Estimation for Wireless Communications – MIMO/OFDM Cellular and Sensor Networks

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Lecture Number 26

Introduction to Orthogonal Frequency Division Multiplexing (OFDM) - Cyclic Prefix (CP) and Circular Convolution

Hello! Welcome to another module in this massive open online course on estimation for wireless communication. In the previous modules we have looked at equalization and equalization to overcome the inter symbol interference of a wireless channel and today we are going to look at a very different technology, a very modern technology in fact to overcome the same problem that is the inter symbol interference in a wireless channel very efficiently.

And this technology is OFDM or Orthogonal Frequency Division Multiplexing so from today, In today's module we will start looking at OFDM. So we are going to start looking at estimations specifically for OFDM that is where, that is our aim, starting this module that is estimation for OFDM where OFDM is an abbreviation. It stands for Orthogonal Frequency Orthogonal, where OFDM stands for Orthogonal Frequency Division Multiplexing and in fact OFDM is a very modern technology and OFDM is a very important technology.

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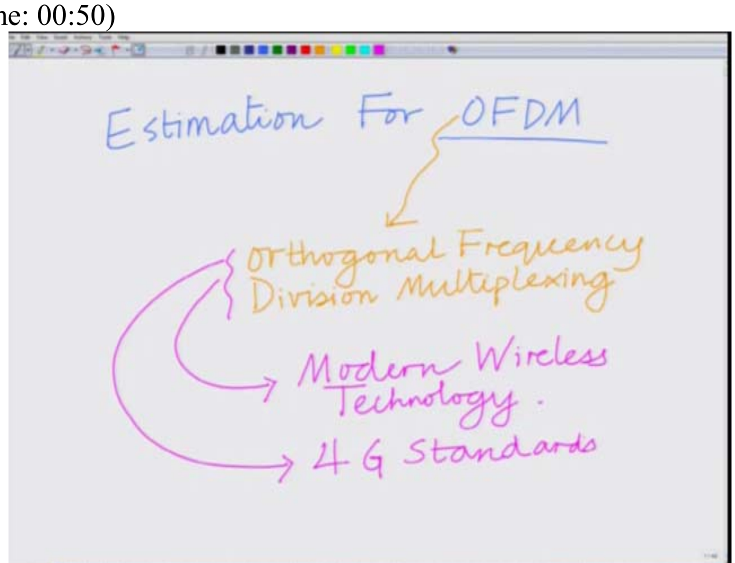

As you might already be familiar OFDM is used in the four G wireless cellular standards such as LTE, WiMAX and it is also used in the modern wireless, Wi-Fi standards such as A two dot eleven N, A two dot eleven AC and so on. So OFDM this is modern very modern wireless technology or very one can say it's the latest wireless technology and this is used in four G standards such as the fourth generation cellular standards.

Such as for example it is used in your LTE which stands for Long Term Evolution (()) (02:36) It's used in the WiMAX standard and it is also used in some other, is also used in other standards such as Wi-Fi for instance eight O two dot eleven N, eight O two dot eleven AC, and so, so, OFDM is a very powerful and a very important technology because its wide applicability in the modern four G wireless standards as well as the Wi-Fi standards.

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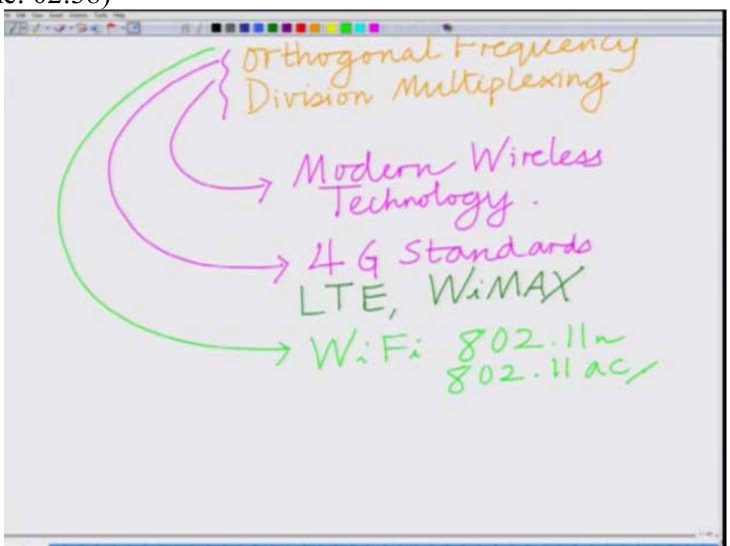

So (understood) to understand estimation for OFDM, we have to first understand what OFDM is about or what Orthogonal Frequency Division Multiplexing is about. So to understand that let us again start with our ISI or inter symbol interference remitted channel and as we already seen many times before, we described it previously.

