Probability and Random Variables/Processes for Wireless Communication
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Module No. 2
Lecture 12
Application: Average Delay and RMS Delay Spread of Wireless Channel

Hello, welcome to another module in this massive open online course on probability and random variables for wireless communications. In the previous model, we have looked at the concepts of mean and variance of a random variable X. Let us now look at a simple application of the

$$mean = \mu_X$$
$$Variance = \sigma_X^2$$

of a random variable X.

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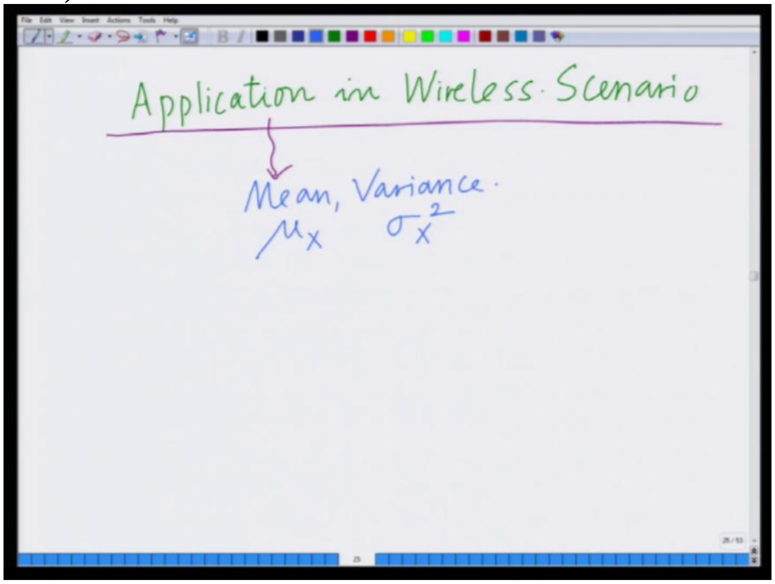

So let us look at an application in a wireless communication for these concepts, mean and variance. Remember, mean was denoted by μ, and variance of random variable X by σ_X^2. We can also show this by μ_X to denote that this is the mean of the random variable, X. Now, as we are already familiar with a typical wireless scenario, let us say I have a base station,

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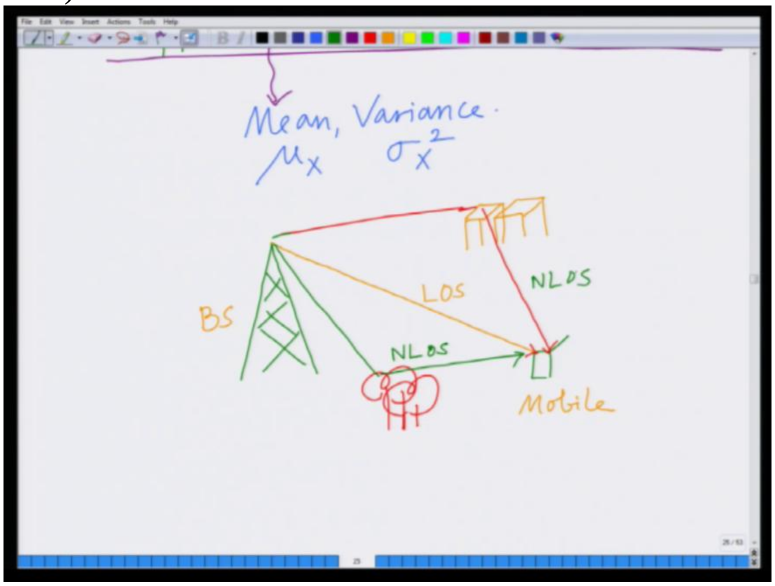

which is transmitting to the mobile. So there is a direct line of sight component, a signal which goes from the base station to the mobile. And because of reflectors, there are several scattered components. Because of obstruction such as buildings, etc in the wireless environment, there are several non line of sight components.

So basically, what we have in a wireless environment that is if I look at the signal that is transmitted from the base station to the mobile, because there is no guiding medium for the propagation of the signal, there is a direct line of sight component between the transmitter and the receiver and several non line of sight components which ~~are~~ arise from the reflections of objects such as the trees, buildings, large objects which are in the wireless environment. Therefore, there are multiple signal copies. Right? Thus at the receiver, there are multiple signal copies. These signal copies are arriving with various delays.

