Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks

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Lecture Number 19

Multiple Input Multiple Output (MIMO) Channel Estimation - Least-Squares Maximum Likelihood (ML) Estimate

Hello, welcome to another module in this massive open online course on estimation for Wireless Communication systems. Today, we are going to look at channel estimation and specifically MIMO channel estimation, where MIMO stands for Multiple Input Multiple Output. So we are going to look at channel estimation of a Multiple Input Multiple Output or MIMO wireless communication system, right.

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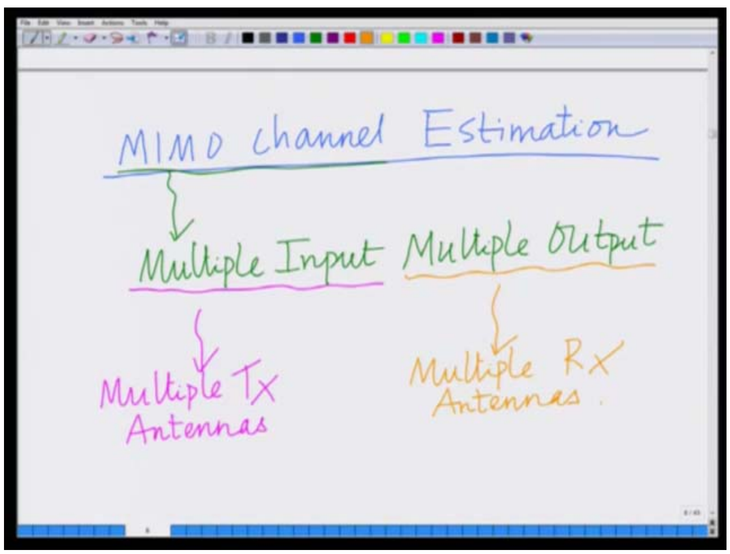

So today let us look at MIMO channel estimation, where MIMO as we have said, it stands for Multiple Input Multiple Output, where as we said MIMO, it stands for MIMO stands for Multiple Input Multiple Output, so MIMO stands for Multiple Input Multiple Output. And in particular, the multiple inputs that are referring, that we are referring to are the multiple transmit antennas.

So through multiple transmit antennas, the multiple input symbols are transmitted and simultaneously the multiple outputs that we are referring to correspond to the multiple received antenna and the multiple output symbols are basically received through the multiple output antennas.

So multiple input multiple output communication system basically means wireless communication system which has multiple transmit antennas at the transmitter and multiple receive antennas at the receiver, all right. So this multiple input is first to the multiple, this refers to your multiple transmit or T x antennas or the multiple output, this aspect refers to the multiple.

When we say multiple, we mean more than 1. Multiple receive, multiple transmit antennas and multiple receive antennas that makes a MIMO wireless system. For instance, let us consider a simple MIMO system. The 2 transmit antennas and 2 receive antennas, so I am drawing here a simple MIMO system, a schematic diagram of a simple MIMO system. This is my T x that is transmitter, this is the R x that is the receiver.

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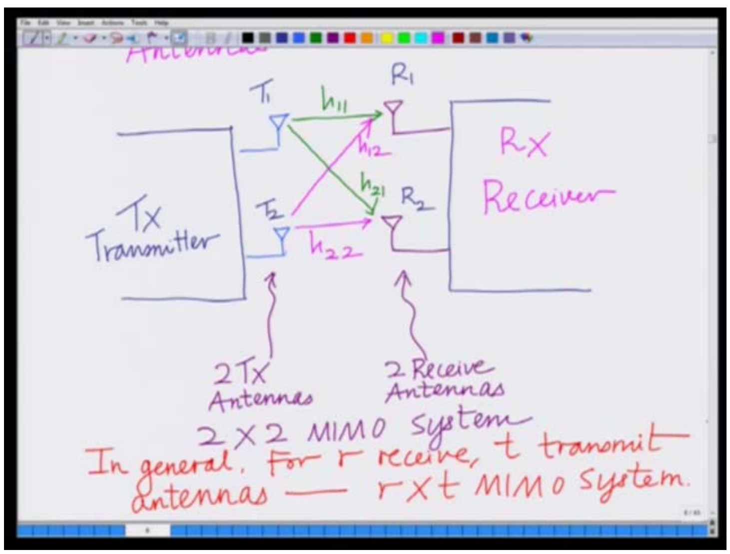

And this basically has let us say to consider a simple example, this basically has 2 transmit antennas and also 2 receive antennas. So we are considering the simple MIMO system, which has 2 transmit antennas and 2 receive antennas, this is known as a 2 cross 2 MIMO system. If we have R transmit antennas and R receive antennas, it is it is known as an R cross T MIMO system in general, right.

So this is a simple example of a 2 cross 2 MIMO system. A simple example or so, this is 2 transmit antennas and these are your 2 receive antennas and this is termed as a 2 cross 2 MIMO system in general for R receive, T transmit antennas we have an R cross T, we have an R cross T MIMO system. So here we are considering a 2 cross 2 MIMO system.

