

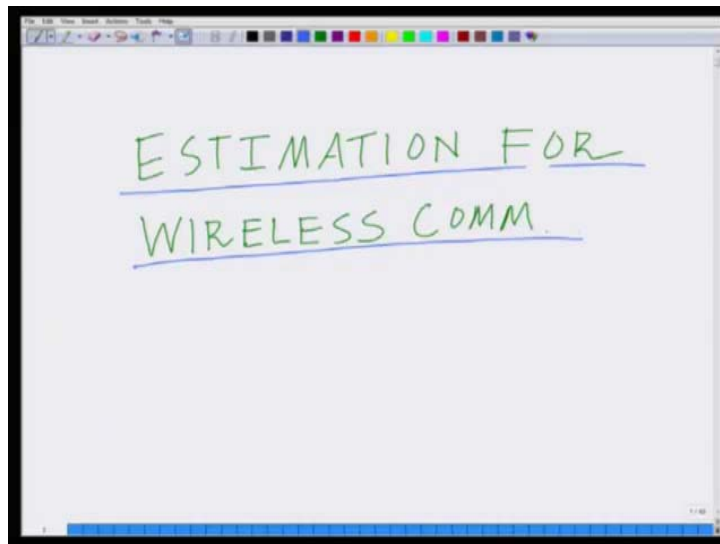
## **Estimation for Wireless Communication –MIMO/OFDM on Cellular and Sensor Networks**

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Lecture Number 1**

### **Basics- Sensor Network and Noisy Operation Model**

Hello, welcome to the first module in this NPTEL message open online course on Estimation for wireless communication. So this is a muke on Estimation or Estimation theory, so we are going to focus on on in this course is Estimation mode generally also know as Parameter Estimation for typically for wireless communication scenarios.

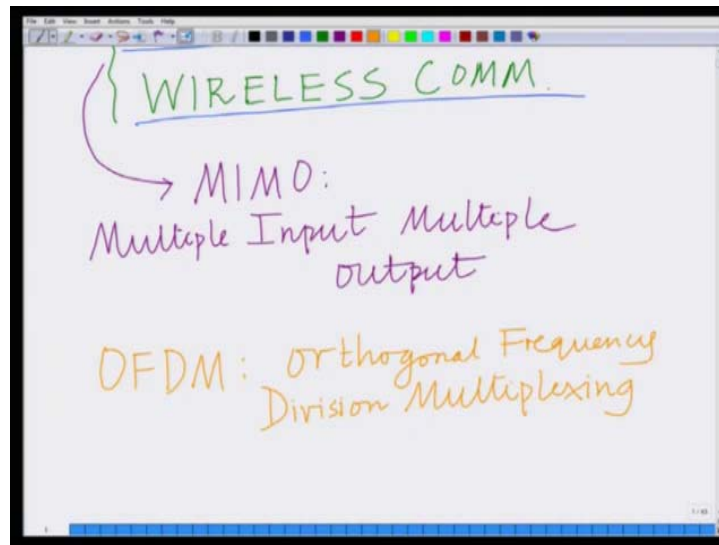
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For wireless indications right, so this is a massive open online course on Estimation theory and specifically estimation theory and as is applicable and applications of estimation theory and wireless communication wireless communication systems. And wireless communication, we are going to focus on are basically MIMO OFDM communication systems and also wireless sensor networks.

So this specific, so this the concept that we are going to focus on and applications that we are going to look at are going to be focused towards wireless communication system, MIMO OMDM wireless communication systems. Many of you must be familiar with this concept, this of MIMO, it stands for Multiple Input Multiple Output wireless communication systems.

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And also another important group of system is OFDM or Orthogonal Frequency Division MIMO OFDM systems, that is, Multiple Input Multiple Output and OFDM that is Orthogonal Frequency Division Multiplexing that is MIMO OFDM based wireless communication systems.

And also wireless sensor networks, we are going to look at applications of Estimation theory in wireless sensor network that is another important area where estimation, concepts of estimation are widely applicable, so we are going to look at Wireless Sensor networks.

