Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture – 48
Cyclic Prefix in OFDM Systems

Hello, welcome to another module in this massive open online course on the principles of CDMA, MIMO, OFDM wireless communication systems. In the previous module, we have looked at the application of the IFFT and FFT operations at the OFDM transmitter and receiver respectively, we said we can use the IFFT operation to generate the transmit signal in an OFDM system, and the FFT operation at the receiver to process the received OFDM signal. Today, let us look at another aspect of OFDM transmission that is a cyclic prefix.

So, today we are going to another important aspect to OFDM, which is also termed as the Cyclic Prefix.

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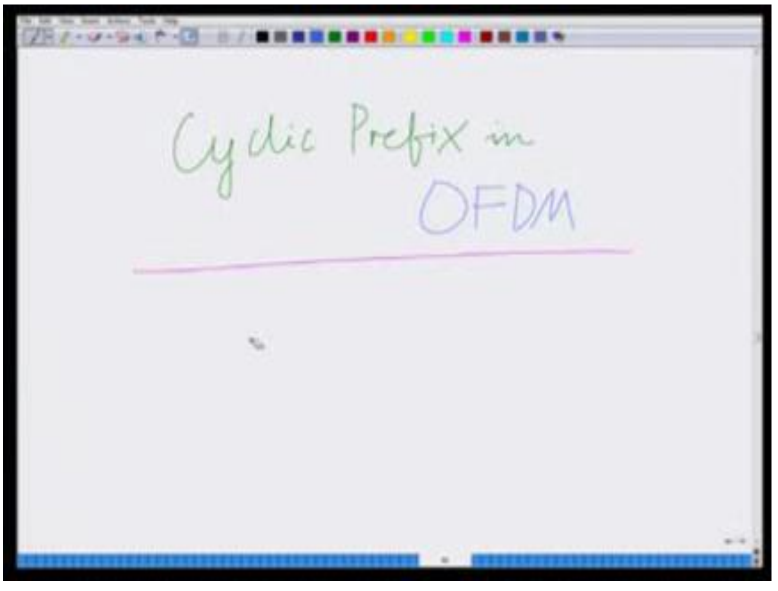

So, we are going to look at the Cyclic Prefix in OFDM or orthogonal frequency division multiplexing systems. So, for this let us start by considering frequency selective channel. So what we want to do is want to start by considering of frequency selective channel.

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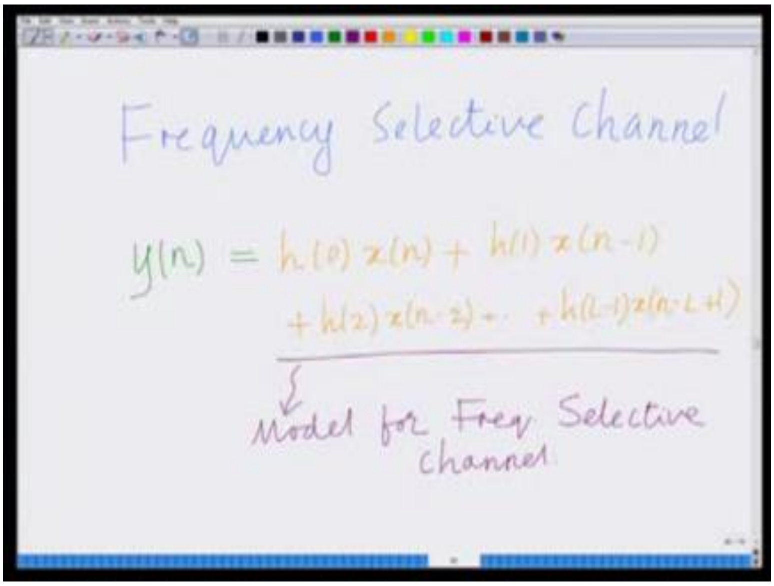

So, let us consider a frequency, of frequency selective channel and as we have said before a frequency selective channel can be modeled as y n equals, the output $y(n)$ at time instant n equals

$$y(n) = h(0)\, x(n) + h(1)\, x(n-1) + \ldots + h(L-1)\, x(n-L+1)$$

this is the model without noise for a frequency selective channel, this is the model for of frequency selective channel remember, we had looked at this in the context of CDMA also.

