Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture – 47
FFT/IFFT Processing in OFDM

Hello. Welcome to another module in this Massive Open Online Course, and the Principles of CDMA, MIMO and OFDM Wireless Communication. In the previous modules we have looked at a basic introduction to a multi carrier modulation system. And we said that this multi carrier modulation system forms a basis or is a precursor, for an OFDM or Orthogonal Frequency Division Multiplex. So, let us now look at two key aspects of OFDM or two key aspects which incorporated in MCM will lead to OFDM. So today we are going to look at the FFT.

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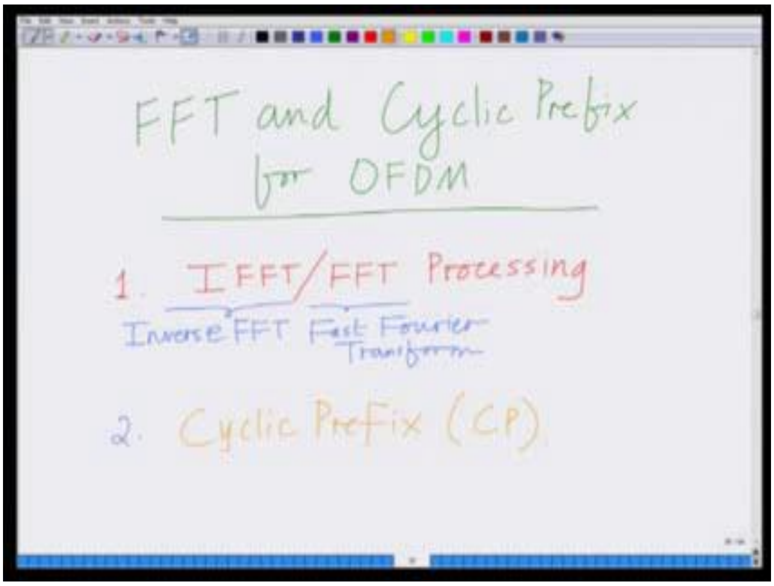

This module FFT and Cyclic Prefix for OFDM that is Orthogonal Frequency Division Multiplex, so we look at two key aspects one is the IFFT / FFT processing. As you all might already be familiar FFT is the Fast Fourier Transform. IFFT is the inverse FFT or Inverse Fast Fourier Transform. And two, the second is what is also known as a Cyclic Prefix which is also abbreviated as C P. So, we are going to look at two key aspects in OFDM one is the IFFT slash FFT processing, where FFT stands for the Fast Fourier Transform, IFFT is the Inverse Fast Fourier Transform. The second aspect is something unique which is known as the Cyclic Prefix. So, we are going to look at these two aspects which are the key ingredient or the key steps in OFDM or orthogonal frequency division modulation based wireless communication.

Now, let us go back to our transmitted signal.

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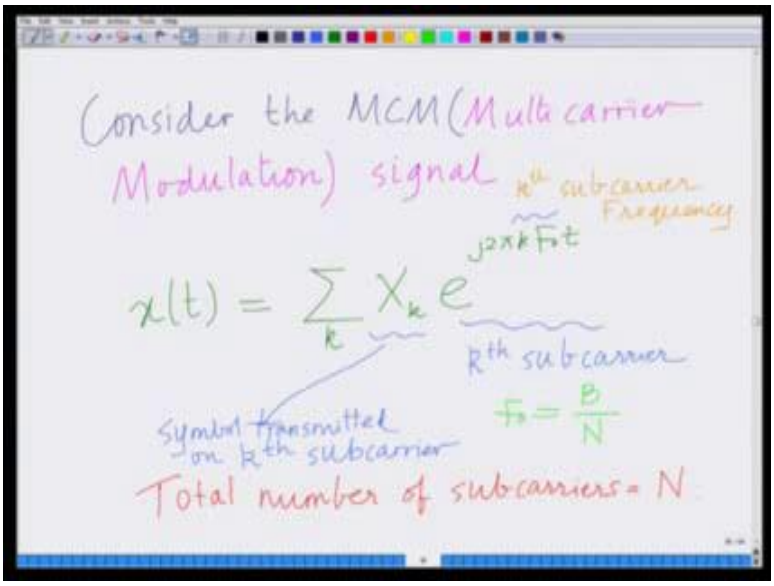

So, consider the MCM signal. Let us go back to our multi carrier modulation signal, I hope everyone remember MCM, stands for multi carrier modulation. So, consider the MCM signal. So, I have the MCM signal which is

$$x(t) = \sum_k X_k e^{j2\pi k f_0 t}$$

Let me refresh your memory X_k is a symbol transmitted on the k-th subcarrier, $s_k(t)$ is the symbol transmitted on the k-th subcarrier, $e^{j2\pi k f_0 t}$ is the k-th subcarrier, this is the k-th subcarrier, kf_0 this is the k-th subcarrier frequency and we have N subcarriers total number of subcarriers.