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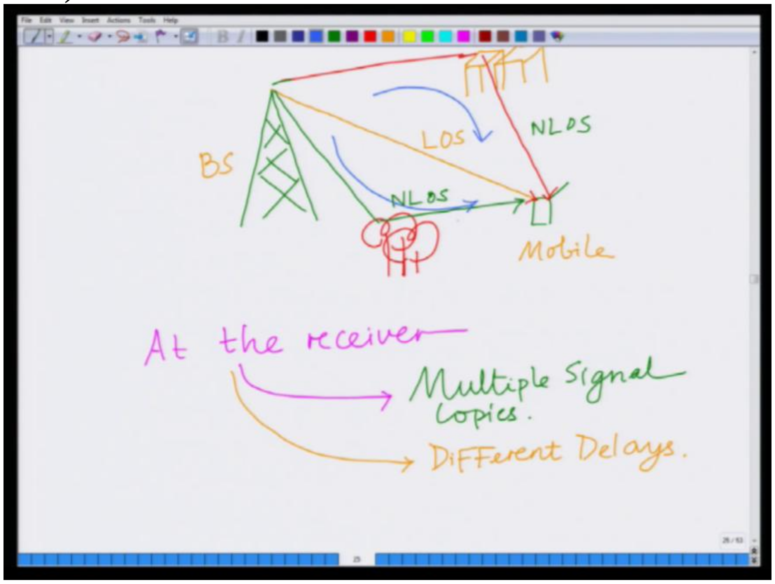

Because if you look at this figure, the distances of propagation of these different paths are different as they traverse different distances. One is reflected from a building, another is reflected from a tree, one is the line of sight which is the shortest distance which means the shortest delay. So these different signal components are arriving at the receiver with a different delays which means the power is going to at the receiver at corresponding to different delays. So the different signals: the different reflected components, reflected signal components arrive with different amounts of power. So the power arrives at the receiver with various delays.

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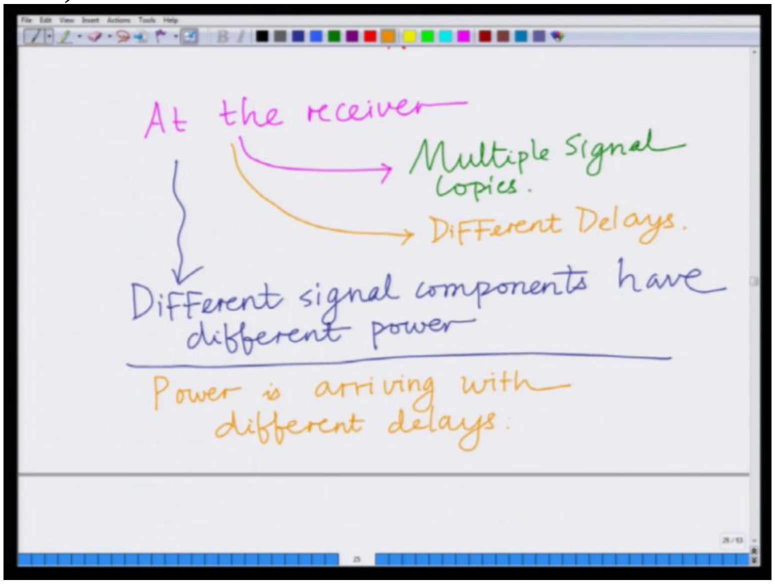

So different signal components arrive at different delays and these different signal components have different power. So basically, the power, one can say that the power is arriving with different delays.

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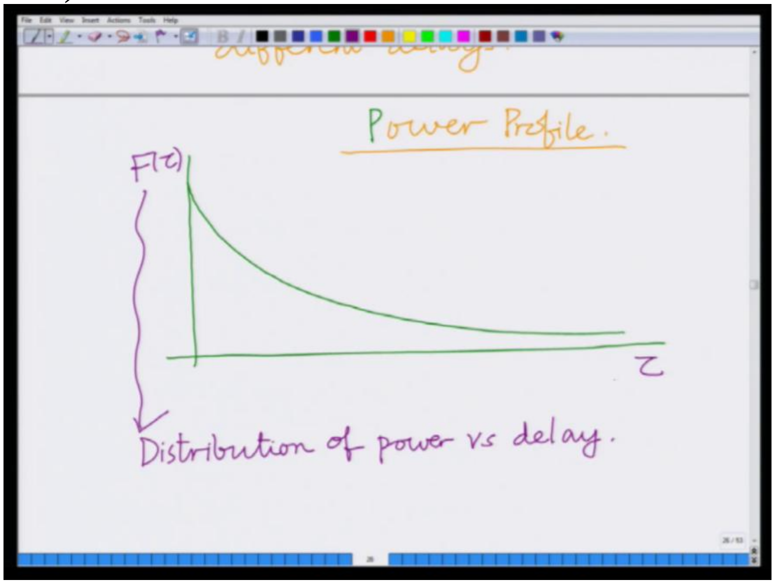

And therefore, in a wireless communication system, we consider what is known as a power profile. So what is my power profile? So this is my power profile or my multipath power profile. This is also known as a multipath power profile which gives a distribution of power vs delay. So if this is my Tao, this is my F of Tao, basically it gives the distribution. So the power in these multipath signal components, multiple signal components is arriving with different delays. Right

? So if I look at the power profile of the wireless channel, what that is, it gives the distribution of the power of the multipath wireless channel as a function of the delay. This is known as the power profile or this is known as the multipath power profile.