So, this has channel tapes, $h(0)$, $h(1)$, $h(N-1)$, that is L channel tapes and we have also seen that the output $y(n)$ depends not only on the input symbol $x(n)$ at time instant n, but it also depends on $x(n-1)$, $x(n-2)$ so on until $x(n-L+1)$, therefore, this channel has inter symbol interference. So, in the time domain this channel has inter symbol interferences, which is the same thing as frequency selective channel or frequency selective fading in the frequency domain therefore, this model of input output symbol, the input output received symbol model with inter symbol interference represents a frequency selective channel.

And now we are going to consider this frequency selective channel in an OFDM system. So, as we said previously at the OFDM system, we transmit we generate the transmitted signal of the transmit signal using an IFFT operation.

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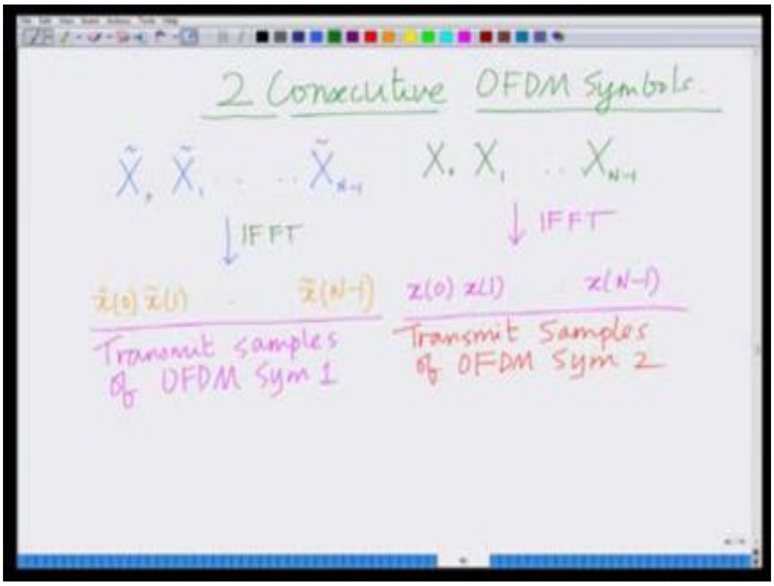

Let us do the following, let us consider the first set of symbols $\tilde{X}_{0,}\tilde{X}_1\tilde{X}_{N-1}$, these are the symbols that are loaded on to the sub carriers and then I process them using the IFFT operation to generate the samples $\tilde{x}(0)$,$\tilde{x}(1)$, $\tilde{x}(2)$, $\tilde{x}(N-1)$. In the 2nd OFDM symbol, let us say I have samples $x(0)$,$x(1)$, $x(2)$, $x(N-1)$, I again use the IFFT operation to generate the corresponding samples of the transmit signal $x(0)$,$x(1)$, $x(2)$, $x(N-1)$, So, these are the samples transmitted samples corresponding to 1st OFDM symbol, the transmit samples of OFDM symbol 1 and these are the transmit samples of OFDM symbol 2, so what do we have? what we are saying is we are considering the transmission of 2 consecutive, we are considering the transmission of 2 consecutive OFDM symbols, the symbols $\tilde{X}_{0,}\tilde{X}_1\tilde{X}_{N-1}$ are the symbols of the first OFDM symbol, which are loaded on to the variance sub carriers, we take the IFFT to generate the transmit samples, which are $\tilde{x}(0)$,$\tilde{x}(1)$, $\tilde{x}(2)$, $\tilde{x}(N-1)$.

Similarly, I have a 2nd OFDM symbol consisting of symbol $X_{0,}X_1X_{N-1}$ of which I am taking the IFFT to generate the transmit samples $x(0)$,$x(1)$, $x(2)$, $x(N-1)$ and I am transmitting these over the channel, so I have, I am considering the transmission of 2 consecutive to OFDM symbols and generating the IFFT that I have using the IFFT, I am generating the sample of the 1st set of symbols transmitting them, then I am generating the transmit samples using the IFFT for the 2nd OFDM symbols and I am transmitting these samples again consecutively.

