Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks

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Lecture Number 18

Least Squares Multi-Antenna Downlink Maximum Likelihood Channel Estimation

Hello, welcome to another module in this massive open online course on estimation for wireless communication systems. So far we are looking at the estimation of vector parameter and we have formulated the Least Squares cost function and we have derived the maximum likelihood estimate of this vector parameter as a solution or as minima of this Least Squares cost function, all right.

And we have explored this vector parameter estimation in the context of a multi-antenna downlink Wireless Channel estimation scenario. Now let us look at the example of this multi-antenna channel estimation scenario to understand this better. So what we are going to do in this module is start looking at a simple example of a multi-antenna channel estimation problem, okay.

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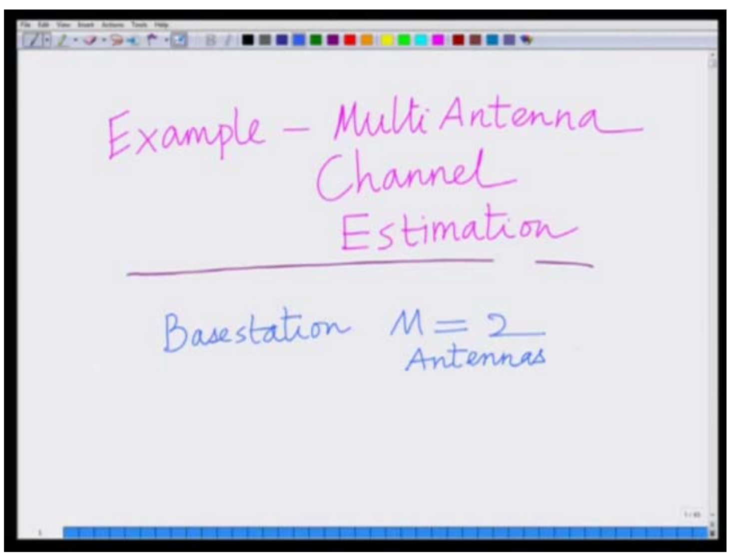

So we are going to start looking at an example of multi-antenna example of multi-antenna channel estimation, correct. So we want to consider a base station remember, we are considering a downlink scenario in which a base station is transmitting to a mobile, so the base station has M equal to 2 antenna, which means there are going to be 2 coefficients or unknown parameters.

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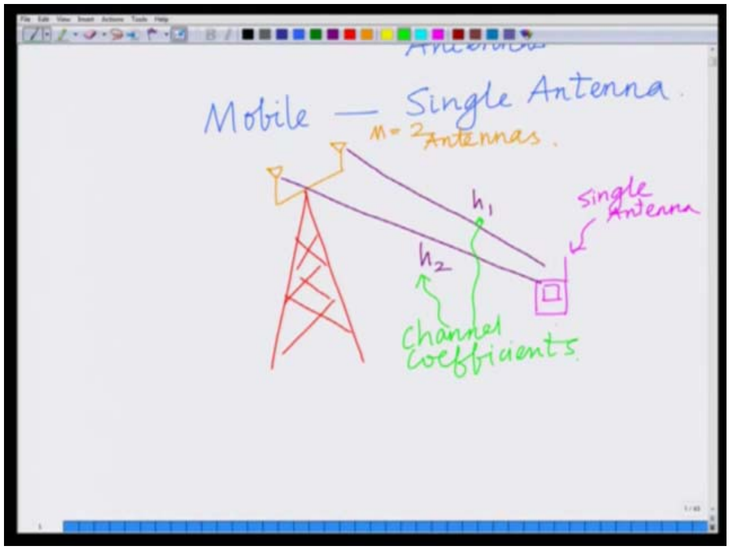

And the mobile has a single antenna, so therefore to draw it depicts it schematically if I have to draw a schematic diagram; it is going to look something like this. I have a base station over here and that has 2 antennas, so this is M equal to 2 antennas. And the mobile station mobile, your mobile has a single antenna, so let us say this is your mobile that has a single antenna.

And therefore naturally I can denote a channel coefficient corresponding to these 2 and enhance as one is h 1, h 2, this h 1 one h 2, these are your multi-antenna, these are basically the channel coefficients which are to be estimated and that is your problem of, so these are the channel coefficients h 1 and h 2 are basically the general coefficient which has to be estimated in this multi-antenna downlink scenario and it is the problem of channel estimation.

In fact, that is the problem of vector parameter or vector or channel vector estimation, all right that is what we are looking at. And the system model for this, again that is something what we have already seen, but let us give it here again so that you can recall it. I have the received symbol y k or the observed symbol y k equals x 1 k times h 1 + x 2 k times h 2 + v k, what is this? This is your is the observed symbol.