Based on this multipath power profile, one can compute parameters similar to what we did for the random variable. That is we can compute the average delay and we can also compute the variance of the delay. These 2 properties are very important for a wireless communication system. So based on the power profile, the multipath power profile, one can compute the average Delay and the variance of the delay.

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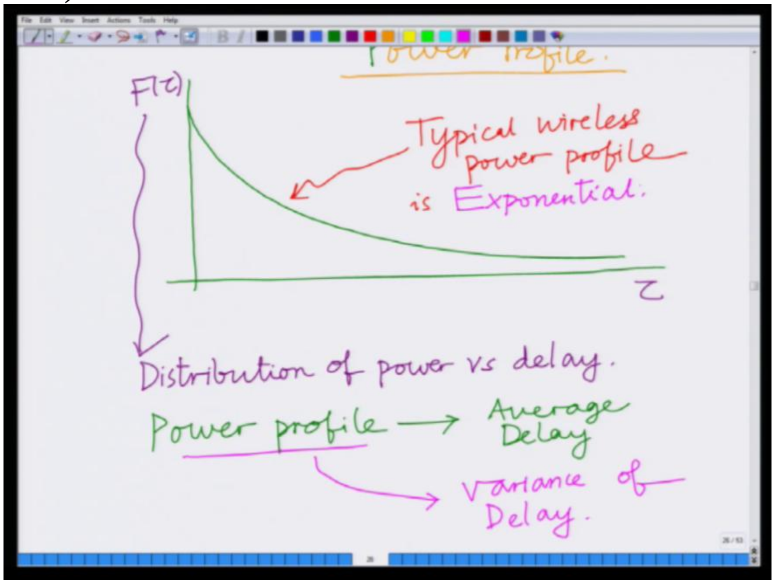

So one can compute, based on the power profile, the average delay and one can also compute the variance of the delay. For instance, let us look at a typical power profile. A typical power profile is given over here. You can see the typical power profile. The typical power profile is exponential in nature. Typical wireless power profile is similar to what we had seen for the exponential random variable. This is exponentially, the typical multipath power profile in a wireless communications, in a multipath wireless communication scenario is exponential in nature. This is the exponential distribution which we had seen in the previous modules.

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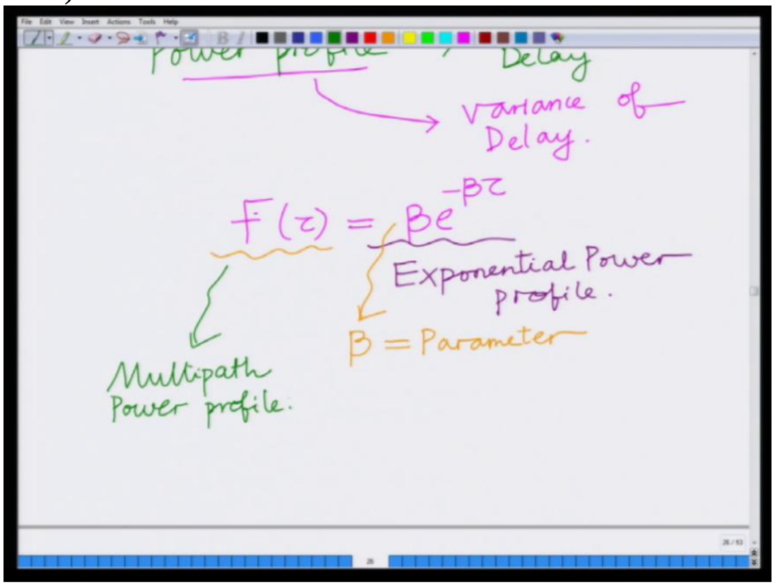

Therefore, if we look at the typical power profile,

$$F(\tau) = \beta e^{-\beta\tau}$$

So this is my exponential multipath power profile and where β is a parameter, and now therefore, from this multipath power profile, one can compute things such as the average delay.

