Probability and Random Variables/Processes for Wireless Communications.
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Lecture -17.
Random Processes and Wide Sense Stationary (WSS).

And now let us start looking at another new concept i.e. random process. So, today's module, let us start looking at random processes.

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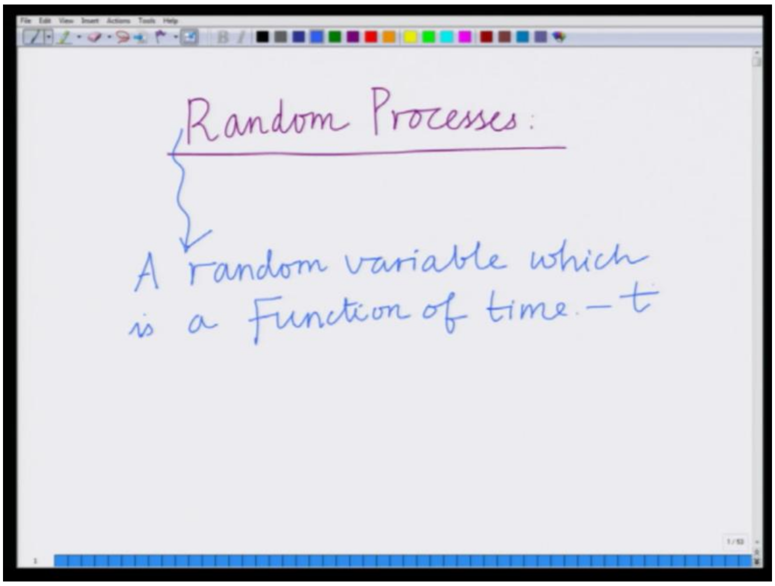

So, we are going to start looking at what is a random process? A random process is a random variable which is a function of time. If we consider a random variable, which is a function of time, i.e. now we are incorporating this time dimension into the random variable, so random variable as a function of time is a random process.

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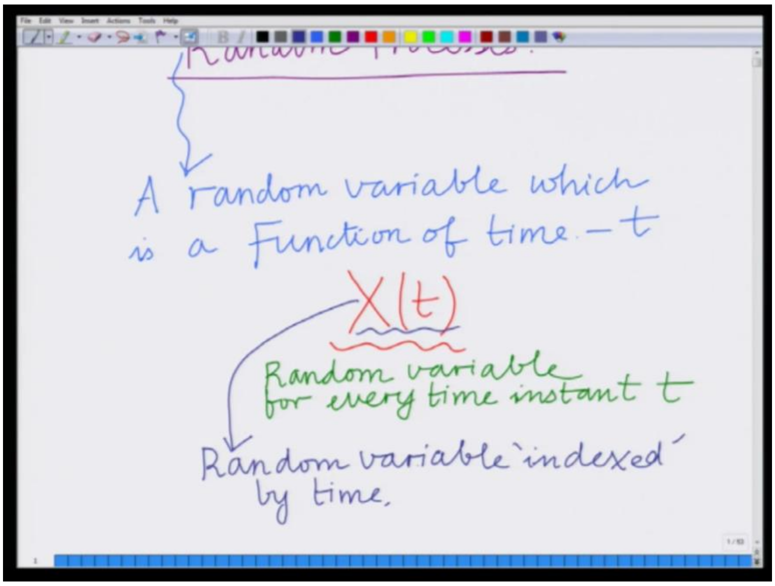

So, random variable as a function of time for instance X is a random variable andtis the times, i.e. X(t) is a random variable, this is what is X(t), X(t) is a random variable, say different random variable for every instant T.

I.e., X(t) is a random variable at every time instant T. This is also known as a random variable which is indexed by time, i.e. random variable which is indexed by time. So, this time is also known as the indexing set. So, this is known as the random variable indexed by time, i.e. a random process. A random variable indexed by time i.e. known as a random variable.

