Probability and Random Variables/Processes for Wireless Communication
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Module No. 2
Lecture 10
Applications: Power of Fading Wireless Channel

Hello, welcome to another module in this massive open online course on probability and random variables for wireless communication. In the previous model but we had seen the concept of a random variable, the random variable X, the concept of probability density function for this random variable, X and we had looked at the exponential random variables. Let us now look at the relevance of this concept of a random variable. Specifically, the exponential random variable in the context of wireless communication.

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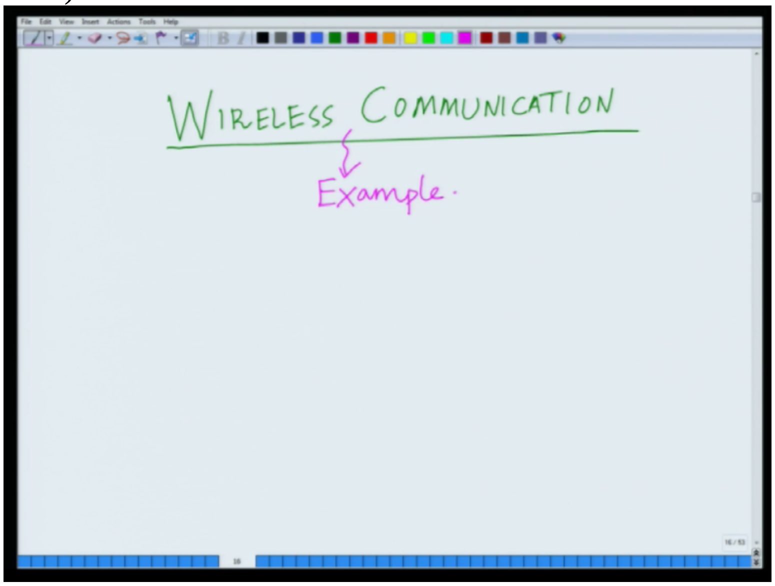

So let us now look at an example, simple example from the context of wireless communication. So let us look at a simple example because remember, this is a course in probability and random variables, the applications of probability and random variables, random processes in communications, specially wireless communication. So let us consider a typical wireless communication scenario with a transmitter and receiver, a cellular scenario in which a base station is transmitting to a mobile.

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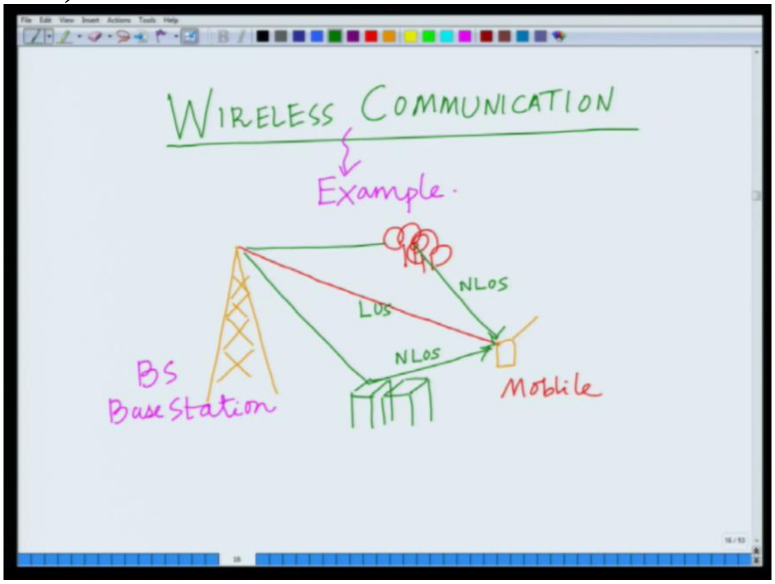

So for instance, I have a typical wireless scenario and we have already seen this to some extent. A wireless transmitter which is transmitting. So I have my base station which is transmitting to the mobile. This is my mobile. And therefore, there is going to be a direct line of sight path. However, there are also going to be several non line of sight paths between the base station and the mobile which arise from the presence of what are known as scatterers. Right? So we have a line of sight path and several non line of sight paths. So we are looking at a wireless communication environment. So in a wireless communication environment, we have a transmitter and a receiver. There is no dedicated medium, there is no guided propagation medium between the transmitter and the receiver. Therefore, there is a signal that is radiated into the space by the transmitter. This propagates along a direct line of sight path from the base station to that mobile. So, there is a line of sight which is also represented by the nomenclature, LOS which stands for line of sight.