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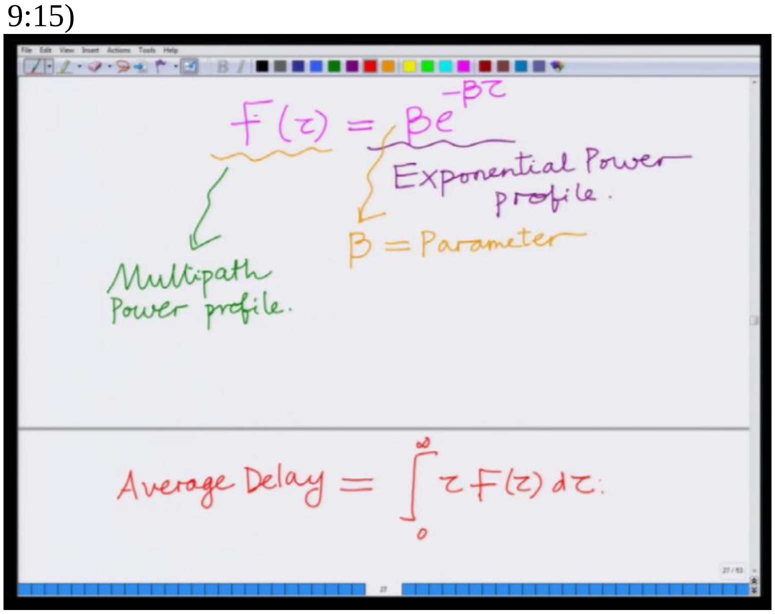

The average delay or the mean delay that as we have said before, is given as

$$Average\ Delay = \int_{-\infty}^{\infty} \tau F(\tau) d\tau$$

However, this power profile, we are assuming, it is exponential which means remember, exponential random variable is nonzero only for

$$F(\tau) \neq 0, \tau \leq 0$$

and,

$$F(\tau) = 0, \tau \leq 0$$

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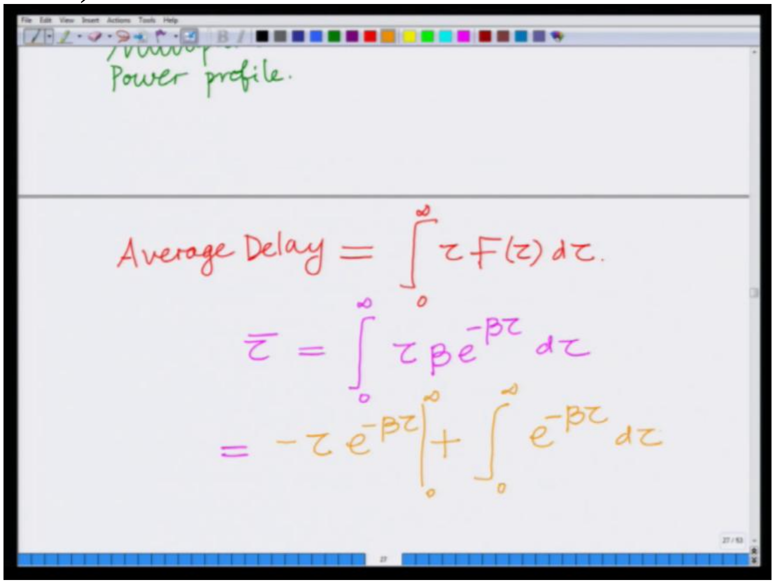