And now therefore, let us denote this transmit antennas by T 1 transmit antenna while, T 2 transmit antenna 2, R 1 receive antenna 1, R 2 receive antenna 2, all right. So let us denote the channel coefficient between transmit antenna 1 and receive antenna 1 by h 1 1, between transmit antenna 1 and receive antenna 2 by h 2 1.

Similarly, let us denote the coefficient channel fading channel coefficient between transmit antenna 2 and receive antenna 1 by h 1 2. And let us denote the coefficient between transmit antenna 2 and receive antenna 2 by h 2 2. So naturally we have 2 transmit antennas, 2 receive antennas, so basically we have 4 wireless links, right because between each pair of transmit and receive, that is between each transmit and receive antenna pair, we have a channel.

So naturally, since we have 2 transmit and 2 receive antennas, we have 4 basically links or basically which are characterized by 4 channel coefficients, that is 2 times 2 that is 4 channel coefficient in an R cross T systems of course you have R times T that is R into T channel coefficients.

And note that we are denoting this channel coefficient using the notation h i j, where h i j is basically the channel coefficient between the received antenna i and transparent antenna j. So this channel coefficient h i j, so h i j is the channel coefficient between the ith received antennas, so this is the channel coefficient or also the flat fading channel coefficient between your ith receive antenna and the jth transmit antenna.

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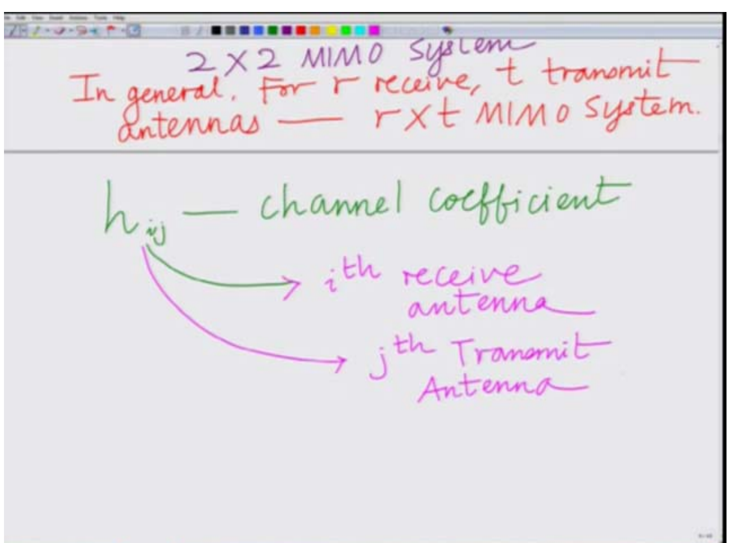

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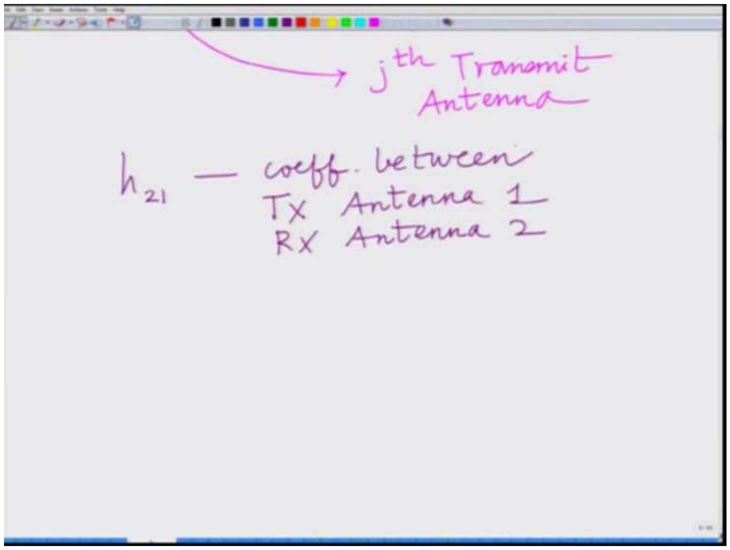

For example, if you take look at the channel coefficient h 2 1, h 2 1 is the channel coefficient between the second received antenna and the first transmit antenna that is h 2 1, okay. So if we take a simple example h 2 1, this is the coefficient between T x antenna 1 and R x antenna 2 and further now if we look at our MIMO system.