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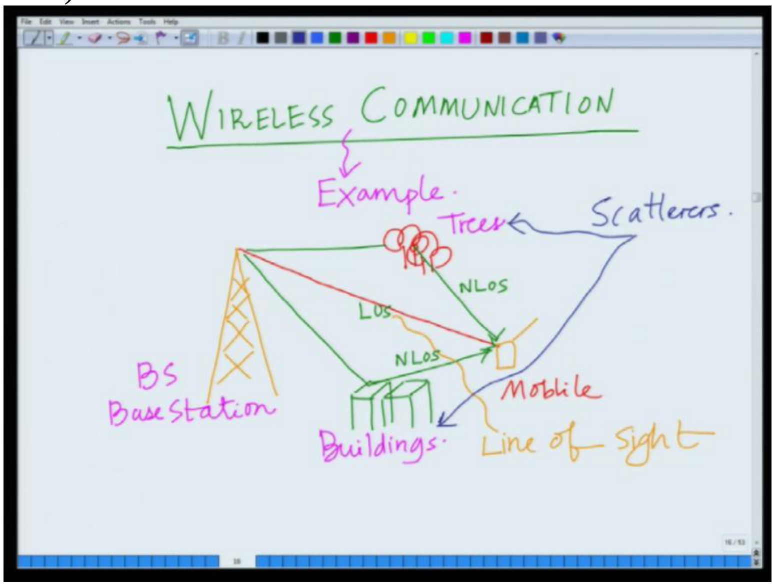

So this LOS, this stands for line of sight, and there are several obstacles such as for instance, trees and buildings in the propagation path. These are known as, these obstacles here, these buildings, trees, etc. these are known as scatterers which basically scatter the ~~towns voted~~ transmitted signal. So, from these scattered, scatterers, there are several scattered signal components or several reflected signal components. These deflect the signal. That is, the transmitted wireless signal and the deflected single component also arrive at the mobile. So there is one line of sight and there are several non line of sight which is denoted by NLOS, several non line of sight components which are deflected or which are scattered by the scatterers such as trees and buildings which also arrive at the mobile.

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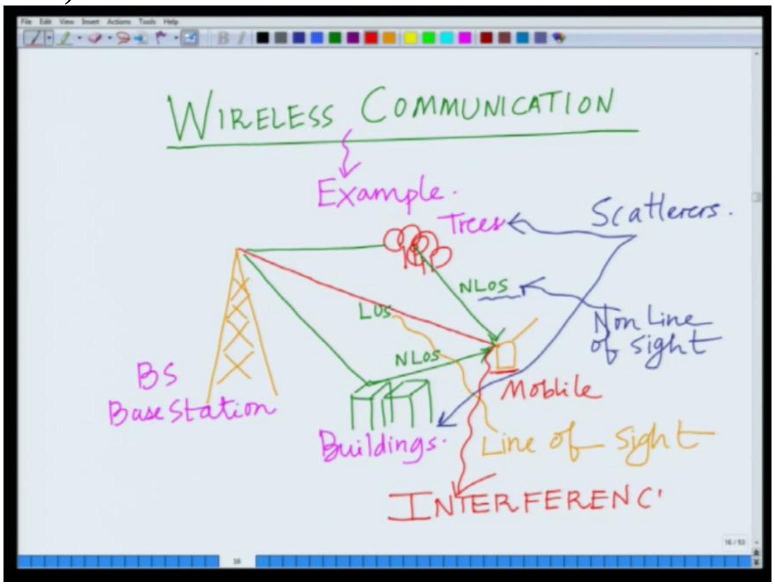

So this NLOS stands for non line of sight. And as a result of this, what is going to happen over here? These several signal components: these are going to cause interference. These interfere. So these are going to cause interference at, so these multiple signal components cause interference at the mobile receiver. As a result of this, the received signal quality, the interference can be constructive or the interference can be destructive. As a result of this, the received signal quality or the received signal level is random in nature. This is known as the fading channel because the received signal power is fading. And therefore, the received signal power is random in nature.

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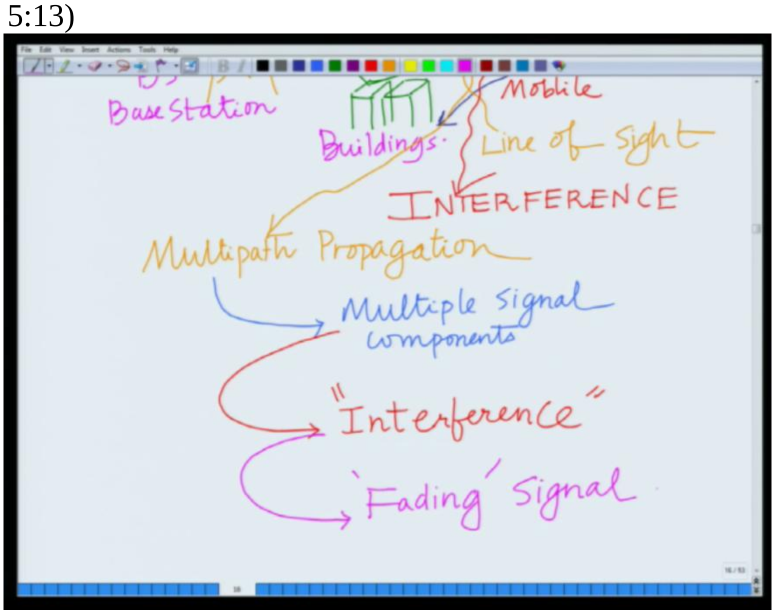

So, interference, so in the wireless channel, we have, the sequence that we said is we have multipath propagation. This multipath propagation leads to, what does this lead to? Multipath propagation leads to interference. It leads to multiple signal components which leads to interference. So, multiple signal components. These multiple signal components in turn lead to interference. This is known as multipath interference. This leads to interference. And because of interference, the received signal is fading. So this leads to a fading signal.