The ISI channel has been described, it has been described as your YK that is received symbol at time K equals H zero XK plus H one XK minus one plus VK; that is, YK this is the received symbol at time K; XK is transmitted symbol at K; this is the transmitted XK is the transmitted symbol at time K and XK minus one is the previous symbol, this is the transmitted symbol, this is the transmitted symbol at time K minus one.

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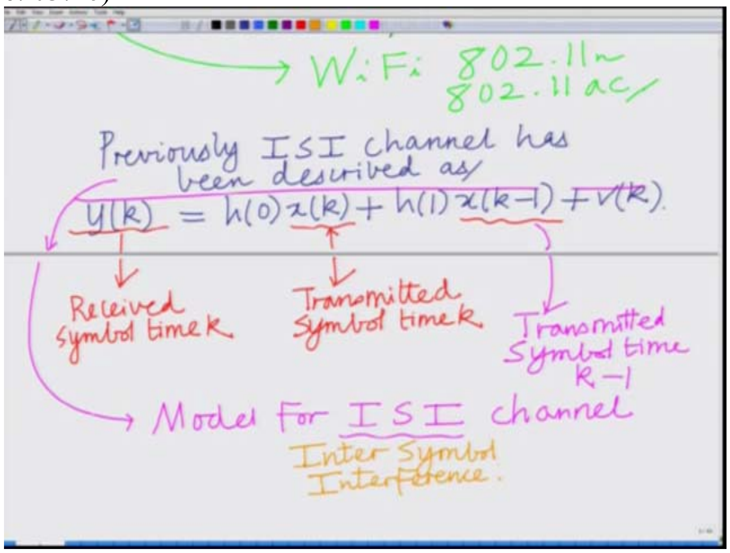

Therefore what we have is that the transmitted symbol at time K minus one, which is XK minus one, is interfering with XK which is the transmitted symbol at time K and this is what we term as inter symbol interference, alright? So what we said is basically inter symbol interference is caused by the fact that XK minus one which is the transmitted symbol at time K minus one is interfering with XK which is the present symbol, so the past symbol is interfering with the present symbol.

This is inter symbol interference, so this is the model for your, Inter symbol this is the model for your ISI channel where ISI stands for, and this is also something that we have seen before, where ISI stands for Inter, Inter symbol, where ISI stands for Inter Symbol Interference. Okay? And previously we have seen that OFDM is a (techno) previously we have seen that equalization is a technology to overcome inter symbol interference, alright?

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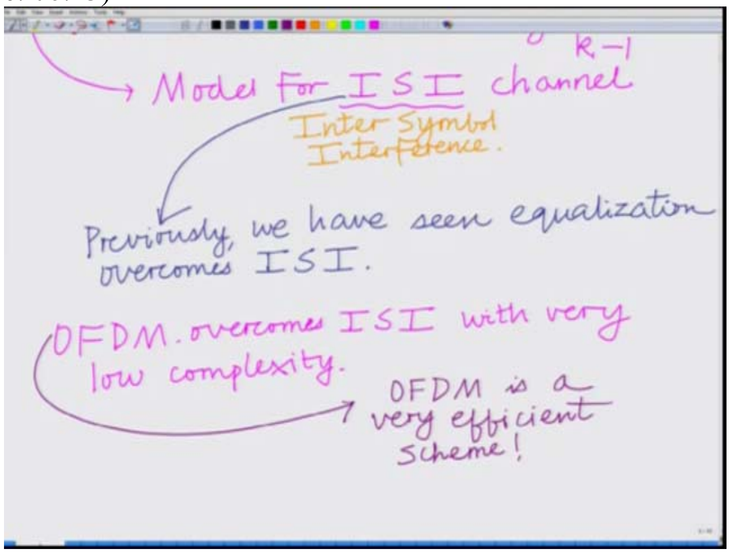

And now what we are saying is that OFDM is another technology which can overcome this inter symbol interference and very efficiently. So we are going to see that. So previously, previously we have seen equalization, overcomes ISI and OFDM is another technology to overcome ISI even more efficiently, more efficiently meaning? Lower computational complexity so that it is easier to implement.

OFDM overcomes ISI, OFDM has many advantages, one is that it overcomes ISI with very low with very low complexity and that is the advantage of OFDM or Orthogonal Frequency Division Multiplexing that its a very low complexity or very efficient scheme to overcome the inter symbol interference in the wireless channel, okay? So this is very low complexity (O) OFDM is a very efficient scheme.

OFDM is a very efficient scheme and that is what has made OFDM such a cutting edge wireless technology for employment in the various four G wireless standards such as LTE, WiMAX and also the Wi-Fi standards that we have talked about.

And now what we are going to do, we are (slo) we are going to first, I'm going to describe the model, the system model, that is how describe how this OFDM based wireless communication works and then we are going to look at how to perform estimation in the context of an OFDM system, Alright?