Now, let us look at the output of this OFDM system, now let when I look at the output corresponding to x of 0, now here at this point when I look I have at the output of the frequency selective channel, I have $y(n) = h(0)\ x(0)\ + \ h(1)$ times the previous sample, but the previous sample is $\tilde{x}(n-1) + \ h(2)$ times the previous sample, so on and so forth, and here at this point you can see, I have a problem because I have inter block interference. Now if look at this OFDM received OFDM symbol, you can see that

$$y(0) = h(0)\ x(0) + h(1)\ \tilde{x}(n-1) + h(2)\ \tilde{x}(n-2) + .. + h(L-1)\ \tilde{x}(n-L+1)$$

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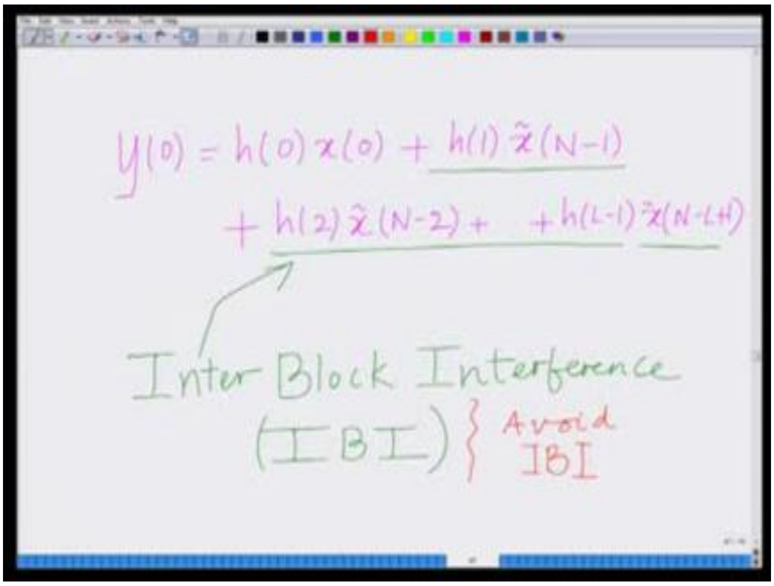

What we have over here, if you can see these are the samples from the previous block so in this there is what is known as inter block so we are seen a new term here, there is inter block interference or there is IBI because of the frequency selective nature of the channel, there is IBI or inter block interferences where the samples in the current block are being interfere by the samples from the previous block in this OFDM or orthogonal frequency division multiplexing system and now our main motive is to basically avoid this inter block interference or remove this inter block interference in the OFDM system. So, we would like to avoid device a mechanism to avoid, the inter block interference in OFDM and for that purpose we are going to use the Cyclic Prefix.

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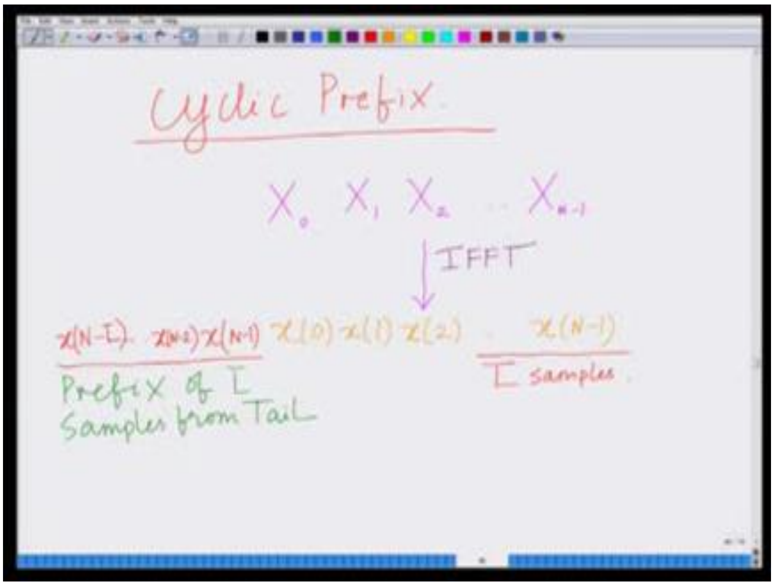

So, the Cyclic Prefix helps avoid the inter block interference in OFDM, what is the Cyclic Prefix now? Let us look at the Cyclic Prefix I am transmitting, let us consider the symbols $X_{0,}X_1X_{N-1}$, these are the symbols that are loaded on to the various sub carriers, I am taking the IFFT to generate the samples $x(0)$, $x(1)$, $x(N-1)$, now instead of directly transmitting the samples I am going to add a prefix $x(N-1)$, $x(N-2)$ so on $x(N-\bar{L})$.

So, what I am taking? I am taking $\bar{L}$ samples and I am prefixing them L prefix of $\bar{L}$ samples from the tail, so what I am doing over here, if you look carefully because this is a key step, what am I doing? I have a block of n OFDM samples that are generated using the IFFT in this N samples, I am taking the last $\bar{L}$ samples that is $x(N-1)$, $x(N-2)$ so on $x(N-\bar{L})$, these last $\bar{L}$ samples and I am prefixing them into to the to the front of this transmitted OFDM samples.