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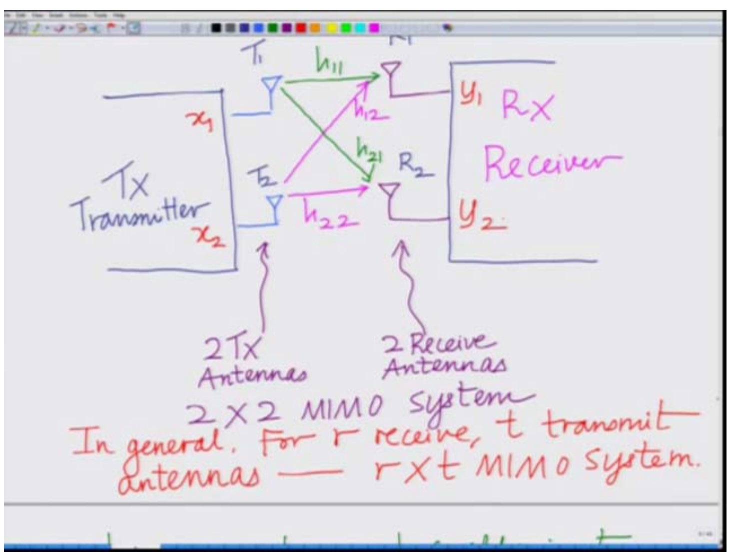

Let us consider the symbol x 1 transmitted from transmit antenna 1, x 2 transmitted from transmit antenna 2, y 1 receive received on received antenna 1, y 2 received on received antenna 2. So generally speaking, x j is basically the symbol or x j is the symbol that is transmitted on transmit antenna j and y i is the symbol that is received on receive antenna i, okay.

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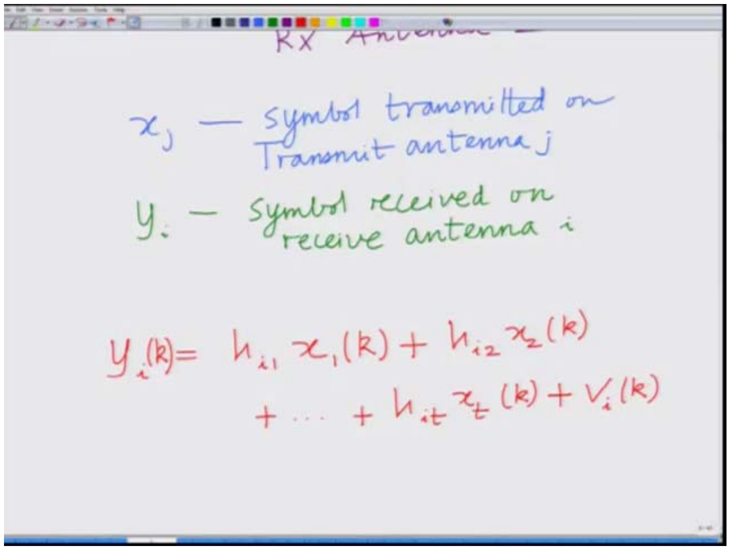

So x j is basically symbol transmitted on transmit antenna j and y i is the symbol received on on receive antenna on the receive antenna i. So y i is given as, now let us write the equation for y i, y i equals h i 1 times x 1, that is y i at time instant k equals h i j times x 1 k + h I 2 times x 2 k, that is symbol transmitted on transmit antenna 2 at time instant k + h i t since there are t transmit antennas h i t times x t k + v i of k.

So this is the symbol received on receive antenna i that is y i k is the symbol received on receive antenna i at time instant k is basically for instance you can see this is h i 1 times x 1 k that is the coefficient between receive antenna i and transmit antenna 1 times x 1 k that is, x 1 k is the symbol transmitted on transmit antenna 1 + h i 2 that is coefficient between receive antenna i and transmit antenna 2 times x 2 k.

The symbol transmitted on transmit antenna 2 so on and so forth till h i t times x t k where x t k is the symbol transmitted on transmit antenna t at time k + v i of k that is the noise sample at receive antenna i at time instant k. For instance, now let us look at this in the context of a 2 cross 2 system specifically for a 2 cross 2 system, so specifically we have y 1 of k on receive antenna 1 is h 1 1 times x 1 k + h 1 2 times x 2 k + v 1 k.

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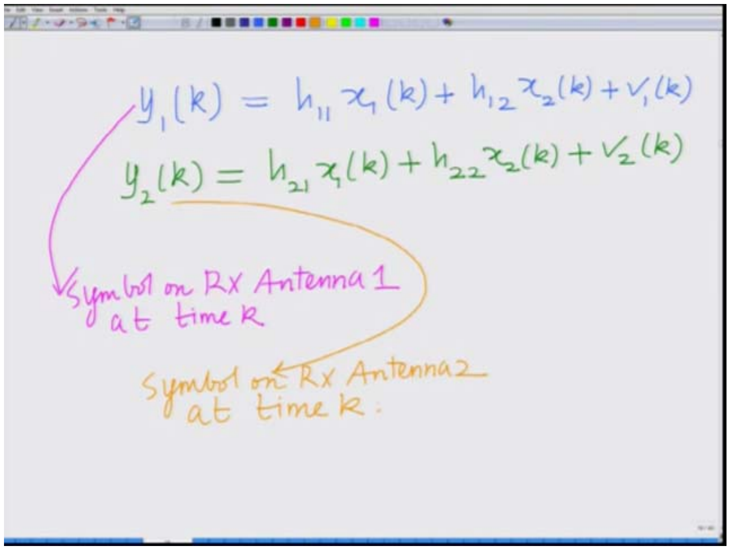

Similarly, y 2 k equals h 2 1 x 1 k + h 2 2 x 2 k + v 2 of k. Now what are these; basically your y 1 of k is symbol on receive antenna 1 at time k and your y 2 k is basically symbol on receive antenna 2 at time k.

