**Probability and Random Variables/Processes for Wireless Communication**
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**Module No. 1**
**Lecture 1**
**Basics -Sample Space and Events**

Hello, welcome to this massive open online course on probability and random variables for wireless communications. Alright! So let us start our discussion with the concept of an experiment.

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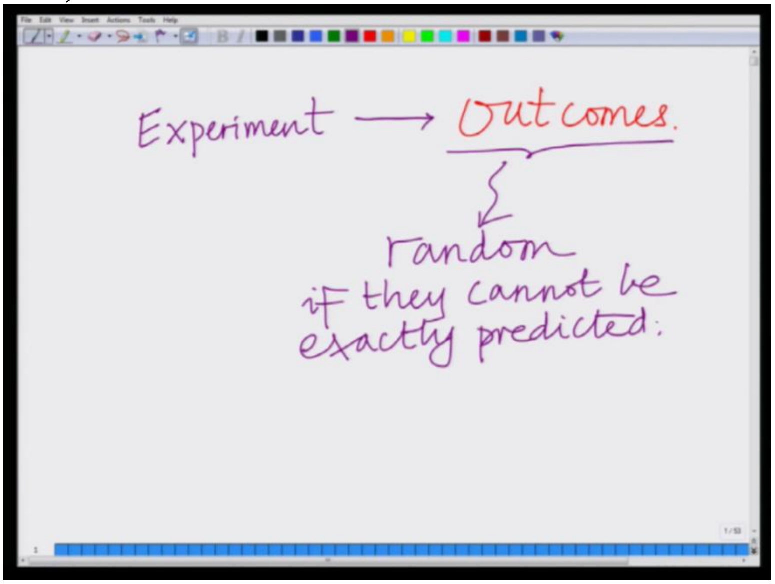


So when we perform an experiment, those experiments lead to outcomes. These outcomes are random in nature. These outcomes are random if they cannot be exactly predicted. So the outcomes of these experiments, we say they are random if they cannot be exactly predicted.

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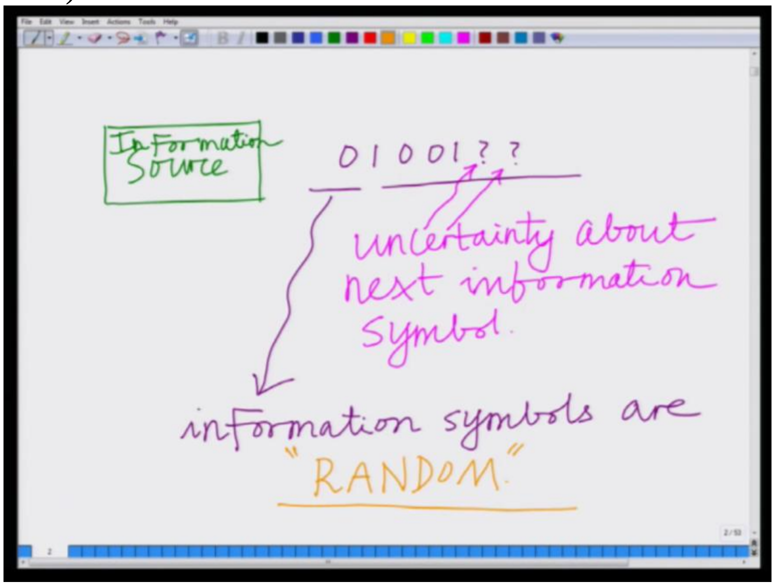


For instance, let us take a simple example. Since this course is about probability for communication systems or wireless communication systems, let's take an information source. In communication, we talk about information source. And let's say, it is generating information symbol 0s and 1s. For instance, 0100 and this is the sequence of information. Now for this information source, at any instant, we do not know exactly what the next information symbol is. It can either be 0 or it can be 1. So for the next information symbol, there is uncertainty about the successive information symbol i.e. the next information symbol. So these information symbols are random in nature.

We say, the information symbols are being generated randomly. So there is uncertainty about the next information symbol. Hence, these information symbols are random. We say, these information symbols are random or they are being generated randomly. There is a qualitative description of randomness. We say, the information symbols are being generated randomly because there is uncertainty regarding the next information symbol that is generated by the source. So let us not develop a framework to formally characterize probability and randomness.

