Probability and Random Variables/Processes for Wireless Communications.
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Lecture -15.
Special Case: IID Gaussian Random Variables.

Hello, welcome to another module in this massive open online course on probability and random variables for wireless communication. So, in the previous module, we started looking at Gaussian random variables and the various key properties of Gaussian random variables. One of the important properties of the Gaussian random variable we said is the following thing that is,

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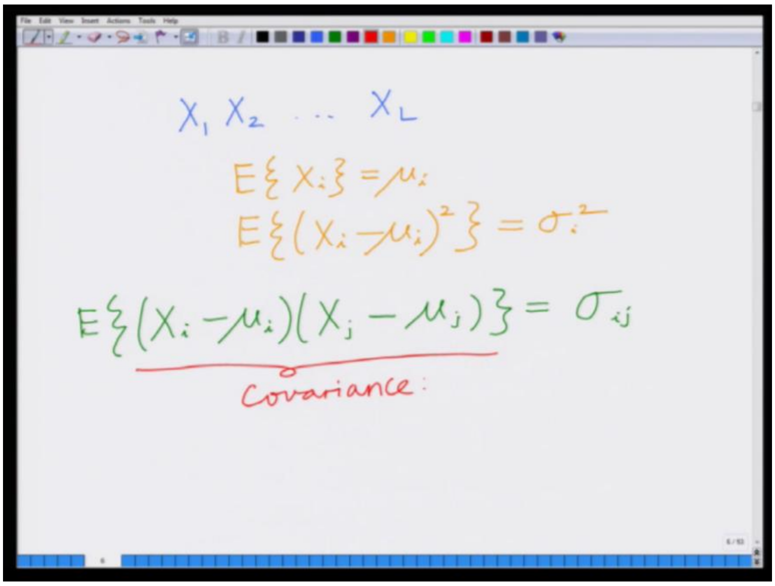

if I have L Gaussian random variables- $\{X_1, X_2, \ldots, X_L\}$ are Gaussian random variables with

$E(X_i) = \mu_i$, and,

$$variance = \sigma_{X_i}^2 = E\{(x_i - \mu_i)^2\}$$

Further if I look at the covariance,

$$Covariance = \sigma_{ij} = E\{(X_i - \mu_i)(X_j - \mu_j)\}$$

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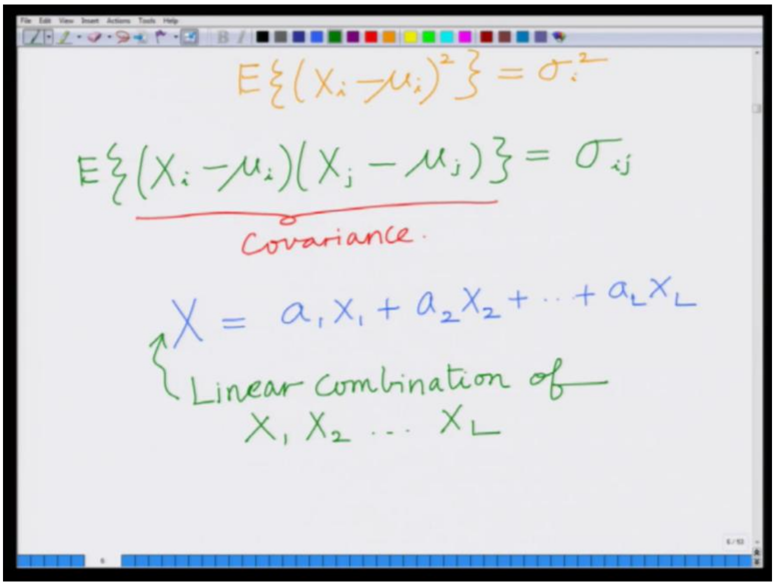

Now, we said if I generate a new random variable X which is generated as a linear combination of these Gaussian random variables as-

$$X = a_1X_1 + a_2X_2 + \ldots + a_LX_L$$

then we said this X is a Gaussian random variable.

