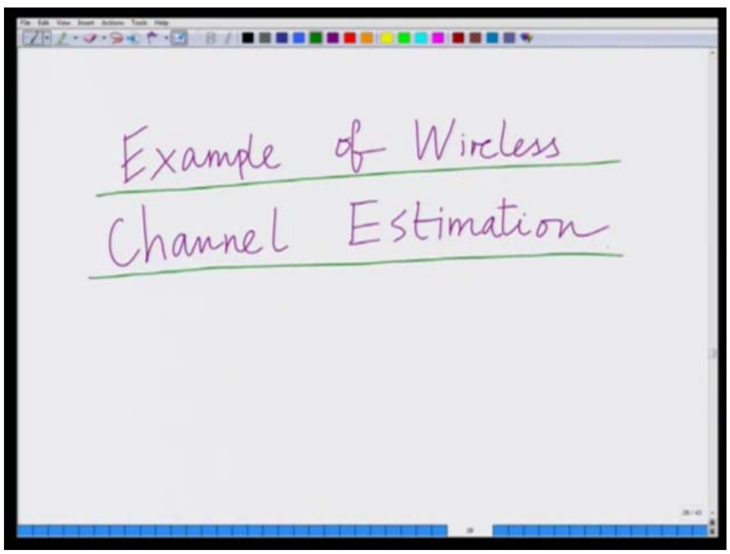
Estimation for Wireless Communications-MIMO/OFDM Cellular and Sensor Networks.
Professor Aditya K Jagannatham.
Department of Electrical Engineering.
Indian Institute of Technology Kanpur.
Lecture -10.
Example-Wireless Fading Channel Estimation for Downlink Mobile Communication.

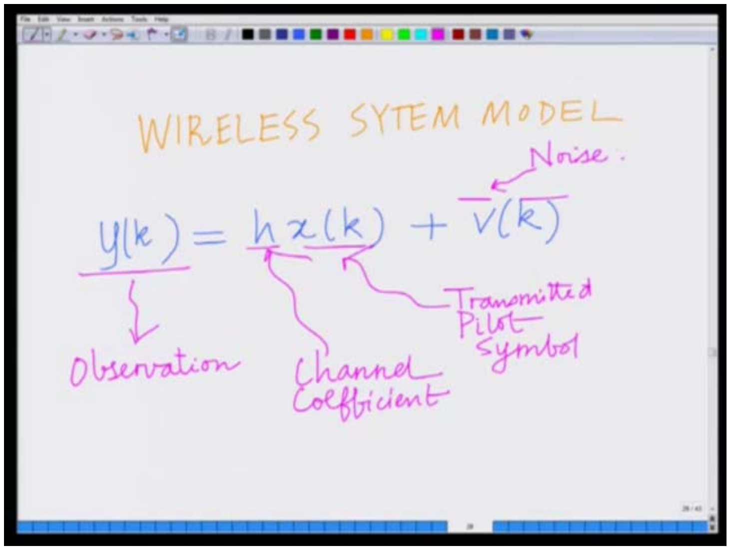
Hello, welcome to another module in this massive open online course on estimation for wireless complications. In the previous modules we have seen the maximum likelihood estimate of the channel coefficient of a wireless communication system as well as the properties of the maximum likelihood estimate that is the mean and the variance. Let us now look at a simple example to compute this estimate of wireless channel coefficient to understand this process better. So, today let us look at a simple example for wireless channel estimation.

(Refer Slide Time: 0:44)



A simple example of wireless, that is wireless channel estimation, as we have seen channel estimation is the process where we compute the estimation of the wireless channel coefficient. We have already seen the model for the wireless channel, that is the model of our wireless channel is given as,

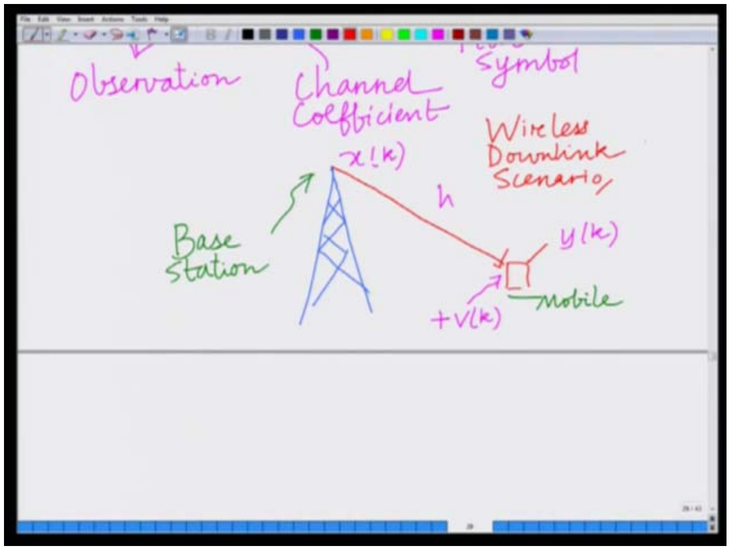
(Refer Slide Time: 1:19)



that is our wireless model or a wireless system model or wireless system model is given as YK equals H times XK + VK where as we have already said and we have described it in the previous model is that YK is your observation, the outputs of, the output at the receiver. H is your channel coefficient X is the transmitted pilot symbol, your X is the transmitted pilot symbol and VK is the noise. VK is the noise. So, we have this model YK equals H times XK + VK in the wireless system.

