Estimation for Wireless Communications-MIMO/OFDM Cellular and Sensor Networks.
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Lecture -13.
Vector Parameter Estimation-System Model For Multi-Antenna Downlink Channel Estimation.

Hello, welcome to another module in this massive open online course on estimation for wireless communication systems. So far we have talked about parameter estimation, the estimation of a scaler parameter H, all right. So, now we are going to start talking about the much more general scenario, that is a vector parameter estimation, that is where the parameter to be estimated is actually a parameter vector which we are going to denote by H bar. Alright, so in today's model, we are going to start talking about vector parameter estimation.

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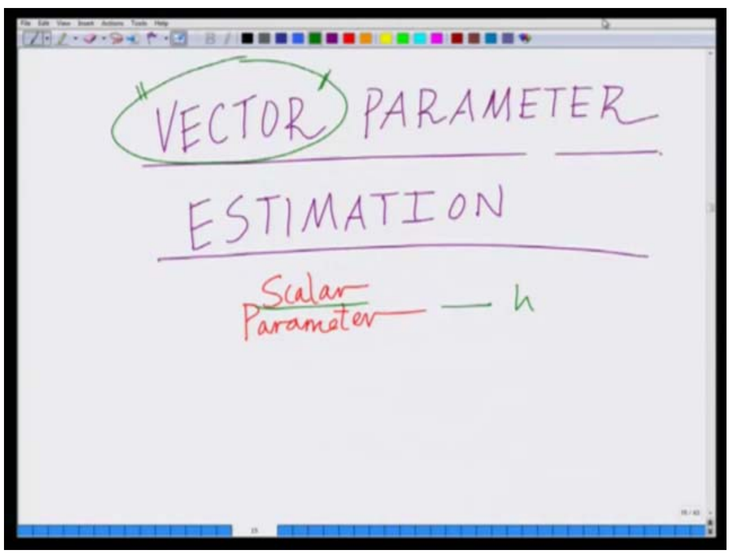

So, what we are going to talk about from today is basically to develop a framework for vector, vector parameter estimation and in fact the keyword here is this word which is basically a vector. So, so far we have talked about a parameter or rather a scalar to be more precise, we have talked about a scaler parameter H, that is estimation of a scaler parameter H. Right now and from this module, we are going to start talking about a vector parameter, estimation of a vector parameter,

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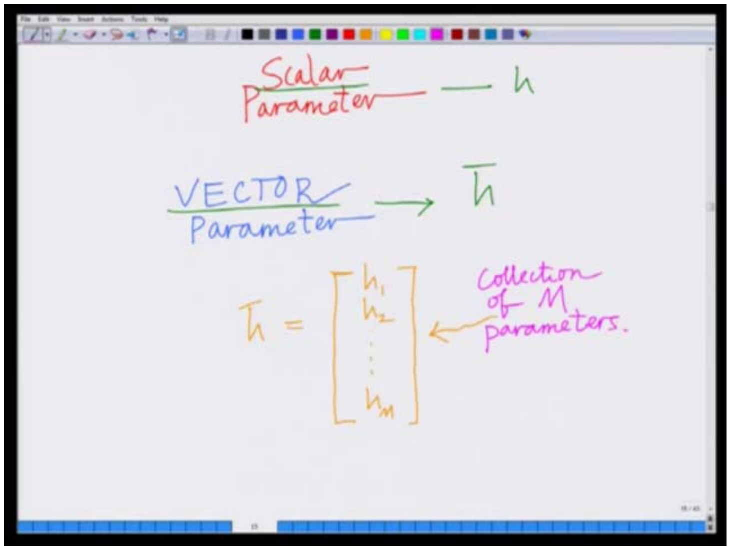

that is let me write it, emphasise this, this is a vector parameter and similar to the notation used for the vectors, we are going to denote this by vector H bar. This is parameter vector H bar and therefore H bar is let us say it is an M-dimensional vector H1, H2 up to HM. Which is basically a stacking or collection of M scaler parameter.

So basically this is a collection of M parameters H1, H2… up to HM. So, what we are going to start talking about from today is basically the estimation of a vector parameter which we are going to denote by H bar and this is an M dimensional vector, which means it contains M parameters which we are denoting by H1, H2… up to HM. So, to understand this better, I mean to understand, to motivate this concept or to motivate why we need to estimate a vector parameter rather than a scaler parameter, let us try to look at it from the perspective of an example. I think an example will clarify this concept and the framework related to it much better.