So first, we are going to look at what is this OFDM or what is an OFDM, that is, how does, how does it work? That is what is the transmission scheme for OFDM and that can be explained as follows. OFDM can be explained as follows.

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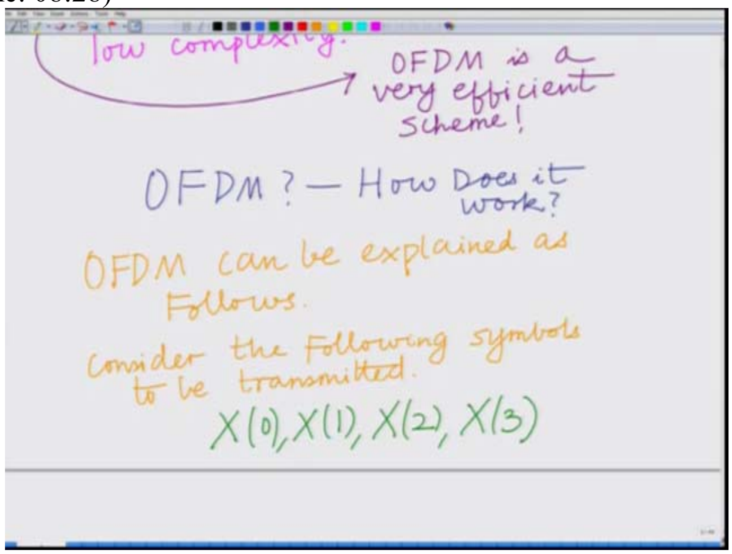

So consider the following symbol, so consider the following symbols to be transmitted in the OFDM symbol. Let us say we have four symbols which we are going to denote by X zero, X one, X two, X three, So these are the four symbols which are to be transmitted so we have capital N equal to four symbols and this is also termed as the number of subcarriers.

Capital N equal to four, subcarriers, so we are looking at, so, this capital N is an important parameter in this OFDM system. We are saying going to use, capital N equal to four symbols. These are denoted by X zero, capital X zero, capital X one, capital X two, capital X three. It is important to note that we are representing them by capital X, because we will use small X to represent something else.

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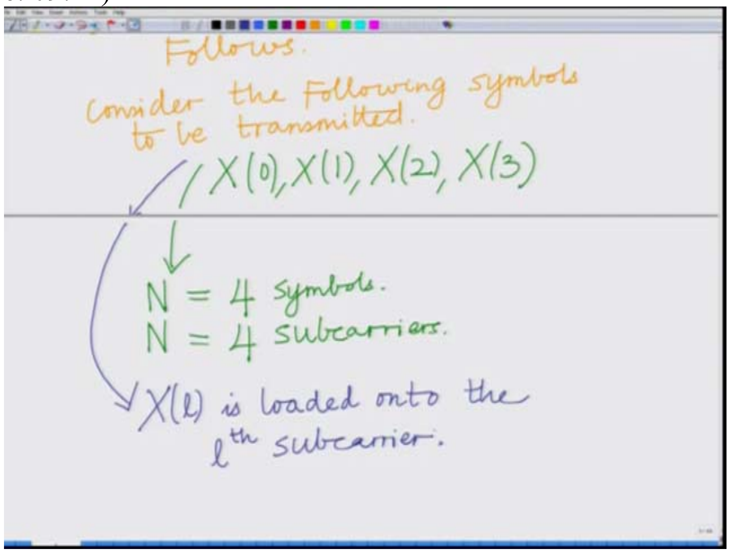

So we have N equal to four, capital N equal to four symbols, correct? And also this is known as capital N equal to four subcarriers, we say that these symbols are loaded on to the capital N subcarriers that is the capital N symbols that is the N symbols are loaded into the, on to the N subcarriers that is capital X zero is loaded on to subcarrier zero, capital X one is loaded on to subcarrier one and so on, capital X four is loaded on to subcarrier four.

So this, let us clarify, this also capital XL is loaded, the terminology used is loaded on to the L^{th}, this is loaded on to the L^{th} subcarrier. There are four symbols; they are loaded on to the four subcarriers, alright? And so in a general scenario you have capital N symbols and they are loaded on to the capital N subcarriers.

Now what does it mean to say they are loaded on to the subcarriers and this is what it means to say they are loaded on to the subcarriers, so we have the symbols X zero, X one, X two, X three. Now to these symbols what we do is basically what we are going to do with this symbols these symbols are, we are going to perform the N point IDFT that is N point IDFT.