So, we have the transmitted signal $x(t) = \sum_k X_k e^{j2\pi k f_0 t}$. Remember we said k, we said f_0 that is the subcarrier fundamental frequency is basically $\frac{B}{N}$ where B is the total bandwidth and N is the total number of subcarriers.

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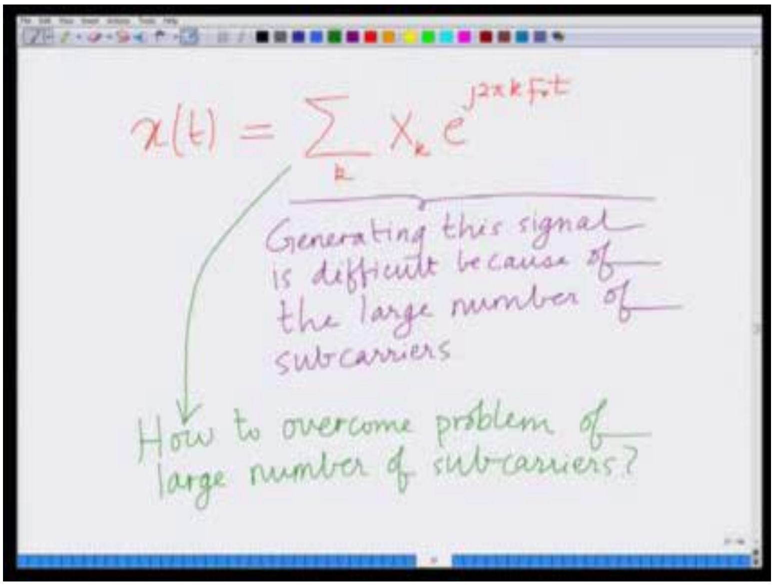

Now if you look at this signal x of t; let me rewrite this signal

$$x(t) = \sum_k X_k \, e^{j2\pi k f_0 t}$$

Generating this signal is difficult, because of the large number of subcarriers. For instance, we are talking about n the number of subcarriers, so the number of subcarriers can be as thousand which means for each subcarrier at is subcarrier frequency you need an oscillator, which means if you have thousand sub carriers you need thousand oscillators. And mind you these thousand oscillators have to be precisely placed at that subcarriers that placing that is 0, f_0, $2f_0$, $-f_0$, $-2f_0$, because only then the orthogonality principle between these subcarriers will hold.

Remember we looked at the orthognality principle between the subcarriers yesterday, that is integral that is if I look at following, which is over 1 fundamental period is equal to

$$f_0 \int_0^{\frac{1}{f_0}} e^{j2\pi(k-l)f_0 t} \, dt \quad = 1 \quad \text{if } k = l$$

$$= 0 \quad \text{if } k \neq l$$

which means the different subcarriers are orthogonal, and this orthogonality which is an important aspect of multicarrier modulation. This will hold only when the subcarriers spacing is maintained precisely.

Therefore, it is it difficult to first have such a large number of oscillators further it is even more challenging to have a precise subcarriers spacing with these large number of oscillators, large number of carriers in a physical system. So therefore, how do we overcome this problem of a large number of subcarriers? Therefore, how does one overcome this problem of this large number of subcarriers and that can be explained as follows.