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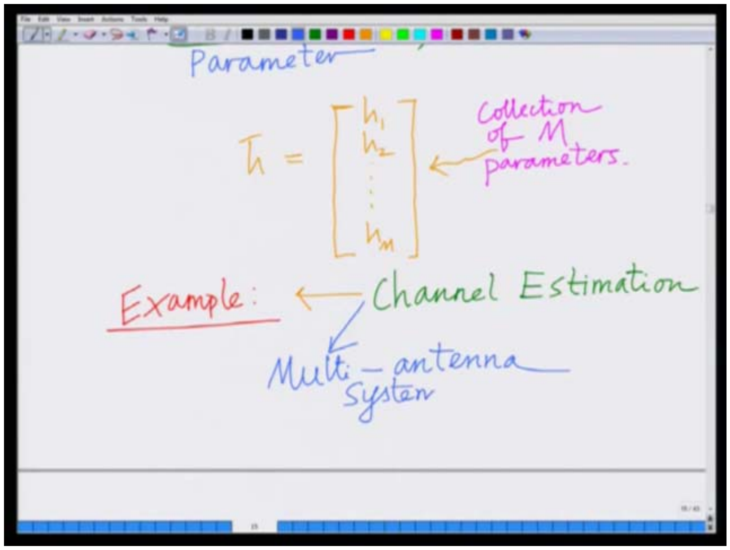

So, let us start with a simple example in the context of wireless communication system. So, let us look at an example and in this example, again similar to our example previously, let us look at channel estimation, let us look at, remember the estimation of the feeding channel coefficient in a wireless system is known as channel estimation. However, let us look at channel estimation in a multi-antenna system. So far we have considered only a single antenna communication system, so let us look at channel estimation in the multi-antenna system. So, what we are saying is let us try to understand this framework of vector parameter estimation to explore and understand that, let us begin with a simple example in the context of channel estimation in a wireless communication system, but channel estimation in a multiple antenna system. That is wireless communication system with multiple antennas system, which means more than 1 antenna, all right.

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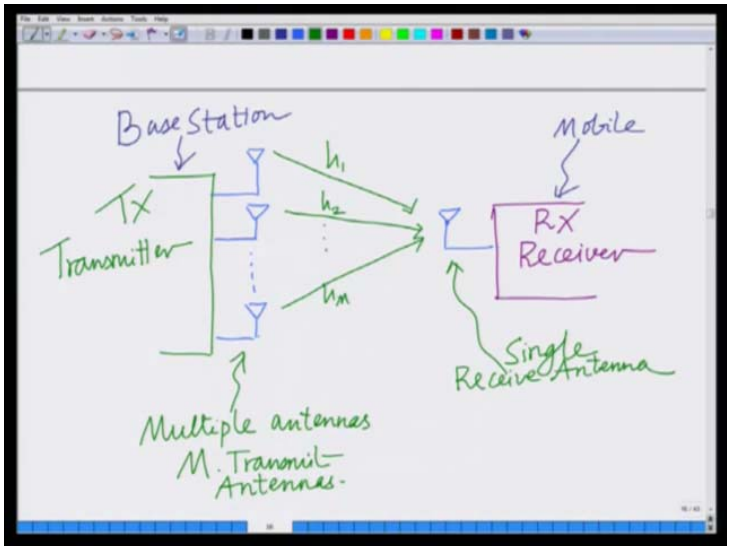

And a simple such scenario is for instance, I have a transmitter, I have my transmitter, my wireless transmitter and I also have my receiver, my wireless receiver, my wireless receiver, so let us say this is my, this is my wireless receiver which I am going to denote by RX and I have let us say multiple antennas at the base station. That is not just a single antenna, but more than 1 antenna at the base station and I have a single antenna… So each of these triangles is basically an antenna and what I am saying is for instance, this is my transmitter and there are multiple antennas, let us say M transmit antennas and there is a single antenna at the receiver, a single receive antenna.

For instance we can consider a simple scenario where this is basically your, the system is something like a base station, for instance your transmitter is a base station and the receiver is let us say, this is simply your mobile or mobile session. So, the transmitter is basically larger in size, so it can have multiple antenna and in fact in current generation, 3G wireless systems have the provision to go up to 2 to 4 antennas, LTE is exploring about 4 antennas, beyond 4 antennas. So, people are already talking about systems with antenna, about 4 antennas and also going up to 8 antennas. Right, so we can have transmitters such as the base stations, wireless base station which is more than 1 antenna, all right.