This Gaussian random variable which is generated as a linear combination of a group of random variables is in turn a gaussian random variable and we also calculated the mean and the variance of this new Gaussian random variable.

$$X = N(\mu_X, \sigma_X^2)$$

Where,

$$\mu = E\{X\} = \sum_{i=1}^{L} a_i\mu_i$$

And,

$$\sigma_X^2 = \sum_i a_i^2 E\{X_i - \mu_i\}^2 + \sum\sum a_i a_j \sigma_{ij}$$

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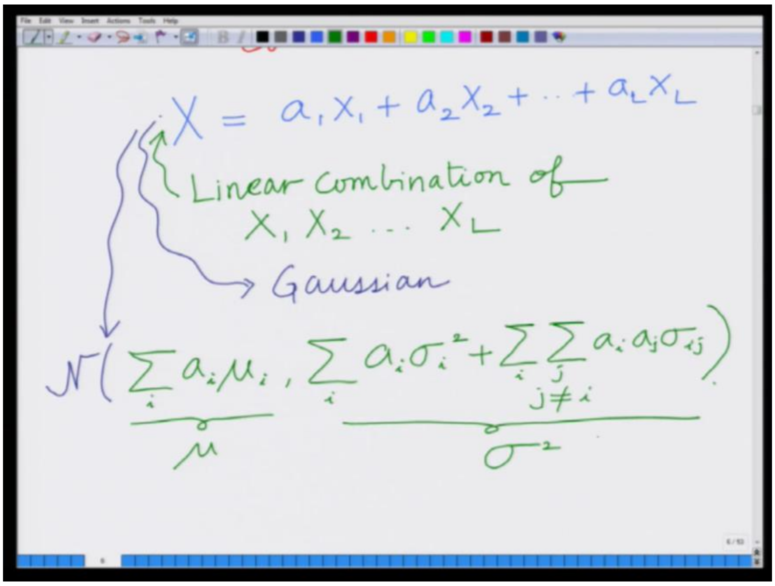

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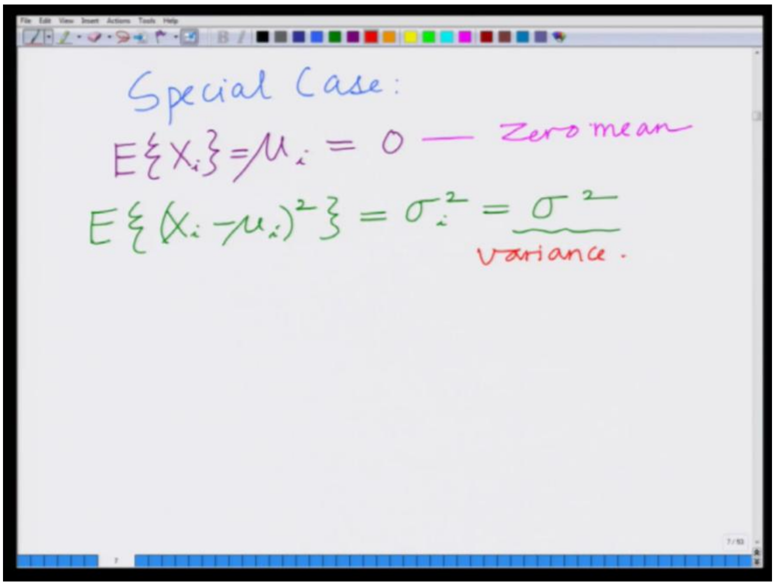

Let us now consider a special case of this linear combination, when

Mean= $E(X_i) = \mu_i = 0$ and,

$$variance = \sigma_{X_i}^2 = E\{(x_i - \mu_i)^2\} = \sigma^2$$

Thus, all the Gaussian random variables have identical mean and in fact the mean is identically equal to 0. So, all the Gaussian random variables $\{X_1, X_2, \ldots X_L\}$ are Identical, i.e.