Alright, and this describe the input output model of a wireless communication system. For instance, let us say we have a simple base station and the mobile receiver that is a mobile, I have for instance, this kind of represents a model where I have a wireless base station which is transmitting on the downlink to a mobile.

(Refer Slide Time: 3:22)

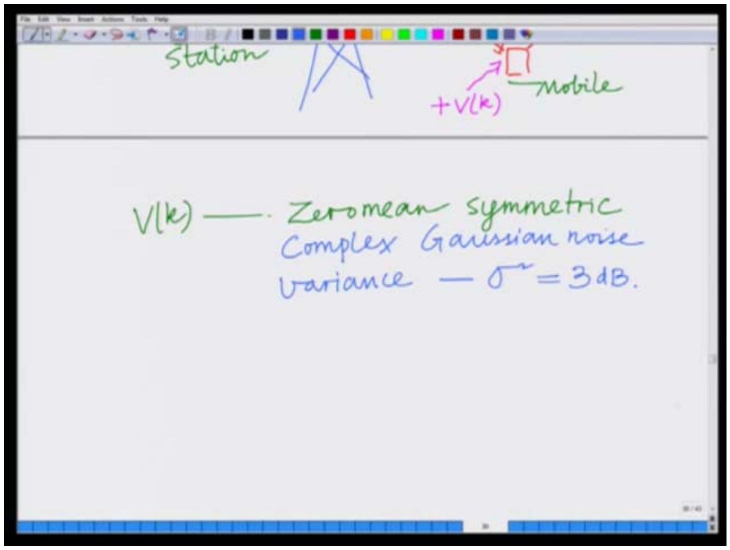


So this is my wireless base station which is the transmitter and the downlink, let us say it is transmitting to a mobile and therefore I have the transmitted signal which is XK, the received signal which is YK at the mobile, channel coefficient which is H and basically also we have additive noise VK at the mobile. So, basically this is my mobile and this is my base station and this is basically a simple wireless downlink scenario.

This is a simple wireless downlink scenario where we have said, we are saying that basically a base station is transmitting to a mobile, the base station transmits the symbol XK that travels through the channel with channel coefficient H which is represented by the channel coefficient H and the received symbol at the mobile is given is denoted by YK and also there is additive noise VK at the receiver, this is known as the downlink scenario because the base station had the is transmitting and the mobile is receiving.

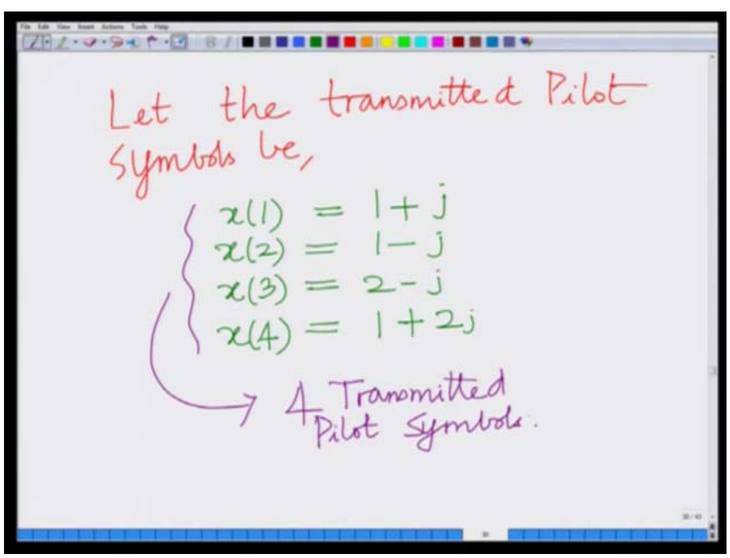
Corresponding when a mobile is transmitting and the base station is receiving, you have the uplink scenario and also the transmission, the corresponding system model for uplink scenario is symmetric where XK is the transmitted symbol by the mobile and YK is the received symbol at the base station. So, basically this can be used to estimate the channel coefficient, the procedure can be used to estimate the channel coefficient both at the mobile in the downlink and also at the base station in the uplink. Alright, so now let us look at a simple example, let us take some simple numbers to understand this paradigm better, alright.

(Refer Slide Time: 5:29)



So, we said VK the additive Gaussian noise and in previous modules, we have also assumed VK to be the 0 mean, if you remember, VK is 0 mean symmetric complex Gaussian noise with variance Sigma square equals 3 DB. So, this is 0 mean symmetric complex Gaussian noise with variance given by 3 DB alright. So, VK we are saying is complex additive complex Gaussian noise is a symmetric and has a 0 mean and it has a variance of 3 DB. Alright. Okay.