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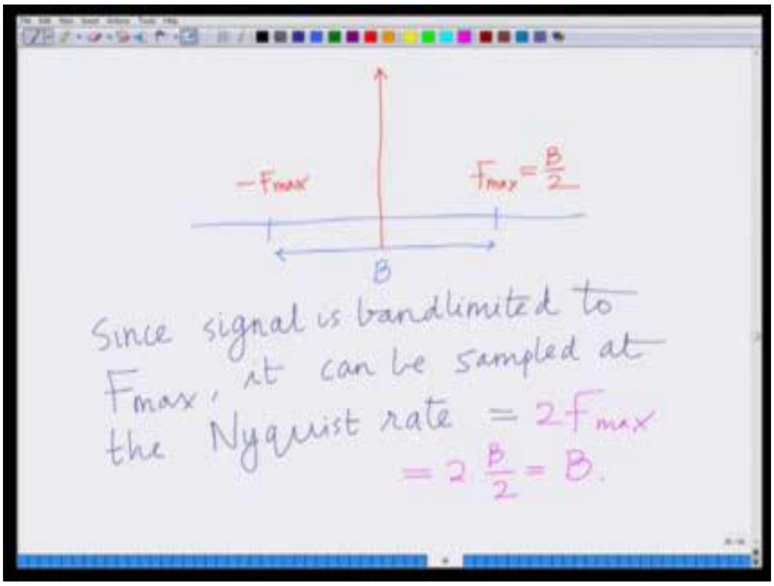

For instance, let us go back to our initial system. In our initial system we said that the signal has a band width of, the total band width of the signal is B. So, let us look at the initial system. We said that initial system has a band width of B, which means my maximum frequency $f_{max} = \frac{B}{2}$. So, what are we saying is our transmitted signal is in the band width B, right? So, the maximum frequency that is $f_{max} = \frac{B}{2}$. So, it lies between $-f_{max}$ and f_{max}, where f max is the maximum frequency that is equal to $\frac{B}{2}$. Now this is a band limited signal therefore I can sample it at the Nyquist rate, the Nyquist rate is $2f_{max} = B$.

Therefore, since the signal is band limited to f max, it can be sampled at $2f_{max}$ that is the Nyquist rate $2f_{max}$ $= B$. So, since signal is band limited to f_{max} it can be sampled at the Nyquist rate. And what is the Nyquist rate, we all know from the Nyquist sampling theory that the Nyquist rate equals $2f_{max}$ which is equal to twice $\frac{B}{2}$ which is equal to B. So, what I am saying is this is a signal that is band limited, the total band width is B, the maximum frequency f_{max} $= \frac{B}{2}$. Therefore this signal can be sampled at the Nyquist rate which is $2f_{max}$ $= B$.

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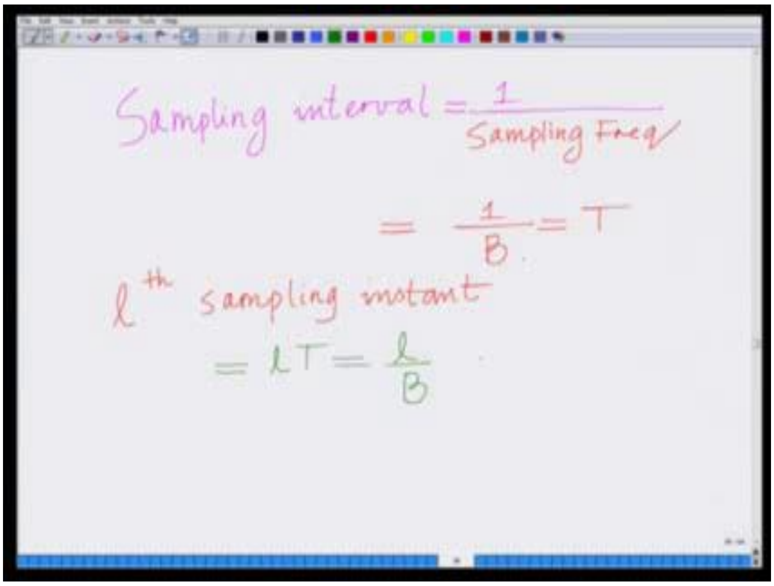

And therefore, the sampling duration; therefore, the sampling interval is 1 over sampling frequency; this is 1 over sampling frequency equals $\frac{1}{B}$. So, the Nyquist sampling interval equals 1 over the sampling frequency which means and this let me denote this by T, which means the l-th, let me call this l-th sampling instant = l T

l-th sampling instant = l T

or the l-th sample is taken at $l\,T = \frac{l}{B}$. So, what we are saying is the sampling frequency is B; the sampling duration is $T = \frac{1}{B}$ the l-th sample is taken as $l\,T = \frac{l}{B}$.