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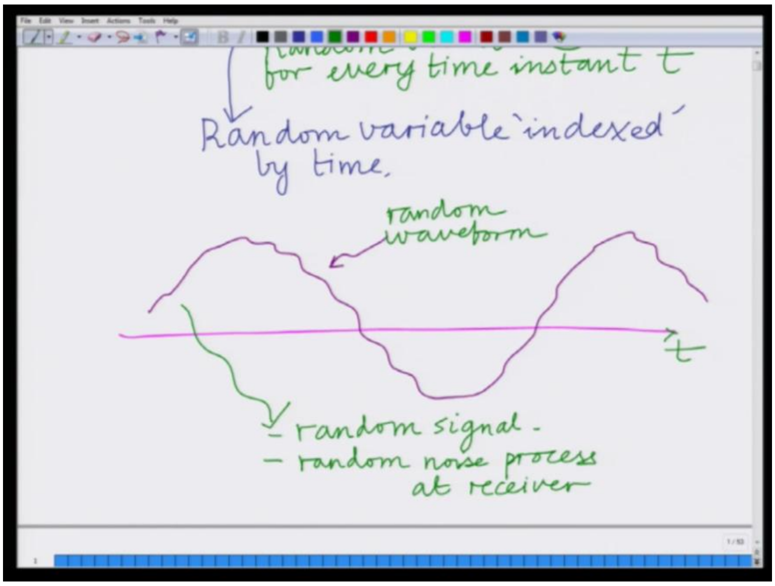

And for instance, we have several random processes, a typical random process is just a random variable i.e. a function of time that looks something like this. So, this is a random for instance, we have here a random waveform and on this axis over here, I have time.

So, I have a random variable which is a function of time, i.e. a random process and random process as you can see is very convenient to handle a large number of random phenomena, especially in wireless communication such as random signals or even the noise process at the receiver which is random. These are variables but at the same time, these are function of time, therefore these are random processes. So, random process like this can be used to model random signal or the random noise process or can be used to model the random noise process at the receiver.

Now what about the probability density function for this random process? Random variable, remember random process is a random variable at every instant of time. So, corresponding to every instant of time, there is going to be a probability density function, so there will be a probability density function which is also therefore indexed by the time.

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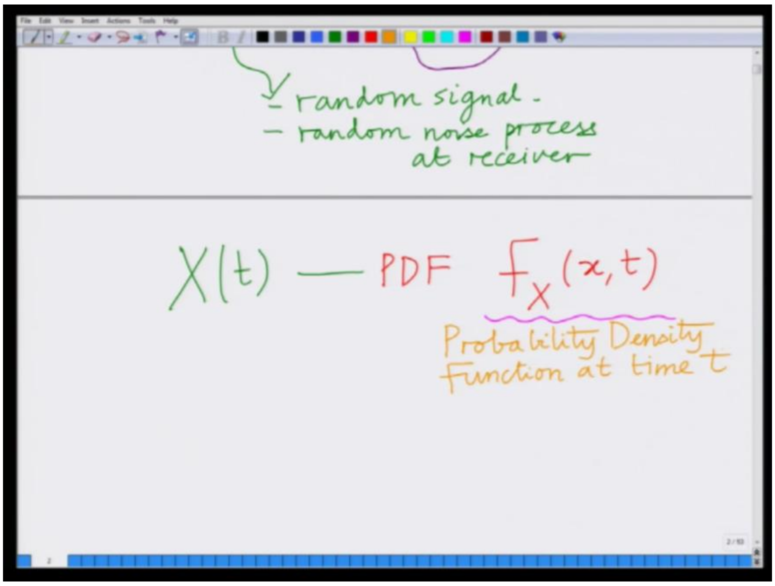

So, random variable X(t) is characterized by the probability density function, by the PDF, probability density function as

$$F_X(x,t)$$

which is the probability density function of this random variable X indexed by time because now random process, because this is a function of time. So, this is the probability density function of X, so what does $F_X(x,t)$ denote?