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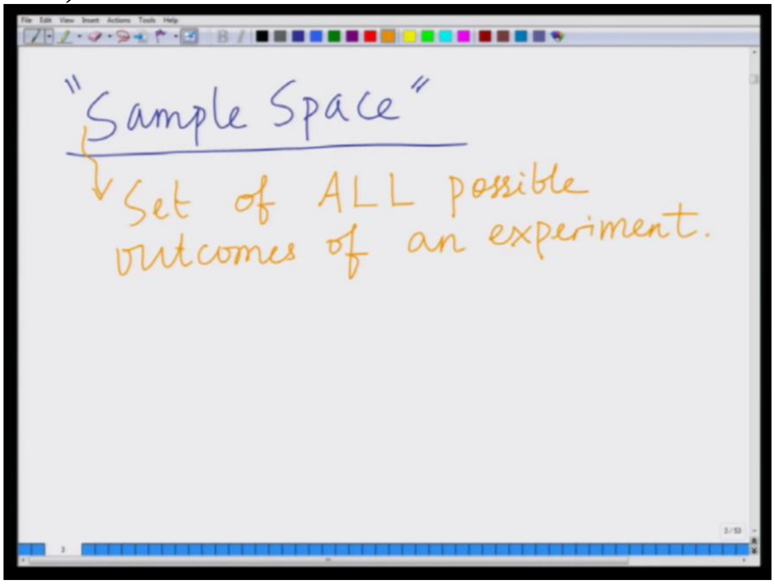


So let us start with building a framework. The 1st thing that we would like to define is what is known as the Sample space. Now, the Sample space is simply put, the set of all possible outcomes of the experiment. This set of all possible outcomes of the experiment, is known as the Sample space. The Sample space is the set of all possible outcomes of the experiment.

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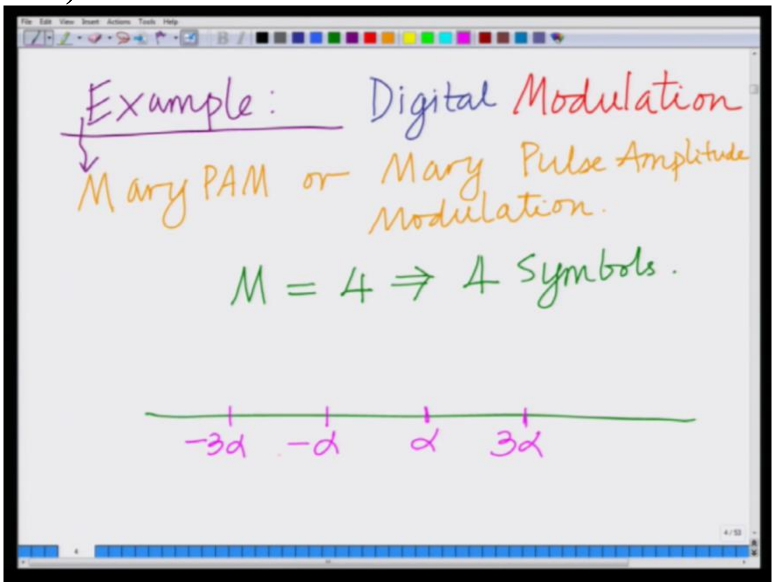


For instance, let us again take a pertinent example from communication, example from the context of communication for Sample space. Let us look at an example and in this example, we are going to look at a digital modulation format. More specifically, we are going to consider what is known as M ary PAM or M ary Pulse amplitude modulation. So I would like to look, illustrate this concept of Sample space, let us look at this digital modulation constellation which I'm terming, as an M ary PAM. Some of you who are already familiar with digital communication must know, this is M ary PAM or M ary Pulse Amplitude Modulation where you choose from one of several amplitude levels to convey information.

For instance, let us choose M is equal to 4. This implies, basically that there are 4 symbols. These symbols are equally spaced on a line. So I have, let's say.

$$-3\alpha, -\alpha, \alpha, 3\alpha.$$

So these are the 4 symbols. Right? So I have 4 symbols, **$-3\alpha$, $-\alpha$, $\alpha$, $3\alpha$** spaced at intervals of **$2\alpha$**. This is the pulse amplitude modulation. That is choosing from one of 4 possible amplitude levels to convey information and these are chosen randomly. That is for the next, there is uncertainty about the next amplitude level.