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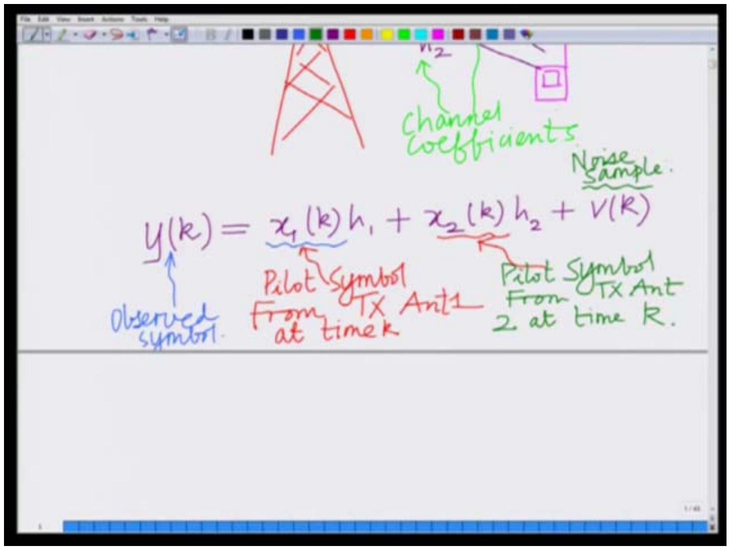

X 1 k is the pilot symbol transmitted from transmit antenna 1 at time instant k. So this is pilot symbol from T x and are now 1 at time k and x 2 similarly is pilot symbol from transmit antenna 2, so what is this? This is your pilot symbol from transmit antenna 2 at time instant at time instant k. So this is the model we are familiar with, so we have the system model y k which is the served pilot symbol at time instant k.

And the mobile equals x 1 k times h 1 + x 2 k times h 2 + v of k, x 1 k is the pilot symbol from transmit antenna 1 at time instant k, x 2 k is the pilot symbol from transmit antenna 2 at time instant k and v k is as usual that is the noise sample, okay. So this is basically again something that we have seen many times before. This is your noise sample or the noise at the receiver, okay.

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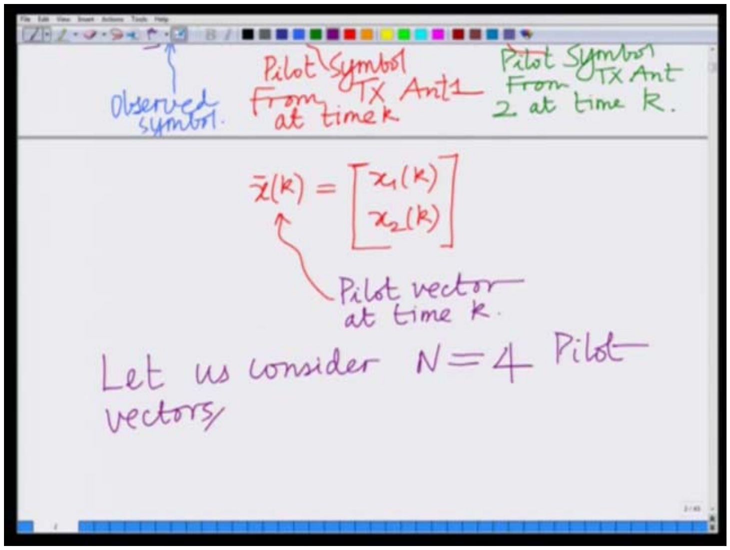

So now let us take a look at this, now our pilot vector going back to our system model, our pilot vector at time instant k, this is x 1 k, x 2 k, this is your pilot vector at time instant k, this is the pilot vector at time k. And now let us consider in this example let us consider N equal to 4, let us consider N equal to 4 pilot vectors. And these pilot vectors for instance can be given as x bar 1 equals 5 comma 4, x bar 2 equals 4 comma 5.

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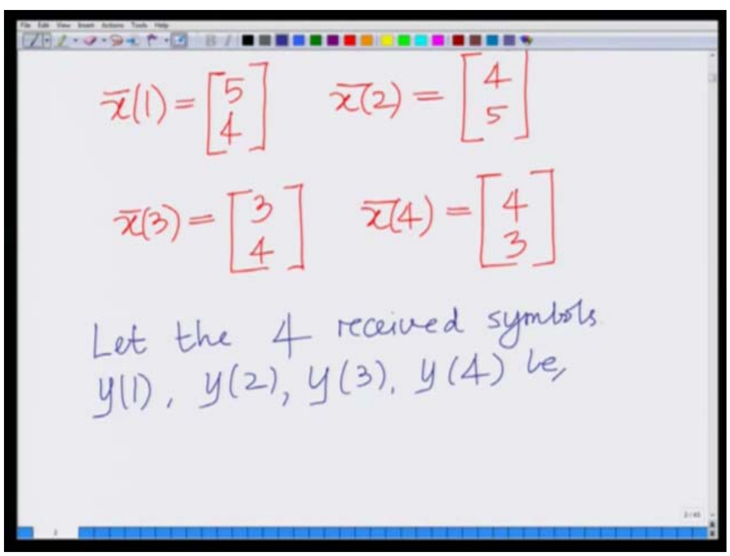

X bar 3 that is the pilot vector transmitted at transmit time 3 is 3 comma 4 and x bar 4 equals let us say 4, 3. So we have 4 pilot vectors, x bar 1, x bar 2, x bar 3 and x bar 4 which corresponds to the pilot vectors transmitted at time instant 1, 2, 3 and 4, all right. So we have we are considering the transmission of 4 pilot vectors, okay.

