raised to minus Jtwo pi KL divided by N, substitute N equal to four and what you have is summation K equal to zero to three small VK.

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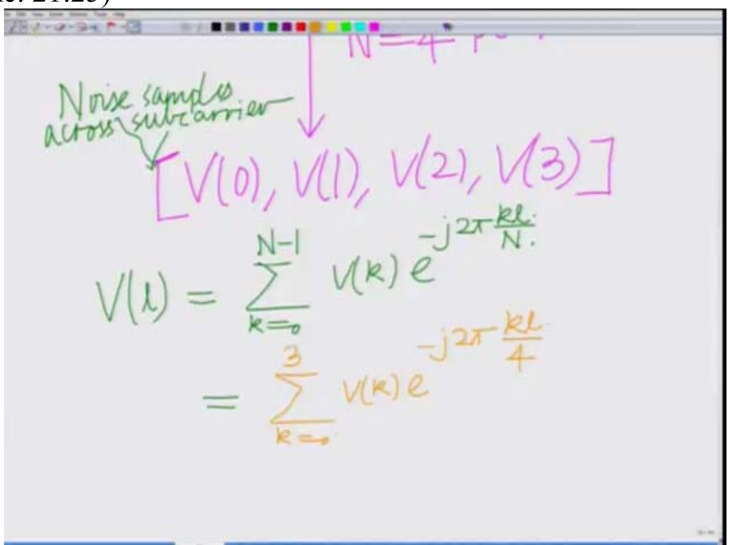

E raised to minus J two pi, KL divided by four which is equal to, so VL, your noise across subcarrier L, that is equal to summation K equal to zero to three, E raised to minus J two pi, or in fact E raised to minus J pi by two because this two and you have the factor of two, so you have pi by two KL and this is basically nothing but your, this is basically your noise sample.

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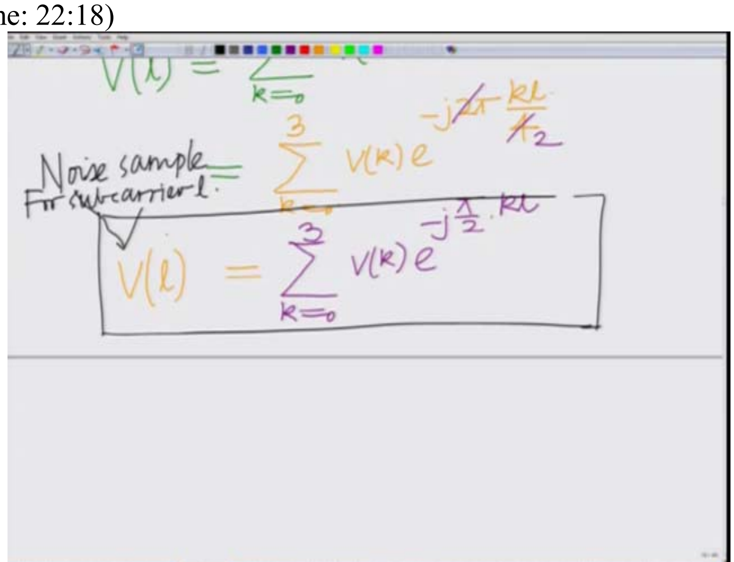

This is basically your noise sample for the subcarrier, for the subcarrier L, and therefore now you have the frequency domain, remember in the frequency domain, we have the FFT of the output Y is equal to FFT of the channel H times the FFT of the samples X which are basically nothing but the symbols loaded on the subcarriers plus the FFT, the noise sample across each subcarrier.

And therefore, now, re-writing it I have FFT of Y equals FFT of the channel filter H, product, remember this is important to remember in the frequency domain it is simply product because circular convolution becomes the product. Yeah, so just to repeat the importance of that in frequency domain is product of the corresponding frequency components across each carrier.