So, the total number of samples that I will transmit is $N + \bar{L}$ that is the total length of the block has now become $N + \bar{L}$ I am adding a prefix of $\bar{L}$ samples from the tail of the OFDM block in the beginning of the OFDM block, so I am cycling samples from the end of the OFDM block towards the beginning of the OFDM block to add a prefix, hence this is known as a Cyclic Prefix. Let me again clarify this important idea once I am taking $\bar{L}$ samples from the end or from the tail of the OFDM block I am cycling them in a cyclic fashion I am moving them to the prefix of the transmitted OFDM block and I am adding this prefix to the transmitted OFDM block hence this is known as a Cyclic Prefix, this I am adding a Cyclic Prefix to the transmitted OFDM samples.

And now, let us look at the corresponding received OFDM signal, so now what do I have? I have the OFDM samples with the Cyclic Prefix.

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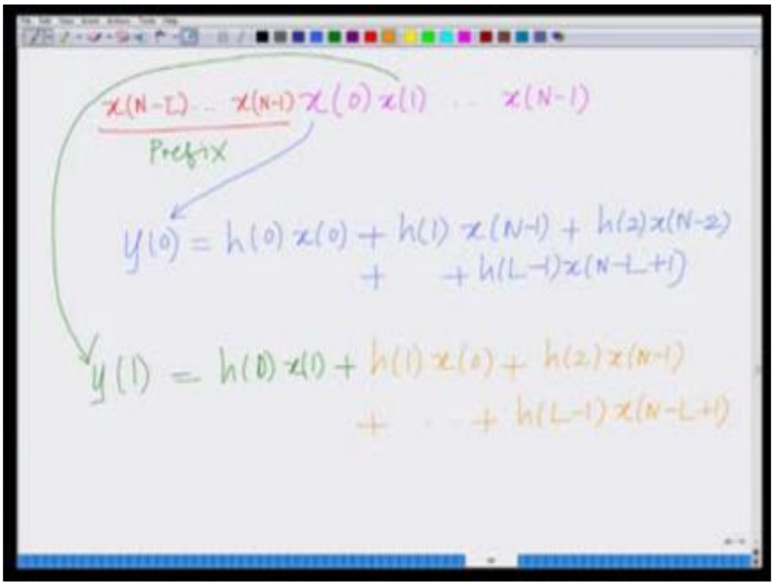

So, I have the samples $x(0)$, $x(1)$, $x(N-1)$ to which I have added the Cyclic Prefix $x(N-1)$, $x(N-2)$ so on $x(N-\bar{L})$, this is my prefix and now if you look at the 1st symbol I have

$$y(0) = h(0)x(0) + h(1)x(N-1) + h(2)\,x(N-2) + ..+ h(L-1)\,x(N-L+1)$$

So, the interference is now restricted to the samples from the same block therefore, we have avoided the inter block interference by adding the Cyclic Prefix, we have been able or we have manage to restrict the inter symbol interference to samples from the same block. So, in this system with the Cyclic Prefix there is inter symbol interference, but there is no inter block interference therefore, we have avoided the inter block interference and therefore, thus the Cyclic Prefix has helped us avoid inter block interference.

$$y(1) = h(0)x(1) + h(1)x(0) + h(2)\, x(N-1) + .. + h(L-1)\, x(N-L+1)$$

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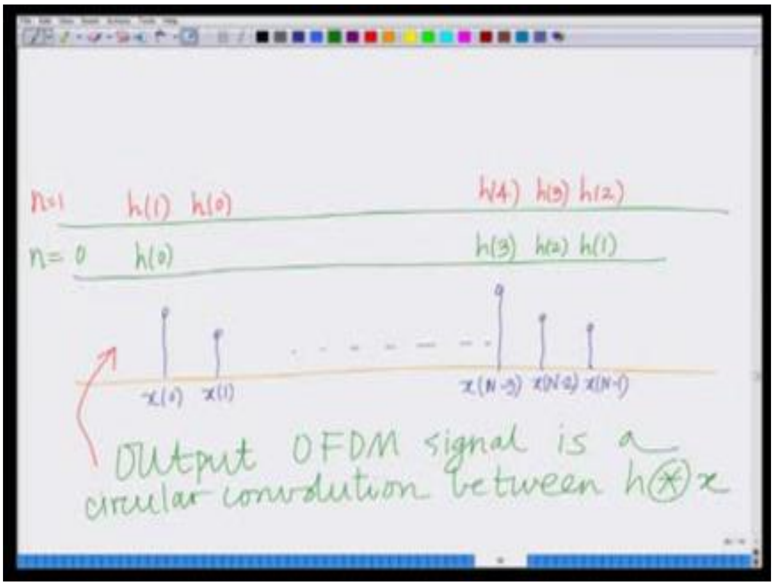