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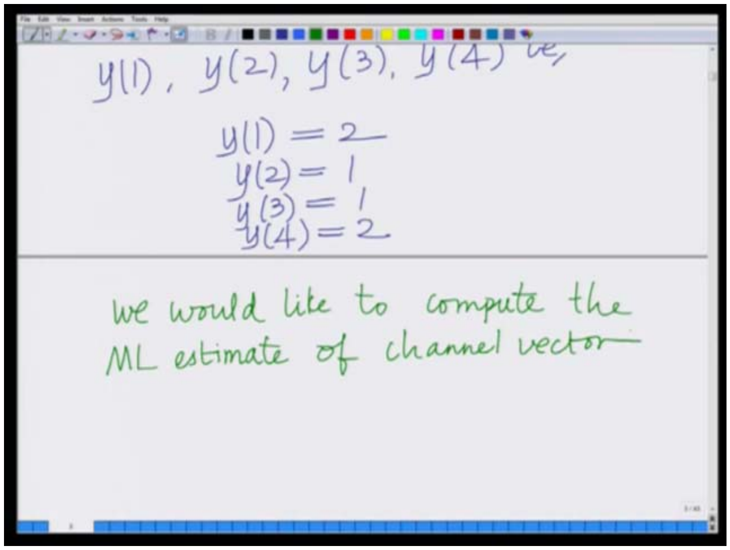

So naturally corresponding to these 4 pilot vectors, there are going to be for received symbols. Let the 4 received symbols y 1, y 2, y 3 comma y 4 be we have in this example, y 1 equals 2, y 2 equals 1, y 3 equals 1 and y 4 is also equal to 2 , okay and now to compute the maximum likelihood estimate of the channel vector h.

So now we would like to, what we would like to do; we would like to compute the maximum likelihood or basically the Least Squares estimate. The ML estimate of the channel vector remember, the channel vector is nothing but basically the vector of the 2 multi-antenna channel coefficients h 1, h 2. So we would like to basically estimate the channel vector h bar which consists of this multi-antenna channel coefficients h 1, h 2.

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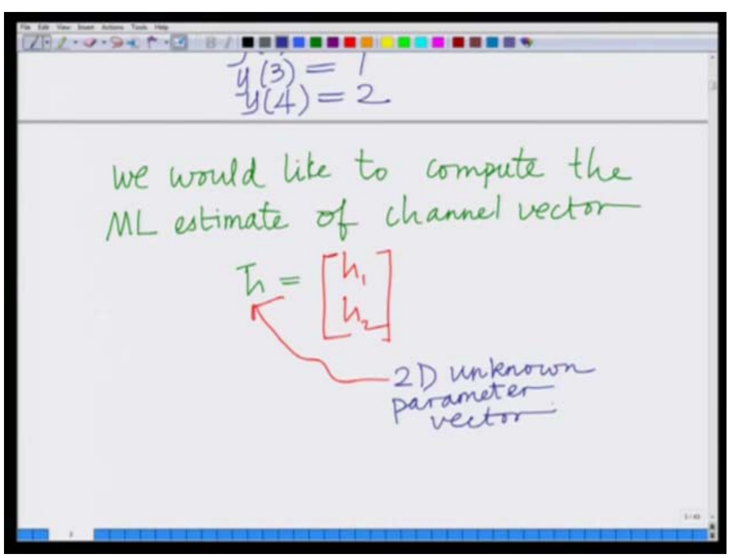

So it is a two dimensional vector which is basically two dimensional channel vector or also basically or also basically two-dimensional unknown parameter vector. So basically, this is a 2 D. This is a 2 D unknown parameter vector, now to find that estimate, let us first give x which is basically your pilot matrix. Remember, this is X which is your pilot matrix, so let me write this clearly again.