This denotes the probability density function of the random process, at the time instant. So, we will now have a probability density function at every time instant.

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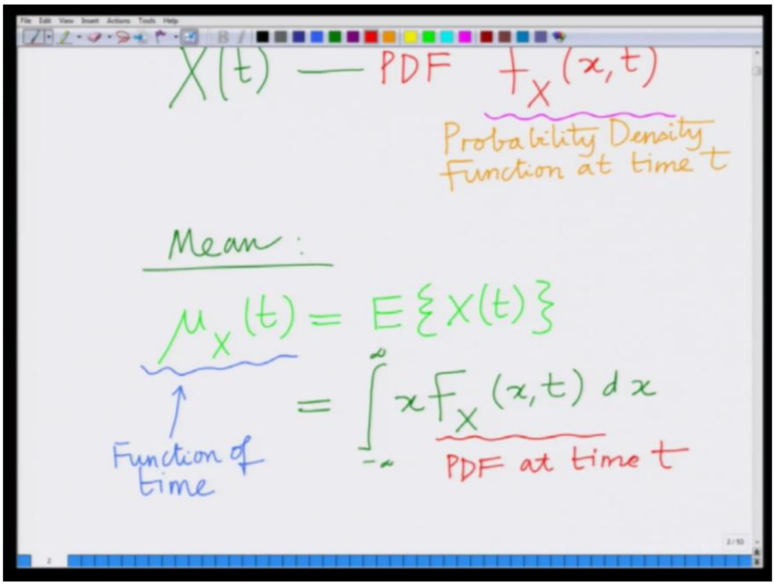

And similarly, therefore now we can define also this statistical parameter such as the mean. Right? The mean, variance etc. but they will also now be functions of the time 't' because since this is a random process, which basically is a function of time. So, one can define the statistical parameters, i.e. a mean, the mean of this random process the mean or the average value of the random process, what is this?

This is basically your expected value, the expected value, so we can write, denote this by

$$\mu_X(t)$$

which will now be a function of time which is nothing but the expected value of this random process X(t) which is basically

$$E\{X(t)\}$$

$$i.e., \mu_X(t) = E\{X(t)\}$$

$$= \int_{-\infty}^{\infty} x\, F_X(x,t)dx$$

$$i.e., \quad \mu_X(t) = \int_{-\infty}^{\infty} x\, F_X(x,t)dx$$

Where, $F_X(x)$ is the probability density function, however now we have the probability density function which additionally is a function of time.

So, we are simply multiplying x by the probability density function which is a function of time and integrating from minus infinity to infinity, so naturally the quantity that you obtain will be a function of time and i.e. the mean which is a function of time i.e. denoted by $\mu_X(t)$.

So, this is our mean and observe that this is a function of time. Since there is a different probability density function, at every time instant T.

So, now let us look at another important property of the random process, which is also known as the autocorrelation.

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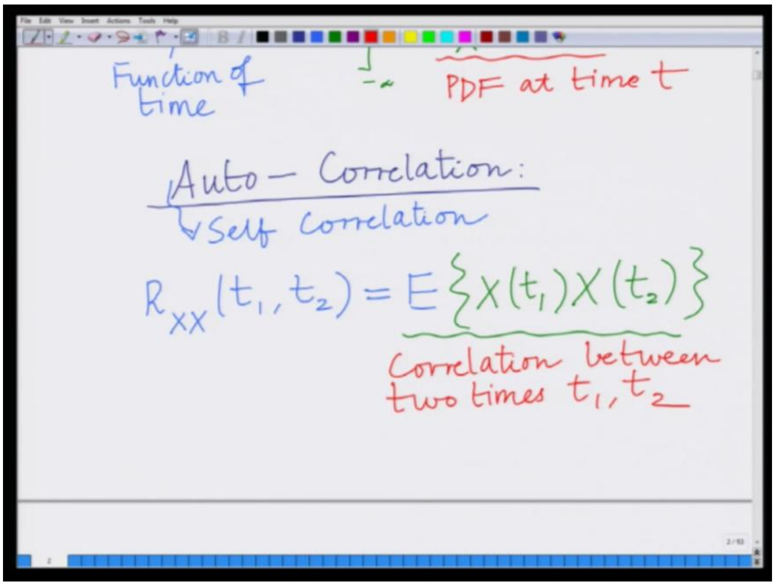