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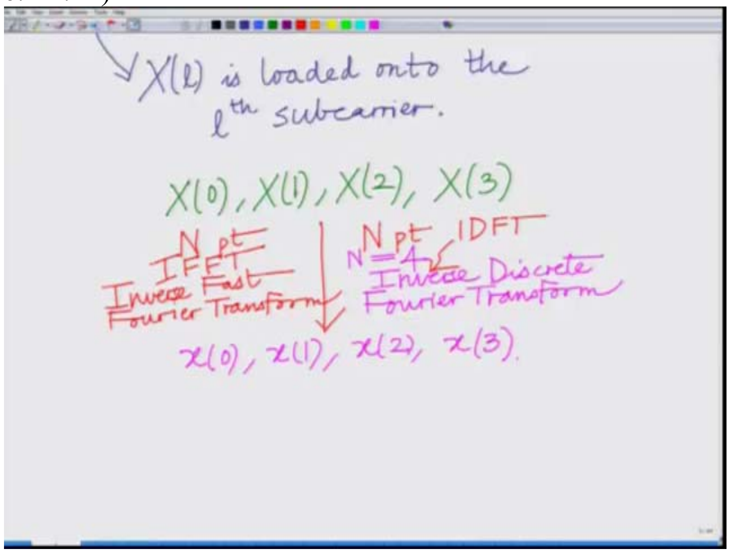

Which is, IDFT stands for as you all (ar) should be familiar with, this stands for the Inverse Discreet Fourier, IDFT stands for the Inverse Discreet Fourier Transform, to get the IDFT samples X zero, X one, X two, X three these are the small Xs, small X zero, small X one, small X two, small X three. So N equal to, N point IDFT or in this case basically you have N equal to four point IDFT. So you take the symbols capital X zero, capital X one, capital X two, capital X three yeah, alright?

And you load them onto the subcarriers which means basically (you form we) take the N point IDFT that is the Inverse Discreet Fourier Transform this can also be performed very efficiently very fast, in a very fast manner using the Inverse Fast Fourier Transform so typically you will also see, basically, it is the N point IDFT which is the same as the N point IFFT.

IFFT is simply a fast algorithm to perform the Discreet Fourier Transform, Inverse Fast, this performs the Inverse Fast Fourier Transform. Therefore now what you have is you have, capital X that is the X of K is the sample, is basically, corresponds to the Inverse Fast Fourier Transform as you know this is given as, one over N, summation L equal to zero to N minus one, capital XL E to the power of J two pi KL divided by N. This is the expression for IDFT.

All of you should be very familiar with this, which is basically now setting N equal to four, or N equal to four; This is basically given as XK equals one over four summation L equal to zero to N minus one which is equal to three X to the (raise) X L E raised to J two pi KL divided by N which is four, which is equal to one over four for this specific example please note that this is only for this specific example corresponding to N equal to four subcarriers.

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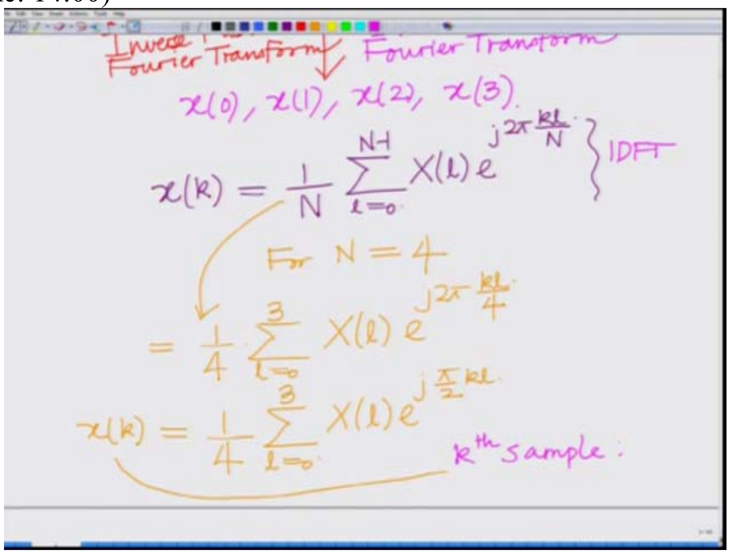

L equal to zero to three XL E raised to J pi y two KL, alright? So this is the expression for the, remember XK is the K^{th} sample. So small X denotes the sample, capital XL denotes the symbol loaded on to the L^{th} subcarrier, small XK is the K^{th} sample, which is generated by the K^{th} IFFT point of the capital Xs, capital X zero, capital X one up to capital X N minus one, Okay?

And now these samples are (tran) are transmitted over the channels subsequently but before transmission over the channel, there is another important operation to be performed and that is as follows so we have the samples X zero, X one, X two, X three and what we are going to do now, is basically, we are going to take the last sample X three and again place it before X zero.

So we are doing a prefix, so what are we essentially doing, is we’re prefixing the samples. So these are your samples and what is this? This is your, this is a very important thing, we are doing a prefix and we are cycling the sample from the end towards the beginning. So this is termed as a cyclic. This is a very important concept in OFDM this is termed as the cyclic prefix simply denoted by the term CP.