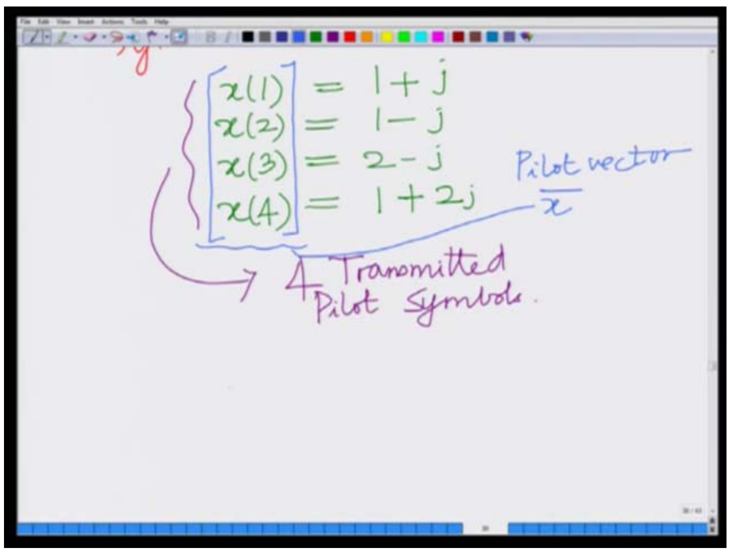
(Refer Slide Time: 6:29)



And also now let the transmitted pilot symbols be given as, let the transmitted… Remember we have pilot symbols being transmitted for channel estimation, so let the transmitted pilot symbols be, the transmitted pilot symbols are X1 equals 1+ J, X2 equals 1 - J, X3 equals 2 - J and X4 equals 1+ 2J.

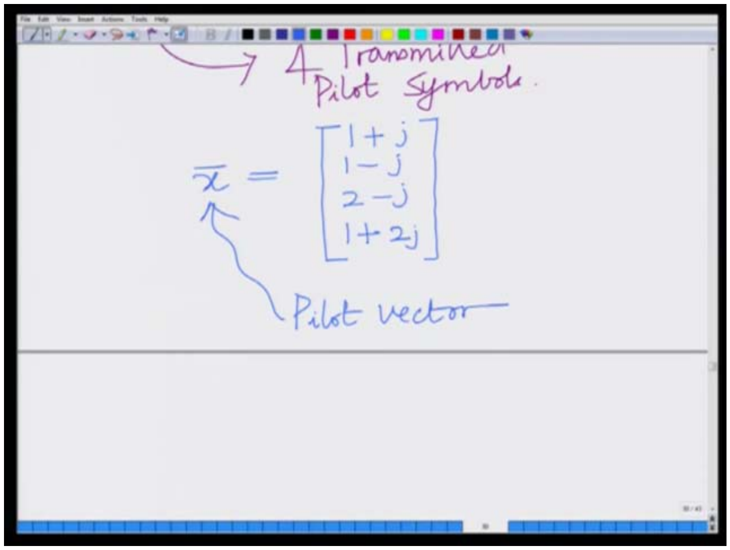
So, these are the transmitted pilot symbols, in fact we have 4 transmitted we have 4 transmitted pilot symbols that is X1, X2, X3, X4, these are the 4 transmitted pilot symbols on the downlink which are transmitted by the base station on the downlink, alright, for the purpose of channel estimation on the downlink, that is the channel, the downlink channel is estimated at the mobile. So, we have 4 transmitted by the symbol and in fact what we did is we stack these 4 pilot symbols as a pilot better.

(Refer Slide Time: 8:06)



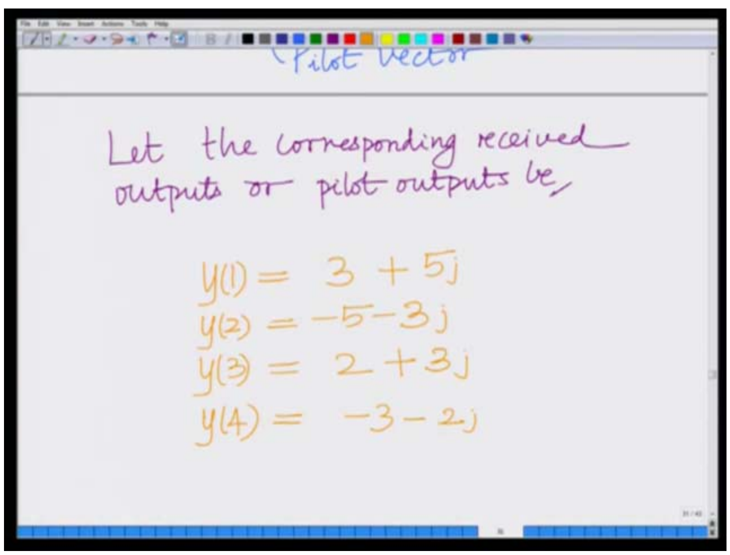
That is we have X bar which is if you remember your pilot vector.

(Refer Slide Time: 8:21)



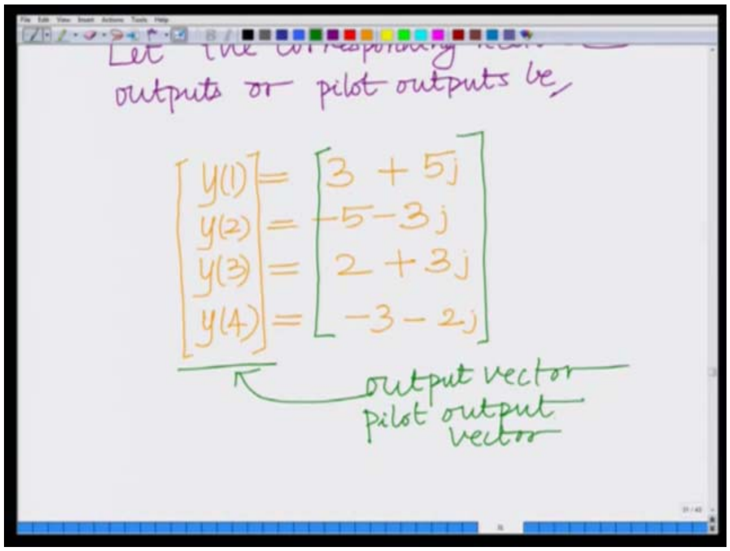
So, the pilot vector X bar is now given as 1+ J, 1 - J, 2 - J, 1+ 2J, these are the transmitted pilot symbols, this is basically the pilot vector. This is basically a pilot vector.