Auto also means self. So this is basically the correlation of the random process X(t) with itself. Thus it is known as autocorrelation. The autocorrelation or the correlation of the random process at 2 time instants T_1, T_2, is equal to

$$R_{XX}(t_1, t_2) = E\{X(t_1)X(t_2)\}$$

So, autocorrelation is the self correlation of the random process corresponding to 2 different time instances, i.e. if you look at the random process $X(t_1)$ at time instant t_1 and $X(t_2)$ at time instant t_2 and we asked what is the, what is the co-relation between these 2, i.e. basically the autocorrelation. I.e. the average value of the product $X(t_1).X(t_2)$ naturally for a general random process, this is going to be a function of both the time instants, i.e. t_1, t_2 and therefore this is denoted by $R_{XX}(t_1, t_2)$.

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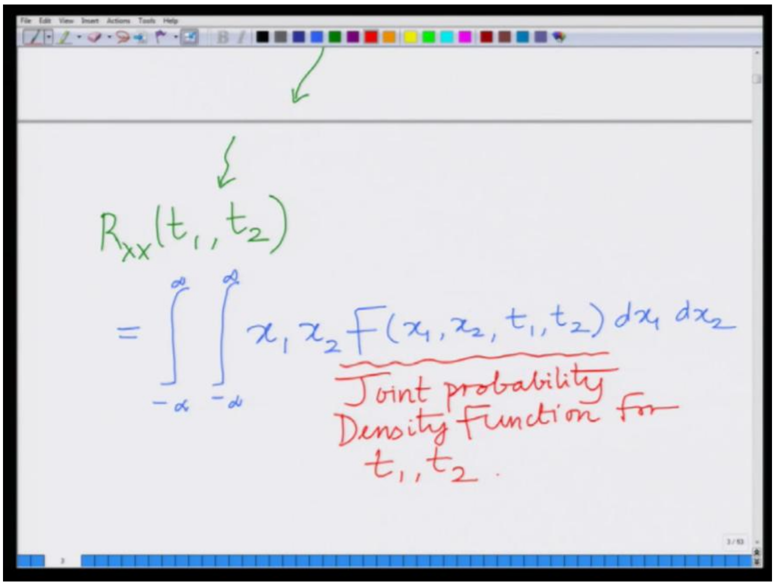

This autocorrelation is defined as

$$R_{XX}(t_1, t_2) = \iint_{-\infty}^{\infty} x_1 x_2 F(x_1, x_2, t_1, t_2) dx_1\, dx_2$$

Here,

$F(x_1, x_2, t_1, t_2)$ *is the joint probability density function for* t_1, t_2

So, we are looking at $F(x_1, x_2, t_1, t_2)$. Remember $F_X(t)$ is the probability density function of this random process at time instant t. $F(x_1, x_2, t_1, t_2)$ is the joint probability density function of this random process corresponding to time instants t_1, t_2. And we are multiplying this by x_1 *and* x_2 and integrating it over a range minus infinity to infinity over both the integrating variables i.e. x_1 *and* x_2, this is basically the autocorrelation, i.e. the self correlation of the random process corresponding to time instants t_1, t_2.

Alright, this is the definition of the autocorrelation, this is the joint probability density function corresponding to t_1, t_2.