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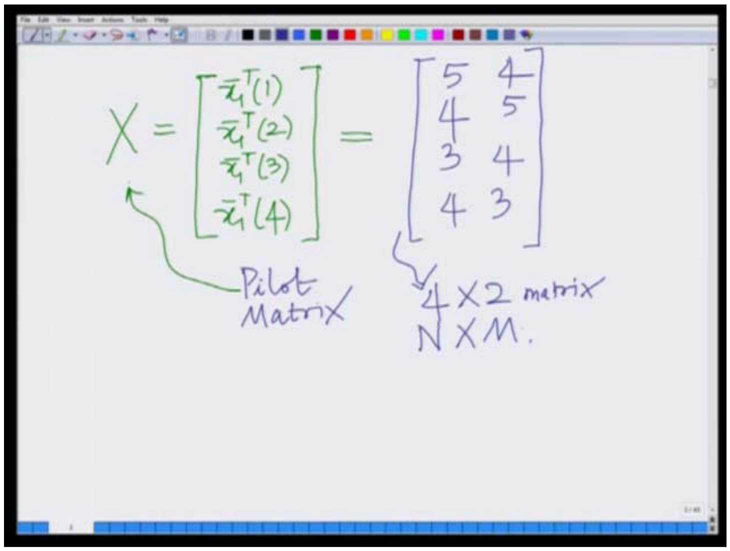

X is x bar transpose 1, x bar transpose 2, x bar transpose 3 and x bar transpose 4, this is basically your pilot matrix. So this is what is this; this is your pilot matrix X from our formulation of the channel estimation or the multi-antenna which is x bar x 1 bar transpose x 5 comma 4, x 2 bar transpose 4 comma 5, 3 comma 4, 4 comma 3 and as you can see, this is basically this is a 4 cross 2 matrix or basically N cross M.

Remember, we said that the pilot matrix is an N cross M matrix, where N is basically your number of basically the number of transmitted pilot vectors which N equal to 4 in this case and M is the number of unknown coefficients, M is equal to 2 in this case. So the pilot matrix is N cross M that is, it is 4 cross 2 matrix X, okay.

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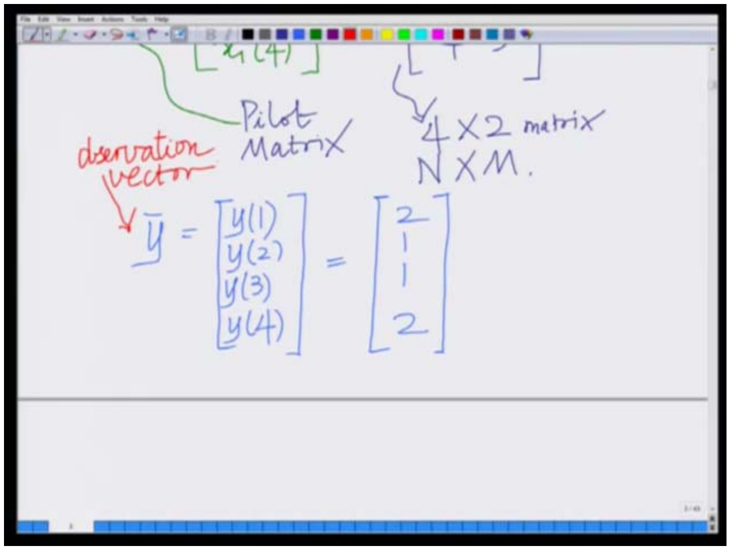

Also, let us now form the observation vector y bar equals y 1, y 2, y 3, y 4, which is basically 2, 1, 1, 2. This is basically this is your recall; y bar is your observation vector. This is basically the observation vector and the ML estimate now is given as ML, recall that ML estimate H hat of channel vector H bar also basically your LS or Least Squares estimate.

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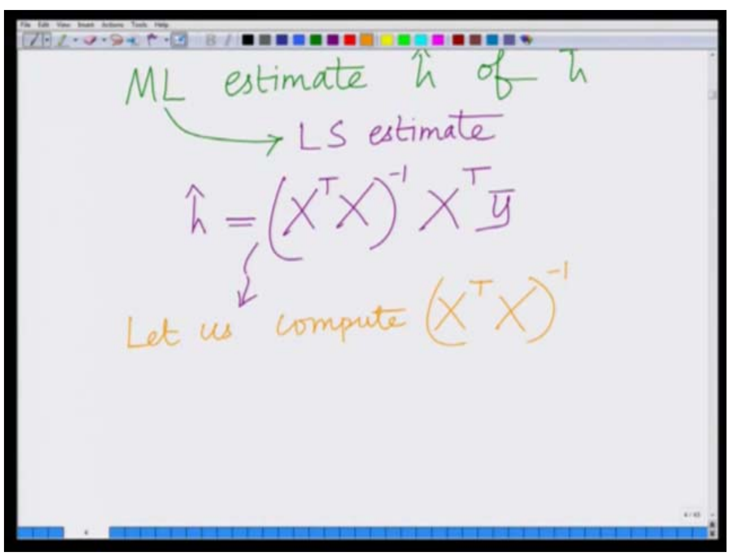

Remember, the ML estimate the maximum likelihood estimate is also the Least Squares estimate is given as H hat equals X transpose X inverse x transpose y bar. So first let us compute x transpose x bar, x is a pilot matrix, let us first compute this quantity X transpose X inverse, okay. So let us first compute X transpose X inverse. First we will compute x transpose x and then compute the inverse naturally.