(Refer Slide Time: 8:54)



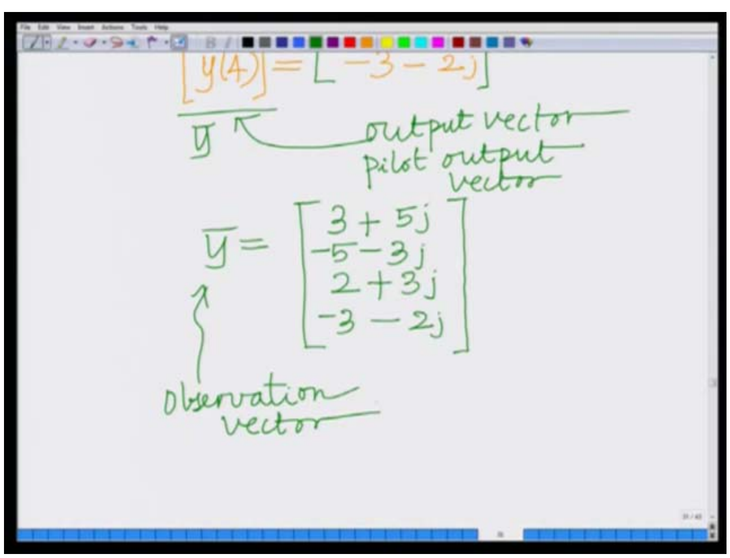
Now let the corresponding received symbols, let the corresponding received outputs or pilot outputs, we said the received outputs or the outputs corresponding to the transmitted pilot symbols which are also known as a pilot outputs are given as Y1 equals 3+ 5J, Y2 equals -5 - 3J, Y3 equals 2+ 3J and Y4 equals -3 - 2J. So, these are the corresponding, 4 corresponding pilot outputs, so what do we have, we have 4 transported pilot symbols X1, X2, X3 and X4 which we have stacked to form the pilot vector X bar corresponding to each of the pilot symbols transmitted pilot symbols on the downlink we have the corresponding received pilot output or the observed pilot output Y1, Y2, Y3, Y4 at the mobile on the downlink and now I can again stack these received pilot outputs as a vector to form the output pilot vector or the output symbol vector Y bar.

(Refer Slide Time: 10:35)



Okay, so now I am going to stack these pilot output Y1, Y2, Y3, Y4 and that gives me, that basically your output vector or basically your pilot output vector. Since these are the outputs corresponding to the pilot symbols, it is automatically understood that this is the pilot output vector Y bar, that is this is denoted by Y bar.

(Refer Slide Time: 10:58)

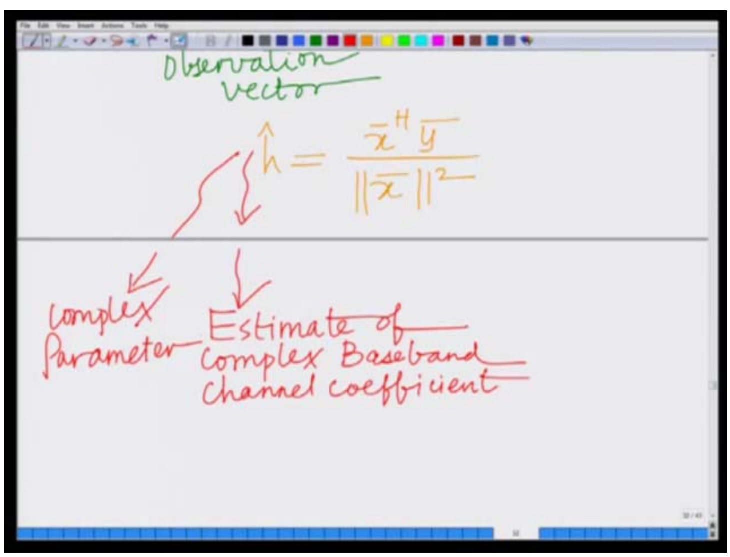


So, writing this explicitly my Y bar is going to be 3+ 5J -5 - 3J 2 + 3J and -3 - , -3 - 2J and this is basically your observation vector, output pilot vector, output vector, also your observation vector. You can call it by any name.

And now therefore we have the transmitted pilot vector X bar, we have the observation vector Y bar, alright. And notice that both these vectors have complex entries, therefore they are complex in nature. Therefore the scenario that we are considering is the estimation of a complex baseband channel coefficient. Remember the framework that we have developed, we said, can also be estimated to a realistic scenario by the channel coefficient that is H is a complex quantity and this arises because we are considering the complex, we are considering a complex baseband channel coefficient H, that basically represents the complex baseband transmission model of the wireless systems of the wireless channel between the transmitter which is the base station and the receiver which is the mobile for the downlink.