So this is integral is-

$$Average\ Delay = \int_{0}^{\infty} \tau F(\tau) d\tau$$

$$\overline{\tau} = \int_{0}^{\infty} \tau \beta e^{-\beta\tau} d\tau$$

$$integrating\ by\ parts, we\ have$$

$$= -\tau e^{-\beta\tau} + \int_{0}^{\infty} e^{-\beta\tau} d\tau$$

$$= 0 + \int_{0}^{\infty} e^{-\beta\tau} d\tau$$

$$= \int_0^\infty e^{-\beta\tau} d\tau$$

$$= \frac{-1}{\beta} e^{-\beta\tau} \, from \; 0 \; till \; \infty$$

$$= \frac{1}{\beta}$$

$Thus,$

$$\boxed{Average\ Delay = \frac{1}{\beta}}$$

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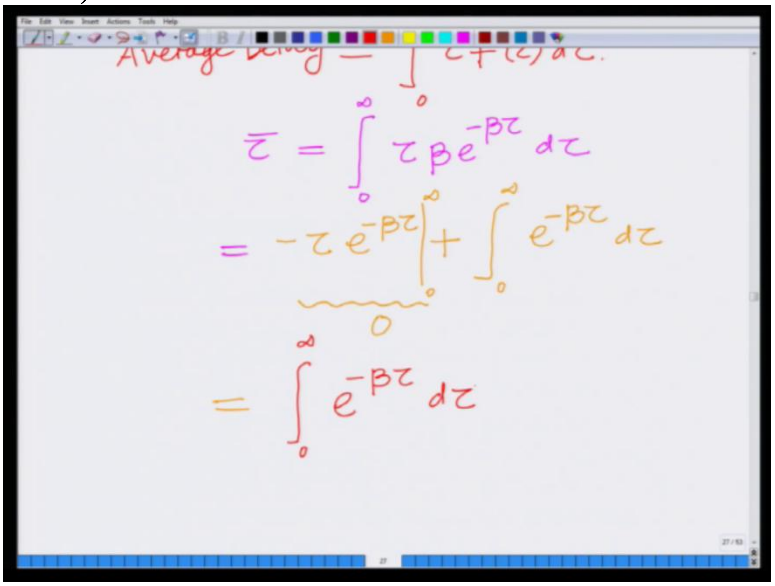

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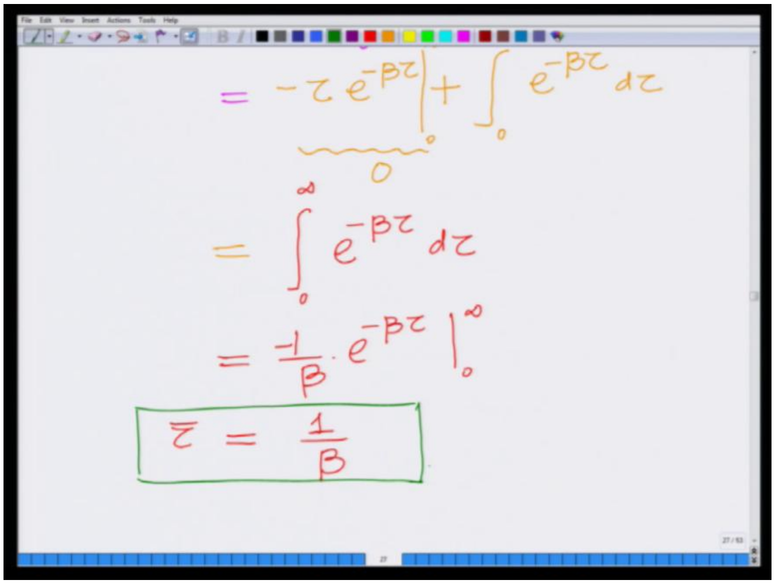

Therefore we can say that the average delay corresponding to this exponential multipath is $\bar{\tau}$.

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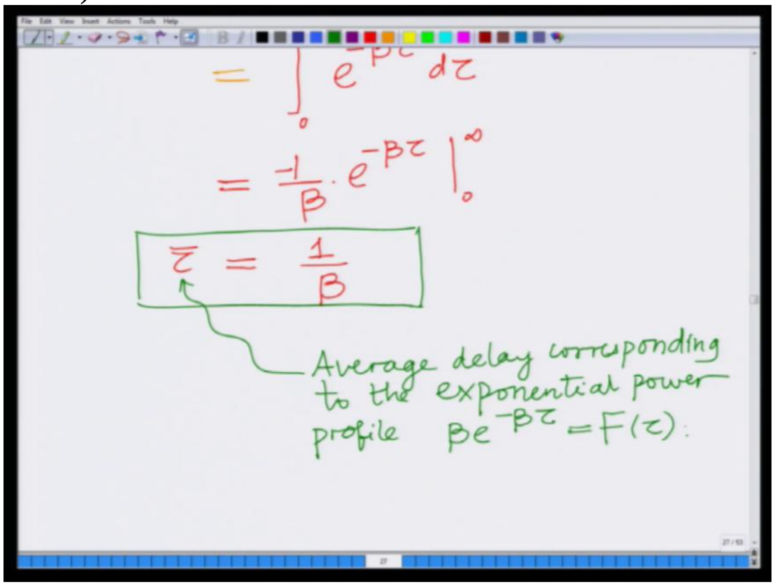

$\bar{\tau}$ is average delay corresponding to the exponential power profile $e^{-\beta\tau}$. So we are saying, corresponding to the exponential power profile in the multipath wireless channel where

$$F(\tau) = \beta e^{-\beta\tau}$$

the average delay $\bar{\tau}$ which is $\int_0^{\infty} \tau F(\tau) d\tau$ is $\frac{1}{\beta}$.

Now, let us look at the variance of this delay.