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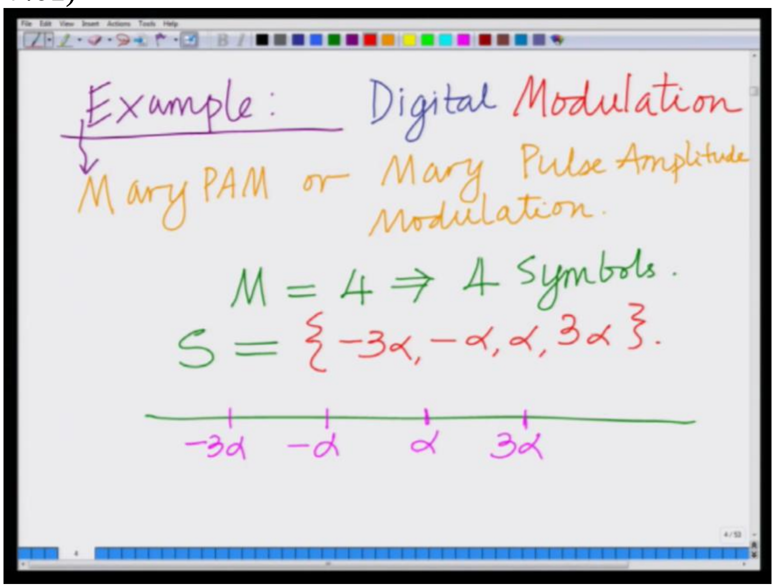


So the Sample space clearly for this example is basically equal to, the set of amplitude levels i.e. **$-3\alpha$, $-\alpha$, $\alpha$, $3\alpha$**.

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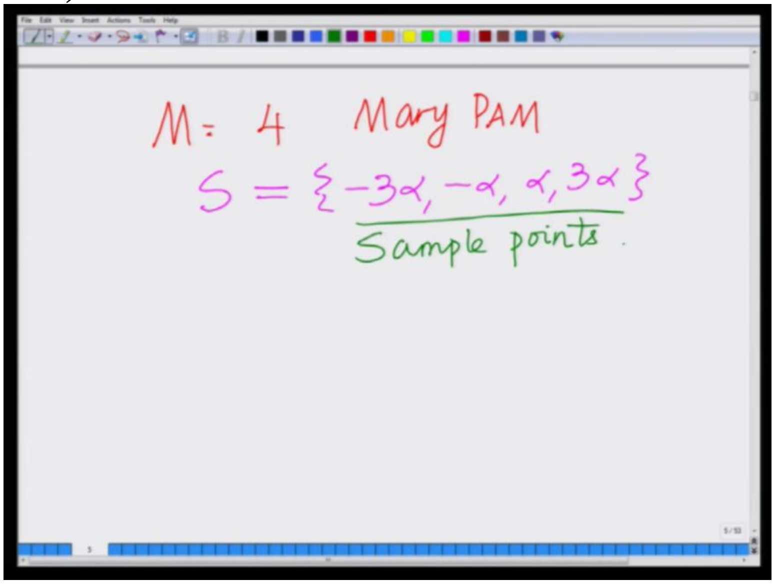


So for this M ary PAM where

$$M = 4.$$

I'm considering an M ary PAM example and my Sample space for this is given as

$$\mathbf{S = \{-3\alpha, -\alpha, \alpha, 3\alpha\}}$$

And each of these elements of this Sample space is known as a Sample point. So these are known as the Sample points. So I have a Sample space which consists of all the possible outcomes of the experiment about which there is uncertainty. Hence these outcomes are random. And each of these possible outcomes i.e. each of these single possible outcome are known as Sample points.