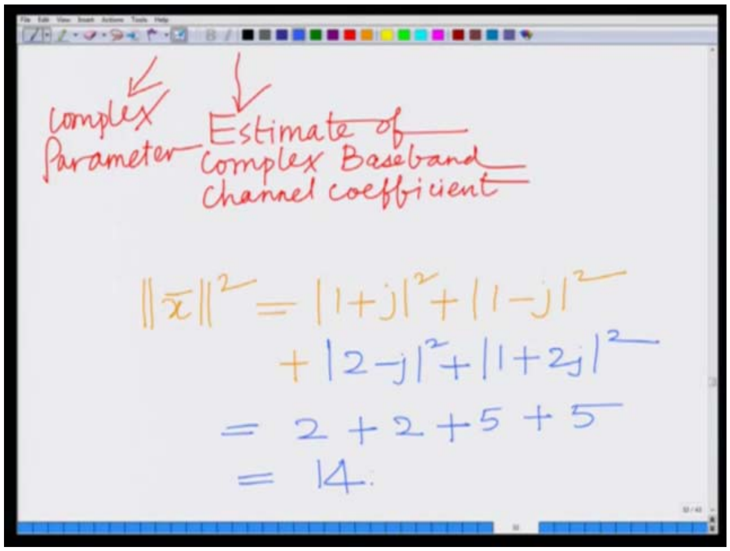
So we are considering a complex baseband transmission model and correspondingly we have the complex channel coefficient H and naturally we also have complex quantity such as the complex transmit pilot vector which is X bar, the complex output vector or the complex observation vector of Y bar and also complex additive white Gaussian noise at the receiver. So, we are considering the framework for the estimation of a complex baseband channel coefficient H. Okay.

(Refer Slide Time: 13:00)



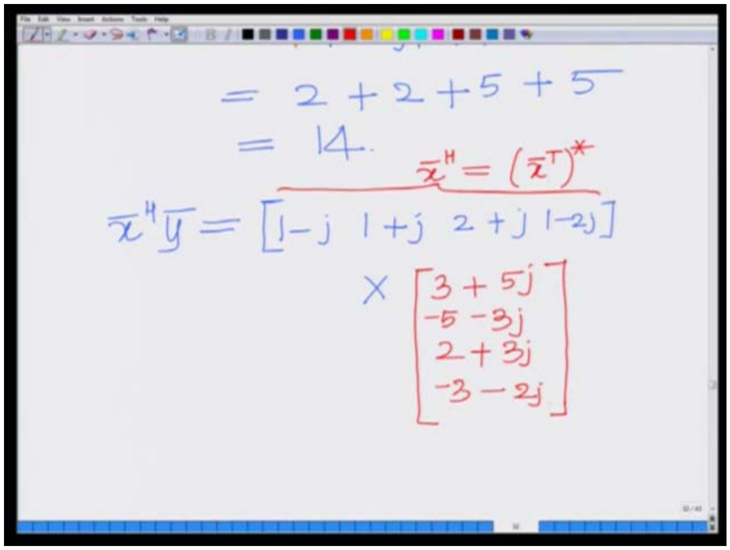
And as we have derived the expression, the estimate of the complex channel coefficient H hat equals X bar Hermitian Y bar divided by Norm X bar square. Alright. So this is the estimate of the complex, remember, this is the estimate of the complex baseband channel coefficient. This is the estimate of the complex baseband channel coefficient and this is a complex parameter. Naturally, this is a complex parameter and therefore let us compute the quantities one by one.

(Refer Slide Time: 13:59)



I have Norm X bar square which is the norm square of the pilot vector that is given as magnitude of 1+ J square + magnitude of 1 - J square + magnitude of 2 - J square + magnitude 1+ 2J square which is basically 2 +2+5+5 which is equal to 14, this is magnitude norm of a bar square that is we have derived the norm of a bar square which is a sum of the magnitude of, sum of the squares of the magnitudes of each of the components of this complex vector, this we have derived as basically equal to 14.

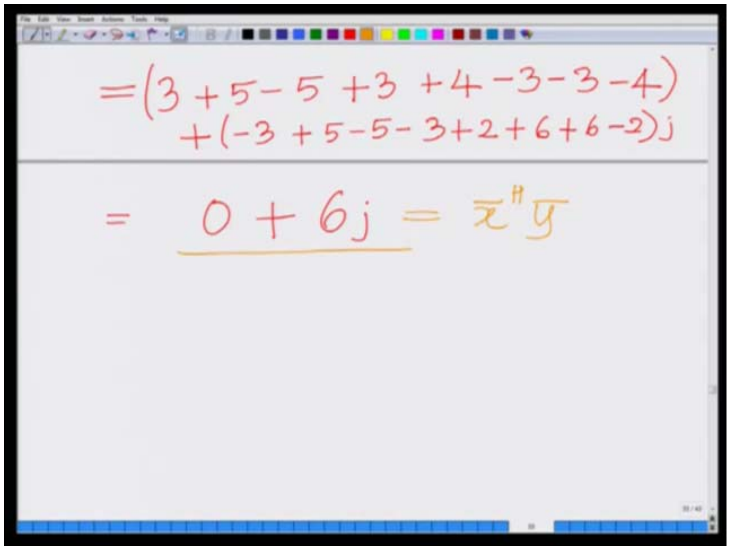
(Refer Slide Time: 14:52)



Now let us derive the other quantity which is X bar Hermitian Y bar which is the numerator, therefore X bar Hermitian Y bar, this is basically equal to the conjugate transpose of X which is 1 - J. That you take X bar the, take the transpose of the vector and take the conjugate of each element because X bar Hermitian denotes the conjugate transpose of the vector 2+ J and 1 - 2J times the product, that is this is X bar Hermitian, this is X bar Hermitian which is basically we said take X bar transpose and then take the conjugate and then take the X bar transpose, transpose of the vector and conjugate.