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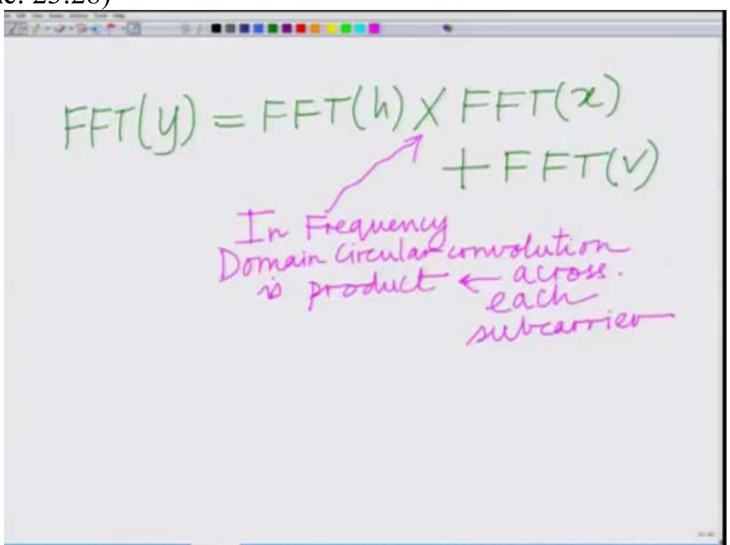

Basically product across each subcarrier, that is the important aspect, because each subcarrier represents a frequency component, and therefore across each subcarrier what do we have? Across for instance, across L subcarrier and we have, remember N equal to four subcarriers across L subcarrier we have YL equals HL times XL plus VL.

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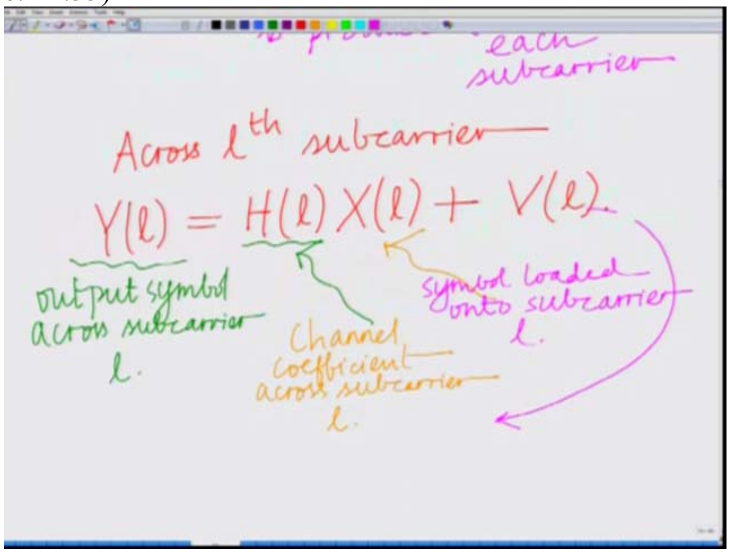

And this is the most important result, where, YL, output symbol, symbol across subcarrier L, HL is basically your, what is HL? It is the channel coefficient, across subcarrier, XL is the symbol loaded onto subcarrier L. And VS, VL, naturally VL not to forget VL, VL is the noise across subcarrier L. So VL is basically your noise, VL is basically the noise across subcarrier L.

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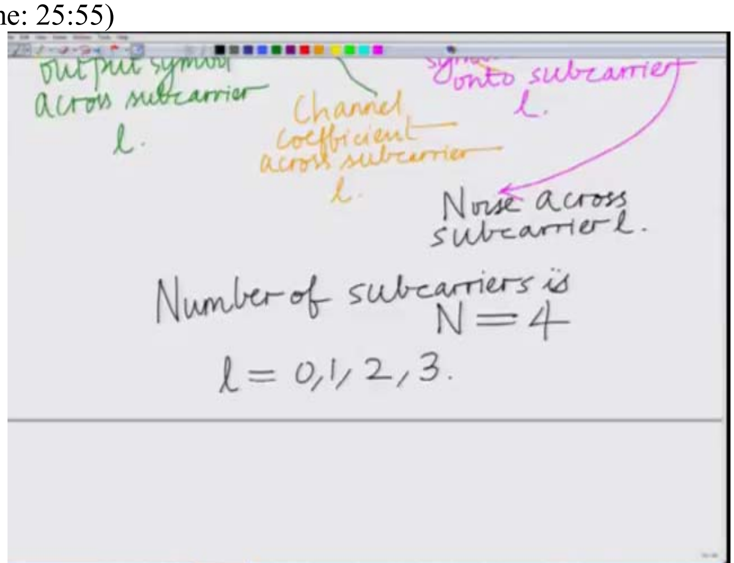

And therefore we said number of subcarriers is basically the same as, basically equal to capital L where N equal to four, alright? So number of subcarriers is N equal to four which means we have L equal to zero, one, two, three, corresponding to the L equal to four subcarriers and therefore for each subcarrier what we have (equ) (equal to) (zee) Y zero equals H zero times X zero plus V zero.