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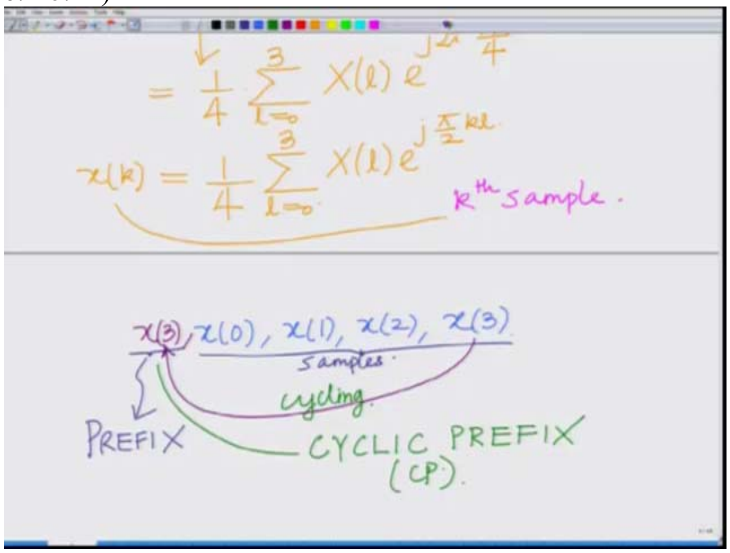

So what we are doing, is basically, we have these samples, small X zero, small X one, small X two, small X three now we are prefixing this by taking X three from the back, that is we are copying X three from the tail of the block and repeating it again at the head that is we're prefixing the block with X three, alright so it's a prefix and since we are cycling the sample from the end of the block towards the beginning, this is known as a Cyclic Prefix.

So we are simple copying some samples from the tail of the block and prefixing them at the head of the block, this mechanism or this step in OFDM transmission is known as a Cyclic Prefix, the addition of the cyclic prefix. So first you take the symbols which are loaded on to the subcarriers perform the IFFT. After the IFFT you add Cyclic Prefix, following the Cyclic Prefix the samples are transmitted over the channel.

So now after addition of the Cyclic Prefix, so we have, the samples, X zero, X one, X two. This is your block, after addition, this is your block after addition of the Cyclic Prefix and this is then transmitted, over the transmitted, up to the addition of the Cyclic Prefix. This is transmitted over the channel and remember our channel is basically the inter symbol interference channel, where YK equals H zero times X K plus H one times XK minus one plus VK, this is our channel.

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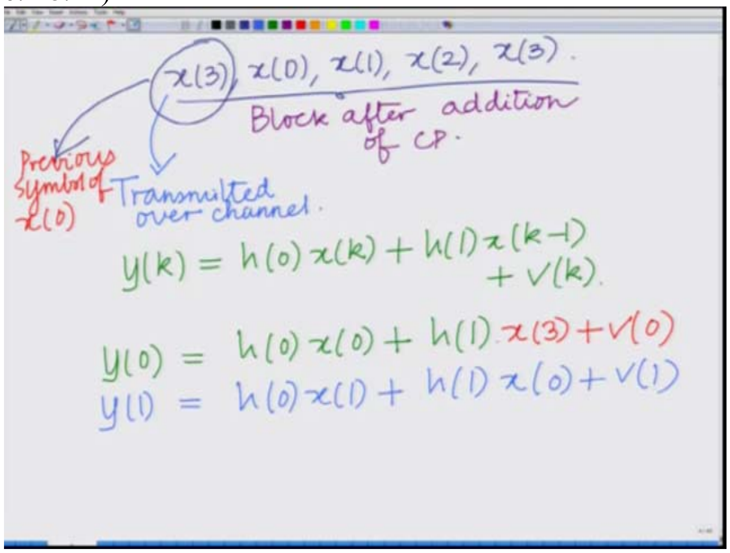

Now if you look at the symbol Y zero we have Y zero equals H zero X zero plus H one times the previous symbol, but the previous symbol you can see is X three, because of the addition of the cyclic prefix X three becomes previous symbol of X zero, so this is the previous symbol of X zero. So this H one into X zero plus V zero (and that is) I'm sorry, H one into X three plus V zero, because X three, because of the addition of the cyclic prefix X three has been moved.

That is copied from the tail of the block and prefixed at the head of the block because of the cyclic nature of the prefix, X three also is the previous symbol to X zero. Therefore you have Y zero equals H zero X zero plus H one times the previous symbol which is X three plus V zero. And of course now for the rest of the symbols we can write it as usual.

That is Y one equals H zero X one plus H one X zero which is the previous symbol of X one plus V one. Y two, equals H zero X two plus H one X one plus V two. Y three equals H zero X three plus H one X two which is the previous symbol to X three plus V three and this now gives us, this now basically gives us the received symbols across the, so Y zero, these are the received symbols, these are the received symbols across the, these are the received symbol (ac) symbols across the channel, alright?