So, we are considering multiple antennas at the transmitter, the number of antennas at the transmitter is denoted by M and we have a single antenna at the receiver which is a mobile and the transmission, let us say the transmission is from the base station to the mobile. So basically you are considering a downlink scenario, that is what we said even in the previous channel estimation scenario. When the transmission is from the base station to the mobile, it is a downlink, when the transmission is from the mobile to the base station, it is the uplink. So, we are considering a multi-antenna downlink wireless communication scenario where is the base station has multiple antennas and the mobile has a single antenna.

Now, naturally since there are multiple antennas, there are going to be multiple channel coefficient corresponding to these multiple antennas. And since there are M antennas, there are going to be M channel coefficients H1, H2… up to HM, right.

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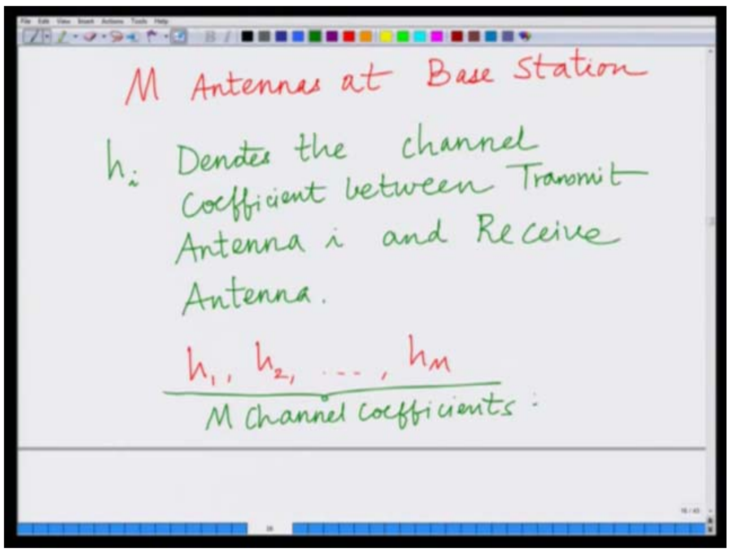

So, basically what we are saying is we have a total of M antennas, that is M antennas, we have M antenna that the base station. And HI denotes, H I denotes the channel coefficient between transmit antenna I and receive antenna. H I denotes the channel coefficient between transmit antenna I and the receive antenna, this is the channel coefficient. For instance, let us say, let us consider H2, H2 is the channel coefficient between the transmit antenna 2 at the base kitchen and the single receive antenna.

Similarly H3 is the channel coefficient between transmit antenna 3 and the single receive antenna. And since we have M antennas, M transmit antennas, therefore we naturally have M channel coefficients corresponding to the M transmit antennas, okay. So, what we have is we have M channel coefficients H1, H2… up to HM, these are the, these are your M, these are your M channel coefficients, correct. In this multi-antenna wireless communication system, we have M channel coefficients and to begin with these M channel coefficients are known, that is the paradigm of the channel estimation. Therefore these M channel coefficients can now be stacked as a vector, this can be considered as a vector H bar and that channel vector H bar has to now be estimated. Therefore this leads to vector parameter estimation.

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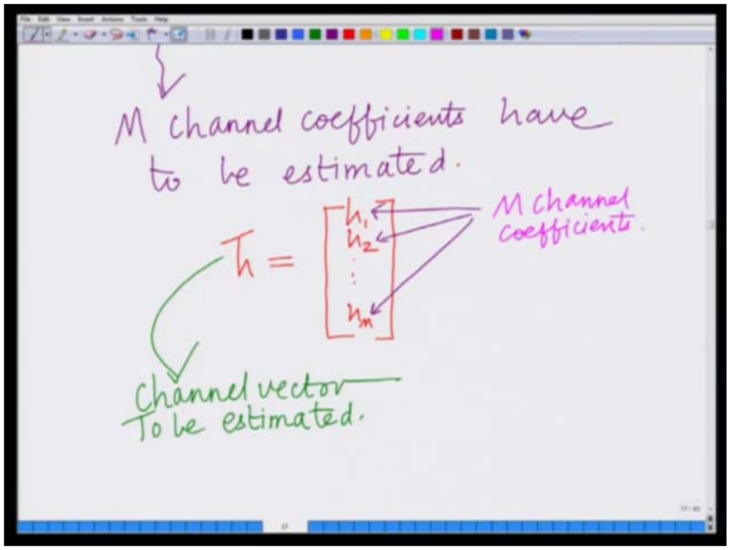