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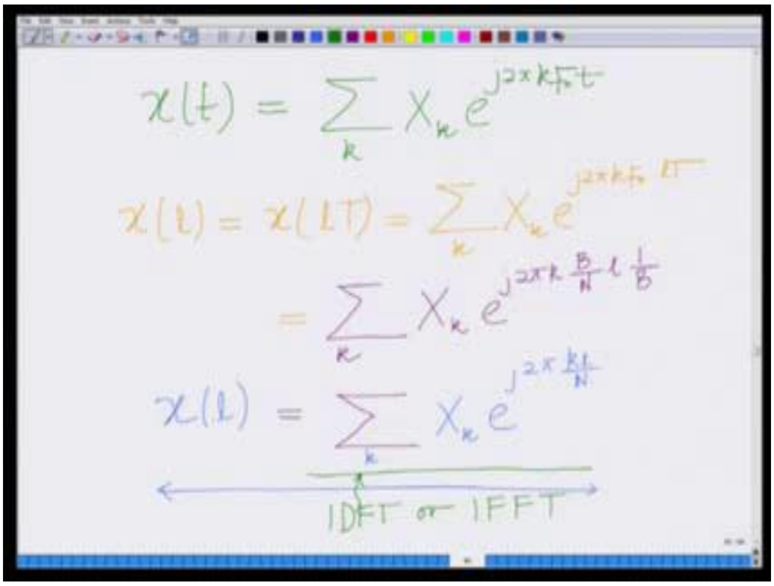

Now, let us look at what the l-th sample is let us go back to our transmuted signal I have

$x(t) = \sum_k X_k\, e^{j2\pi k f_0 t}$

$x(l) = x(l\,T) = \sum_k X_k\, e^{j2\pi k f_0 l T}$

$= \sum_k X_k\, e^{j2\pi k \frac{B}{N}\frac{l}{B}}$

$= \sum_k X_k\, e^{j2\pi k \frac{l}{N}}$

and this is exactly the expression for the IFFT, it is precisely the l-th IFFT that is the Inverse Fast Fourier Transform, or IDFT the Inverse Discrete Fourier Transform or IFFT, IFFT simply a fast algorithm to evaluate IFFT the IFFT simply a fast algorithm to evaluate IDFT. So, x l is the l-th FFT point.

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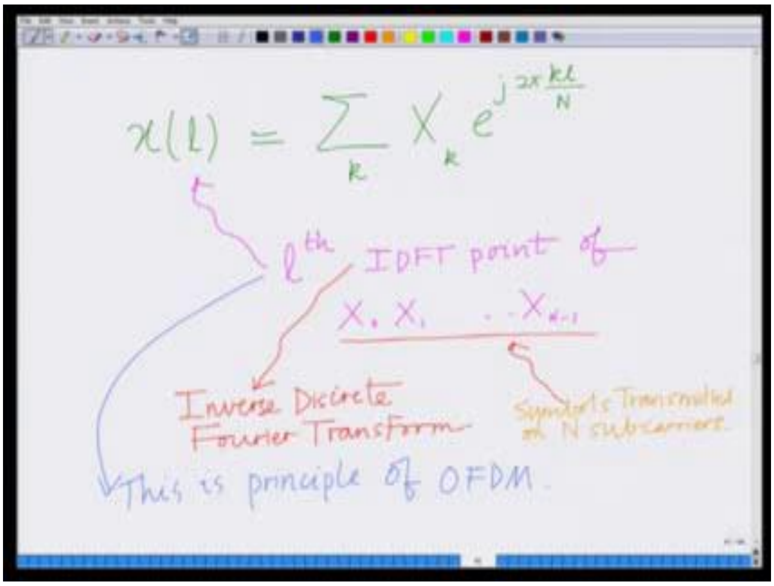

So, if you look at this

$$x(l) = \sum_k X_k \, e^{j2\pi k \frac{l}{N}}$$

is simply the l-th IDFT point inverse discrete Fourier transform point of $X_{0,} X_1 X_{N-1}$, where this IDFT this stands for inverse discrete Fourier transform right. These are the symbols transmitted on the different subcarriers as we already seen symbol transmitted on N subcarriers. Remember we started out with the problem the problem is that the transmit signal is difficult to generate, because it requires n subcarriers. Therefore, N oscillators and N modulator what we are saying is we do not need this requirement if we consider the sampled transmit signal that is the samples of the transmit signal, because the transmit signal is a band limited signal. These samples can be generated by the simple IFFT operation or the Inverse Fast Fourier Transform operation, and this is much simpler than employing N modulators a bank of N modulators corresponding to the N subcarriers. So, the samples of the multicarrier modulation signal can be generated by the IFFT operation rather than employing modulators. This is precisely the key principle behind OFDM this is one of the key principles behind OFDM that is employing.