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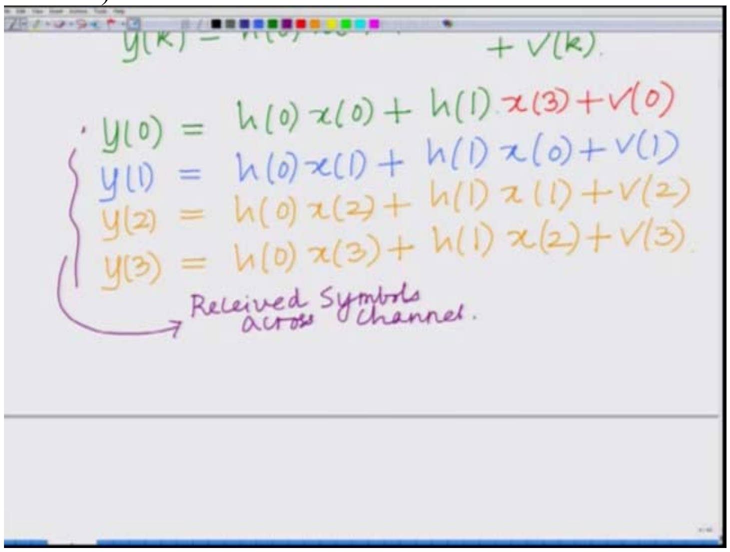

So we have the small Y zero, small Y one, small Y two, small Y three which are the received symbols. Now let us observe an important property of this small Y zero, small Y one, small Y two, small Y three which are the N received symbols. Now what I am going to do is I am going to illustrate with a figure, because with a figure it is best illustrated pictorially with the aid of a figure.

So let us use, I think we let us have here what I am going to draw is the time axis with X zero, let me draw it clearly. I have X zero, X one, X two, X three and let's say this is your X zero, let's say this is your X one, now you say this is your X two, and this is your X three. Now look at this, at time K equal to zero, that is for YK at time K equal to zero.

What do we have, at time K equal to zero if you can look at the expression above you have H zero times X zero plus H one times X three, so I can write it as H zero times X zero. Plus H one times X three plus zero times X one plus zero times (ex) So what I get is H zero times X zero plus H one times X three, that is the effect of the channel on the symbols.

Now let us look at time, let us again now look at time K equal to one, if time K equal to one I have H zero times X one plus H one times X zero, so this is H zero times X one plus H one times X zero plus zero times X two plus zero times X three. I can also include X two X three by simply putting zeroes, correct?

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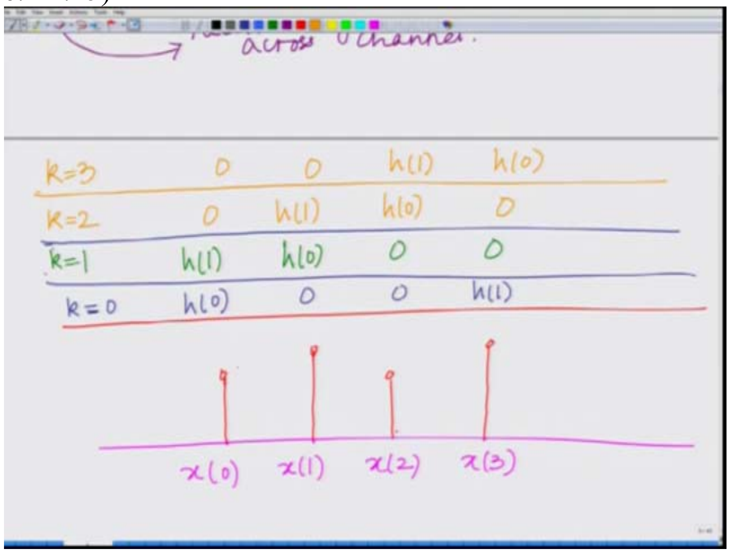

And at time K equal to two, now look at something interesting at time K equal to two, I have H zero times X two plus H one times X one So H zero times X two plus H one times X one and now if you can see at time K equal to three I have H zero times X three plus H one times X two plus zero times X one plus zero times X zero.

And now as you can see, as a result of this, and if you observe closely this figure what you can see is basically, I have the symbols X zero, X one, X two, X three and I have the channel tabs H zero, the channel filter H zero, H one, and you can see is that the channel filter is rotating over the transmitted symbol.

So you can see in the first step we have H zero H one, in the next step H one has circularly shifted to the left, so H one has circularly shifted, so if you can observe this, this is basically a circular shift, H one has circularly shifted to the left, H zero has moved to the right the next step, both H one and H zero have moved to the right, and then in the next step H one and H zero have moved to the, so this is basically a circular shift of the filter, channel filter over the symbols X zero and X one.