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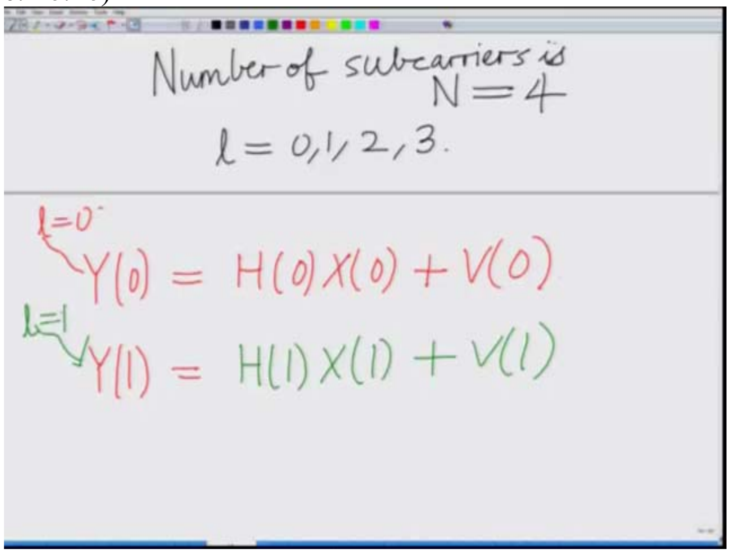

This is corresponding to subcarrier L equal to zero, Y one equals H one times X one plus V onethis corresponds to subcarrier L equal to one, then we have naturally again the same thing, Y two across subcarrier two Y two equals H two, X two plus V two, across subcarrier L equal to two and just to finish this I have Y three equals H three X three plus V three across subcarrier, L equal to three, okay?

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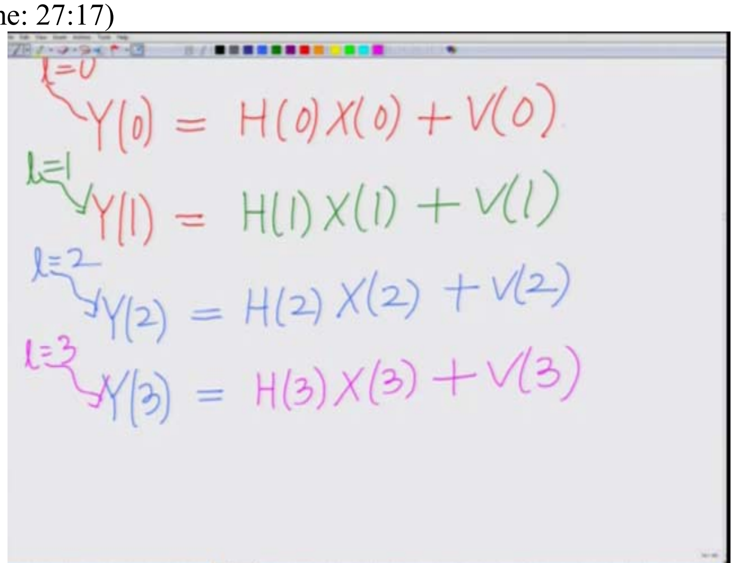

So basically now what we have done is we have written down explicitly relation corresponding to each subcarrier that is YL equals HL times XL plus VL. Now if you look at this system what you can observe is that for each subcarrier the output YL is simply the channel coefficient times the input symbol XL. There is no inter symbol, there is no inter symbolinterference from the previous symbol on each subcarrier.

Therefore now if you can look at this model, the incredible thing across this about this model YL equals HL XL plus VL. This is only current symbol that is current symbol loaded on to subcarrier L. Current symbol and this is in current output across subcarrier L. and therefore what we have, therefore there is no inter symbol interference from the previous subcarrier, no (inter) ISI or basically your inter, no inter symbol interference from previous symbol on each subcarrier.