So, what do we have, is to put it more explicitly, M, M channel coefficients have to be estimated okay. So now these M channel coefficients can be denoted by the vector H bar equals H1, H2… up to HM, so this is a channel vector which has to be estimated, so this is your channel vector which has to be estimated. And these, these are basically your M, all right. So, naturally a multiple antenna system in which there are multiple antennas and therefore multiple channel coefficients, is naturally leads to a necessity and it naturally leads to a paradigm of vector parameter estimation. So, we can no longer be satisfied by simply considering a scale parameter, that is individual parameters but we have to look at a group of parameters.

That is parameters which are grouped as a vector that naturally leads to the requirement and the necessity of vector parameter estimation. Now, how do we estimate this channel coefficient vector, let us formulate this problem for estimation of this channel vector H bar and that can be seen as follows. Similar to the single input, similar to the single transmit antenna and single receive antenna, we are going to transmit pilot symbols from the transmitter, which in this case, since it is a downlink scenario, the transmitter is the base station. So, let us consider the transmission of pilot symbols.

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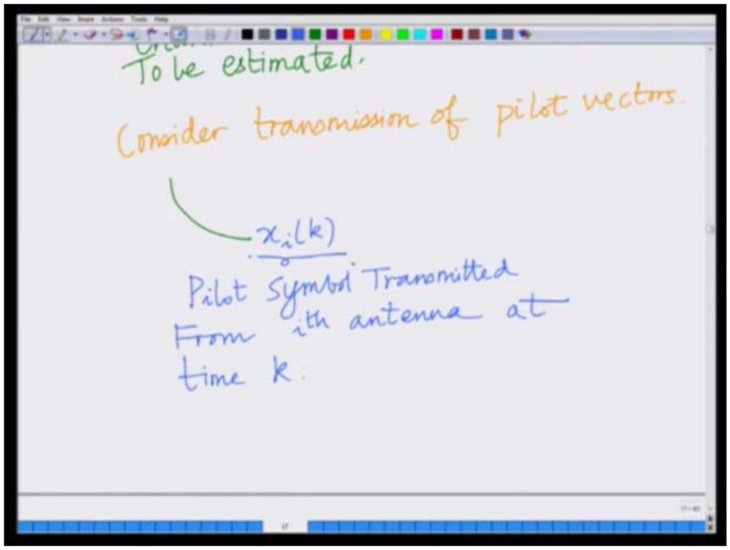

However, I have been slightly inaccurate because we have multiple antennas, so we need the transmission of one pilot symbols from each transmit antenna, so rather than pilot symbols, we will actually be transmitting pilot vectors where the vector of transmitted symbols corresponds to one transmit symbol from each transmit antennas. So, we talk about pilot vectors, right. So let us consider the transmission of pilot vectors, and in fact there will be multiple such vectors. So, what will we have, let us say now we denotes by XI of K, that is XI of K is the pilot symbol transmitted from the XI of K, this is the pilot symbol transmitted from your Ith antenna at time K. So, we have multiple antennas, we cannot the transmission of a single pilot symbol. Previously we had a single transmit antenna, so we said XK the pilot symbol transmitted at time instant K, however now we have multiple antennas, so I am using the subscript I to denote by X I K, the pilot symbol transmitted at time instant K on antenna I.

So, X I K denotes the pilot symbol transmitted on transmit antenna I at time instant K by the transmitter which is the base station in the downlink.

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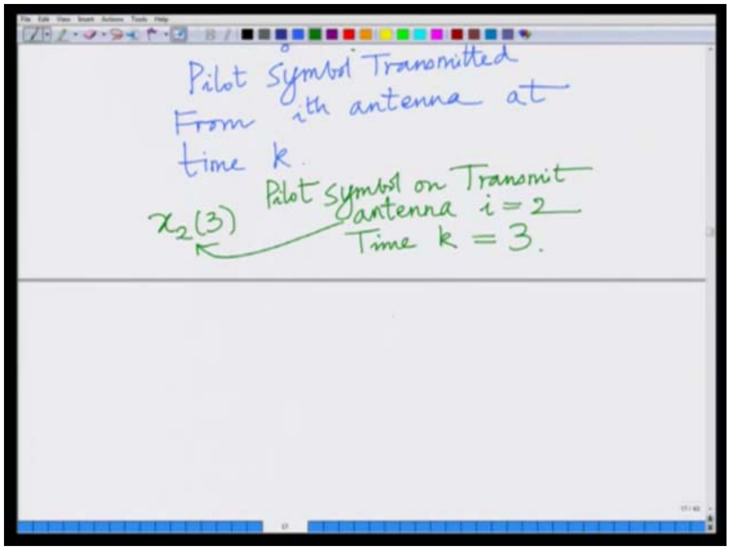