$$X_i = N(0, \sigma^2), \quad i = 1,2, \ldots, L$$

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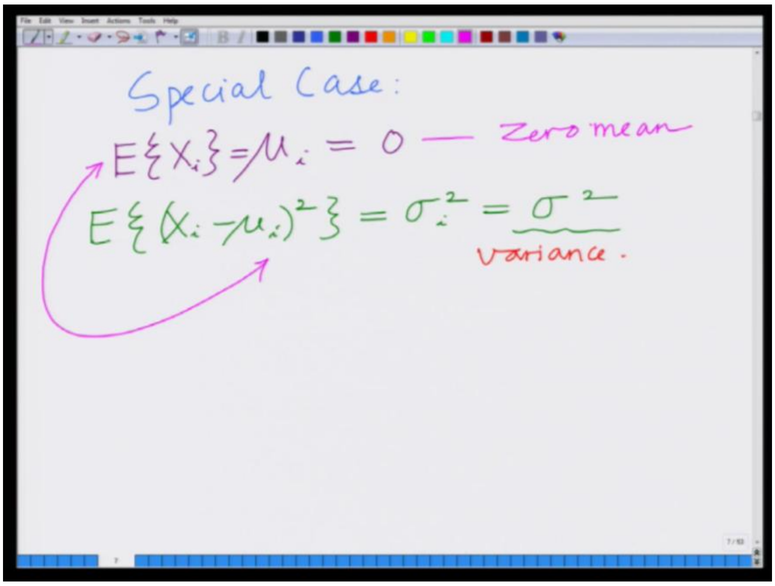

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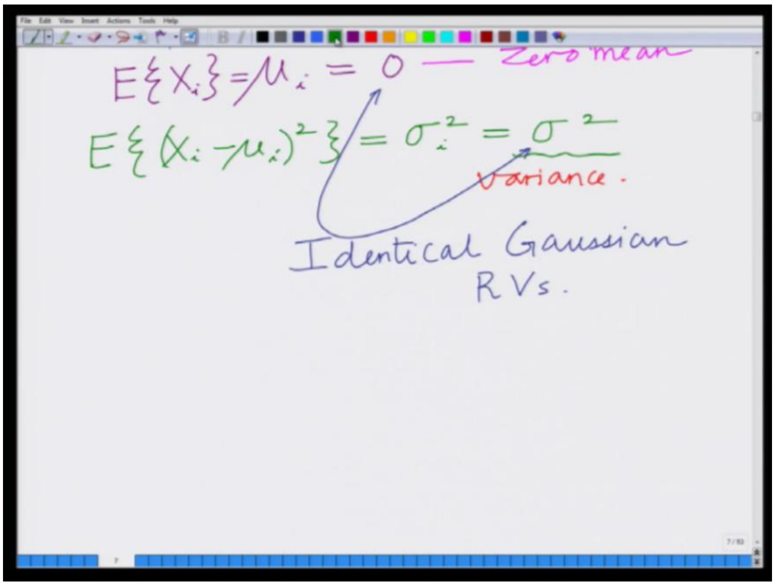

So, these are bunch or a group of identical Gaussian random variables.

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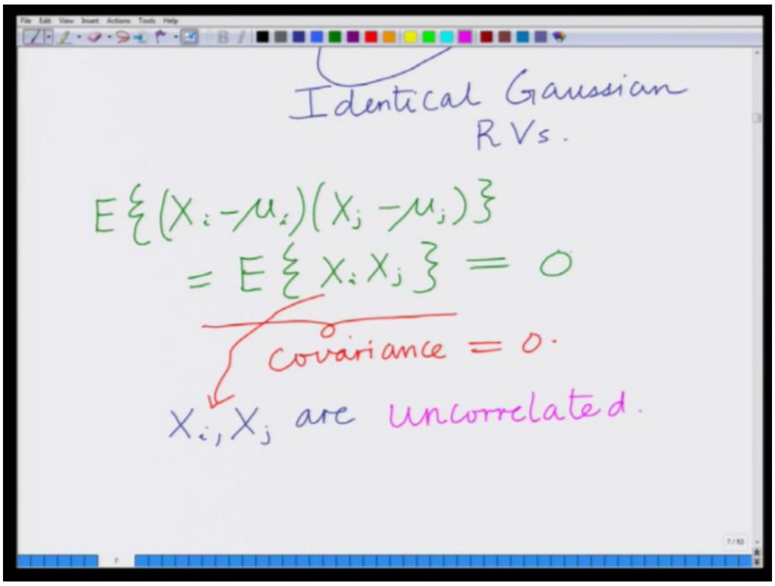