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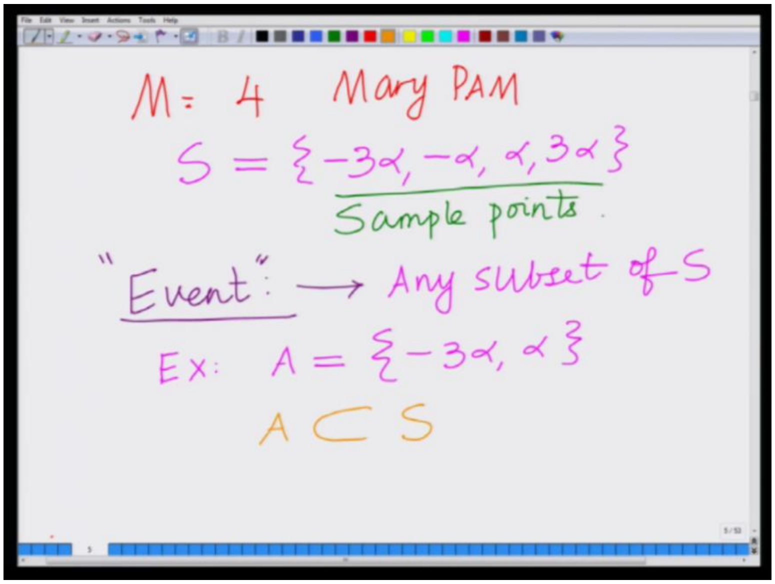


Now, also let us define what is known as an Event, an event **A** in this.

Let us define an event, the notion of an event which is the other important thing. An event is any subset of S. For example, consider A, the set A is equal to

$$A = \{-3\alpha, \alpha\}$$

Clearly we can observe that A is the subset of S. So, A consists of the symbols **- 3α, α** . So A is clearly a subset of S. Therefore A is an event. A is a random event of this experiment. So, A is an event.

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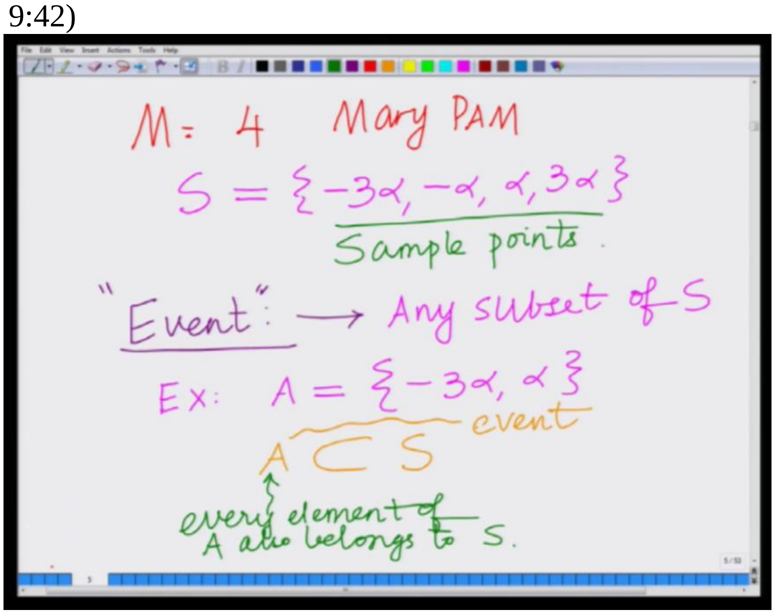


And also from elementary set theory, you must be familiar with definition of subset.

$$A \subseteq S \Rightarrow \forall\, a \in A, a \in S.$$

So A is a subset of S implies that every element of A is also present in S. Such a set A is considered which consists of, which is a subset of Sample space, S. Therefore it consists of some sample points in S. That is known as an event.

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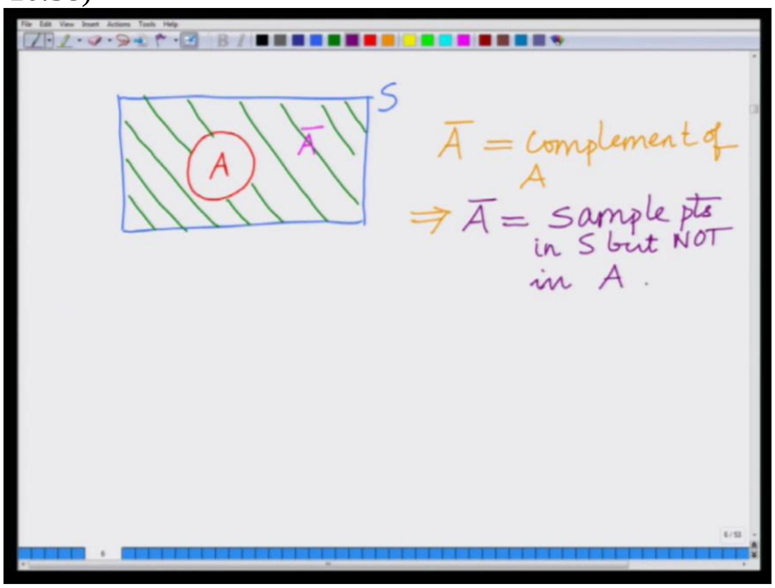