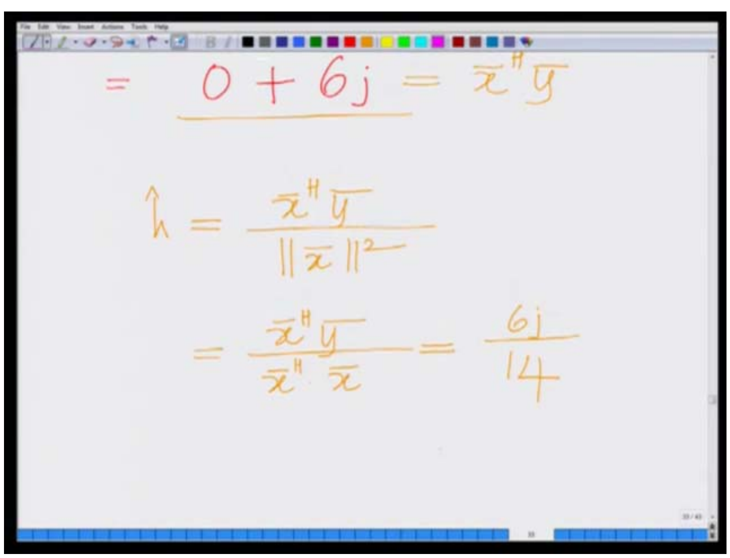
And then of course we have the observation vector Y bar which is 3+ 5J, 3 + 5J -5-3 - 3J, 2+ 3J -3 - 2J, which I can write as…

(Refer Slide Time: 16:08)



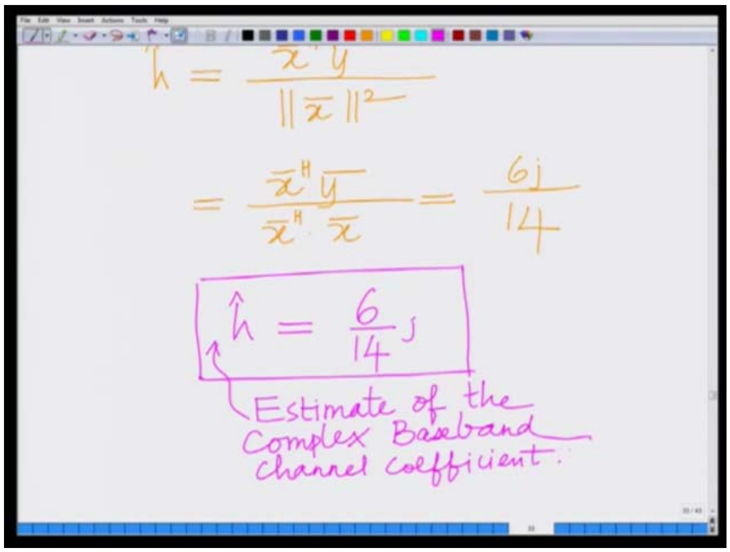
now taking the sums, well 1st I am going to write the real part, the real part turns out to be 3+5-5+3+4-3-3-4, this is the real part + the imaginary part which is -3+5-5-3+2+6+6-2 times J, you can check this through computation of X bar Hermitian Y and therefore this is equal to 0, real part is 0+ imaginary part 6J. So, this is basically X bar Hermitian, this is basically your X bar Hermitian Y bar and therefore now we have computed both the quantities, we have computed the numerator, that is X bar Hermitian Y bar and we have also computed the denominator, that is norm X bar square. Which is also basically X bar Hermitian X bar, remember, Norm X bar square is also X bar Hermitian X bar.

(Refer Slide Time: 17:21)



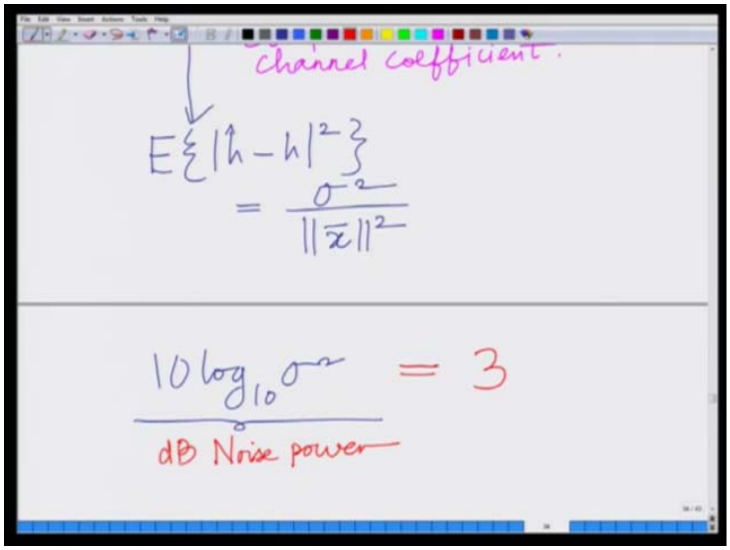
Therefore the estimate of the complex channel coefficient H hat equals X bar Hermitian Y bar divided by Norm X bar square which is basically equal to your X bar Hermitian Y bar divided by X bar Hermitian X bar and this is basically equal to, well X bar Hermitian Y bar equals 6J Norm X bar square is basically we have already calculated this, this is equal to 14, so basically the channel estimate **X** H hat is equal to, that is the complex baseband channel estimate H hat equals 6 divided by 14 times J.