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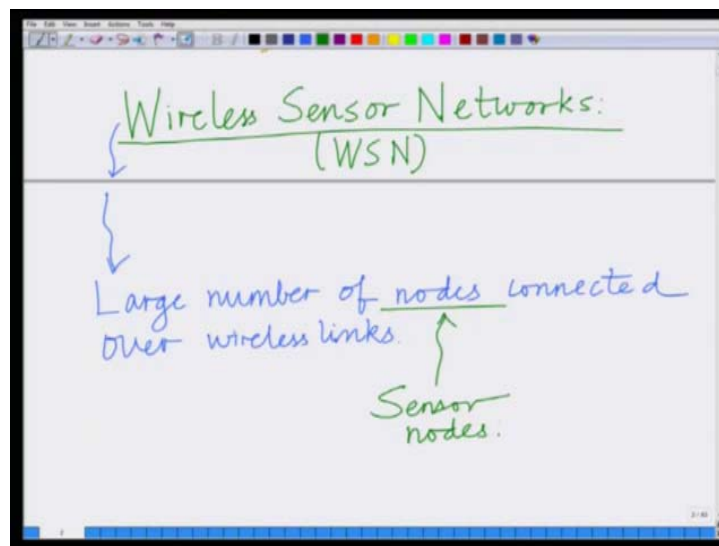


Or these are also basically abbreviated as W S N Wireless Sensor Network is basically group of sensor loads connected over wireless links that is these wireless sensor networks contains large number of sensor nodes which form a network and these sensor nodes are connected or can communicate over wireless link.

They typically estimate they typically measure various parameters such as temperature, pressure, humidity, et cetera and then these measurements are communicated over the wireless sensor nodes typically to a cluster head, typically to a fusion Centre or a cluster head from which they are disseminated to basically other devices and so wireless sensor networks have also found large number of applications for a various things such as health monitoring, et cetera.

And wireless sensor networks an important area in wireless communication, so basically wireless sensor networks consist of a large number of nodes; it consists of a large number of nodes connected over wireless links.

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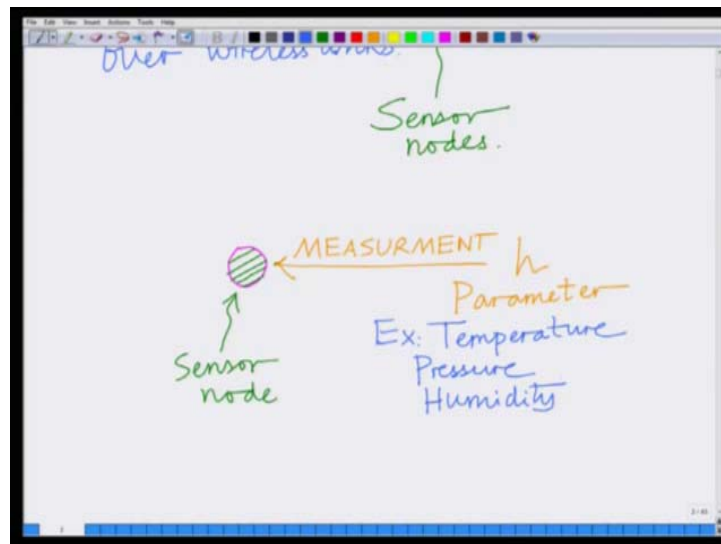
These nodes, these are also known as your sensor nodes. In the Sensor network, these are the sensing nodes and these are the Sensor nodes.

And Estimation theory, the concept of Estimation that we are going to have look at in this course have applications in both MIMO OFDM wireless communication systems and also wireless sensor networks which are currently evolving paradigms or current research areas, active areas of interest and current research in wireless communication.

So let us basically start looking at Estimation theory or concept of Estimation. To start with, let us consider a simple scenario in a wireless sensor network with a single sensor node. So let us start exploring this area of Estimation or Estimation theory by starting with what happens in a simple sensor network with a single sensor node.

So I have a sensor node, so let us say this is my sensor node which is sensing a phenomenal, this is your sensor node and each sensor node is basically what it is doing, it is sensing or it is sensing a phenomenal by making appropriate measurement.

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So this sensor node is sensing the phenomenal through making the measurement of a parameter which are going to denote by  $h$ , this  $h$  is also called, we are going to called this  $h$  as a parameter.