Further we are also going to assume that the covariance,

$$Covariance = \sigma_{ij} = E\{(X_i - \mu_i)(X_j - \mu_j)\} = 0$$

Such random variables are known as uncorrelated random variables, which means the covariance of 2 random variables is 0. So, we are assuming that all Gaussian random variables are identical and further, they are uncorrelated, and specifically for the case of Gaussian random variable, uncorrelated also implies independence.

This is not true for any random variable but specifically for the Gaussian random variable, the property of uncorrelated random variables imply that they are independent.

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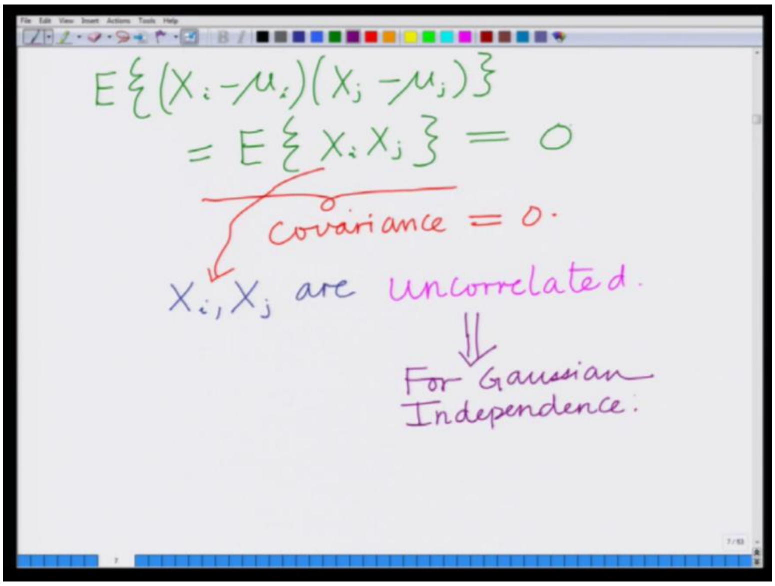

So, therefore we are considering a group of L independent and identically distributed Gaussian random variables $\{X_1, X_2, \ldots, X_L\}$.

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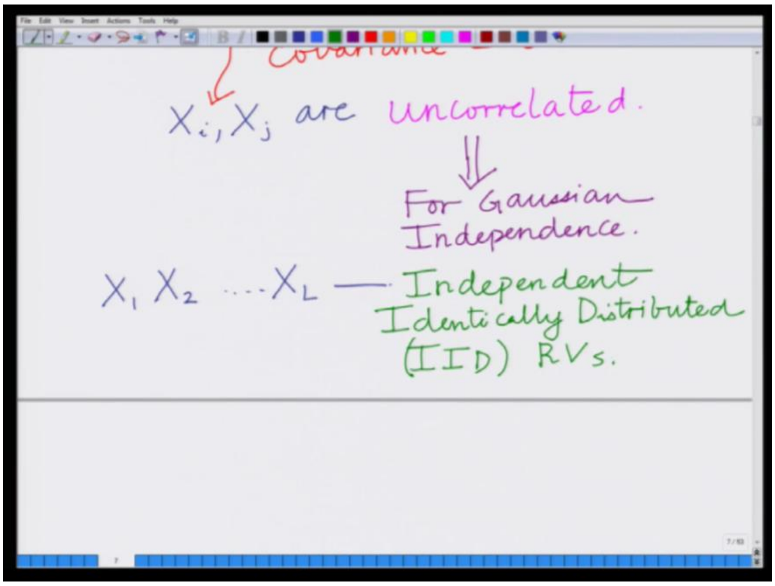

So, we are considering group of L random variables, i.e. $\{X_1, X_2, \ldots, X_L\}$. In this context, these random variables are independent. Remember we have seen this nomenclature before, independent and identically distributed random variables.