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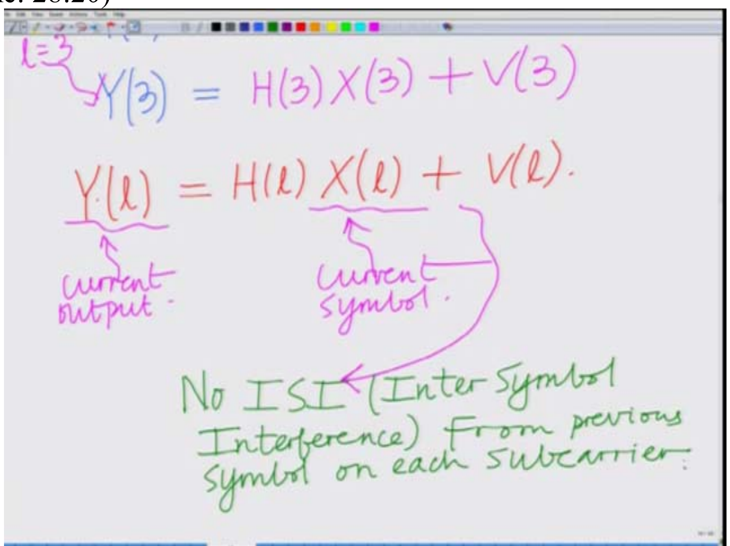

Remember in the time domain you still have the inter sample interference, the time domain, the inter sample interference is there, but the intelligently working in the frequency domain by adding the cyclic prefix and converting it back into the frequency domain at the receiver, you have eliminated or one has eliminated the inter symbol interference in the frequency domain across each subcarrier.

This is the important aspect of OFDM, that is it eliminates the inter symbol interference in the frequency domain. And what is efficient about this? This is efficient because it is based on IFFT and FFT, so IFFT at the transmitter and FFT at the receiver and since, IFFT and FFT, that is the (fa) Inverse Fast FourierTransform and Fast FourierTransform can be performed in a very fast fashion.

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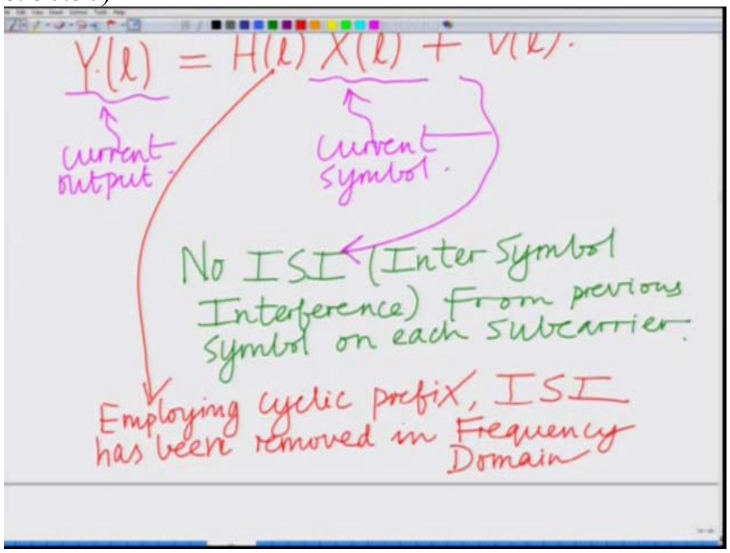

That is, these are efficient algorithms, as a result the entire OFDM architecture, the entire OFDM system; transmission scheme is very efficient since it does not employ any matrix inversion anywhere. It is simply based on IFFT and FFT which are fast operations, so basically to summarize employing the cyclic prefix it has been converted into a, employing cyclic prefix ISI has been removed, has been removed in the, ISI has been removed in the frequency domain.