Let us draw a diagram to represent this to clarify in greater detail what is happening in this scenario, now if you look at this let me draw the various samples $x(0)$, I have here $x(1)$ so on and so forth, I have $x(N-2)$, I have $x(N-1)$ and now let me draw a line for time instant 0, I have $h(0)x(0)$, $h(1)x(N-1)$, $h(2)\, x(N-2)$, so this is for time instant or this is for $N = 0$. Now, look at $N = 1$, we have something very interesting, we have $h(0)x(1)$, $h(1)x(0)$, $h(2)\, x(N-1)$, so what you can see is the whole channel the coefficient of the channel are moving in a circular fashion over the transmitted symbols. And if you can look at that, what you can see is that the channel coefficients are moving with these time instant, the channel coefficients are moving circularly over the transmit OFDM samples therefore, this output operation is a circular convolution, this operation that is the output OFDM signal is a circular convolution, this is a very important idea output OFDM signal is a circular convolution between the channel h and the input x.

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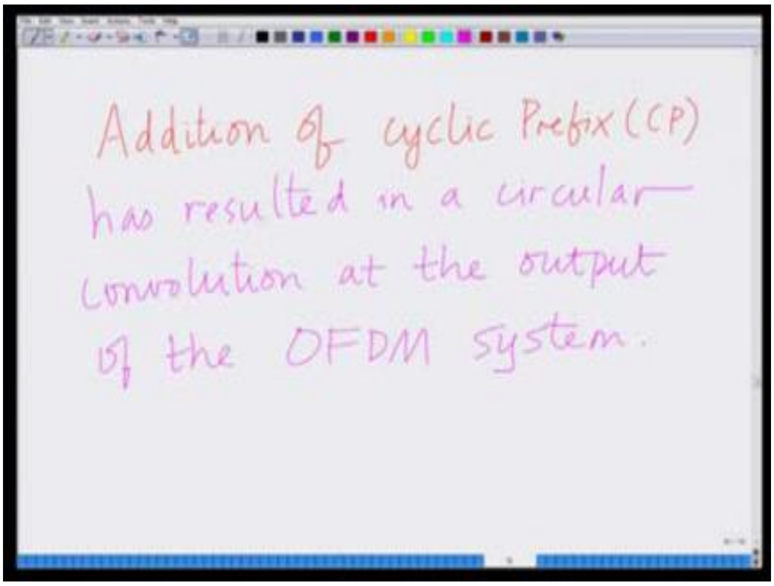

And this is a very important idea so let me repeat that, adding the Cyclic Prefix after addition of the Cyclic Prefix, so addition of Cyclic Prefix which is abbreviated as CP has resulted in a circular convolution at the output of adding a Cyclic Prefix. So, what is happened we have added a Cyclic Prefix at the beginning of the transmitted OFDM signal and as a result what we are observing at the output is surprisingly, we are observing that the generated samples $y(0)$, $y(1)$, $y(N-1)$ are obtained from a circular convolution of the channel coefficient $h(0)$, $h(1)$, $h(N-1)$with the transmit samples of the OFDM signals that is $x(0)$, $x(1)$, $x(N-1)$ therefore, I can now represent it mathematically as interestingly.

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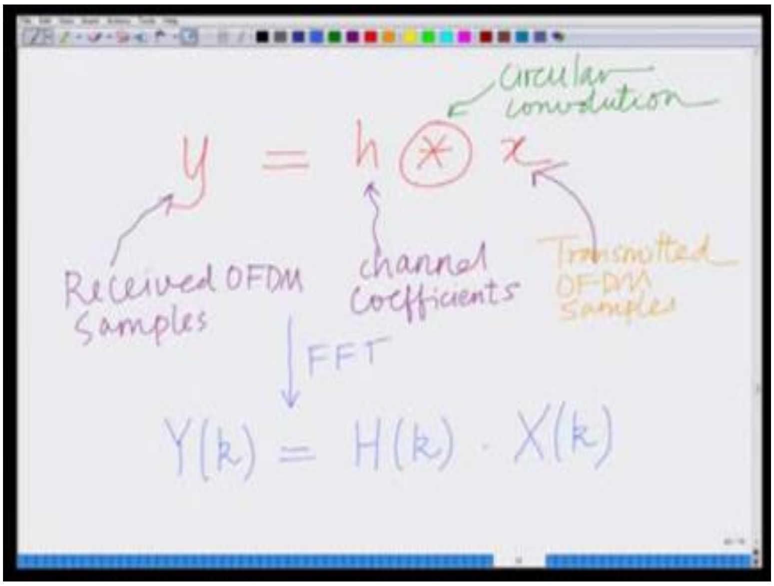