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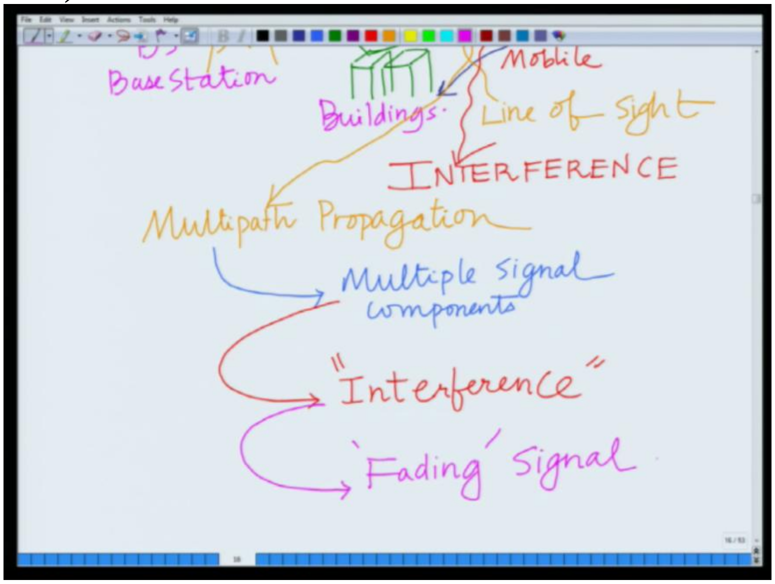

And as the signal is fading, the received power, therefore the received signal level or the received signal power is random in nature. So there is multipath propagation arising because of the scatterers or reflectors in the wireless environment that leads to several non line of sight components. These non line of sight components together with the line of sight components when they impinge on the mobile, they cause interference between these multiple components, between these multiple signal components.

The interference can be constructive or destructive depending on the amplitude and phases of these different signal components. And therefore, the received signal power is fading. That is, the received signal power at the mobile is random, it is fading in nature. It is varying with time. Correct? And therefore the received signal power is random in nature. This wireless channel is a fading Wireless Channel. The received signal power is random in nature. And the power of the channel coefficient, the power of the fading channel is characterized by the probability density function.

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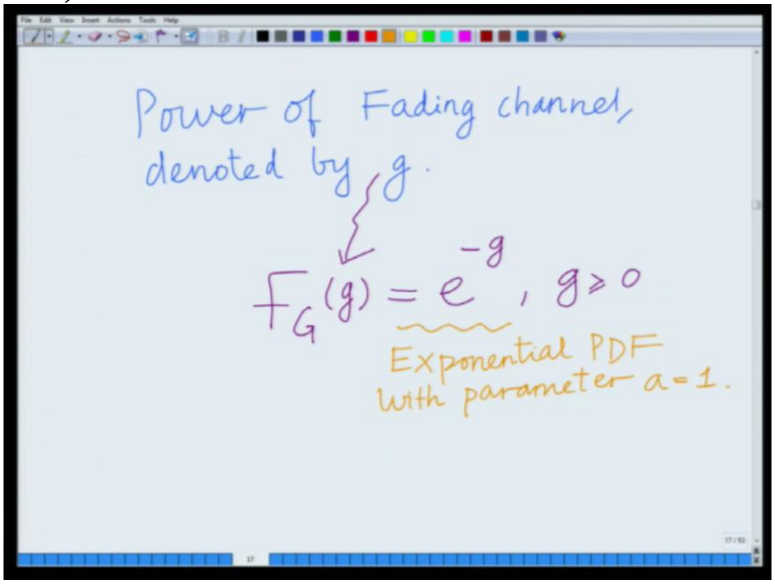

Therefore, the fading power, the power of fading channel which is denoted by G. This is characterized by the fading channel distribution. This is characterized by the probability. This is random. As we said, the received signal power is random, this is characterized by the exponential probability density function –

$$F_G(g) = e^{-g} \quad , \quad g \geq 0$$

which is basically nothing but as we have seen in the previous module, this is the exponential probability density function. What is this? This is the exponential PDF with parameter A is equal to 1. So this is a very interesting example from wireless communication in application of the probability theory that we have learnt so far in wireless communication. We have said that the fading channel is random in nature.