Also further from elementary set theory, you should be familiar with the concept of a complement of a set or the complement of an event. Now let us consider, to illustrate it pictorially. If this is my Sample space, S and this is my event A, all the events, all the Sample points that are in S but not in A, for instance, this shaded region over here represents the complement.

So

$\bar{A}$ ,from elementary set theory. $\bar{A}$ equals the Sample points in S but not in A

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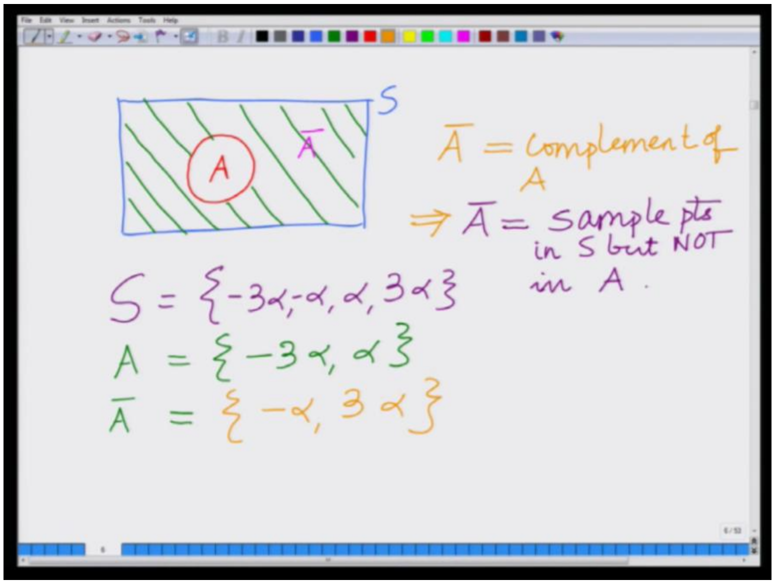


For instance, let us go back to our example. I have

$$S = \{-3\alpha, -\alpha, \alpha, 3\alpha\}$$

I have my set A which we have defined as

$$A = \{-3\alpha, \alpha\}$$

Therefore A complement is the set of elements that are in S but not in A. So

$$\bar{A} = \{-\alpha, 3\alpha\}$$

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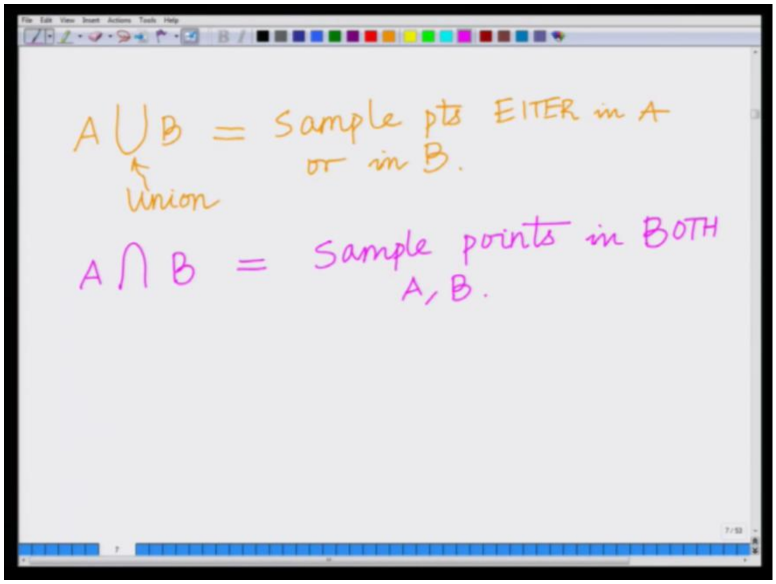


Now similarly, let us come to the next aspect. Again from the elementary set theory, this is also something that you should be familiar with, $A \cup B$ equals sample points, that is the union of 2 sets, union of 2 events, A and B. It is denoted by $\cup$, so this is the symbol for union, equals Sample points which are either in A OR in B.