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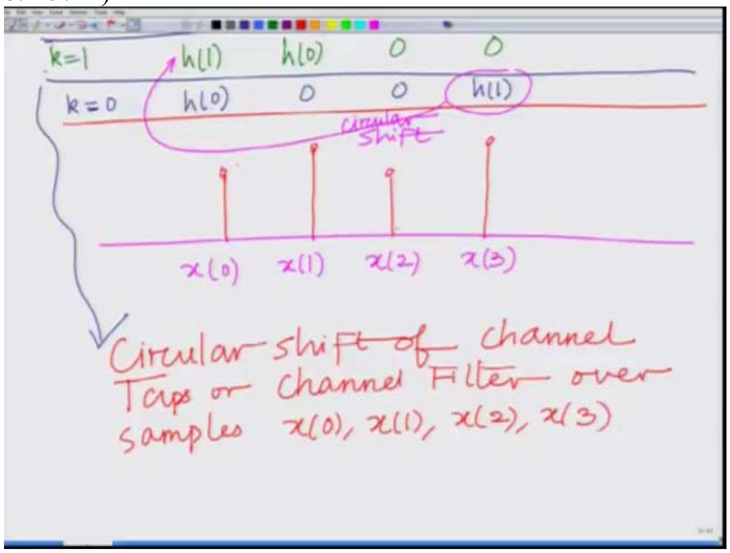

So if you look at this what this represents is something very important that is a circular shift of channel filter, or basically your channel taps or basically channel filter over the samples, over the samples X zero, X one, X two, X three and therefore the important point that it is a circular convolution. It represents a circular convolution so the output Y equals H circularly convolved with X plus of course there is the noise with V.

So this represents your Circular, this represents your circular convolution, so what we have is you can look at this is basically the action, because of the addition of the cyclic prefix, the action of the channel filter on the input symbol is such that basically at every point and time you're rotating the, that is your circularly shifting basically the channel filter, right?

So H zero is first to the left, H one is to the right and the next step, H one has circularly shifted to the left, H zero has shifted to the right, and then they are shifting, then (at with) every time instant they are shifting one step to the right and of course towards the end. There is the end circularly shifts back to the beginning and that is very basically the definition of a circular convolution.

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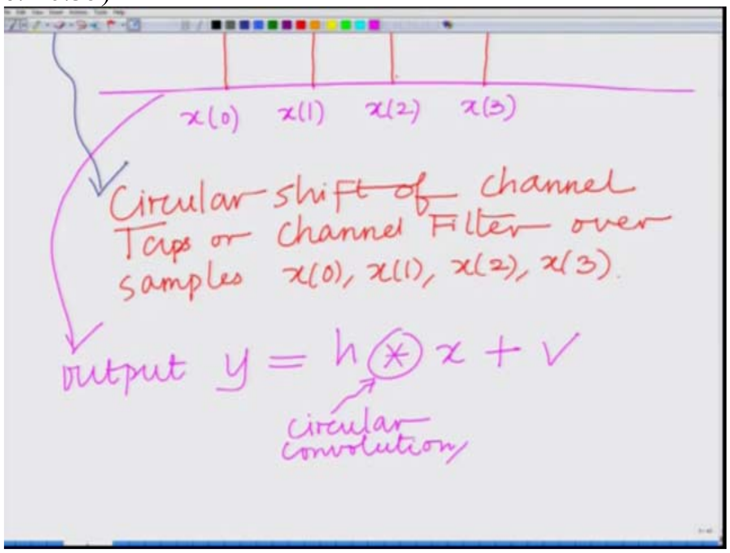

Therefore the action, because of the addition of the cyclic prefix, the action of the channel on the input symbols is such that it represents the circular convolution. And therefore the output Y is basically the circular convolution of the channel filter H with the samples X plus the noise. And this is the important point, what is this?

This is the circular convolution by addition of cyclic prefix we have converted the output of a channel into a circular convolution, so addition of, and this is also important to realize why is the cyclic (pre) why is the circular convolution arising? Addition of cyclic prefix has converted the linear channel or linear convolution, has converted a linear convolution and otherwise a linear convolution to a circular convolution and that is the important point.

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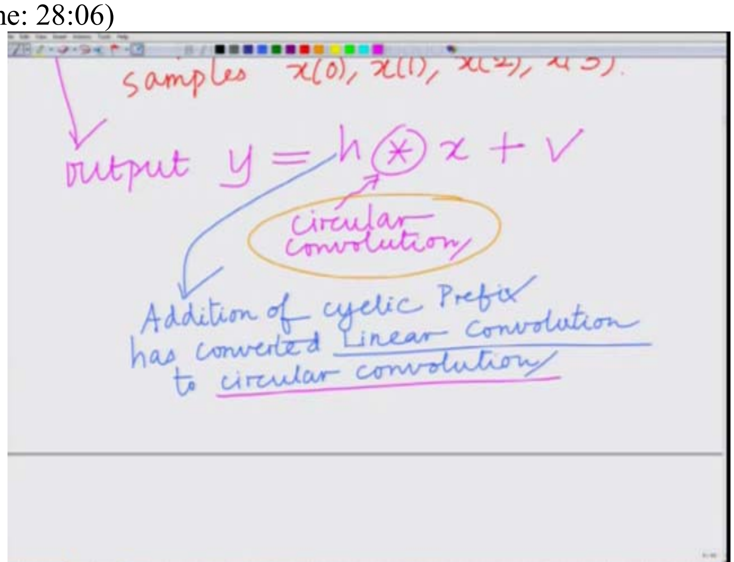