That is the signal level and hence the power level in the fading wireless channel because of the unpredictable nature of interference arising from these random scatterers in the environment, the signal power is random in nature. This signal power, if we denote this power of the fading channel coefficient by G, G is a random variable. The probability density function F(G) is denoted by –

$$F_G(g) = e^{-g} \quad , \quad g \geq 0$$

because it is a fading power. The power cannot be negative. So this probability density function is defined only for g greater than or equal to 0. This is characterised by e^{-g} which we are saying is the exponential probability density function. Remember, in the previous module, we had seen the exponential probability density function

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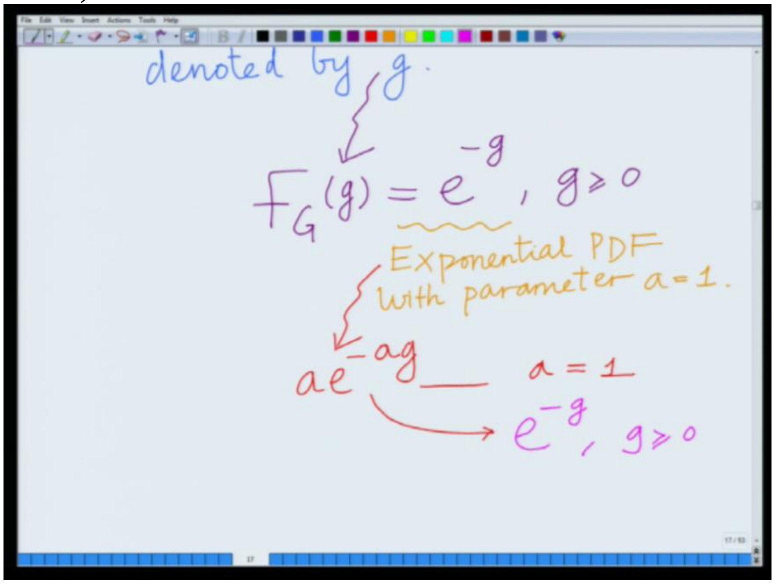

which is basically exponential probability density function is a parametric distribution which is **ae^{-ag}**. If we set a=1 , then we get basically what we get is **e^{-g} for $g \geq 0$.** So therefore, what we are saying is, in a fading Wireless Channel, the power is distributed as an exponential random variable with the parametric a=1. And this example is about specific fading wireless channel which is known as the Rayleigh fading Wireless Channel.

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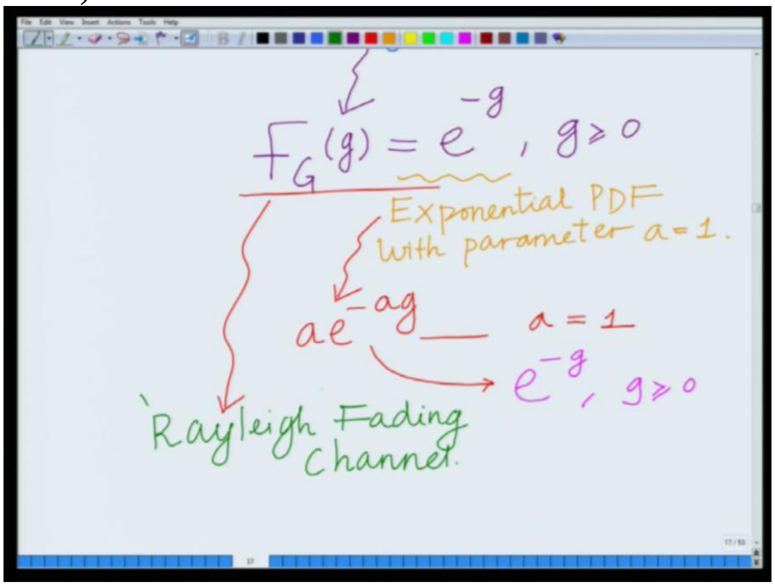

This is valid for a specific wireless channel which is known as the Rayleigh Fading which is a very popular model for wireless channel. This is known as a Rayleigh Fading channel in which the amplitude of the fading coefficient is distributed as a Rayleigh Fading channel. So we are looking at a fading wireless channel, we are saying that the amplitude of the channel coefficient of this random channel coefficient is distributed as a Rayleigh Fading channel. The power is distributed as an, power which is the square of the amplitude is distributed as an exponential random variable, e^{-g} with parametric which is an exponential probability density function with the parameter $a=1$. And therefore, this probability density function can be used to answer several interesting questions. For instance, we can use this to characterize the behavior of the wireless channel.