And $A \cap B$ equals sample points in both A AND B. So we have 2 operations, $A \cup B$ and $A \cap B$ of 2 events, A, B. This is very simple and from elementary set, these are similar to elementary set theory. $A \cup B$ is the set of all Sample points which are either in A OR in B. And the event $A \cap B$ is the set of sample points which are both in A AND B.

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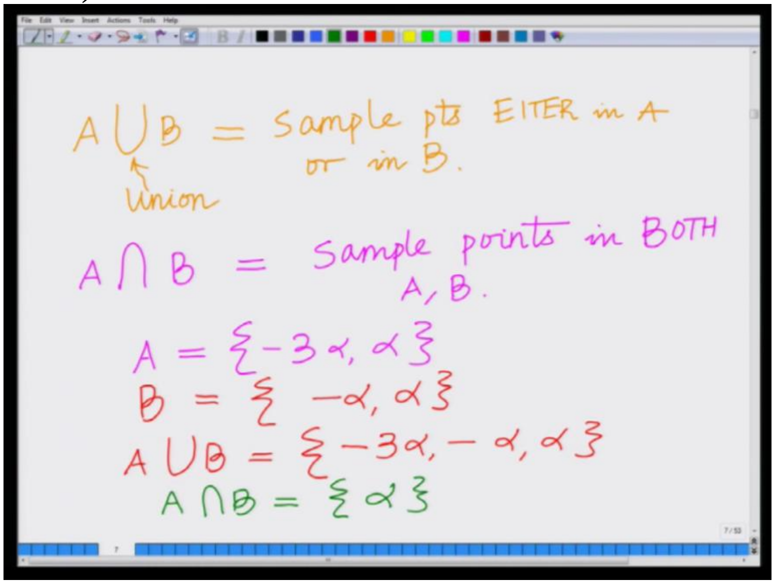


Again, let us go back, let us take a simple example. We have our example in which

$$A = \{-3\alpha, \alpha\}.$$

Let's say the event B is defined as

$$B = \{-\alpha, \alpha\}$$

Then,

$$A \cup B = \{-3\alpha, -\alpha, \alpha\}$$

And

$$A \cap B = \{\alpha\}$$

So we have defined the union of 2 events and the intersection of 2 events.

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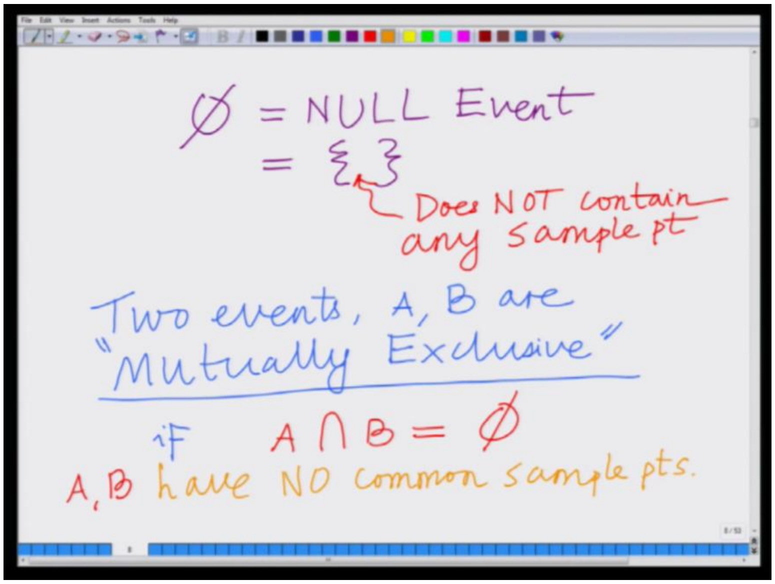


And let us also define now a special event that is also known as the null event , $\phi$ . Also denoted by the empty set. This is the null event. This contains no Sample points. Further, we can say 2 events are mutually exclusive if

$$A \cap B = \{\phi\}$$

That is to say, they have no common. A, B have no common sample points. That is we can see, , that 2 events, A and B are mutually exclusive if A intersection B is the null event. That is A and B have no common sample points.