For instance again, let us take a simple example X2 of 3, if I look at X2 of 3 other symbol or rather pilot symbol on transmit antenna 2, on transmit antenna 2, that is I equal 2 transmit antenna, your I equals 2 at time or discrete time K equals 3. That is X2 of 3 is a symbol transmitted, pilot symbol transmitted on transmit antenna 2 at time instant 3. Therefore at time instant K, we have, we are going to have,

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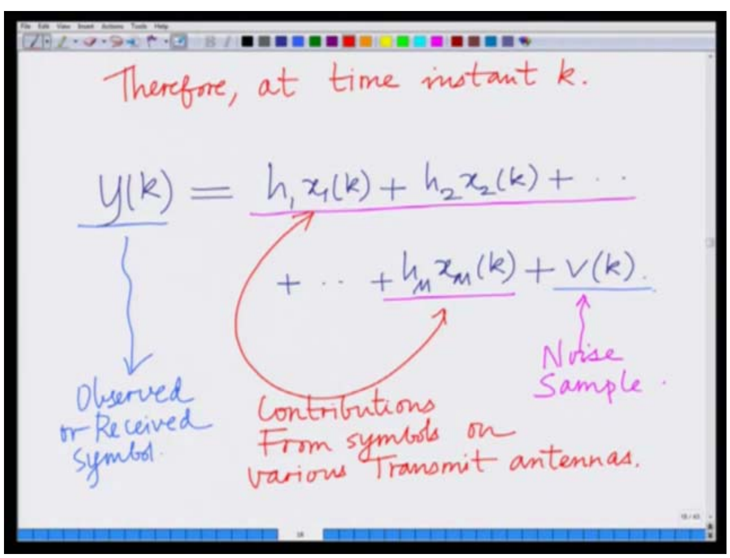

therefore at time, therefore at time instant K, if we have the observed symbol YK, let me write this clearly, YK which is the received symbol, this is equal to H1 X1 K + H2 X2 K + so on + HM XM K + VK.

Let me explain this, YK is the observed symbol at the time instant K, VK is the corresponding noise sample at time instant K, these 2 things are simple. Now, in between the rest of the things are contributions from the various channel coefficients. For instance, H1 X1 K corresponds to X1 K, that is a symbol transmitted at time instant K on transmit antenna 1 and that goes through the channel coefficient H1, all right. And that, in addition to that we have H2, channel coefficient H2 times X2 K which is the symbol, pilot symbol transmitted on transmit antenna 2 at time instant K and all these contributions from the various transmit antennas add up at the receiver and therefore we are going to see the composite, the sum signal.

Okay so let me describe that H1 K is the observed symbol at the receiver, this is the observed or one can say your received symbol, VK is basically your noise sample and this quantity here is basically these are the contributions from the various receive antennas or various transmit antennas. Contributions from symbols on transmitted on various, transmitted on the various transmit antennas. For instance, X1 K is the symbol transmitted on transmitted antenna 1 at time instant K, that goes through channel H1 K, for instance H2 X2 K basically refers to symbol X2 K transmitted on transmit antenna 2 at time instant K going through channel H2 and so on until HM XM K that is what we have. Okay, and now therefore, this is the system at time instant K, this corresponds, this is the model corresponding to the received symbol YK at times it K, I write it succinctly again using vector notation YK as follows.