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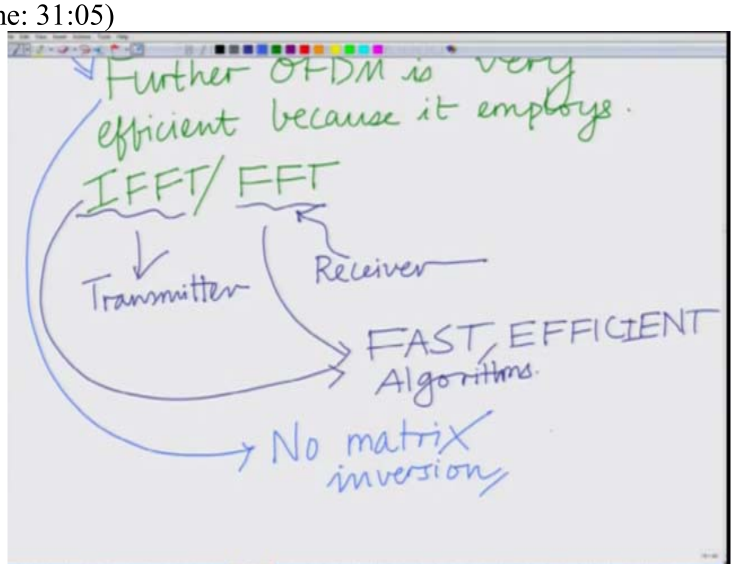

Further this is a very efficient, OFDM is very efficient, to highlight that point further OFDM is very efficient, OFDM employs IFFT slash FFT that is, IFFT at the transmitter and FFT at the receiver which are, and both of these algorithms are very fast and efficient, which are very fast and efficient algorithms and to note that there is no matrix inversion, though inversion which is a very, it just adds to the complexity.

There is no matrix, there is no matrix inversion in OFDM and so basically that is the, so basically if you look at this equation, this equation here summarizes OFDM, which is basically, it is no ISI across each subcarrier, each subcarrier is ISI free. There is no ISI across the subcarriereach subcarrier is ISI free and basically which are very efficient algorithms in such based only on IFFT and FFT which can be done in a very fast and efficient manner.

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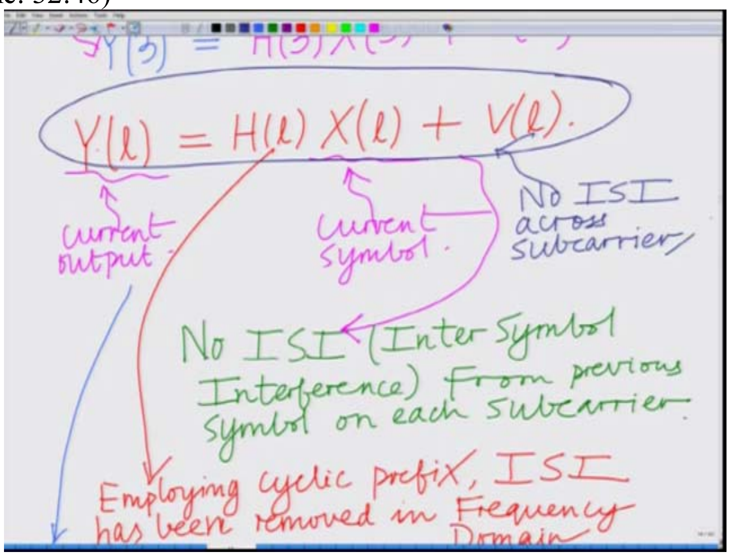

So basically what we have done in this module is basically we have completed description of the OFDM system model which is based on loading the symbols under the subcarriers that is basically performing the FFT, IFFT operation in the transmitter followed by addition of the cyclic prefix that results in a circular convolution at the output of the channel.

When you take the FFT of the output, basically what you get is basically in the time domain, in the frequency domain it is basically it can be represented across each subcarrier has the FFT of the channel times the FFT product times the FFT of the samples which is basically nothing but the symbols plus the FFT of the noise. Therefore across each subcarrier you have YL equals the channel co-efficient HL times the symbol XL loaded on to the subcarrier plus the noise VL.

And therefore this transmission across each subcarrier is inter symbol interference free and this is done very efficiently by use of the IFFT and FFT algorithms which makes OFDM an overall very efficient scheme for transmission in modern wireless communication systems. In fact as we talked about in the previous module, several four G standards basically such as LT and modern Wi-Fi standards such as A two dot eleven N and A two dot eleven AC are all based on OFT.Alright?

So we will stop here in this module and in the next module subsequent modules we will look at the estimation aspect of OFDM, that is how do you estimate these channel coefficients in an OFDMsystem, Alright? So thank you, we will conclude this module here, Thank you very much.