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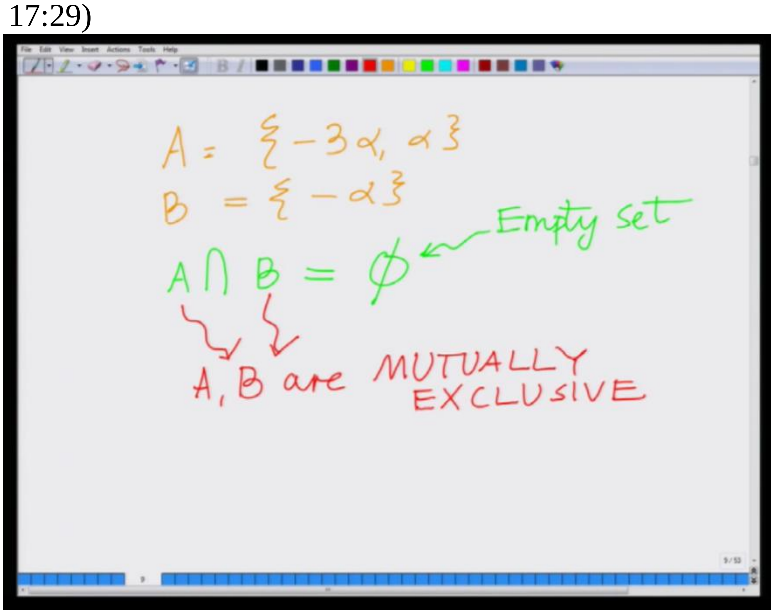

For example, let's take a simple example from our MPSK. If A contains the symbols **$-3\alpha$**, **$\alpha$** and B contains the symbol **$-\alpha$** . Then you can clearly see that we have

$$A \cap B = \{\phi\}$$

where this phi is the empty set. And we say that A and B are mutually exclusive. Further, it is easy to see some properties which can be easily clarified with a diagram.

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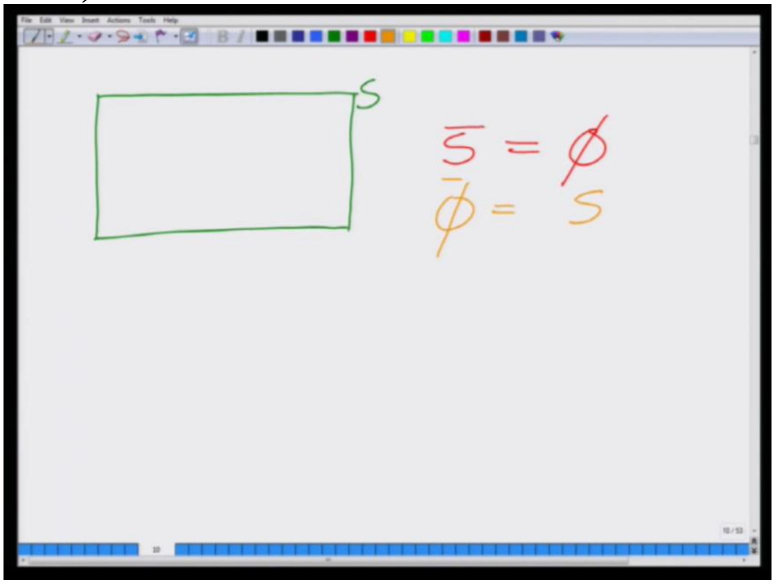


Let us see some properties which can be clarified with a diagram, a picture representation. Let us say, this is my sample space, S. We can clearly see that

$$\bar{S} = \Phi$$

Naturally, if you look at the Sample space, the complement of the Sample space, that is all the events which are not in the Sample space, which is the empty set phi.

And similarly,

$$\bar{\Phi} = S$$

Now, phi complement is the complement of empty set. All the Sample points not in the empty set. Empty set does not contain any Sample points. Therefore the complement of the empty set is basically all the Sample points in the Sample space which is the Sample space itself. So $\bar{\Phi}$ is equal to S.