So, we were considering L independent identically distributed Gaussian random variables. Let us now again consider X which is generated as a linear combination of these Gaussian random variables.

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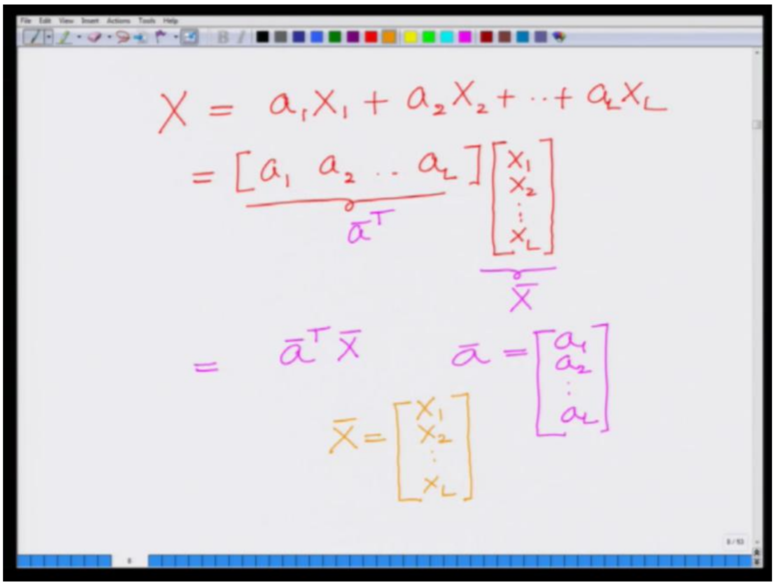

$$X = a_1X_1 + a_2X_2 + \ldots + a_LX_L$$

which we can now write using vector operations as

$$X = [a_1\ a_2\ \ldots\ a_L]\begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ . \\ X_L \end{bmatrix}$$

Denoting,

$$\bar{a}^T = [a_1\ a_2\ \ldots\ a_L] \quad \text{and}$$

$$\bar{X} = \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ . \\ X_L \end{bmatrix}$$

Therefore,

$$X = \bar{a}^T\bar{X}$$

This is the new random variable X which we saw previously is Gaussian in nature.

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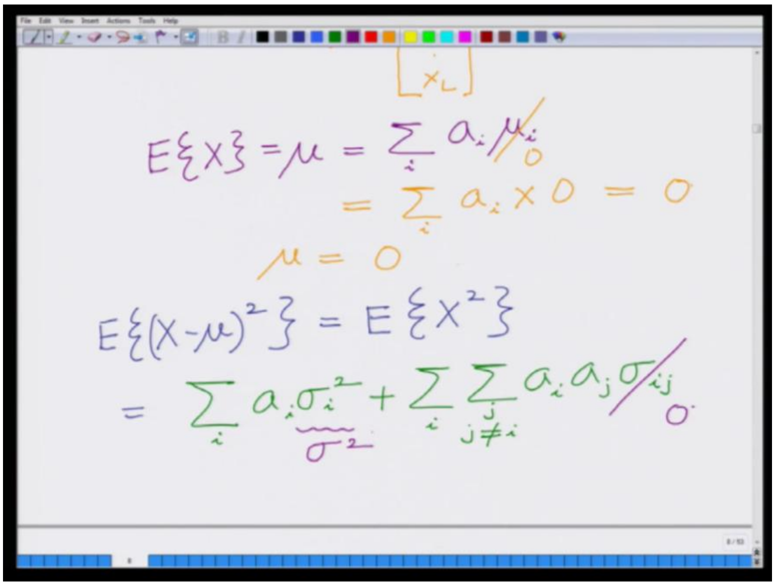