And therefore now we know from the properties of the circular convolution, that is if you take the FFT of H and X, that is circular (conva) then the FFT, the FFT domain it is simply the product of the FFTs of H and X so what we have is because we have circular convolution if we take the FFT of Y that is equal to FFT of H circularly convoluted with X plus V which is the noise.

Now the FFT operator is linear, so that is simple the FFT of H circularly convolved with H plus the FFT of V, which is, now if you look at this the FFT of a circular convolution is the product of the FFT so this is basically your FFT for H times the product remember the circular convolution in the FFT domain becomes the product plus the FFT of V, and this is a very important.

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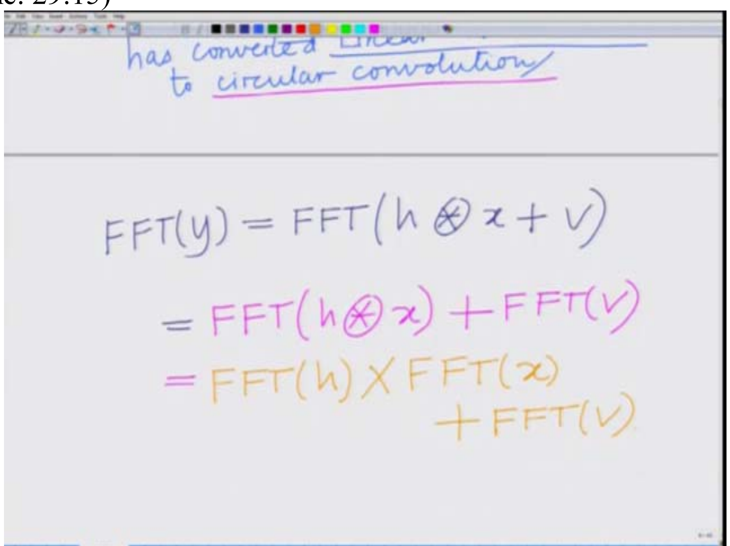

So the circular convolution has now been converted into, circular convolution becomes the product in the FFT, FFT domain or basically your frequency domain, alright? So basically that is the addition, that is the interesting aspect of OFDM. Where we have taken a block of symbols, the capital Xs, capital X zero, capital X one, capital X two, capital X three performed the IFFT or the IDFT to generate the samples, with the samples we have added the cyclic prefix.

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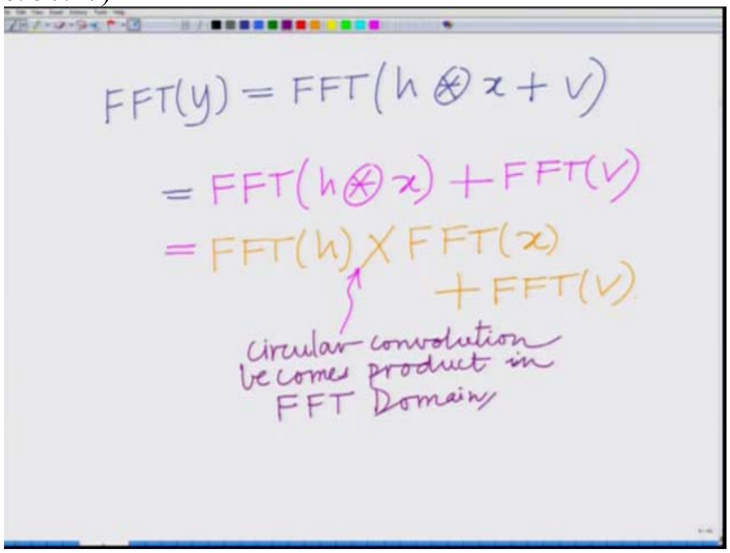

And now once you look at the output of the inter symbol interference channel, you see that the, that basically what the channel is performed is (eq) is (each) performing is equivalent to the circular convolution. Therefore if you take the FFT at the receiver, in the FFT domain channel is basically the product, the FFT of Y is basically equal to the FFT of the channel filter H times FFT of the samples, that is the small X plus of course the FFT of the noise that is V, alright?

And this is the important, this is the key equation in the OFDM is described so OFDM system model. So with this model we will stop here and we will continue to explore this aspect and implications of this aspect in the next module, alright? Thank you, thanks very much.