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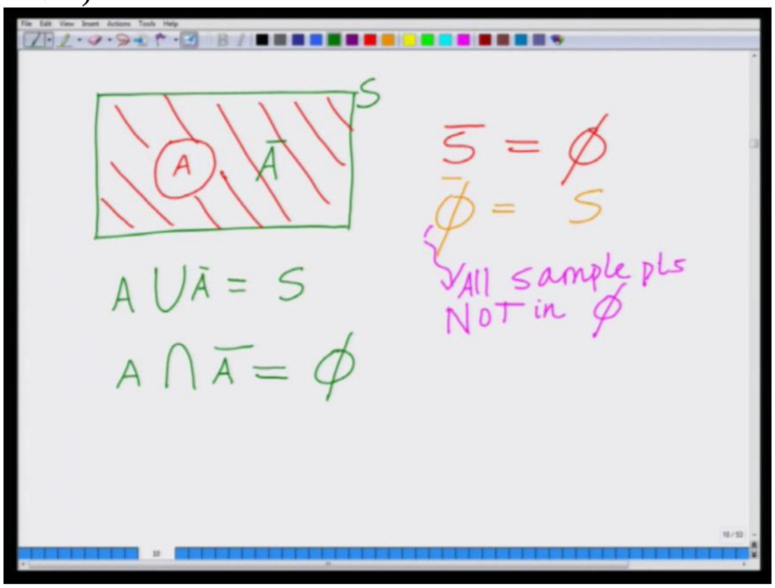


Further, if I have any set **A** in this and this is of course $\bar{A}$. Then,

$$\bar{A} \cup A = S$$

As, $\bar{A}$ contains all the elements in **S** that are not in **A**. Therefore naturally, $\bar{A} \cup A$ is the Sample space S.

Further as **A** and $\bar{A}$ do not have any elements in common. Therefore,

$$\bar{A} \cap A = \Phi$$

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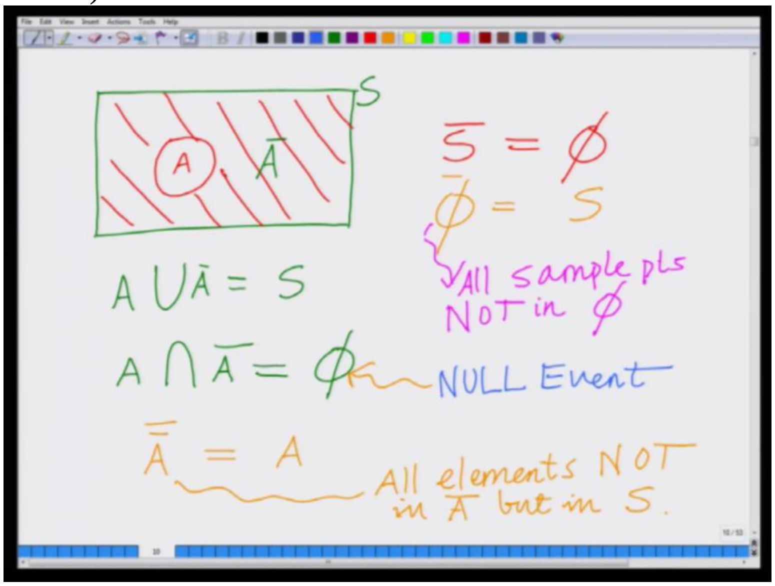


Further, the last property which is also obvious. That is,

$$\bar{\bar{A}} = A$$

if I take the complement of A complement, all the elements which are not there in A complement but rather in the Sample space is A itself. So, complement of $\bar{A}$, is A itself. So A complement complement is **A**.

So we have some key properties. That is

$$\bar{S} = \Phi$$

$$\bar{\Phi} = S$$

$$\bar{A} \cup A = S$$

$$\bar{A} \cap A = \Phi$$

$$\bar{\bar{A}} = A$$

These are some properties which I will not go into the details of and which are fairly obvious from both, from elementary set theory and also from this simple diagrammatic representation that is given here.

So we will end this basic module here about experiment, outcomes, events which are basically subset of the Sample space and the various operations on events, that is the union, intersection, the complement and some properties of these events. So we will end this module here. Thank you very much.