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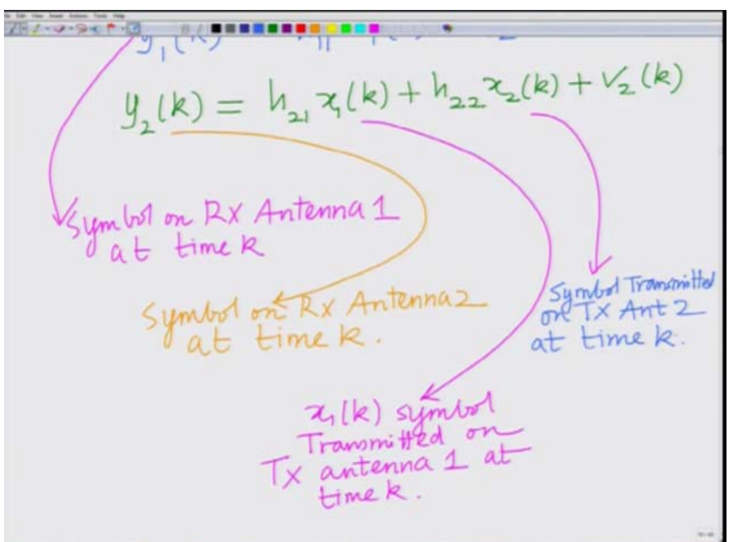

Similarly, x 1 k this x 1 k again just to repeat it, x 1 k symbol transmitted on transmit antenna 1 at time k and x 2 k is basically symbol transmitted on transmit antenna 2 on T x antenna 2 at time k. And similarly, your v 1 k and v 2 k, basically if you look at v 1 k and v 2 k, there is the noise samples on receive antennas 1 and 2 at time instant k, okay. So v 1 k, what are these?

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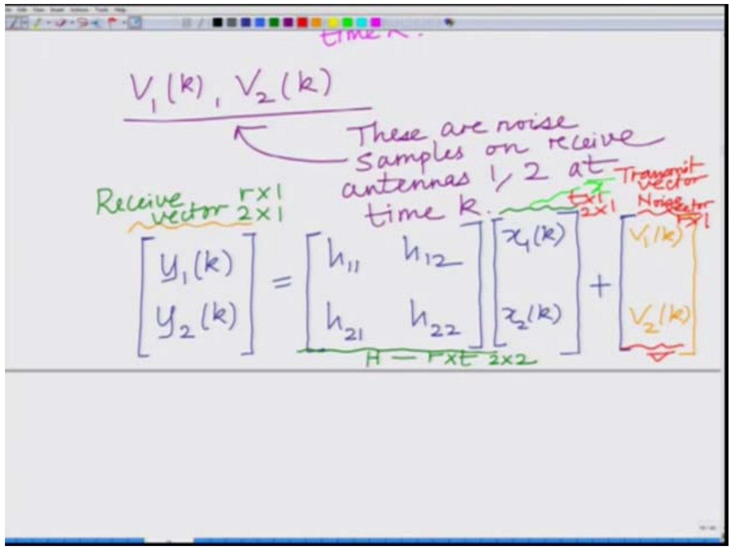

These are noise samples on receive antennas 1 comma 2 at time k. These are noise on receive antennas 1 comma 2 at time k.

And therefore now what I am going to do, I am going to write this 2 cross 2 that is this input output model for this 2 cross MIMO system using vector notation. And I can write this as follows using vector notation I can write this as basically your y 1 k equals h 1 1, h 1 2, h 2 1, h 2 2 times x 1 k, x 2 k + the noise vector v 1 k v 2 k, where y 1, y 2 this is the received vector which is R cross 1 or in this case since R equals 2 basically 2 cross 1.

This is your channel matrix, which is R cross T, in this case 2 cross 2, this is your transmit vector which is x bar, which is your transmit vector which is T cross 1 in this case 2 cross 1 and this is our noise vector v bar, this is the noise vector v bar, which is basically R cross 1. So if I am just write it a little bit more clearly below. So this is Y bar of k which is your received vector equals h.