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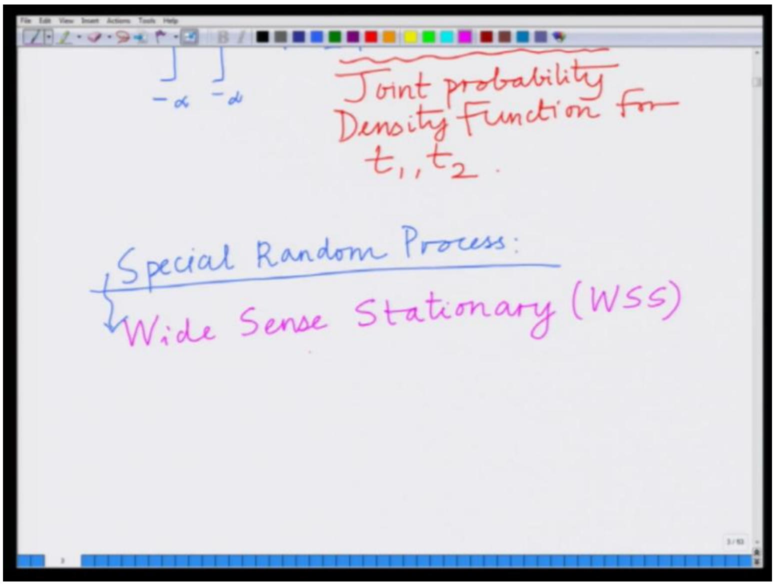

Let us now consider a very special kind of random process which will come in handy when we talk about wireless and especially communications system. So, we can also call this as a subclass of the general class of random processes which we term as a wide sense stationary WSS. So this a WSS random process. Let us now consider a specific subclass of the general class of random process which we term as wide sense stationary or WSS random process.

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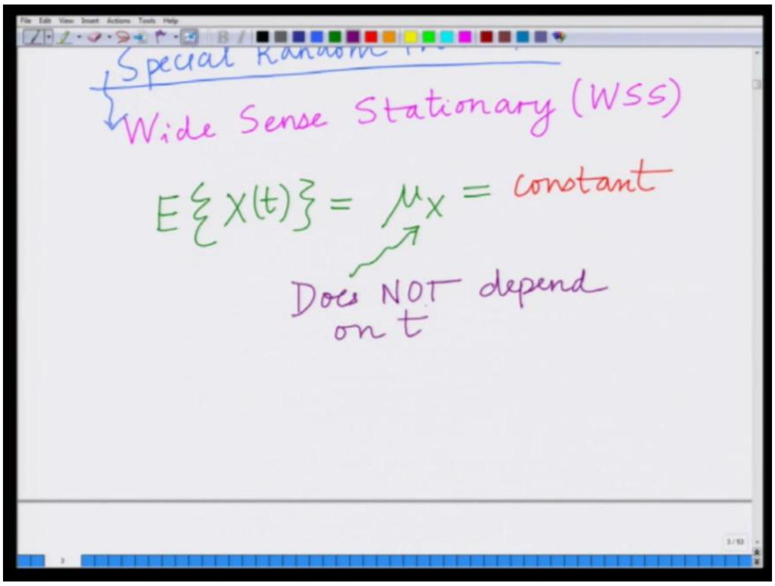

Now what is a wide sense stationary random process? For a wide sense stationary random process, we must have the expected value of the random process i.e. that average,

$$E\{X(t)\} = \mu_X = constant$$

i.e. μ_X does not depend on time.