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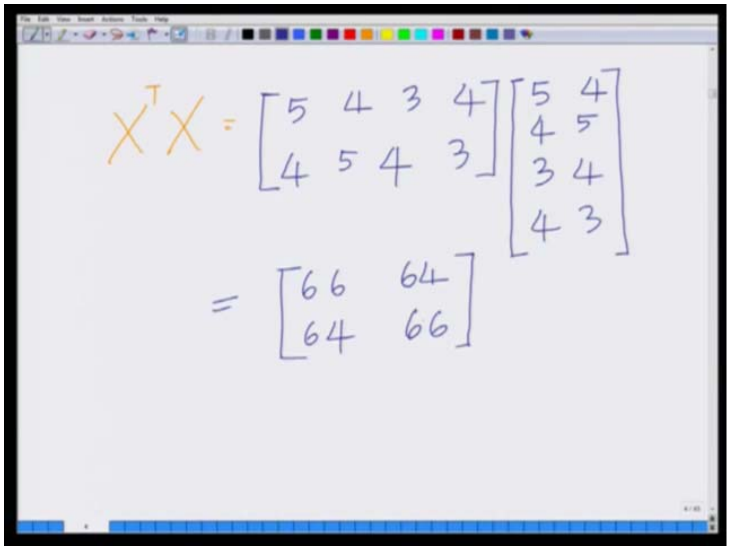

So proceeding in steps, I would like to first compute X transpose X inverse. Now x transpose x, x transpose is the transpose of the pilot matrix that is equal to 5 4 3 4, 4 5 4 3 times 5 4 4 5, 3 4 4 3. This is equal to, you can simplify this, this is 66, 64, this is a 2 cross 2 matrix. Note that this is basically your M cross M matrix, this is 66 I am sorry, this is 64, 64 66.

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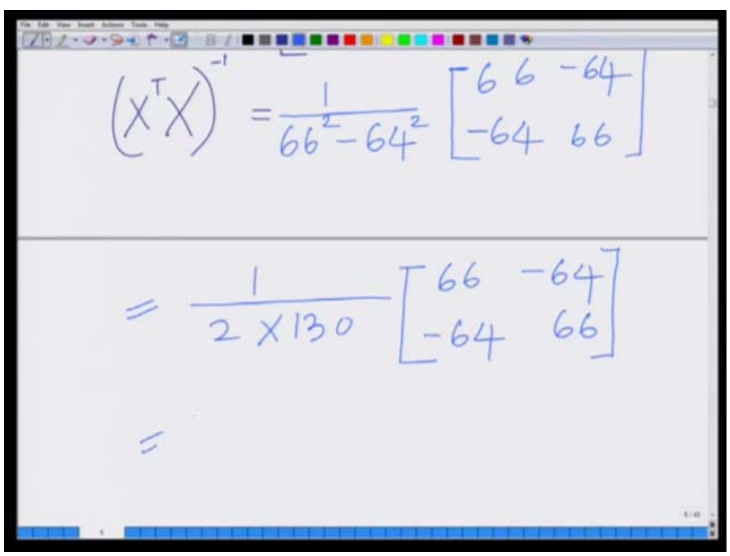

Therefore, the inverse of this 2 cross 2 matrix, the inverse of a 2 cross 2 matrix has a standard formula that is 1 over the determinant which is basically 66 square - 64 square times interchange the diagonal elements and negatives of the off diagonal elements. And 1 over 66 square - 64 square is 1 over 66 - 64 that is 2 times 66 + 64 that is 130 times 66 - 64 - 64 66 which is now cancelling the 2, I have 1 over 130 times 33 - 32 - 32 33 okay.

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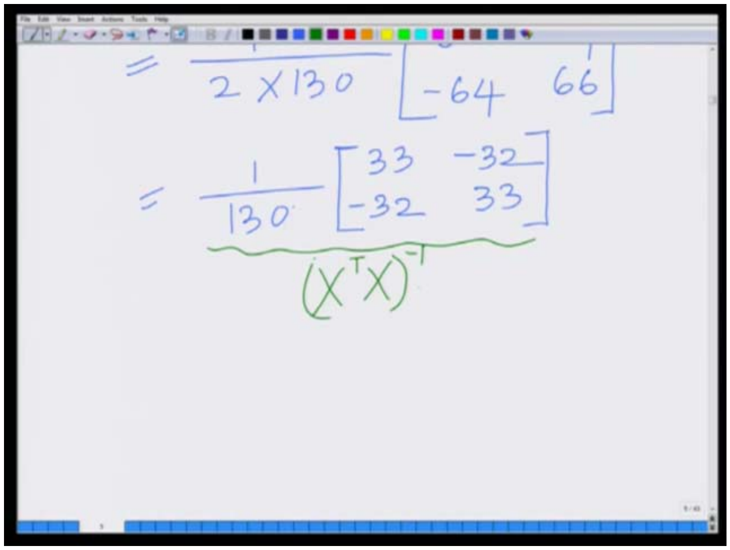