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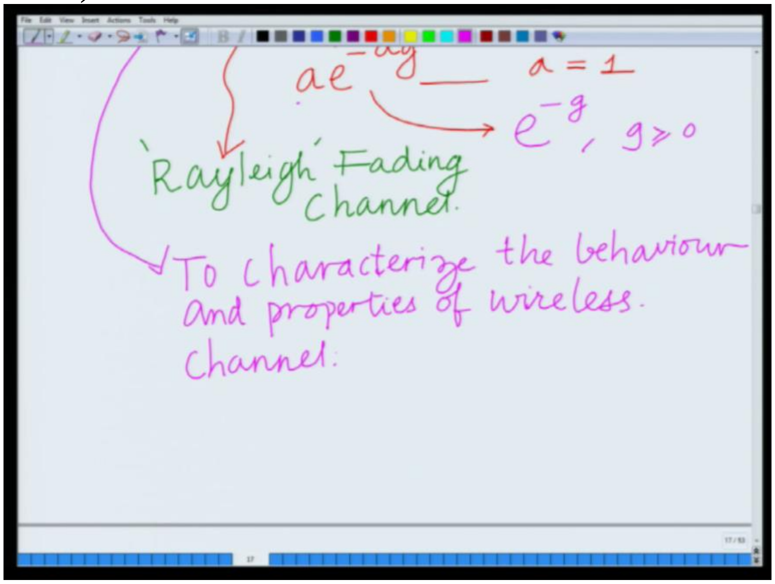

So we can now use this probability density function to characterize the behavior and properties of the wireless channel. We can use this to characterize the behavior and properties of the wireless channel. For instance, let us look at this simple example. Since we are talking about the characterization of the behavior of wireless channel, let us look at this simple example.

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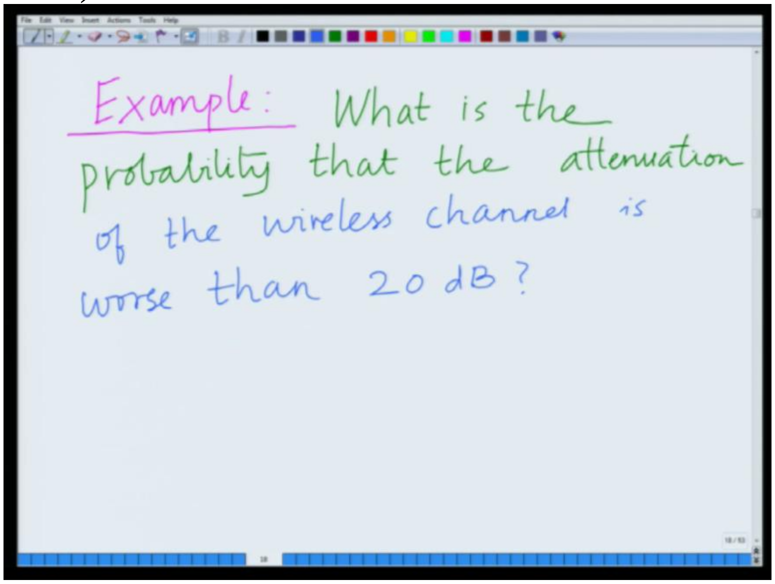

Let us consider this Rayleigh Fading channel, Rayleigh Fading wireless channel we are considering so far. And let us ask the question, what is the probability that the attenuation of the wireless channel that is the signal attenuation of the wireless channel is worse than that is we are asking the question, what is the probability that is given this fading wireless channel in which the power is distributed as an exponential random variable, what is the probability that the attenuation of this fading wireless channel is worse than 20 dB. That is, if I transmit a signal, X(T) with a certain power P, the received signal power is 20 dB attenuated with respect to the transmitted signal power., What is the probability that the attenuation of the signal, of the wireless signal is worse than 20 dB? And we can answer this as follows.

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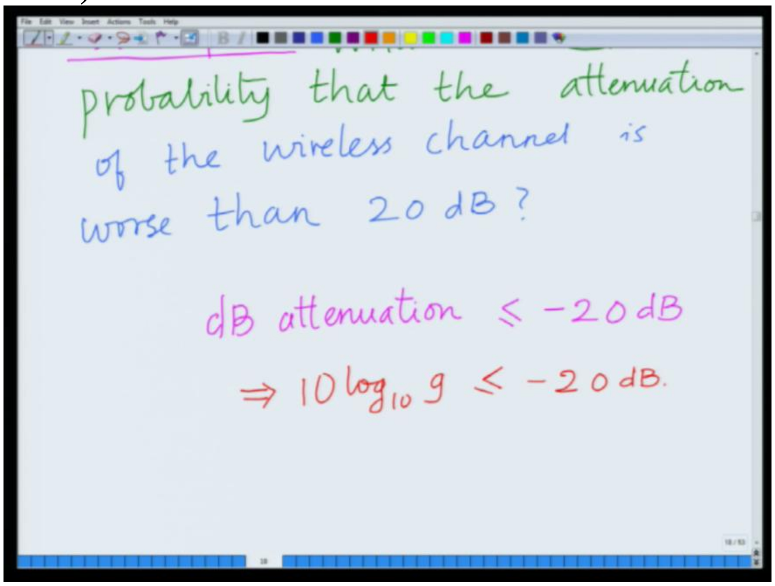

The answer to this is as follows, since the attenuation, the power attenuation, the dB attenuation has to be, since the attenuation is 20 dB, the dB attenuation has to be less than or the amplification or the power gain has to be less than or equal to - 20 dB which implies –

$$10 \log_{10} (g) \leq - 20 \text{ dB}$$

Remember, the power gain is G. The power gain in dB is **$10 \log_{10}(g)$.** And for the attention to be worse than 20 dB, this has to be less than or equal to -20. So, **$10 \log_{10} (g) \leq - 20$** which implies….