Now what is the mean of this Gaussian random variable? The mean of this Gaussian random variable is

$$\mu = E\{X\} = \sum_{i=1}^{L} a_i \mu_i = 0,$$

$$\text{as, } \mu_i = 0$$

Similarly, we have

$$E\{(X-\mu)^2 = E\{(X)^2\}$$

$$= \sum_i a_i^2 \sigma^2 + \sum \sum a_i a_j \sigma_{ij}$$

Now as, we assumed, $\sigma_{ij} = 0$, as the random variables are uncorrelated

Therefore,

$$E\{(X-\mu)^2\} = \sum_i a_i^2 \sigma^2$$

$$= \sigma^2 \sum_i a_i^2 \quad \text{(as variance is same for all } X_i \text{)}$$

$$= \sigma^2 \|\bar{a}\|^2$$

Therefore,

$$\boxed{E\{(X-\mu)^2\} = \sigma^2 \|\bar{a}\|^2}$$

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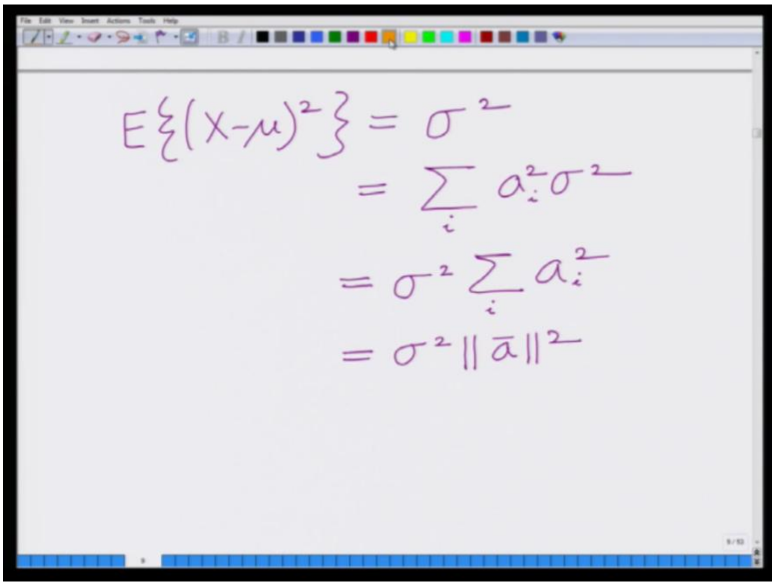

So, basically variance is equal to $\sigma^2\|\bar{a}\|^2$.

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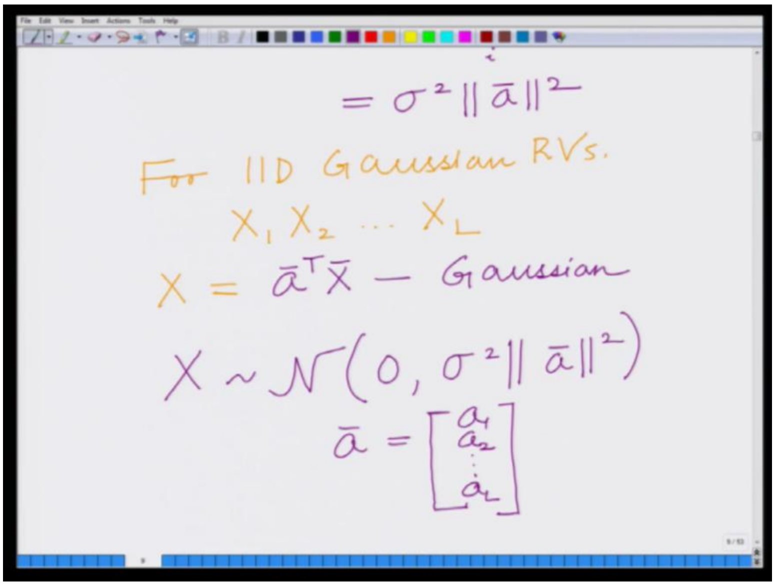