I can now represent this mathematically as

$$y = h \oplus x$$

y equals h circularly convolved with X, where y is the received OFDM samples, this is the channel coefficients or the channel filter x is the transmitted OFDM samples. And this symbol here the circle with star in between of course, denotes the circular convolution and we know from DSP that if the input if y is given as a circular convolution between h and x, if I take the FFT or the DFT of this, I will have

$$Y(k) = H(k) . X(k)$$

because, circular convolution in the time domain is equivalent to a product of the DFT coefficients a product of the FFT coefficient in the frequency domain, that is if y is h circularly convolved with x, then if I take the corresponding DFT, then the DFT coefficient where $Y(k)$ is the DFT coefficient of the signal y, $H(k)$ is the DFT coefficient of the channel h and $X(k)$ is the DFT coefficient of the transmitted a signal x.

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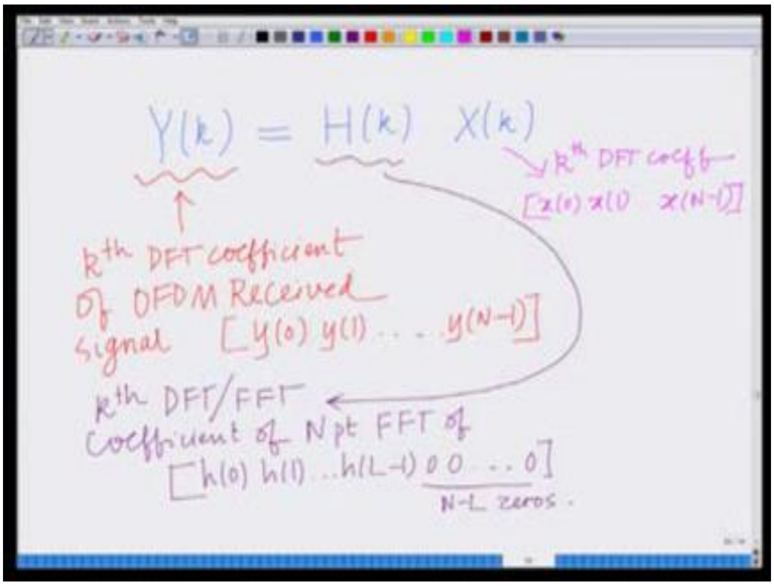

Therefore, what we have now in the DFT domain, we have

$$Y(k) = H(k) . X(k)$$

and what we have is the k-th DFT coefficient of the OFDM received signal. In fact, the received signal is $y(0)$, $y(1)$, $y(N-1)$, so I am taking the k-th DFT coefficient from the N point DFT or FFT. Now for $H(k)$ is the k-th DFT or FFT coefficient of N point FFT because h is of length l I have to 0 pad it so $h(0)$, $h(1)$, $h(L-1)$ and I have; obviously, $X(k)$ is the FFT or the k-th FFT coefficient k-th, this is k-th DFT coefficient of the samples $x(0)$, $x(1)$, $x(N-1)$.

And this is a very important step in the OFDM, If I look at the FFT which means I am going back to the frequency domain therefore, I am looking at the sub carriers over the k-th sub carrier very interestingly what I have is $Y(k)$ that is the k-th DFT coefficient of the received OFDM signal equals $H(k)$, which is the k-th DFT coefficient of the zero padded channel coefficient $h(0)$, $h(1)$, $h(L-1)$times $X(k)$ were $X(k)$ is the k-th DFT coefficient of the OFDM samples $x(0)$, $x(1)$, $x(N-1)$.