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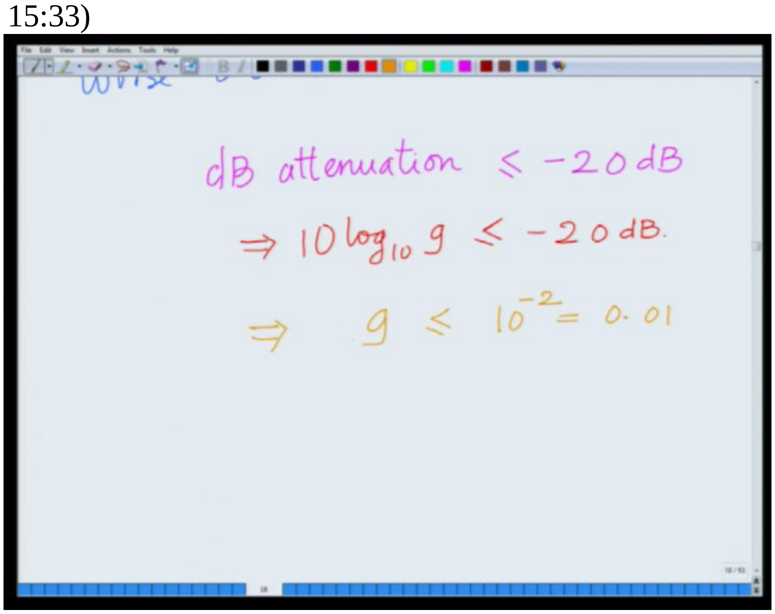

Which basically implies that,

$$g \leq 10^{-2} = 0.01$$

And we already know that **$g \geq 0$**, Because this is the power gain, power cannot be negative which means we are asking,

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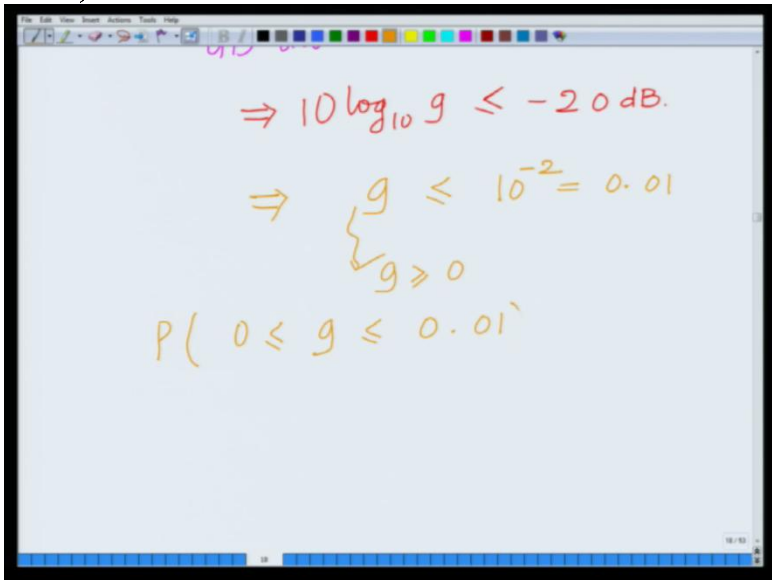

…what is $P(0 \leq g \leq 0.01)$? So the probability that the attenuation is worse than 20 dB is the probability that the power gain of the wireless channel G belongs to, lies in [0 , 0.01], it is greater than or equal to 0 but less than 0.01 which means it belongs to the interval, (0 , 0.01).

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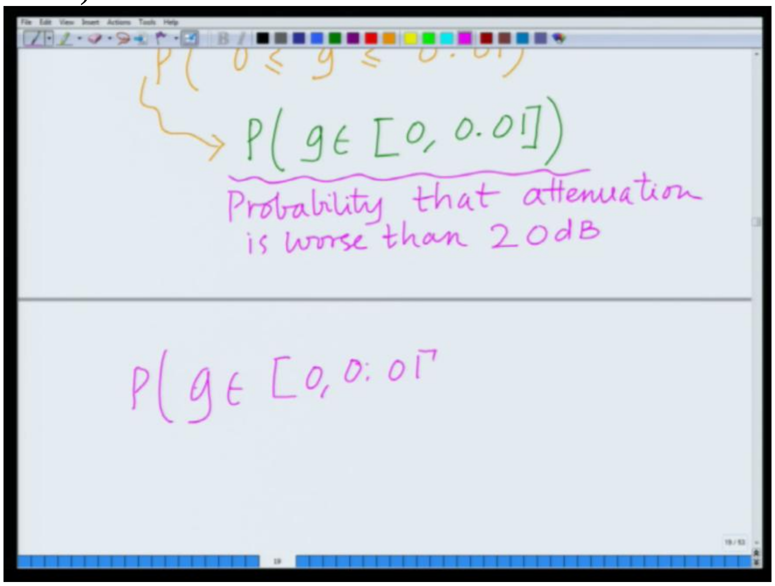

So this is the same as the probability, it belongs to [0 , 0.01]. What we are saying is, this is the probability that attenuation: this is equal to the probability that attenuation is worse than 20 dB.