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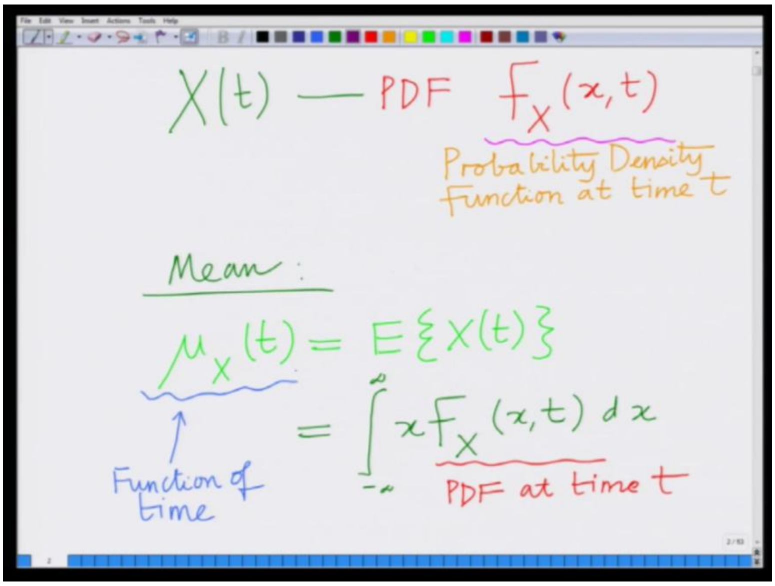

As you can clearly see in the general definition the main $\mu_X(t)$ of the random process depends on the time t. I.e. we said random process is a random variable for every time instant t, therefore naturally, the mean can also be a function of the time instant t.

However if that mean is specifically for a wide sense stationary process that means that average i.e. the expected value of the random process of X(t) is μ_X, i.e. for every time instant, irrespective of time instant t, it is a constant and this is basically a prerequisite. This is one of the required properties of a wide sense stationary property. And now you can see where nomenclature stationary comes from, stationary comes from the fact that the mean is the constant, it is not varying, it is stationary, this random process is also said to be stationary in the mean.

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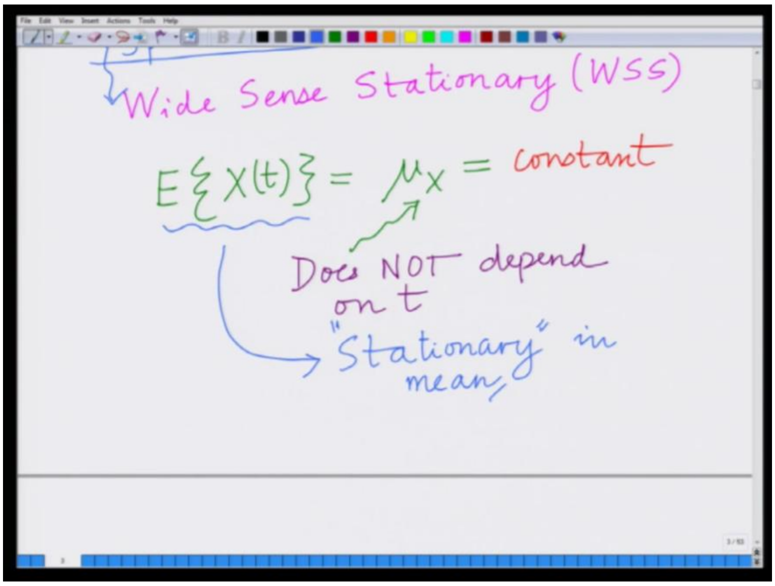

So, one can also say that such a random process has to be stationary, this is stationary in the mean in the sense that this random process, the mean of this random process is not changing with time, it is a constant or it is stationary with time. Now let us look at the autocorrelation for this wide sense stationary process.