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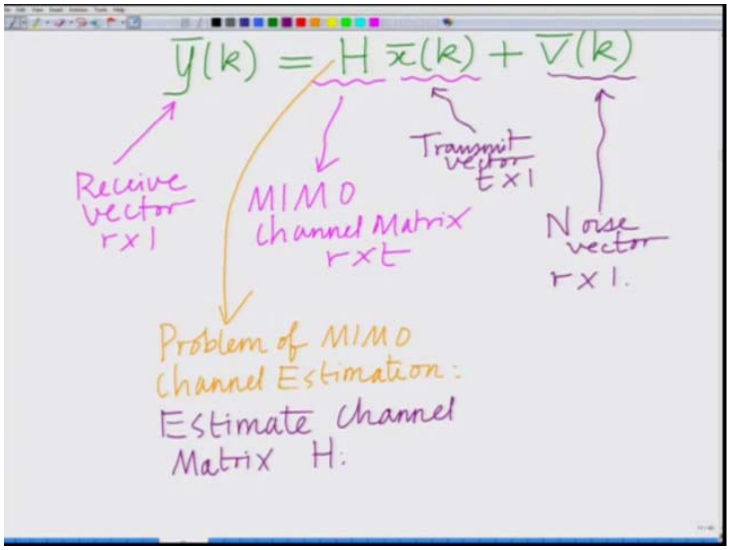

The channel matrix the MIMO channel matrix H times x bar k + V bar k, so Y bar k is this is your receive vector, which is R cross 1, this is the MIMO channel matrix, which is R cross T, this is the transmit vector which is T cross 1 and this is the noise vector which is basically R cross, V bar is the noise vector at time instant k, which is R cross 1.

So we have the MIMO system model which is Y bar k that is the received vector, received symbol vector at time instant k is equal to h that is the R cross T MIMO channel matrix times x bar k where x bar k is the transmitted symbol vector at time instant k + V bar k, which is the noise vector at time instant k, okay.

This corresponds to a single time instant that is the transmission of the symbol vector or in our case of estimation that is the pilot symbol vector x bar of k and the reception of the corresponding receive vector Y bar of k. Now since we have T transmit antennas, we are transmitting T symbols, therefore x bar of k has the T transmit symbols therefore; it is a T cross 1 vector or a T dimensional 1 vector.

And since we have R receive antennas basically, we are receiving R output symbols, right. Therefore, the output vector Y bar of k is R dimensional; it has the R output symbols, all right. And this corresponds to the off and R cross T MIMO system that is Multiple Input Multiple Output since we have multiple inputs from the T transmit antennas, multiple outputs from the R receive antennas.

This is the model of the R cross T MIMO system, which is captured basically by this R cross T MIMO channel matrix will consist of the channel coefficients between each pair of transmit and receive antenna, correct. And now what we want to do in MIMO channel estimation, we want to estimate this MIMO channel matrix. Therefore, the problem of MIMO channel estimation, now if I can motivate it better, is to estimate this channel matrix.

Problem of MIMO channel estimation is to basically estimate the channel matrix H. And similar to channel estimation that we have seen earlier, we will consider transmission of pilot symbol. However, we are now considering a MIMO channel therefore; we have to transmit pilot symbol vectors, right. At each instant we are transmitting a T dimensional vector; we are receiving an R dimensional vector.

So for MIMO channel estimation, we will transmit T dimensional pilot vectors and we will receive corresponding R dimensional pilot output vectors that are the Y bars, correct. So let x 1 bar, so let us denote the pilot vectors, let us first let us consider the transmission of N pilot vectors. Consider transmission of N pilot vectors, these N pilot vectors, these are given as x bar of 1, x bar of 2 so on x bar of N.

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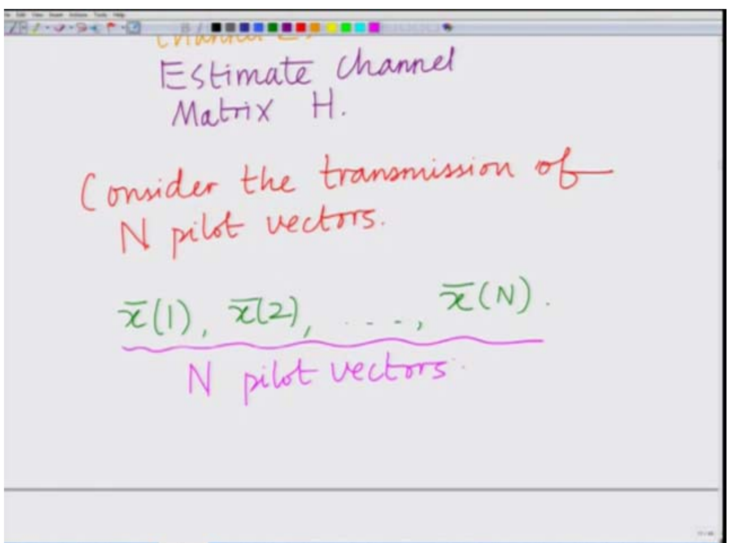

What are these? These are basically your N pilot vectors. And corresponding to these transmitted N pilot vectors, we are going to have N output vectors. Each pilot vector results in an output vector, or it. So corresponding to each of these pilot vectors we are going to have an observed vector y bar, all right. So naturally, let us now denote the system model by y bar of 1 equals h times x bar of 1 + v bar of 1.