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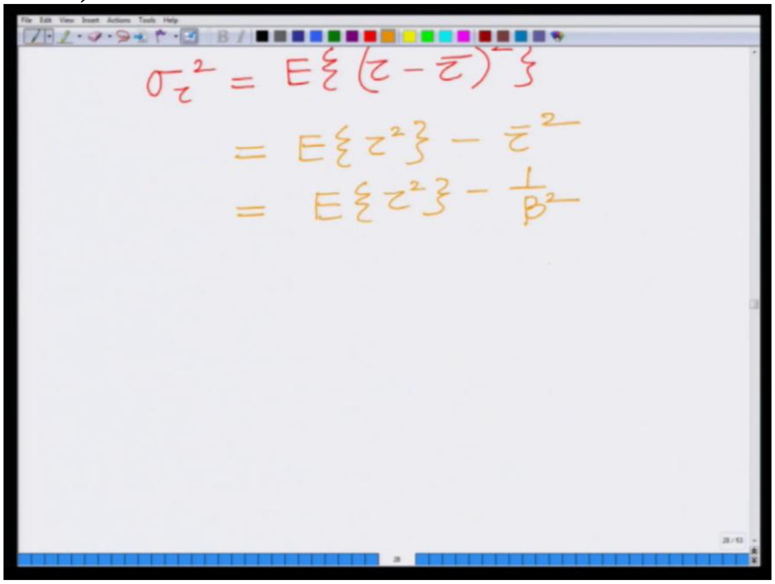

The variance is given as,

$$\sigma_\tau^2 = E\{(\tau - \bar{\tau})^2\}$$

And it is also equal to as we had remember, proved in the last lecture.

$$\sigma_\tau^2 = E\{\tau^2\} - \bar{\tau}^2$$

Also, we had derived the expression of mean

$$\bar{\tau} = \frac{1}{\beta}$$

So, the expression becomes

$$\sigma_\tau^2 = E\{\tau^2\} - \frac{1}{\beta^2}$$

Now let us evaluate $E\{\tau^2\}$ that is average of the Square of the delay.

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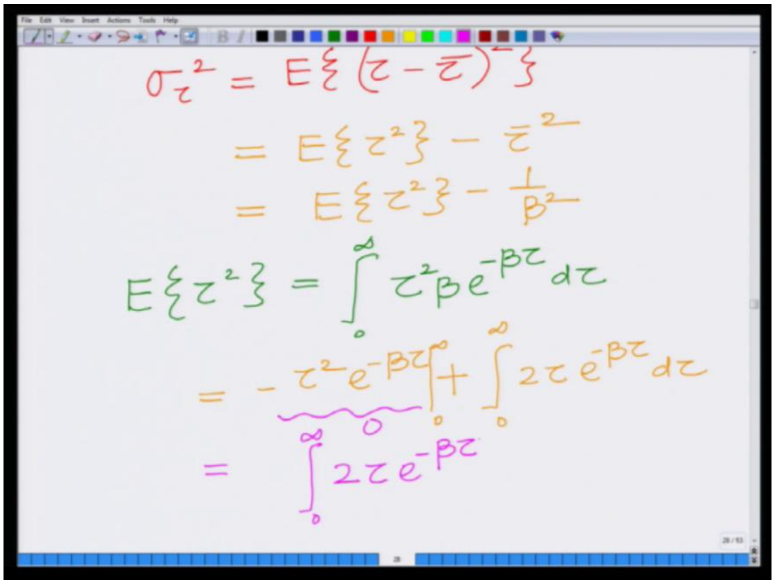

So the average of the Square of the delay, expected value of Tao square is nothing but

$$E\{\tau^2\} = \int_0^\infty \tau^2 \beta e^{-\beta\tau} d\tau$$

Once again, using integration by parts, I have,

$$= -\tau^2 e^{-\beta\tau} + \int_0^\infty 2\tau e^{-\beta\tau} d\tau$$

Now if I look at this part again, at ∞, $e^{-\beta\tau}$ is 0. At 0, τ^2 is a 0. So this is basically 0. So I have, what remains is integral between 0 to infinity of $2\tau e^{-\beta\tau} d\tau$.