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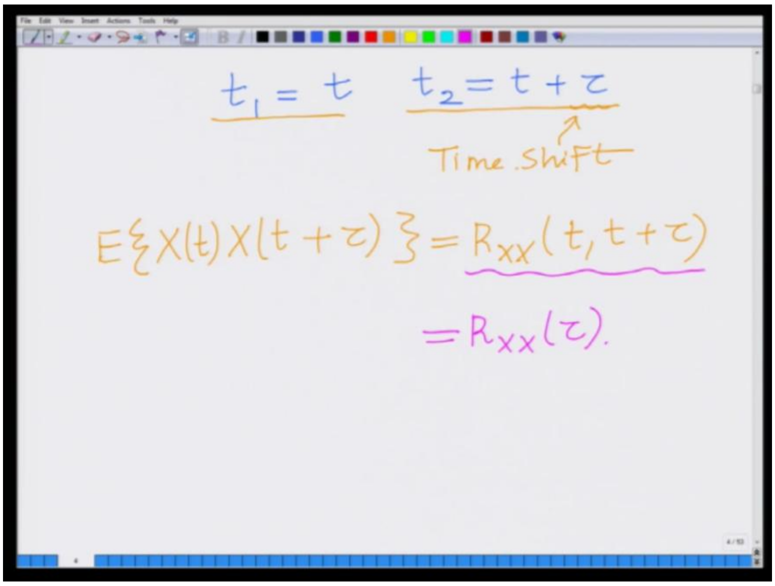

If we look at the autocorrelation, i.e. let us consider

$$t_1 = t,$$

$$t_2 = t + \tau$$

$$where, \tau\ is\ the\ time\ shift$$

And if we look at the autocorrelation of the signal corresponding to these 2 instants, we have

$$E\{X(t)X(t+\tau)\} = R_{XX}(t, t+\tau)$$

$$and\ for\ a\ wide\ sense\ stationary\ process, that\ should\ further\ be\ equal\ to$$

$$= R_{XX}(\tau)$$

i.e. it only depends on the time difference τ, i.e. autocorrelation only depends on the time difference τ and not the specific time instants $t_1\ and\ t_2$

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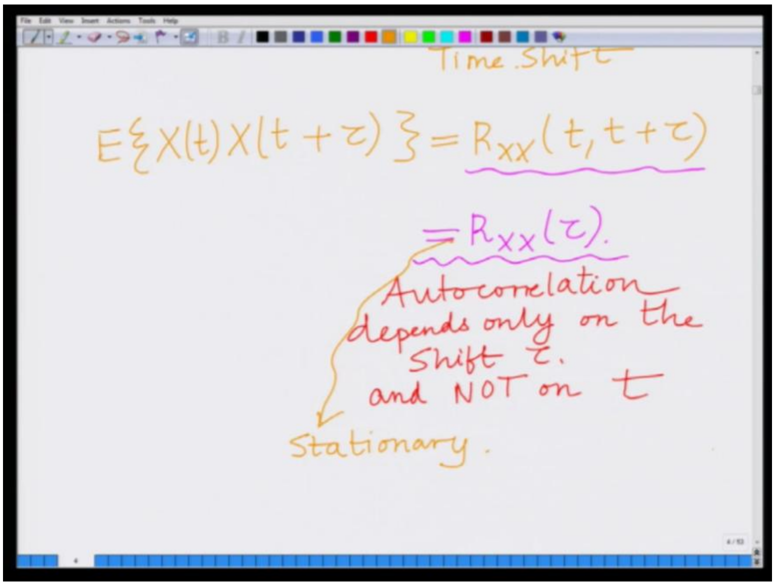

Alright, so, for a wide sense stationary process R_{XX} autocorrelation depends only on the shift τ and not on t_1, t_2 or the specific time instants, therefore this is stationary. Basically one can say this autocorrelation is stationary. Alright, so what are we saying, for a wide sense stationary process the autocorrelation, it has to be stationary in both the mean and it has to be stationary in the autocorrelation.

So, there is such a process which is stationary in both the mean and the autocorrelation is known as a wide sense stationary random process. So, basically, in this module, what have we seen, we have looked at the various properties of random we have looked at the definition of random process, we said random process is a random variable which is a function of time, we have defined the concept of a mean of a general random process and the autocorrelation of the general random process and we have also seen for a special kind of random process, the specific subclass of random process which is wide sense stationary process, the mean has to be constant, does not depend on the time and the autocorrelation at time instants and $t + \tau$ depends only on the time shift τ and does not depend on the specific time instant t. So, basically with this let us conclude this module, we look at the other aspects in the subsequent models. Thank you very much.