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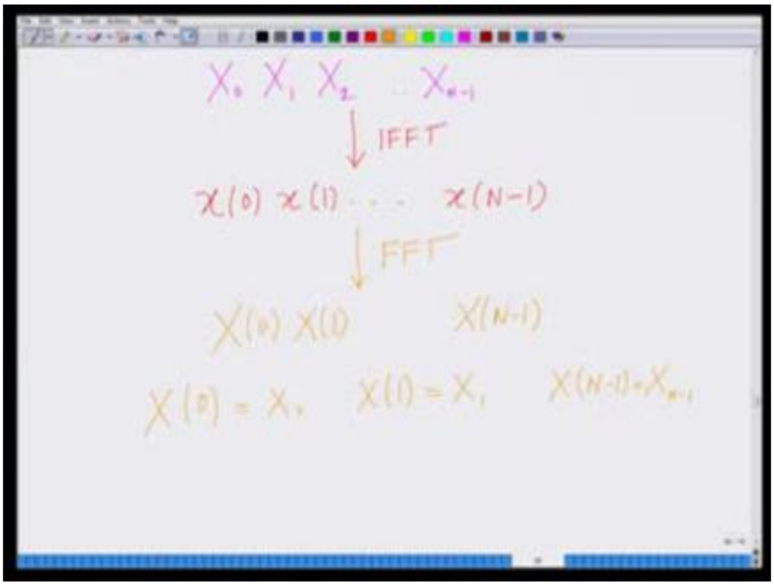

And now also realized that the $x(0)$, $x(1)$, $x(N-1)$, these are nothing but, these are generated by an IFFT of the OFDM symbols $X_{0,}X_1X_{N-1}$.. So, therefore, when I do the FFT or the DFT, when I do the FFT of these, I get back the symbols $X(0)$, $X(1)$, $X(N-1)$. and therefore, it means that $X(0)$ = $X_{0,}$, $X(1)$ = $X_{1,}$, $X(N-1)$ = $X_{N-1,}$. So, what I am saying is the following thing $x(0)$, $x(1)$, $x(N-1)$ are generated by an IFFT of $X_{0,}X_1X_{N-1}$ therefore, once I take the FFT of the x, $x(0)$, $x(1)$, $x(N-1)$, I basically get back I basically get $X(0)$, $X(1)$, $X(N-1)$, which are basically nothing, but respectively equal to x, $X_{0,}X_1X_{N-1}$, initially taking the IFFT to generate the OFDM samples and then I am generating the FFT again so the FFT and FFT cancel and I therefore, get back the symbols that are loaded on to the k-th on to the each sub carrier.

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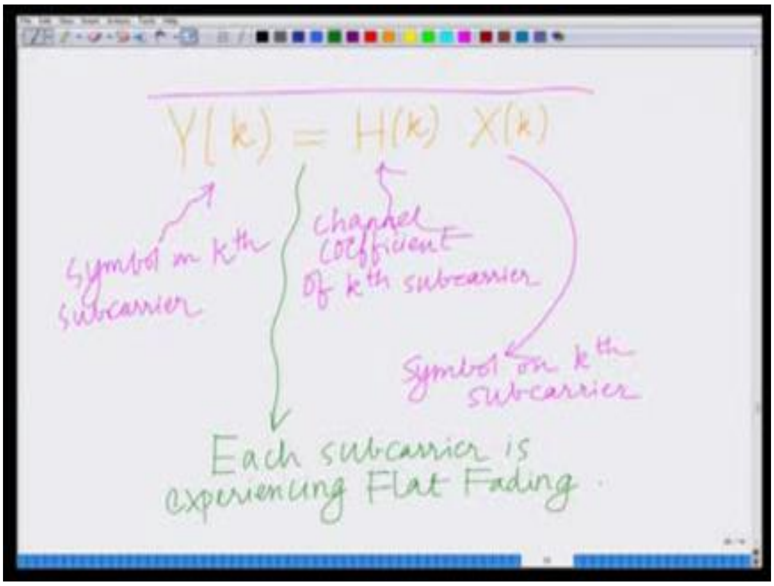

Therefore very interestingly what I have is $Y(k)$ in our OFDM system after the FFT

$$Y(k) = H(k) . X(k)$$

$Y(k)$ is symbol on k-th sub carrier, $H(k)$ is the channel coefficient of the k-th subcarrier and $X(k)$ is the symbol, $H(k)$ is the symbol that is transmitted on the k-th subcarrier and therefore, now you can see that each subcarrier is experiencing flat fading. Because, what do we have? We have $Y(k)$ equals $H(k)$ times $X(k)$, there is no frequency is electivity the symbol $X(k)$ in the subcarrier is multiply by the channel coefficient $X(k)$ to generate the output symbol $Y(k)$, off course we have ignored effect of noise, but you can see there is no inter symbol interference once we consider each subcarrier, there is no inter symbol interference and in fact, this is the motivation with which, we set out for OFDM; remember orthogonal frequency division multiplexing has an aim to overcome the frequency selective nature of the wireless channel.