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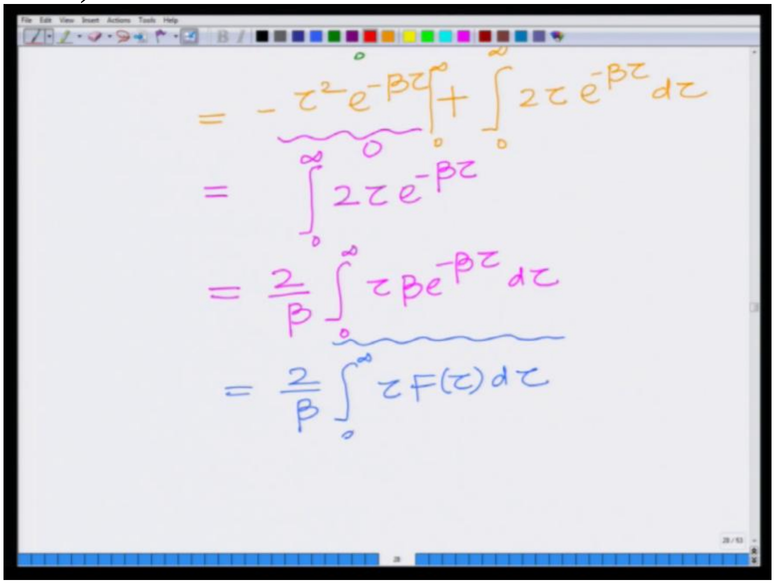

Now what I am going to do here is I am going to divide and multiply by beta. So I have

$$E\{\tau^2\} = \frac{2}{\beta}\int_0^{\infty} \tau\beta e^{-\beta\tau} d\tau$$

Now if you look at this, this is $\tau F(\tau)$. Here, $\beta e^{-\beta\tau}$ is $F(\tau)$, and this integral is nothing but the mean…

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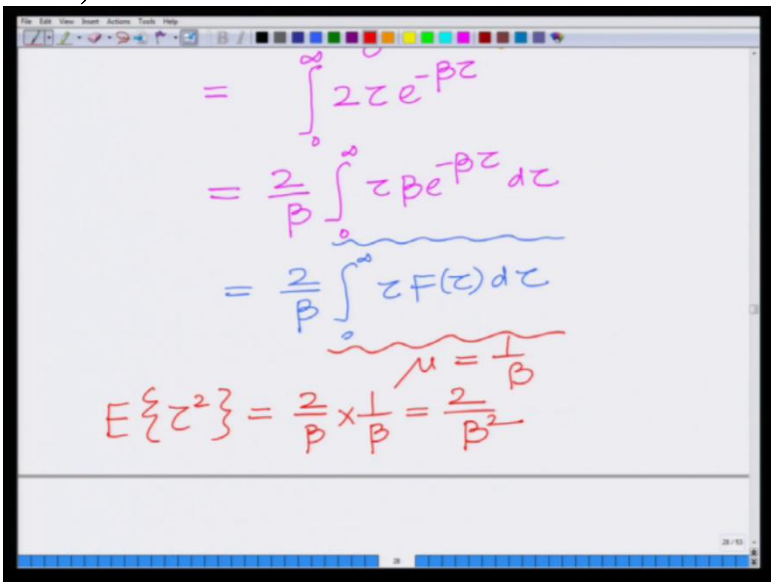

… which is myu equal to

$$\mu = \frac{1}{\beta} = \int_0^\infty \tau\beta e^{-\beta\tau} d\tau$$

Therefore I have,

$$E\{\tau^2\} = \frac{2}{\beta^2}$$

Okay? So that is what we have derived. Now one can derive the variance as follows.

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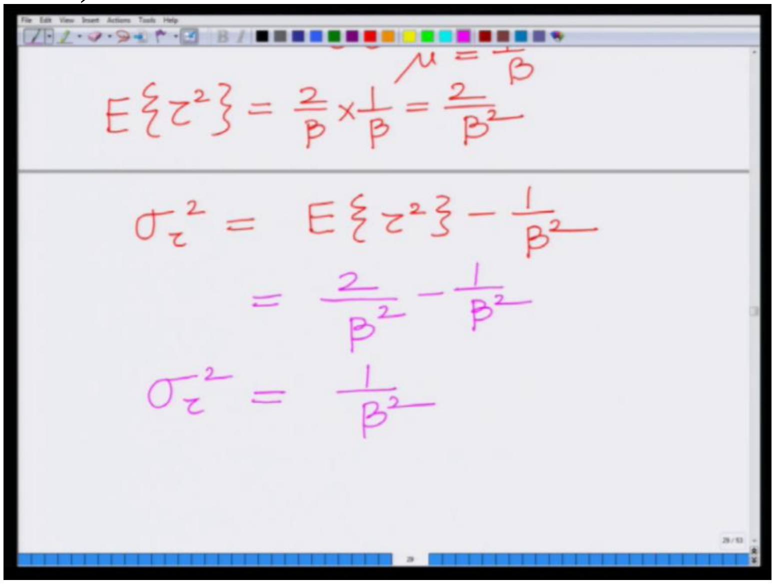

$$\sigma_\tau^2 = E\{\tau^2\} - \frac{1}{\beta^2}$$

So substituting the value of $E\{\tau^2\}$, we have

$$\sigma_\tau^2 = \frac{2}{\beta^2} - \frac{1}{\beta^2}$$

Therefore,

$$variance = \sigma_\tau^2 = \frac{1}{\beta^2}$$

So the variance of the delay is $\frac{1}{\beta^2}$.