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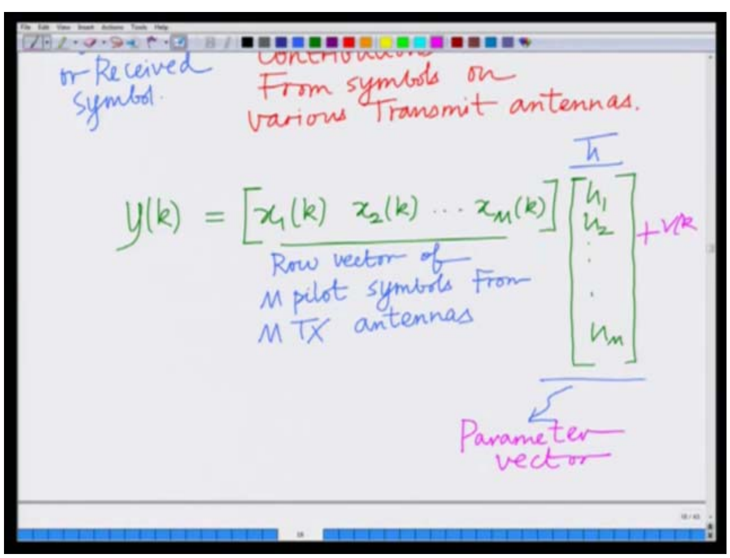

We are already familiar with vector notation, YK equals H1, H2 let us write it this way… X1 K, X2 K, XM K times the row vector times H1, H2 up to HM. So, this is the row vector of M pilot symbols transmitted from the M transmit antennas. Row vector of M pilot symbols from M transmit antennas. This of course is H bar, this is basically your unknown parameter vector, this is your, + of course this is going to be your noise as usual, that is VK. So, we have YK is basically the receive symbol which is equal to X1, X2 up to XK which is the row vector of pilot symbols transmitted at time instant K from the M transmit antennas times the column vector of the channel coefficients H1, H2… up to HM which we are denoting by the vector H bar + VK which is the noise sample.

Now let us say we have N such pilot vectors that the transmitted, that is pilot transmissions corresponding to N time instants, that is multiple pilot symbols over multiple transmit times 1, 2, 3 up to N and multiple transmit antennas, 1, 2, 3 up to N. Now we can concatenate all this and that the composite system model as follows.

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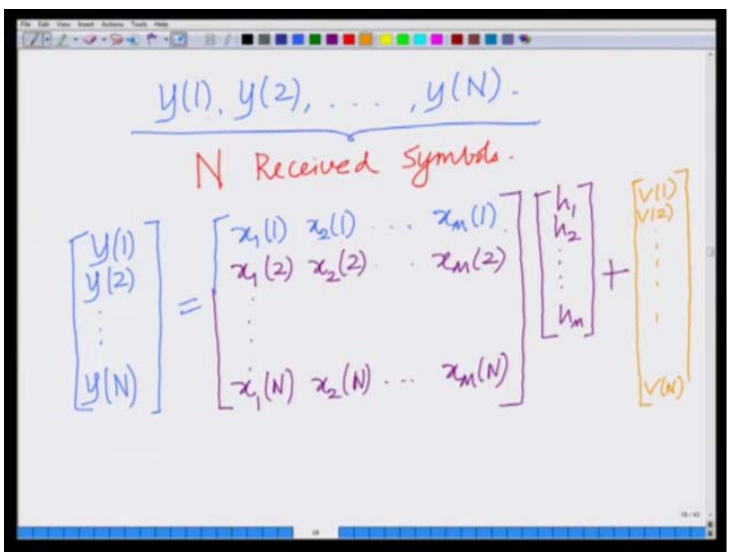

So, therefore, consider now, N pilot symbols, consider now, consider now N pilot symbols, therefore I can now, I have N, I have N observed symbols Y1, Y2, YN, these are the, what are these, these are your N received symbols. In fact I should say consider N pilot vectors because at each time instant, we are transmitting a vector from the M antennas.

Now if I look at and I am going to explain this again, let me write the system model, I will have Y1, Y2… Up to YN, that is received symbol at time instant N which is equal to this matrix, that is X11, X12 up to X1 X1M or let me write it this way. I think we have to write it as X11, X 21 up to XM1, letter the pilot symbols transmitted at time instant one and X2 or X12 transmitted at time instant 2, X22 at time instant 2, so on, XM2 at time instant 2 and XN1, XN2, sorry X1N, X2 N so on X XMN times your parameter vector, let me just draw it a little bit shorter, I will explain the reason. H1, H2… up to HM, + your noise vector, noise vector is naturally going to be of dimension N.

That is V1 or noise sample at time instant one V1, V2… So on up to VN and this is basically your input output pilot symbol model.