And the probability that this belongs to [0 , 0.01], this is nothing but the probability, $g \in [0, 0.01]$. Remember, the probability that any random variable belongs to a particular interval, A to B is…

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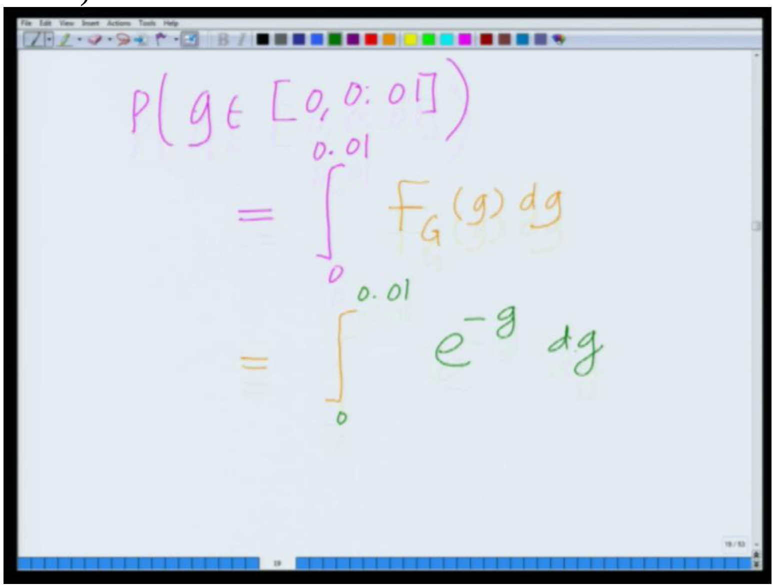

Integral A to B, integral [0 , 0.01], integral of the probability density function which is –

$$P(g \in [0,0.01]) = \int_0^{0.01} \mathrm{FG(g)}\, dg$$

We already know that this probability density function $\mathrm{FG(g)}$ is distributed as the exponential random variable width parameter 1. So this is,

$$= \int_0^{0.01} e^{-g}\, dg$$

That is the probability that it lies in the interval [0 , 0.01]. It is the integral of the probability density function e^{-g} between these limits, [0 , 0.01] which is basically nothing but…

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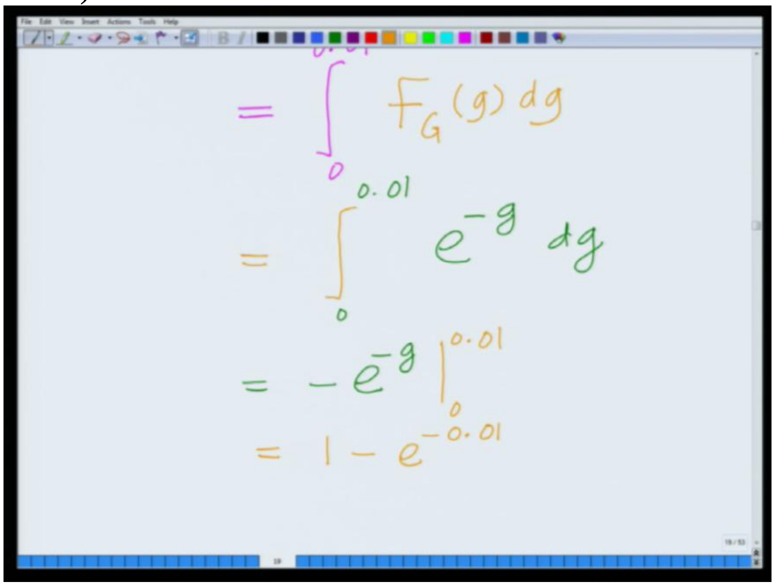