Now, if we look at the σ_τ, that is the standard deviation, we have

$$\sigma_\tau = \frac{1}{\beta}$$

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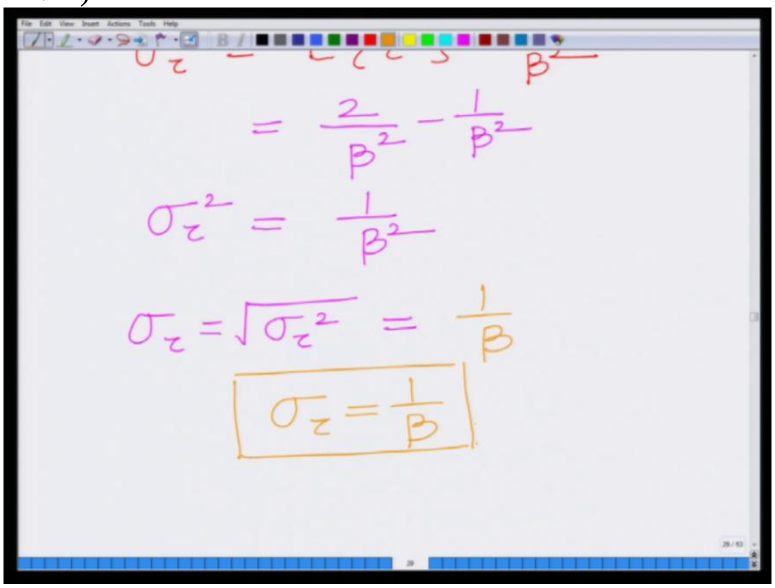

So we have the standard deviation equals 1 over β and this is an important parameter in wireless communication.

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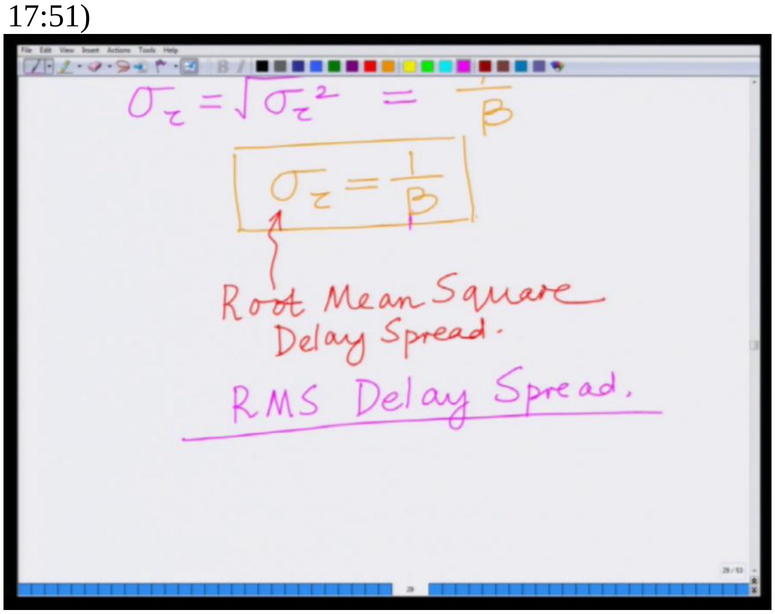

This is also known as the root mean square delay spread or the RMS delay spread of the wireless channel. This equal to 1 over β.

So what we have derived is that the σ_τ which is the standard deviation of this delay which is the square root of the variance is equal to 1 over β.

This is an important parameter in wireless communication because it denotes the RMS delay. It tells you what is this delay spread, what is the spread of time over which the signal energy is arriving at the receiver. Also known as RMS delay spread. It is a metric to measure the delay spread.

So what we have seen in this module is we have seen an important application of the mean and variance of the random variable in the context of wireless communication for a given multipath power profile.

That is, the power, spread of power, the distribution of power vs the delay. We have seen what is the average delay which is given by the mean, the variance of the delay and the square root of the variance of the standard deviation of the delay which is known as the RMS delay spread of the wireless channel. So we will stop this module over here and we will look at other aspects in the subsequent modules. Thank you very much.