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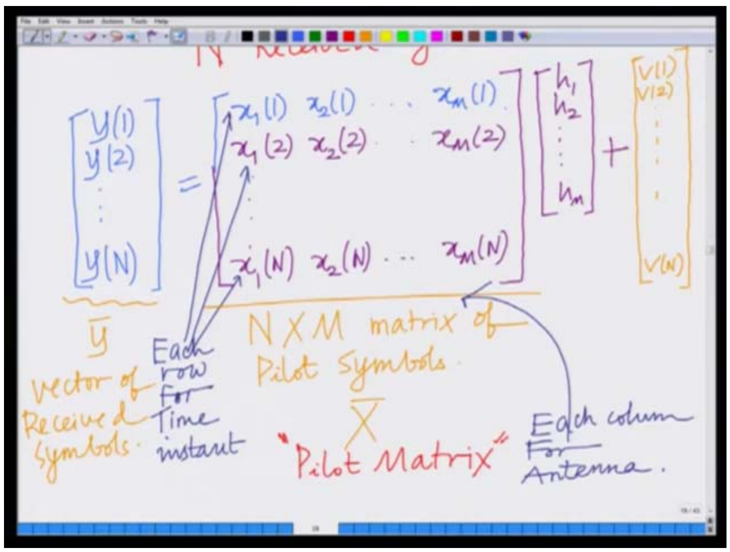

Now, let me denote this vector, vector of the received symbol as your vector Y bar, this is your vector of received symbols, let us write this down. What is this, this is an N x M matrix of pilot symbols. This is N x M matrix, this is N x M matrix of pilot symbols, let us denote this by X bar. This is now your pilot matrix, this is a pilot matrix and look at this, in this pilot matrix, each row corresponds to a particular time instant. Each row corresponds to a particular time.

And each column corresponds to a, let me write it here, each column corresponds to an antenna. Each column corresponds to an antenna, right. So, for instance, at time instant 2, the row is basically X12, X22 so on XM2, basically which means that X 11 is a symbol transmitted at transmit antenna 1 at time instant 1 X2 X1 X X22 is basically a symbol transmitted from transmit antenna 2 at time instant 2 and so on until XM2 which is basically symbol transmitted from transmit antenna M at time instant 2. So, each row basically, if you observe this matrix, each row of this pilot matrix corresponds to the pilot symbols transmitted from the M transmit antennas at time instant K. Kth row corresponds to the M transmit M pilot symbols transmitted from the M transmit antennas at time instant K.

Therefore we have letter M entries in a row, which means we have basically M columns and similarly we have N such rows, each corresponds to a one particular time instant and therefore we are saying we have N, N such time instants, that is basically corresponding to the transmission of N pilot vector. So, this matrix, pilot matrix is N x M. And also, now we are going to, I said this, I am going to draw this column vector, this H bar is slightly smaller because typically we assume N is larger than or equal to M. The reason being very simple, because we have M unknown channel coefficients and N, N is the number of operations, to estimate M unknown coefficients, we need at least N observations or more.

This is a simple property from equations, that is basically to solve, to have a unique solution for M unknown quantities, we need at least M equations or more. So, basically we have N pilot vectors, the N pilot of observations, these observations, that is number of transmitted pilot vectors has to be at least equal to or basically equal to or greater than M where M is the number of unknown channel coefficients. So, we have N x M pilot matrix where N is greater than equal to M, that is at least going to be the assumption going forward. Alright, so let me also write this down. It is, even though it is a subtle point, it is important,

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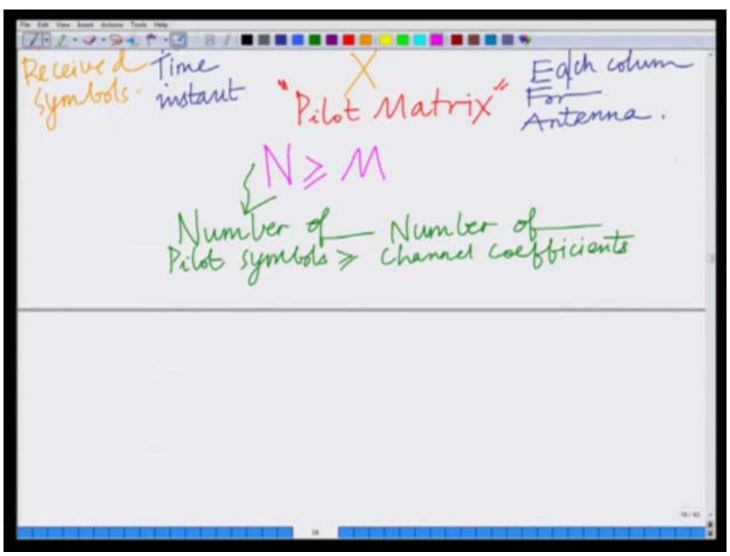