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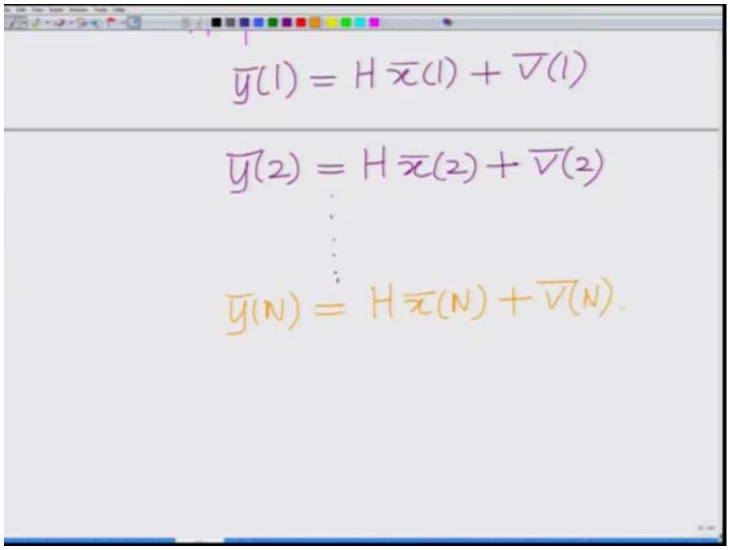

y bar of 2 equals H times x bar of 2 + v bas of 2 and so on so forth we have y bar of N equals H times x bar of N + v bar of N.

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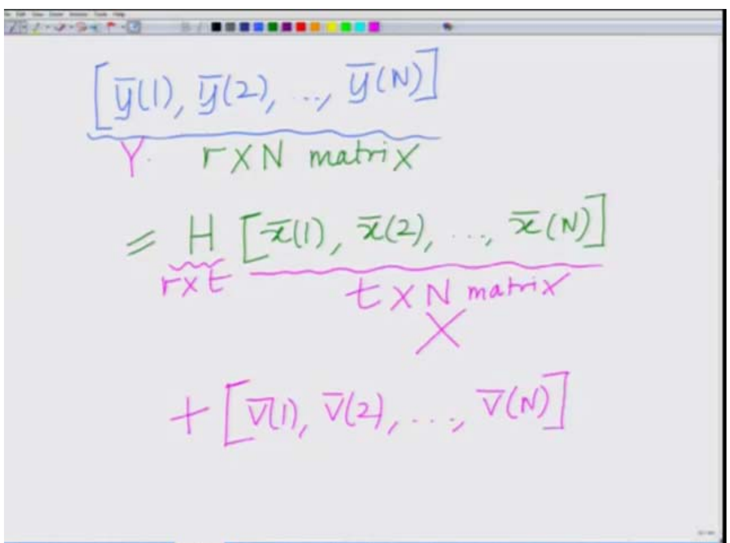

Therefore, now if I look at the output vectors y bar 1, y bar 2, if I stack I can stack these output vectors. What I am doing is basically I am stacking them basically row wise, right y bar of 1, y bar of 2, so on y bar N. So basically now what you can see is basically you will get each vector is R dimensional and have N such vectors.

So this is an R cross N matrix, correct. And this is basically equal to your H times correspondingly x bar of 1, x bar of 2, x bar of N and naturally what is this matrix; this is basically each vector is T dimensional, I have N such vectors, so this is a T cross N matrix. Let us denote this by X, this is your pilot matrix let us denote this by Y, this is the received vector matrix.

Of course, your H is your channel matrix which is still your R cross T matrix + of course I have noise matrix which is v bar 1, v bar 2, v bar N, this consists of R N vectors which are R dimensional, so R dimensional vectors N of them, so this is R cross N matrix again. And let us call this as V.

And therefore now after we concatenate this, that is we look at these N transmitted pilot vectors and we look at the corresponding N received output vectors y bar 1, y bar 2, y bar N corresponding to the transmitted pilot vectors x bar 1, x bar 2, x bar N. And now I can write the system model as Y equals H X + V, correct.

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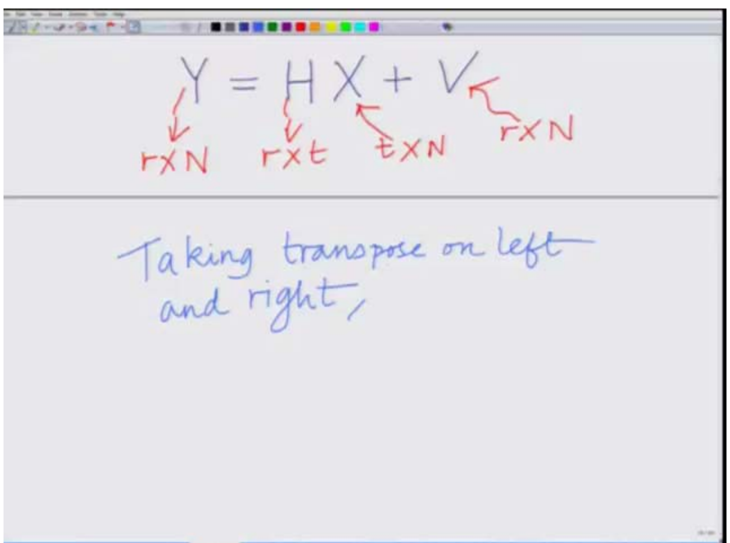

And therefore, so now as we have shown this our just to repeat this is R cross N, this is R cross T, this is T cross N consisting of the N pilot vectors and this is basically your R cross N matrix. Now this is similar to your Y, your Y equals remember the kind of system model that we had for multi antenna channel estimation except there we had Y equals X H, now here we are saying we have Y equals H X, this is most standard notation for MIMO.