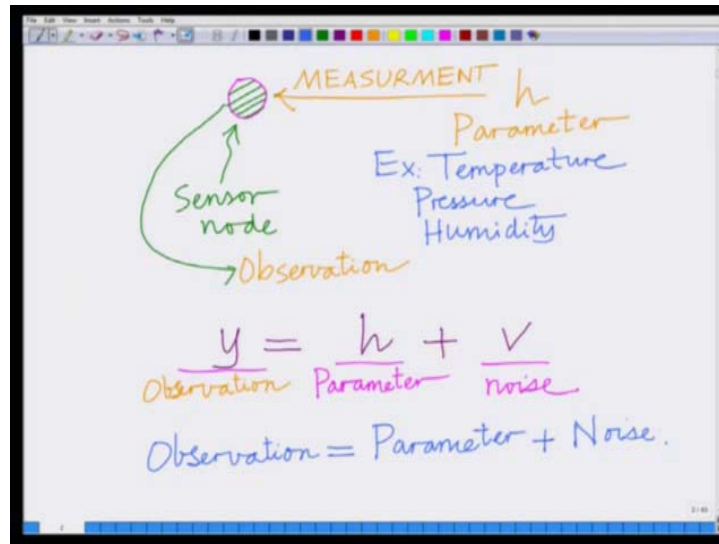
This is a parameter of interests and it can represent any component that is being sensed, any phenomenal that is being sensed, just for instance the ambient temperature, the pressure, the humidity, et cetera, so on, so this parameter  $h$  represents any phenomenal that is being sensed. Example, your temperature, pressure, humidity, atmospheric content of various gases, various pollutant and so on.

So this parameter that it is being sensed can be any particular parameter that is being sensed by the Sensor network, so the Sensor node is making a measurement of this parameter, that is  $h$  at these measurements and then subsequently processed and then can be sent to a Fusion Centre or then can be directly sent to a Fusion Centre, all right.

So this is denoted by the parameter  $h$ , so the Sensor node, this is making a measurement for what we will also call as an observation. So this is making an observation of the parameter  $h$ , now frequently these observations cannot measure the exact parameter  $h$ , but these observations are going to be affected by the noise, so these observations are typically noisy, so these are called the noisy observations.

So let us say, I denote my observations by  $y$ , so this is my observation  $y$ .

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The observation of the parameter  $h$  is typically going to be affected by the noise  $v$ , so this  $v$  represents your noise,  $h$  represent the parameter as we are already seeing and basically  $y$  is your observation which is also your noisy observation.

So the model that we have which we are going to employ throughout is that the observation equals parameter that is your parameter + noise. Your observation is basically a noisy observation and the observation is basically equals the parameter, that is noisy observation  $y$  equals the parameter  $h$  + the noise which is  $v$ .

We are going to use this model and basically the refined version of this model as we go through this course on Estimation for Wireless communication, so the observation is typically modeled as the parameter  $h$  in the presence of the noise  $v$ . And typically this noise  $v$ , this noise  $v$  is typically, this is Gaussian noise, typically this is Gaussian noise. Again a very common model for the noise that is the noise has is Gaussian in nature or normal.

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$$y = h + v$$

Observation = Parameter + Noise.

Gaussian noise

Gaussian which is normal that is the noise has a probability density function that is noise has the P D F that is Probability Density Function which is Gaussian or normal probability, this is the probability, density. Probability Density Function the P D F of this noise  $v$  is Gaussian and any Gaussian that is this noise  $v$  is basically a Gaussian random variable.

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Gaussian noise  
Normal

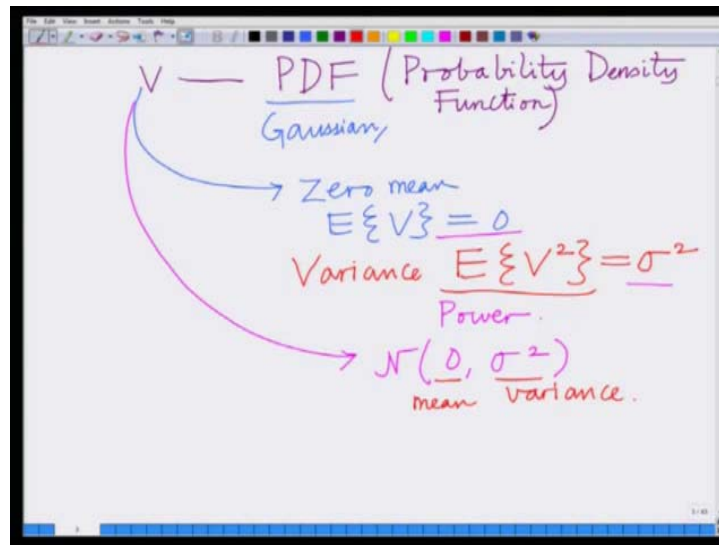
$v$  — PDF (Probability Density Function)  
Gaussian

→ Zero mean  
 $E\{v\} = 0$

Variance  $E\{v^2\} = \sigma^2$

And any Gaussian random variable is characterized by a mean and a variant and typically this noise is going to be, we are going to this as mean 0 and variant Sigma square. This noise we is 0 mean that is the expected value of  $v$  equal to 0.