(Refer Slide Time: 18:04)



This is the estimate of the complex baseband channel coefficient. This is the estimate of the complex baseband channel coefficient. So, we have derived the expression for the estimate, we have computed the estimate of the complex baseband channel coefficient and H hat equals X bar Hermitian Y bar divided by Norm X bar square which is equal to 6J divided by 14, therefore the estimate is 6 divided by 14 times J. And now let us also compute the the variance of the channel estimate,

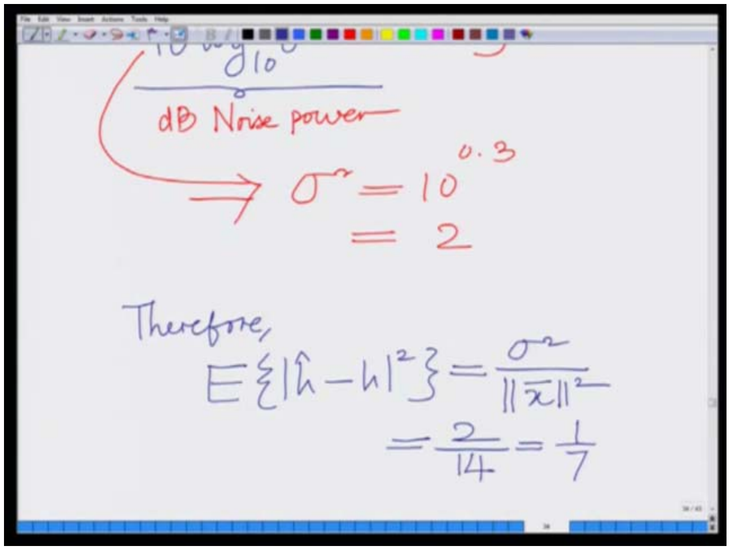
(Refer Slide Time: 19:27)



of course we know that the expected value of the general estimate is basically the true channel underlying underlying true unknown channel itself because we have said that the ML estimate, the ML channel estimate is unbiased.

Let us now compute the variance of this computed channel estimate and we have said that the variance of the general estimate, that is what we have computed the previous module, that is expected value of magnitude H hat - H whole square, that is average value of the square of the deviation, remember this is equal to Sigma square divided by Norm X bar square, it is already been given that the DB noise variance, that is 10 log 10 Sigma square, what is this, this is your DB noise variance. Noise variance or noise power DB noise power or DB noise variance equals 3 DB,

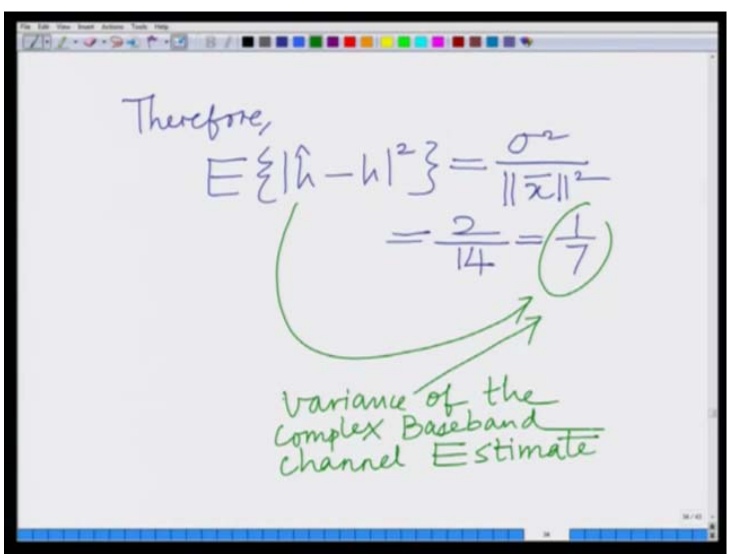
(Refer Slide Time: 20:09)



so 10 log to the base 10 Sigma square equals 3 which basically implies that Sigma square equals stand to the power of 0 .3 which is equal to 2.

So, the noise variance Sigma square equals 2 which means the variance of the channel estimate, therefore we have expected value of magnitude H hat - H whole square which is equal to Sigma square divided by Norm X bar square which is equal to 2 divided by 14, which is basically 1 divided by 7, so this quantity is basically 1 divided by 7.