Therefore for IID Gaussian random variables therefore for IID Gaussian random variables $\{X_1, X_2, \ldots, X_L\}$. we have X which is defined as

$$X = \bar{a}^T\bar{X}, \text{ is a Gaussian random variable}$$

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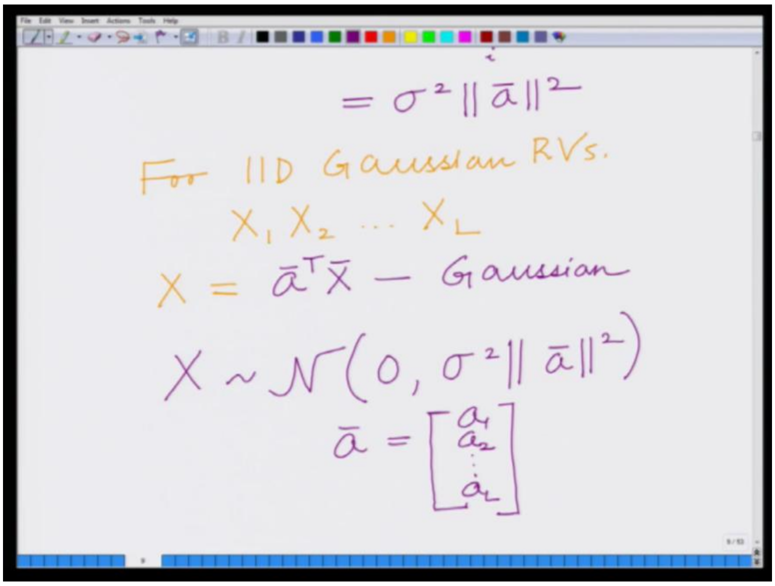

Further X can now be represented as

$$X \sim N(0, \sigma^2 \|\bar{a}\|^2),$$

$$\text{Where, } \bar{a} = \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ a_L \end{bmatrix}$$

So, what we have done is we have considered the special case that, all the Gaussian random variables $\{X_1, X_2, \ldots, X_L\}$ are 0 mean, they have identical variance σ^2 and further they are uncorrelated, that is their covariance σ_{ij} is equal to 0, and for the Gaussian random variables, we said uncorrelated also means that these Gaussian random variables are independent, therefore were considering a group of IID (i.e. independent identically distributed) Gaussian random variables and then we said if we generate a new Gaussian random variable X which is a weighted combination of these IID Gaussian random variables, that is

$$X = a_1X_1 + a_2X_2 + \ldots + a_LX_L$$

then the mean of X is 0, the variance is $\sigma^2 \|\bar{a}\|^2$.

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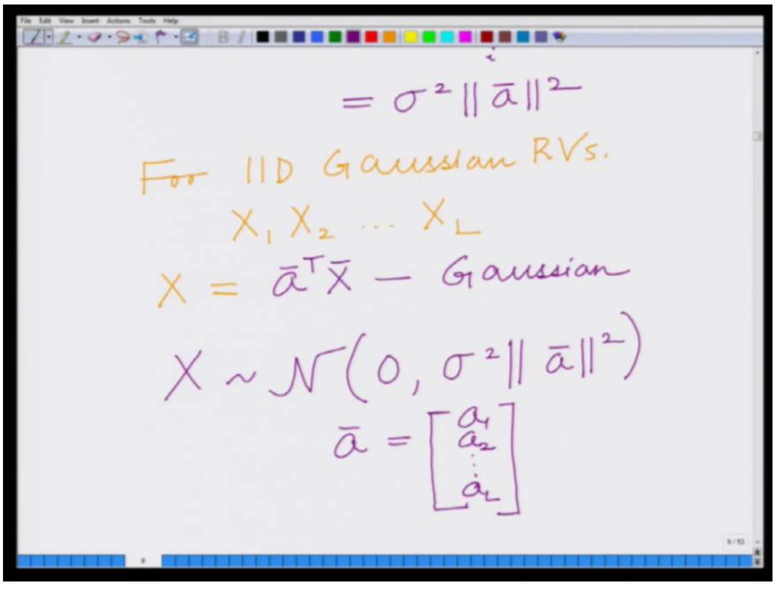

So, this is an interesting property which will in fact come very handy, which is in fact very useful when we consider, when we analyze various communications, the behavior and the performance of various communication system as well as wireless a medication systems. So, we have seen a special case of linear combination of a group of random variables. So, we will end this module here and proceed with other topics in subsequent modules. Thank you very much.