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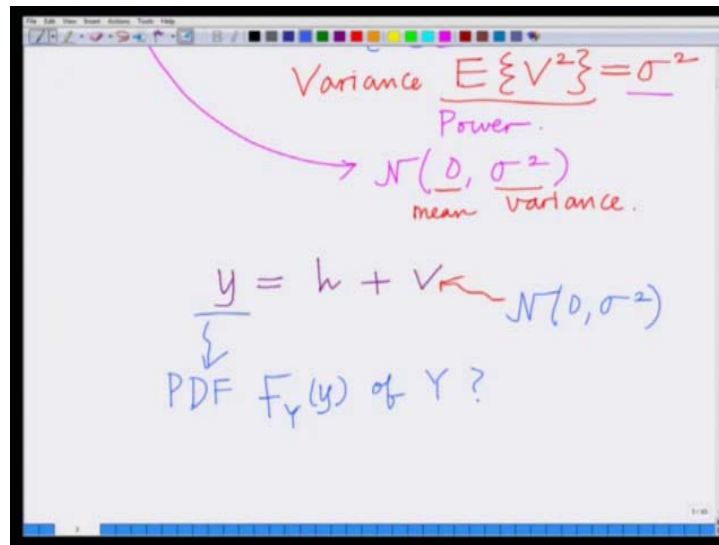


And further the variance of this noise that is expected  $v$  square equals Sigma square and since it is expected  $v$  square, this is also basically the power, the variance or the noise the variance or the noise power is Sigma [v] Sigma square. And therefore this is Gaussian with mean 0 and variance Sigma square, this is denoted as  $\mathcal{N}(0, \sigma^2)$  where basically this 0 denotes the mean, this Sigma square denotes the variance of the noise.

So what are we saying, we are saying we have this noisy observation model of the parameter  $h$ , the observation is  $y$  equals  $h + v$  and therefore and we also assume that this noise  $v$  is basically Gaussian noise that is the noise has a P D F or a Probability Density Function which is Gaussian with mean 0 at variance or basically power Sigma Square.

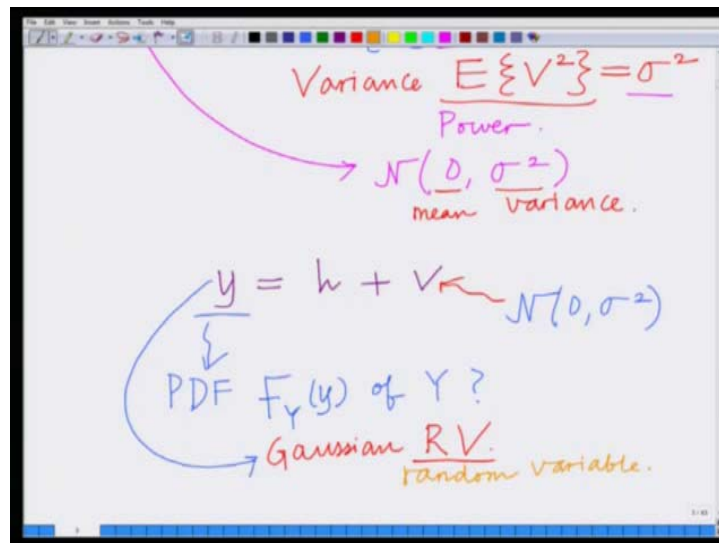
Now, one can ask the question, if the noise  $v$  is Gaussian, what is the Probability Density Function of the observation  $y$ ? So, we have  $y$  equals the parameter  $h + v$  and we said we are going to assume that this  $v$  is Gaussian noise with mean 0 and variance Sigma square. Now what is  $y$ , what is the Probability Density Function, what is the P D F,  $f_Y(y)$  of the random variable  $y$  where  $y$  is basically, recollect  $y$  is the observation.

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So what is the PDF or Probability Density Function of  $y$ ? Now you can see basically  $v$  is Gaussian and the parameter  $h$  is a constant, it is an unknown constant, therefore when  $v$  is Gaussian and you are adding a constant to a Gaussian, the output, the Probability Density Function of the output or the observation is going to remain Gaussian therefore,  $y$  also has a Gaussian, it is a Gaussian Random Variable.