N is greater than equal to M which basically means that number of pilot symbols greater than or equal to your number of channel coefficients. It is greater than or equal to the number of channel coefficients. And of course the save already seen before, this is H1, H2… up to HM, this is which is the pilot, which is the channel coefficient, the channel vector H bar, this is the noise vector V bar which contains the N noise coefficients V1, V2… VN corresponding to the N received samples Y1, Y2… Up to YN.

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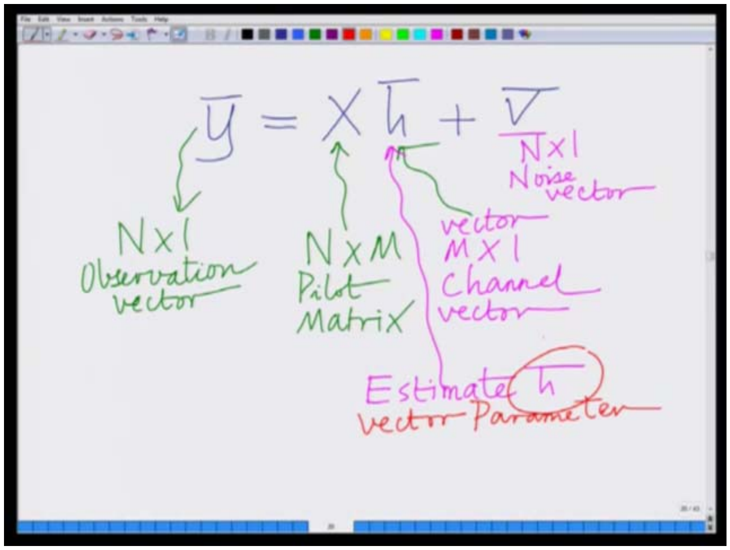

And now therefore if I write this entire thing together, succinctly in the vector notation, let me write that clearly, I have Y is equal to, let me just write this clearly because this is important to understand. Understanding this is very important because we are going to use this notation going forward in vector parameter estimation H X bar + V bar where let me again for the sake of clarity defines this thing. This is N x 1 observation vector or received symbol vector, this is X which is the N x M pilot matrix corresponding to M pilot vectors, vectors of M pilot symbols is transmitted over N time instants, this is the vector of M channel coefficients. M x 1, this is the channel coefficient vector or simply called the channel vector and this V bar is your N x 1 noise vector.

So, this is your N x 1 noise vector and this is your input output, this is the matrix, input output vector, input output system model and estimation we need to estimate, estimate H bar which is basically your vector channel vector which is a vector parameter. So, this basically H bar is the vector parameter which we are interested in estimating. So, basically now I formulated this, very succinctly and very… In a very compact and innovative very tractable fashion where Y bar is the observation vector which is basically Y1, Y2… Up to YN corresponding to the N transmitted pilot vectors which is equal to X, which is the N x M pilot matrix, right, N rows corresponding to the N time instants and M columns corresponding to

M transmit antennas times H bar which is the channel vector, which contains the channel coefficients, the unknown channel coefficients H1, H2… up to HM and that denotes the parameter vector, the unknown parameter vector H bar + V bar where V bar is the noise vector.

So, this is a compact representation of this vector parameter estimation model and therefore what we have done in this module is simply to motivate the necessity, the need for the estimation of a vector parameter because we have, previously we have only considered the estimation of a scaler parameter and also through an example, basically considering the example of a channel vector estimation for a multiple antenna downlink wireless communication scenario, we have built up the framework and motivated the necessity or illustrated the need to develop a framework for vector parameter estimation and we have also developed this succinct, this elegant, compact vector model, vector system model, we would also call it which is Y bar equals X times H bar + V bar which we are going to subsequently explore towards the estimation of this vector parameter H bar. Alright, so we will stop with this problem formulation here and how is the vector parameter estimation done, that is how do we come up with the likelihood function and how do we do the exact vector parameter estimation, that we are going to explore in the subsequent modules. Thank you very much.