This is one of the key principle of OFDM, that is I look at the symbols $X_{0,} X_1 X_{N-1}$,, which correspond to the N subcarriers I generate the IFFT points. These are the samples of the transmit signals I transmit these samples over the channel I get the corresponding receive samples from the receive samples naturally I can take the FFT the Fast Fourier Transform to recover back the transmitted symbols. So, I do not longer need to modulate on the n subcarriers and demodulatives in the match filter or the coherent demodulation. I can simply replace the transmission by the FFT IFFT at the transmitter I can take the IFFT. I can transmit the IFFT samples at the receiver on the receive samples I can take the FFT operation to recover back my transmitted symbols.

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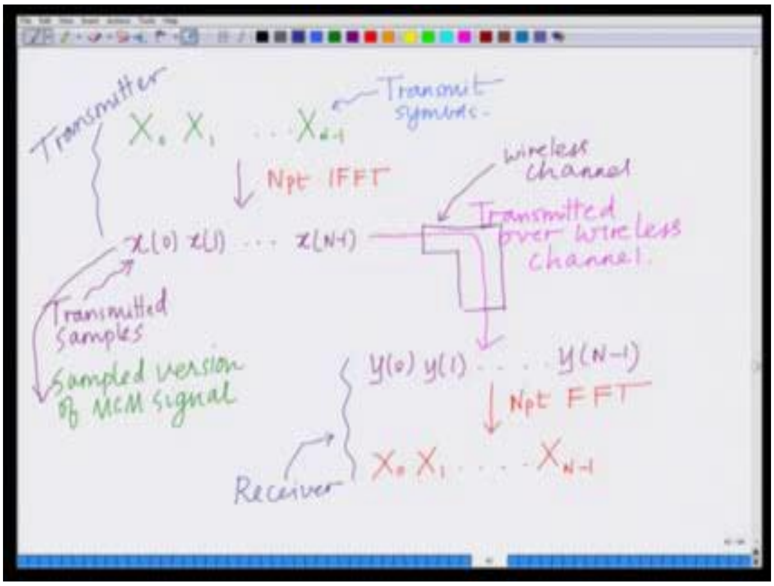

So, I can schematically illustrate this process as shown below, what I am doing is I have N symbols $X_{0,}X_1X_{N-1}$, at the transmitter. So, this I use to take the IFFT that is the samples of this is IFFT this. In fact, N point IFFT. So, these are these capital $X_{0,}X_1X_{N-1}$ these are the transmit symbols on the N subcarriers, this $x(0)$, $x(1)$, $x(N-1)$. These are the transmitted samples over the channel these are the sample version or the transmitted symbols, transmitted samples the actual samples transmitted or these are also the sampled version of the MCM signal. This is the sampled version of the multi carrier modulated signal. Now these I can transmit over the channel, now these samples $x(0)$, $x(1)$, $x(N-1)$, I transmit these over the channel.

So, these are transmitted over the channel. In fact, let me clearly write this these are transmitted over the wireless channel. In fact, if I look at this part, this is my this box is basically my wireless channel, and this at the receiver what do I receive, at the receiver I receive corresponding samples $y(0)$, $y(1)$, $y(N-1)$. Now on these samples I will take the FFT operation at the receiver, I will perform the FFT operation more precisely the n point FFT operation and then I will regenerate my transmit samples $X_{0,}X_1X_{N-1}$.

So, this is exactly. So, this processing is at the transmitter and this processing is at the receiver. So, in a nut shell what we are doing is basically is very simple, we are taking the symbols $X_{0,}X_1X_{N-1}$. These are the symbols that are to be transmitted on the n subcarriers we are performing the IFFT operation and generating the samples of the transmit signal $x(0)$, $x(1)$, $x(N-1)$.