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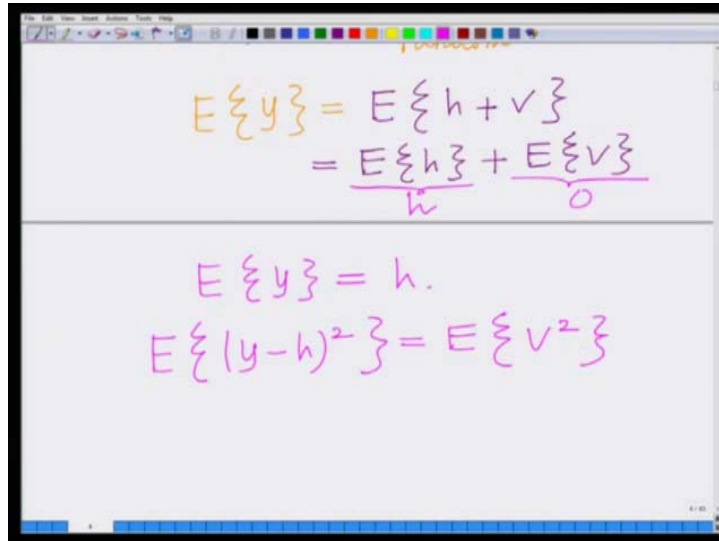


The observation  $y$  is a Gaussian Random Variable. I am going to denote this by  $r v$  stands for,  $r v$  stands for random variable, so output is a Gaussian random; output is a Gaussian random variable.



And what is the PDF, what is the mean of  $y$ , now look at this we have Gaussian noise  $v$  which is 0 mean with a constant  $h$  is being added, therefore the mean is shifted by the value of this constant which is  $h$  therefore, the mean of  $y$  is basically nothing but  $h$  because  $v$  the noise is 0 mean.

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$$\begin{aligned} E\{y\} &= E\{h + v\} \\ &= \underbrace{E\{h\}}_h + \underbrace{E\{v\}}_0 \\ E\{y\} &= h. \\ E\{(y-h)^2\} &= E\{v^2\} \end{aligned}$$

Therefore, the expected value of  $y$ , remember the expected value of  $y$  equals expected value of  $h + v$  which is  $y$  which is equal to the expected value of  $h +$  the expected value of  $v$ . We have already seen that this is basically 0 mean, so expected value of  $v$  is 0,  $h$  is a constant, so expected value is  $h$ . Therefore, we have expected value of  $y$  equals  $h$  yeah.

Now, what is the variance of  $y$  and that is also easy to see, the variance of  $y$  is the expected value of  $y - h$  that is the quantity  $y -$  its mean whole square that is  $y -$  the mean of the expected value for random variable  $y$  is expected value of  $y -$  the mean that is  $h$  whole square.

But  $y - h$  is again  $v$  that is nothing but the noise because  $y$  equals  $h + v$ ,  $y - h$  equals  $v$ , therefore,  $y - h$  equal to  $v$ , therefore expected value of  $y - h$  square is expected value of  $v$  square. But from above, we already know that noise variance is  $v$  Sigma square, which means expected value of  $v$  square equals Sigma square.

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The image shows a whiteboard with handwritten mathematical derivations in pink and green ink. At the top, there are some faint, partially visible equations:  $E\{y\} = h$  and  $E\{v\} = 0$ . The main derivation consists of the following steps:

$$E\{y\} = h.$$
$$E\{(y-h)^2\} = E\{v^2\}$$
$$= \sigma^2$$

The final result is enclosed in a green rectangular box:

$$Y \sim \mathcal{N}(h, \sigma^2).$$

Therefore, this is the variance of  $y$  is also Sigma therefore, the variance of  $y$  is basically also is basically a Sigma square. Therefore,  $y$ , we have  $y$  is also of Gaussian Random Variable. It is denoted by script  $n$ .

The mean is  $h$  and the variance is Sigma square and this is an interesting result that is the noisy observation  $y$ , which is basically the parameter  $h$  affected by the additive Gaussian noise, the noise  $v$  is additive and is Gaussian in nature with mean 0 and variant Sigma square.

The noisy observation  $y$  also has a Gaussian Probability Density Function, it is Gaussian in nature but its mean is  $h$ , where  $h$  is a parameter, the variance remains unchanged that is the variance is Sigma square and we know from the expression of from the knowledge of Probability and Random Variables, that Probability Density Function of a Gaussian Random Variable with mean  $h$  and variance Sigma square is given as...