And now, we are seeing with this IIFT FFT operation at the transmitter in receiver respectively and the addition of the Cyclic Prefix that the transmitter, what we have been able to do is we have been able to convert this wireless communication system, into a frequency flat fading channel across each subcarrier and how many such subcarriers?

Precisely we have N subcarrier remember, we said we have N sub bands in each of these sub band we have a subcarrier therefore, we have N subcarriers, let me draw this schismatically to conclude this module and therefore, if I look at this in a schematic fashion.

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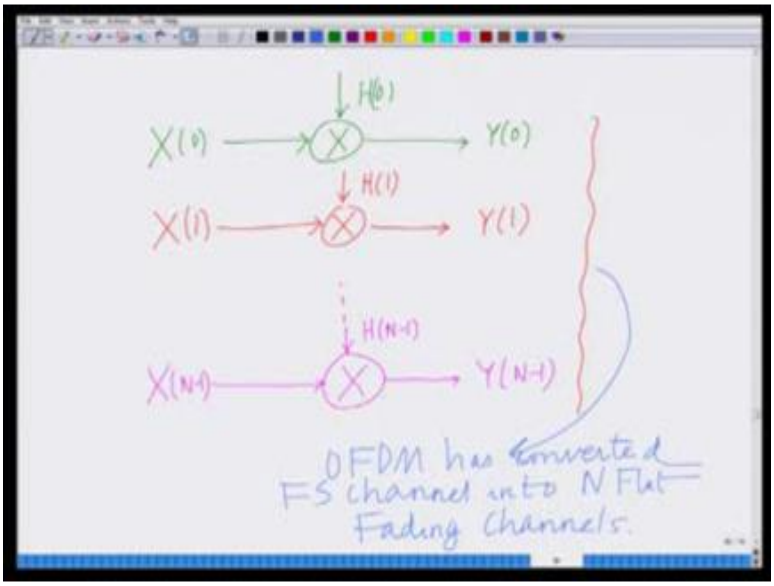

I have $\mathrm{X}(0)$ which is multiplied by the channel coefficient $\mathrm{H}(0)$ to generate the symbol $\mathrm{Y}(0)$ across sub carrier 0, I have $\mathrm{X}(1)$ which is multiplied by the channel coefficient 1 to generate the symbol $\mathrm{Y}(1)$ and finally, I have also this continuous and finally, I have $\mathrm{X}(N-1)$ multiplied by $\mathrm{H}(N-1)$ to generate the subcarrier $\mathrm{Y}(N-1)$

So, what I have is basically OFDM has converted frequency selective channel into N flat fading channels OFDM, has converted frequency selective channel into N flat fading, so what we are saying very interestingly with the fallowing thing, that is after I take the FFT at the receiver it is as if I have N parallel flat fading channels across the N subcarriers, that is each sub carrier can be thought of as a flat fading channel with input so if I look at the k-th channel sub carrier. I have input $\mathrm{X}(k)$ multiplied by the channel coefficient $\mathrm{H}(k)$ and there is generating the received signal $\mathrm{Y}(k) = \mathrm{H}(k) \cdot \mathrm{X}(k)$ and therefore, across each sub carrier k, I have a flat fading channel total number of sub carriers is N, I have a collection of N parallel flat fading channels, remember we said OFDM is a multi carrier transmission system.

Whereas, in the single carrier transmission system we are transmitting 1 symbol every time instant $\frac{1}{B}$ where B is the net band width of the system in an OFDM system, we are transmitting n symbols in parallel over N sub carriers and that time of each OFDM symbol is $\frac{N}{B}$., we are expanding the time of each symbol there by removing the frequency selective nature of the channel of removing inter symbol interference. Now we are seen basically OFDM converts frequency selective channel into a set of N parallel flat fading channels 1 flat fading channel across each sub carrier so these are 2 very important ideas in OFDM, 1 is the FFT IFFT processing in OFDM and 2 is the addition of the Cyclic Prefix, so these are the 2 key steps in OFDM.

With this let us conclude this module here and let us look at other aspects of OFDM in the subsequent module.

Thank you very much.