So let us just recast it in a form that we are already familiar with, so I am going to take the transposed of each quantity. So I can write taking the transposed of taking the transposed on left and right.

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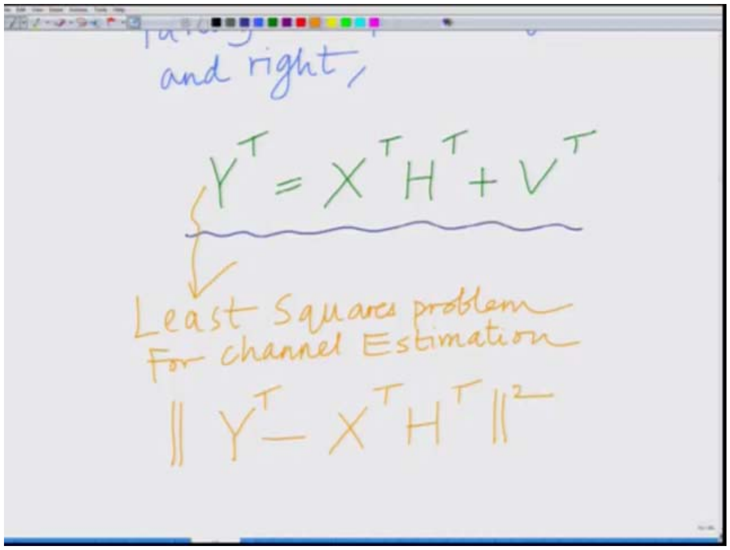

Taking the transpose on left and right what do we have; we have Y transpose equals X transpose H transpose + V transpose. And therefore, now you can see this is of the form Y equals H X.

And now I can formulate my least-squares problem for channel estimation similar to the multi antenna case. Least-squares problem for channel estimation that is given as Y transpose - X transpose H transpose whole square, this is my least squares problem for channel estimation.

And except here, there is a slight change, this is the matrix frobenius norm of the matrix because now we are dealing with matrixes, we have to consider the frobenius norm of the matrix, but do not worry too much basically this can be formulated as a least-squares problem. And the corresponding solution can now be given as the pseudo inverse of X transpose times Y.

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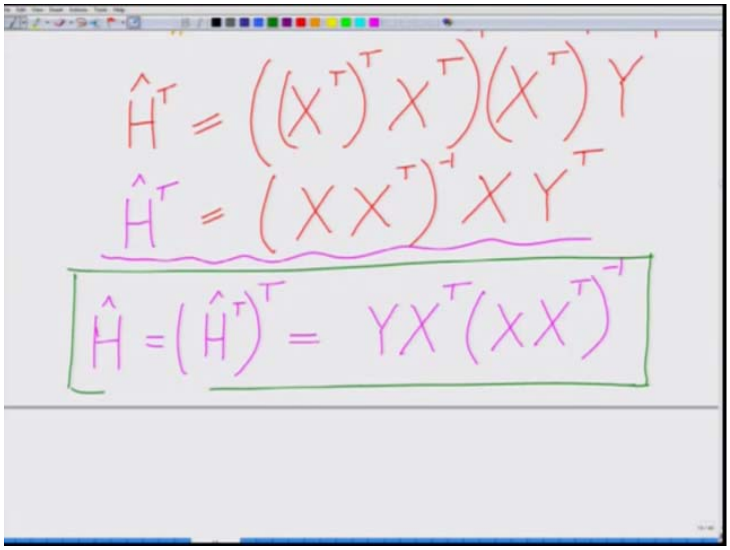

So the corresponding least-squares estimate of H transpose remember, we have formulated the problem in terms of the transposes is that is the pseudo inverse of X transpose, which is basically X transpose transpose times X transpose inverse, X transpose transpose times basically your Y transpose and that is equal to basically X X transpose inverse X times Y transpose that is the estimate of your transpose of the MIMO channel matrix.

Therefore, now taking the transpose of this, the estimate can be opted as H hat equals the transpose of the estimate, transpose which is basically Y X transpose into X X transpose inverse and that is basically your MIMO channel estimate. In fact, this is the least-squares MIMO channel estimate.

So what we have shown is basically when you concatenate this corresponding receiver vectors, received pilot vectors y bar 1, y bar 2, y bar n as matrix Y as the R cross N matrix Y. And the corresponding transmitted pilot vectors as the T cross N matrix x bar 1, x bar 2 up to x bar N, all right and call it the matrix X.