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$$= \sigma^2$$
$$Y \sim \mathcal{N}(h, \sigma^2)$$
$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(y-h)^2}$$

PDF of Gaussian with mean  $h$ , variance  $\sigma^2$

That is this Probability Density Function  $f_Y(y)$  of  $y$  with mean  $h$  and variance  $\sigma^2$  is given as  $\frac{1}{\sqrt{2\pi\sigma^2}}$ ,  $e$  raised to  $-\frac{1}{2\sigma^2}(y-h)^2$ . That is the Probability Density Function of a Gaussian. Remember this is a Probability Density Function of a Gaussian, PDF of Gaussian with mean  $h$  and variance  $\sigma^2$ , so that is a Probability Density Function  $f_Y(y)$ .

Which is given as  $\frac{1}{\sqrt{2\pi\sigma^2}}$ ,  $e$  raised to  $-\frac{1}{2\sigma^2}(y-h)^2$ , that is the Probability Density Function of the observation  $y$  of the parameter  $h$  in the presence of that additive Gaussian noise  $v$  of mean 0 and variance  $\sigma^2$ . However, now we come to the framework of Estimation, now we come to the motivation for estimation.

Realize that this parameter, this mean which is  $h$  is unknown. This parameter  $h$  is unknown, this is the parameter, remember this parameter  $h$  corresponds to the temperature or pressure. This parameter  $h$  is unknown that is the parameter; let me write this clearly, the parameter  $h$  is unknown.

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$$Y \sim \mathcal{N}(h, \sigma^2)$$
$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(y-h)^2}$$

PDF of Gaussian with mean  $h$ , variance  $\sigma^2$

Parameter  $h$  is: Unknown

Therefore, we would like to estimate this parameter that is one would like to find the output of this parameter  $h$  that is what the idea behind the sensor network. That is we would like to know what is the ambient temperature or what is the ambient pressure therefore, one would like to estimate this unknown parameter  $h$  or one would like to compute this unknown parameter  $h$  that is known as Estimation of the unknown parameter.

So this is basically one would like to estimate, so basically we would like, we are motivated from this noisy observation. Estimate the unknown the unknown parameter  $h$  and this is known as this is basically known as, let me just write it in block letters, this is basically the central idea behind this course. This is basically known as **Parameter Estimation**.

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The image shows a whiteboard with handwritten mathematical notes. At the top, the probability density function (PDF) of a Gaussian distribution is written as  $f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}}$ . A blue arrow points from the text 'PDF of Gaussian with mean  $\mu$ , variance  $\sigma^2$ ' to the equation. Below this, it is noted that 'Parameter  $\mu$  is Unknown'. In green, the text 'Estimate the unknown parameter  $\mu$ ' is written, with a red arrow pointing to the word 'PARAMETER ESTIMATION.' written in purple at the bottom.

So these frameworks, where you are trying to estimate this unknown parameter  $h$  from the noisy observation that why remember  $y$  is a noisy observation of this parameter  $h$  that is made by the sensor. So trying to get estimate or trying to get knowledge of this unknown parameter  $h$  from the noisy observation.

This is basically the framework of parameter estimation and we are going to focus and basically this is going to be the focus of the rest of the modules in this course. That is how to estimate this parameter  $h$  that is estimation of this parameter edge from the noisy observations that is the have our noisy observations  $y$ .

How to estimate this parameter is from the noisy observation  $y$  that is going to be the focus that is going to be the focus for the rest of this course and this broadly, basically this broadly forms the framework or the area of parameter estimation and as we said in the beginning of this of this module that it has wide variety of...

It has wide application in basically both cellular network in the context of 3G and 4G third-generation and fourth-generation cellular networks which employ MIMO that is Multiple Input Multiple Output and OFDM Orthogonal Frequency Multiplexing Communication Systems.

And also wireless sensor networks which employ large number of sensor nodes. Basically which are measuring these different parameters, that is temperature, pressure, et cetera and

are communicating these measurements over the wireless links to either the cluster head or the fusion centre?

So this model basically sets up the initial model, the initial basic, the preliminary model for estimation which you are going to define, develop and look at different estimation different models as well as different techniques strategies for estimation of these parameters and as well as the performance of the various schemes that we are going to use for estimation in wireless communication.

Both in Wireless Cell Network and also in Wireless Sensor Network, so let us stop this module over here. We will continue other aspects in the subsequent modules. Thank you very much.