So now basically what we have found is we have found this quantity X transpose X inverse, x transpose x is a 2 cross 2 matrix, we have computed the inverse of this matrix, okay. The x is the pilot matrix, okay. So what is this; this is basically your X transpose X inverse. Now the other quantity that we need to compute is basically x transpose y and that can also be computed as follows.

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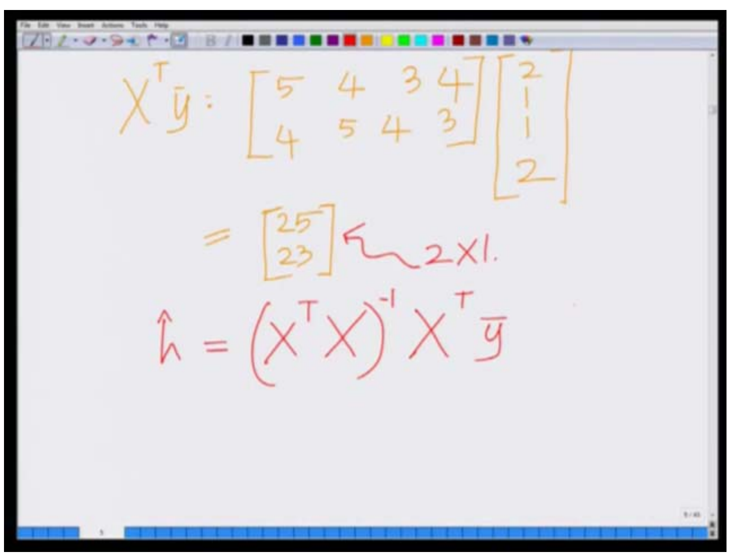

X transpose y bar equals 5 4 3 4, 4 5 4 3 times 2 1 1 2, which is basically equal to this vector 25 23, this is 2 cross 1 vector. You can also see clearly, this is your M cross 1 or basically your 2 cross 1 vector. And now the channel estimate H hat equals X transpose X inverse x transpose y bar.

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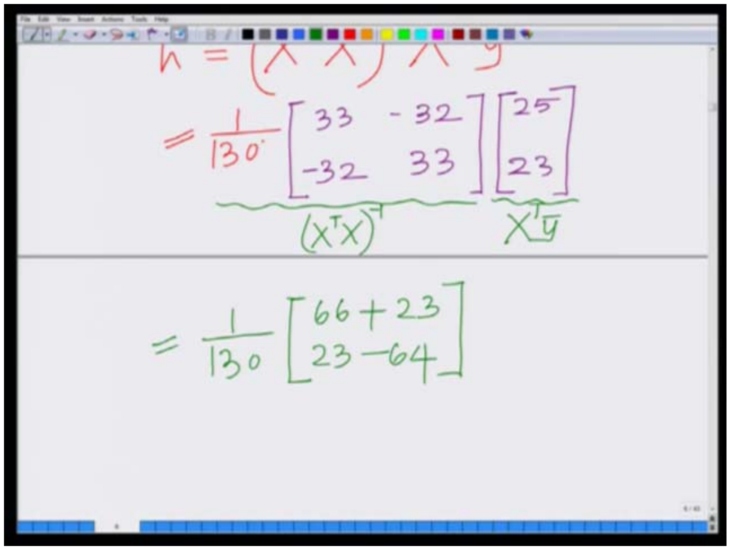

Remember, this is your maximum likelihood estimate which is basically equal to, now substituting for X transpose X inverse, I have 1 over 130 times 33, - 32, 33, - 32, 33 times the vector y bar, which is 25, 23 and this is basically what we are saying is this is your X transpose X inverse and this is your x transpose y bar and therefore now I can simplify this further this I can show.