Then the least-squares estimate of the MIMO channel matrix based on minimising the least-squares norm of the least squares norm or in this case the least-squares Frobenius of the error that is given as Y times X transpose times X X transpose inverse, okay so that is the least squares estimate of the MIMO channel matrix. So let me repeat that, just write it down again.

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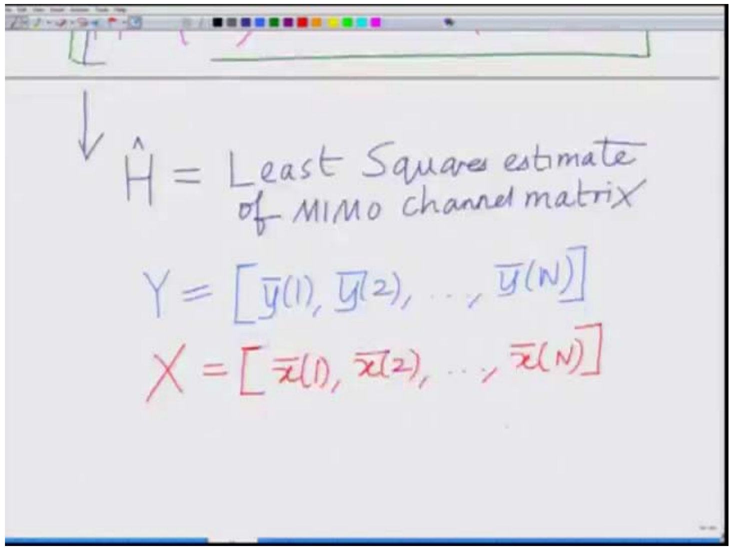

H hat equals least-squares estimate of least-squares estimate of the MIMO channel matrix, I have Y, which is equal to y bar 1, y bar 2 so on up to y bar N, this is an R cross N matrix, I have X which is basically the matrix of pilot vectors that is x bar 1, x bar 2 so on up to x bar N and H hat the least-squares estimate or basically also the maximum likelihood estimate is therefore equal to Y X transpose times X X transpose inverse.

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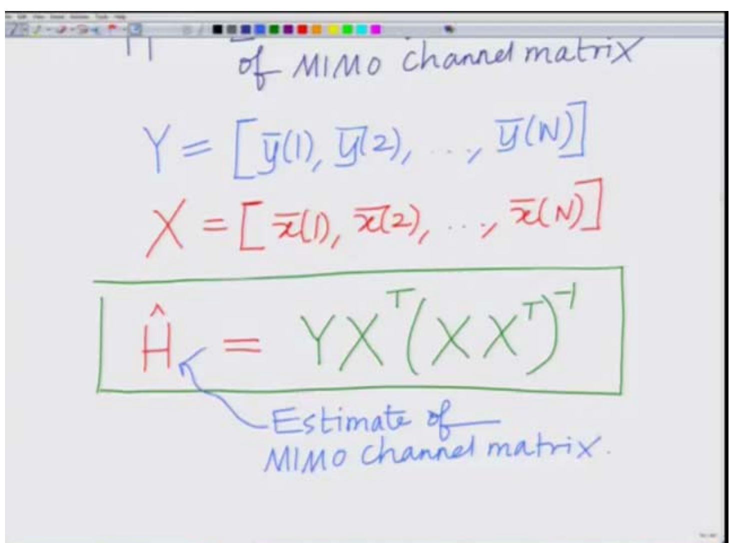

This is the estimate of the MIMO channel matrix. And so what we have done is, we have achieved something significant in this module. What we have started with if we have now considered a very general scenario of a multiple input multiple output wireless channel.

Remember, we first started basically with a single input signal output wireless channel, the very first example where we had a single transmit antenna.

And a single receive antenna, slowly extended we extended it to a multi antenna communication system that is we have multiple transmit antennas. Remember, we considered a downlink transmission scenario, where the base station has multiple antenna and the Mobile had a single receive antenna, we termed that is a multi antenna channel.

Now we are extended it to very general Multiple Input Multiple Output scenario where there are multiple inputs or multiple transmit antennas at the transmitter as well as the multiple outputs or multiple receive antennas at the receiver. We have formulated this system model for this MIMO Wireless system in terms of the matrix channel, in terms of the channel matrix H.

And now we have also given or derived a least the least-squares or maximum likelihood estimate of this MIMO channel matrix H based on the transmission of N pilot vectors x bar 1, x bar 2 up to x bar N and the corresponding receiver vectors y bar 1, y bar 2 up to y bar N and if the receiver matrixes denoted by Y, the transmitted pilot matrix is denoted by X. We have said that the MIMO.

We have derived that the MIMO channel matrix estimate H hat is given as Y X transpose times X X transpose inverse. And in the next module we will look at a simple example to basically solidify this concept sort of make this even more clearly, all right. So we will stop this module here, we will continue in the subsequent module, thank you.