$$\int_0^{0.01} e^{-g}\, dg = -\, e^{-g} \,\Big|_0^{0.01}$$

$$= 1 - e^{-0.01}$$

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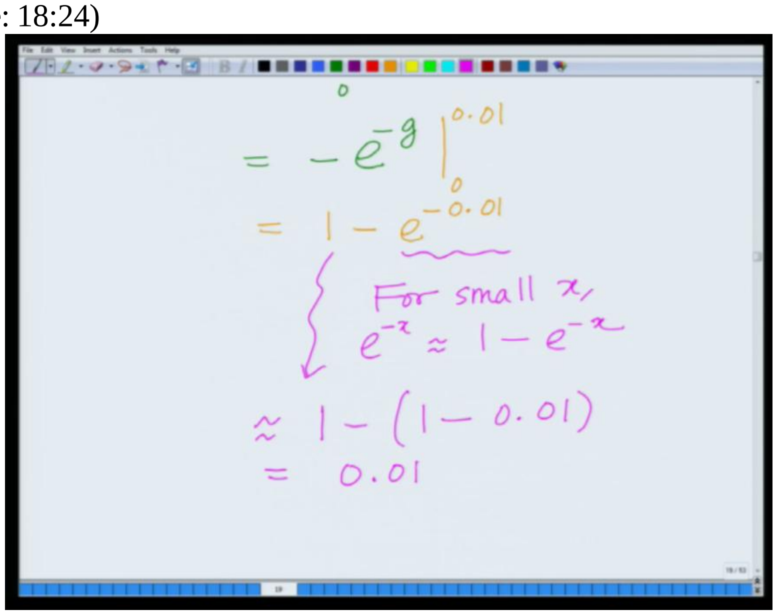

Now, I am going to use an approximation for small X,

$$e^{-x} \approx 1 - e^{-x}$$

$$\approx 1 - (1 - 0.01) = 0.01$$

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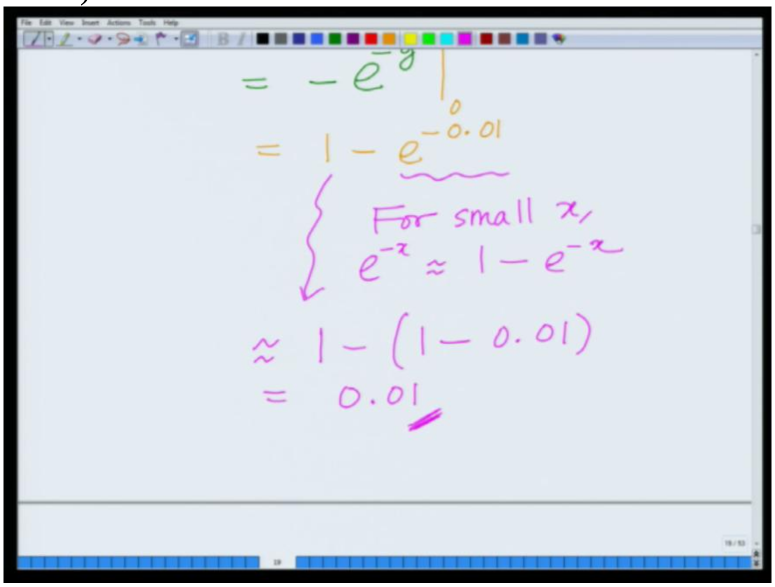

And therefore, the probability that the attenuation is worse than 20 dB or the probability that the gain $g \leq 10^{-2}$ i.e. 0.01. That is approximately 0.01. Therefore the probability that the attenuation is worse than 20 dB of the fading wireless channel is 0.01 or a 1% chance that the received wireless signal is attenuated by more than 20 dB.

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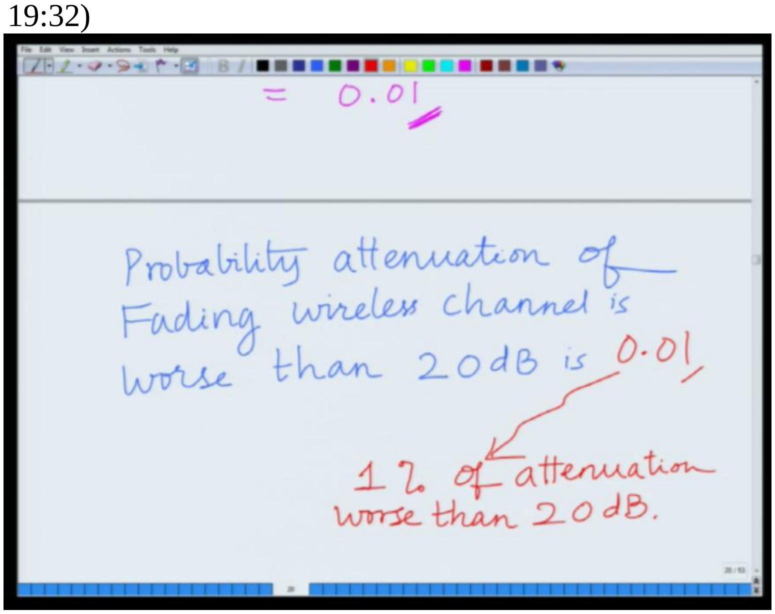

So this probability, attenuation of fading wireless channel is worse than 20 dB is 0.01 or basically, there is a 1% chance of attenuation. There is a 1% chance of attenuation being worse than 20 dB. So this is a practical example in the context of wireless communication. We have said that the wireless channel his fading in nature. We said that this can be characterized by an exponential random variable with parameter a = 1 and we said, this can be used to characterize the behavior of the wireless channel and we did a simple example in which we found what is the probability that the attenuation of the received signal is lower than - 20 dB. So we will stop this module here. We will continue with the other aspects in the subsequent modules. Thank you, thank you very much.