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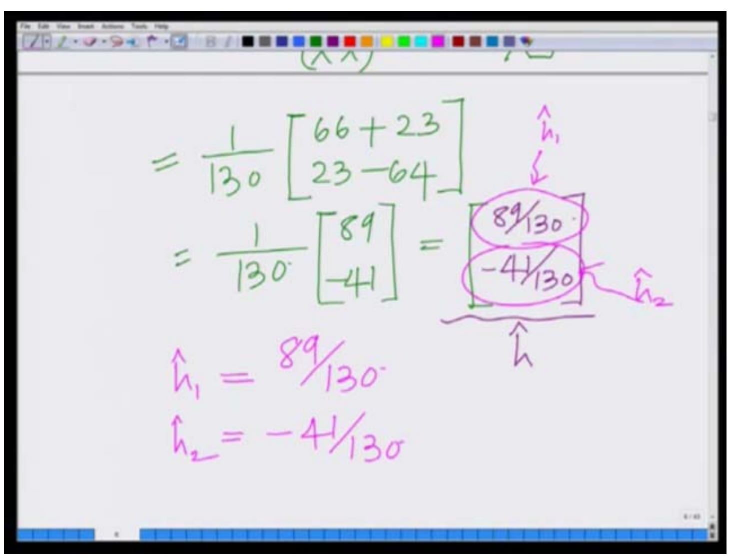

This is basically 1 1over 130, 66 + 23, 23 - 64, this is basically 1over 130. I am simplifying your estimate of the channel vector 1 over 130, 89 - 41, which is basically now if I have to write this, this is 89 by 130 - 41 by 130 and this is basically your H hat. So what we have computed if we have computed basically this quantity H hat which is the maximum likelihood estimate of the channel vector H bar.

For this multi antenna downlink channel estimation scenario with 2 antennas Centre base station and a single antenna at the Mobile, okay. And therefore, now this quantity you can see this is basically your h 1 hat, estimate of the first channel coefficient and this is basically your h 2 hat, so therefore h 1 hat equals 89 by 130, h 2 hat equals - 41 by 130. Now further if the noise is IID.

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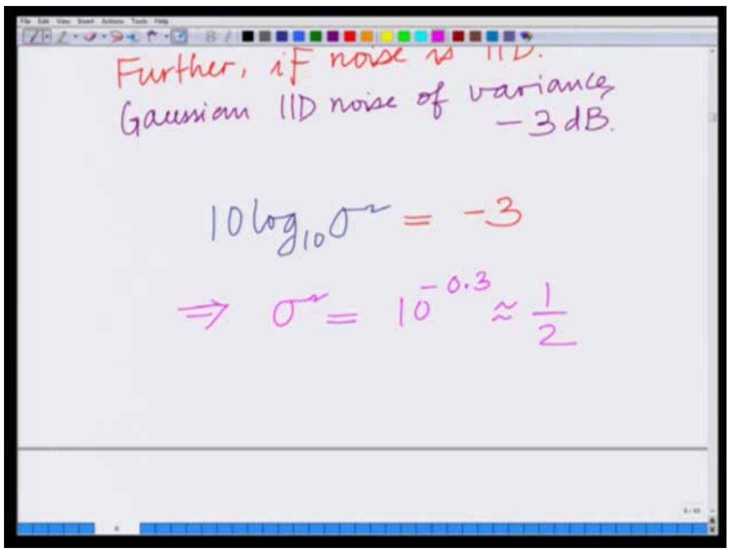

Let us say, let us consider Gaussian IID noise of variance - 3 dB, so we are considering independent identically distributed Gaussian noise samples of variance. So the variance is - 3 dB implies 10 log 10 sigma square equals - 3, which basically implies sigma square equals 10 to the power of - point 3, which is approximately equal to half. So - 3 dB noise variance basically implies that sigma square is approximately equal to half.

And remember, from this I can compute the covariance matrix, the covariance matrix for this multi antenna channel estimation scenario is sigma square X transpose X inverse, right. So the covariance, remember the covariance matrix of channel estimation, that is your expected value of h hat - h bar times h hat - h bar transpose.

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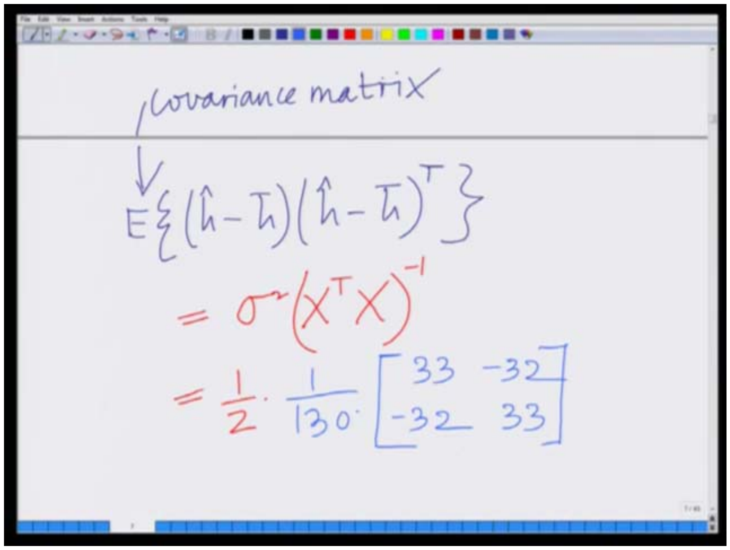

That is basically equal to, we have already derived the expression for that in the behavior of maximum likelihood estimate sigma square X transpose X inverse, sigma square we have calculated is half, X transpose X inverse also is something that we have calculated before that is 1 over 130, 33, - 32, - 32, 33 and this is basically equal to 1 over 260 33, - 32, - 32, 33 and therefore now I can write this as the 2 cross 2 matrix.