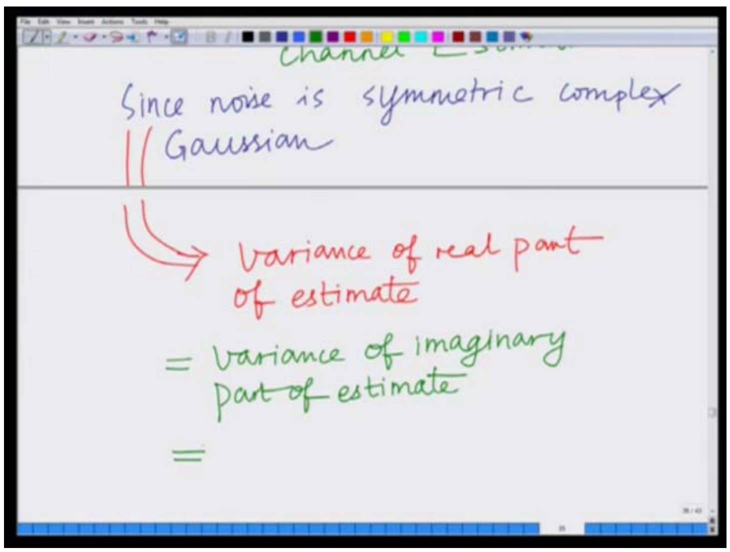
(Refer Slide Time: 21:06)



This quantity is basically the variance of the complex baseband channel estimate. This is variance of the complex baseband channel estimate. Further, so we have computed the variance of the complex baseband channel estimate and we are saying that is 1 over 7, further remember we are assuming 0 mean symmetric complex Gaussian noise of various 3 DB, that is of power 2. Which means because the noise is symmetric, a complex Gaussian symmetric, therefore it also means as we have proved in the previous modules that the estimate of the real and imaginary parts are basically uncorrelated, estimate that is errors in the estimate of the real and imaginary parts are uncorrelated. Alright.

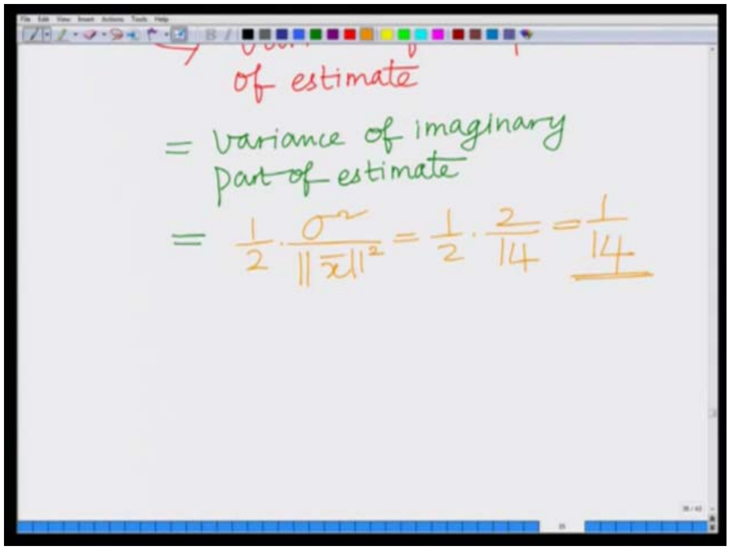
Also both of them have an equal variance which is half of the total variance of the complex parameter.

(Refer Slide Time: 22:15)



Therefore, since and finally this also implies, further we can say, since noise is symmetric complex**,** Gaussian, this also basically implies that you are variance of real part of the channel estimate is equal to the variance of the imaginary part, variance of the imaginary part of the estimate and that is equal to basically half of the total variance,

(Refer Slide Time: 23:19)



half Sigma square divided by Norm X bar square which is equal to half 2 divided by 14 equals 1 divided by 14, so basically this is the you are variance of the real and imaginary part.

So, what we have done, we have seen a simple example in which the variance of the estimate of the complex parameter, the net variance of the estimate of the complex parameter is basically 1 over 7 and the variance of each of the real and imaginary, the variances of each of the real and imaginary parts are equal, are equal to basically half of this net variance, that is half of basically 1 over 7, that is basically 1 by 14.

So, what we have seen in this module is we have seen a simple example of a channel estimation procedure in action when considering the downlink Wireless scenario with the base station is transmitting pilot symbols, the mobile is observing the corresponding pilot symbols, pilot outputs, with knowledge of the transmitted pilot symbols and also with the corresponding received pilot outputs, one can compute the channel estimate at at the receiver which in this case is the mobile which is receiving the symbols on the downlink.

Alright, so we have seen and in fact we are considering a complex baseband transmission model in which all the quantities are complex, that is the transmitted pilot symbols, the received pilot outputs and the noise samples at the receiver, all the quantities and also for importantly the channel coefficient H is a complex baseband channel coefficient.

So, corresponding to the simple example with the transmission of 4 pilot symbols and 4 correspondingly received pilot outputs, we have computed the channel estimate, that is estimate of the complex baseband channel coefficient H which is denoted by H hat and also not only that, more importantly we have characterised this computed estimate in terms of the variance. We have computed the variance of the estimate of this complex baseband channel coefficient and also the variances of each of the real and imaginary components of this complex baseband channel coefficient. So this example sort of comprehensively illustrates this process of channel estimation, the scheme for maximum likelihood channel estimation in a wireless system. So, we will stop this model here and continue with other aspects in subsequent modules. Thank you very much.