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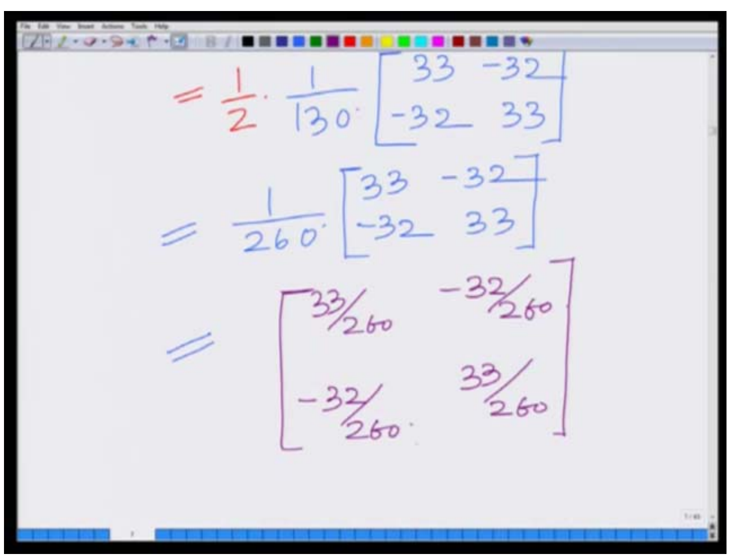

This is your covariance which is basically equal to 33 over 260, - over 260, - 32 over 260 and 33 over 260, what is this?

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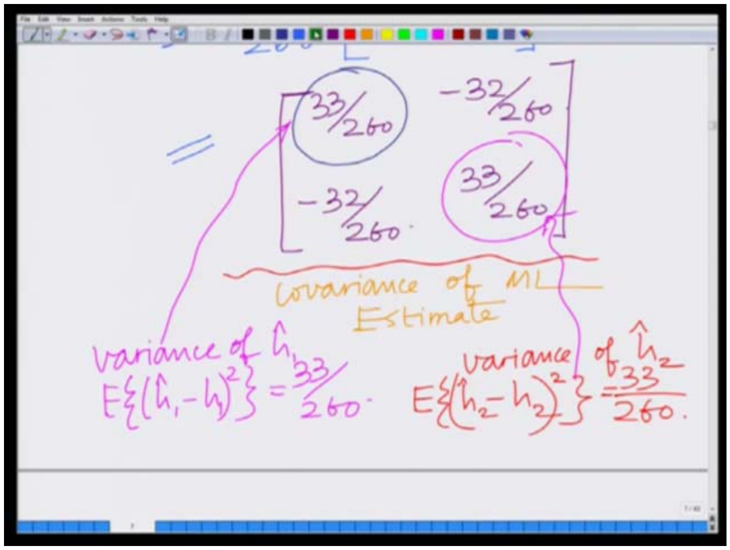

This is basically the covariance matrix. This is the covariance of the maximum likelihood this is the covariance of the maximum likelihood estimate, which we have calculated as Sigma Square X transpose X inverse, okay. And now we also said, from this covariance I can extract the variances of the elements.

Remember, recall this is variance of h 1 this quantity, this is variance of h 1 hat, that is expected value of h 1 hat - h 1whole square equals 33 by 260 and this in fact that the second the I comma I diagonal element is the variance of the Ith channel coefficient. So this is the variance of h 2 hat, which is the 2 comma 2th diagonal elements.

So that implies that expected value of h 2 hat - h 2 whole square is also equal to 33 divided by 260, all right. So basically we have calculated the covariance and we have also calculated the variances of the individual channel estimation, all right. So this is basically a simple problem, all right.

A simple problem which illustrates the application of the maximum likelihood estimation procedure or the Least Squares estimation procedure that we have developed in the context of basically a vector parameter estimation scenario, all right. So we have developed the Least Squares parameter estimation problem and we have shown that the Least Squares solution is given as

h hat equals X transpose X inverse X transpose y bar where x is the pilot matrix, y bar is observation vector and we have considered the simple channel estimation scenario for 2 antenna at the base station and a single antenna at the mobile and considering N equal to 4 transmitted pilot vectors.

We have derived the maximum likelihood channel estimate and leave also derived the covariance of the channel estimate and the variances of the individual channel coefficient, all right. So this example illustrates, simple example illustrates basically comprehensively the computation of the Least Squares channel estimate as well as the covariance and the respective variances, all right.

So we will stop this module here and continue in subsequent module, thank you very much.