These samples are transmitted over the wireless channel at the receiver you have the samples $y(0)$, $y(1)$, $y(N-1)$ which are the corresponding received samples on these received samples you perform the FFT operation that is the Fast Fourier Transform operation and then you can reconstruct back the transmitted symbols that is $X_{0,}X_1X_{N-1}$.

So therefore, you are performing the IFFT operation at the transmitter and the FFT operation at the receiver, these together this IFFT, FFT operation at the transmitter and receiver respectively. This is one of the key steps 1 of the key principles of OFDM. This is the difference between OFDM and multicarrier modulation, in multicarrier modulation we need to generate all the symbols and subcarriers using modulators, but in OFDM system, we replace these modulators and demodulators by the simple IFFT operation at the transmitter and the FFT operation at the receiver and this is the key step in OFDM this is the IFFT processing.

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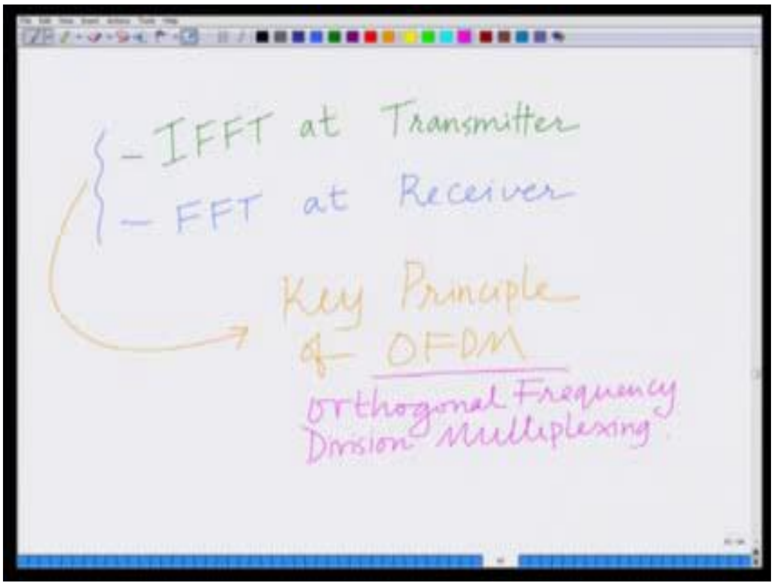

So, what is the key step in the in OFDM that is IFFT at transmitter, correct and the FFT at receiver and what is this is the key principle of OFDM which is also, which is short for as you might all ready remember again orthogonal frequency division multiplexing. Now, why is this orthogonal frequency division multiplexing first of all remember the different subcarriers are orthogonal, right let us go back to the previous lectures to look at these thing if you remember we look precisely at this orthogonality property of the previous subcarrier we said.

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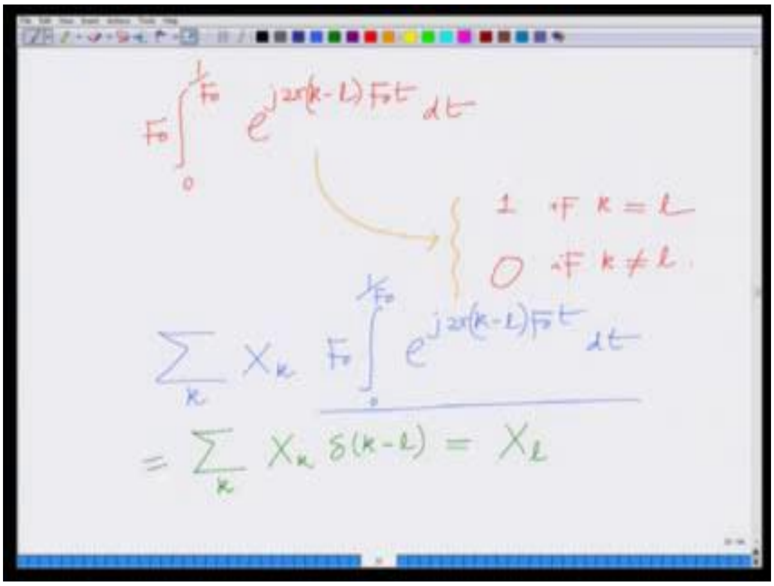

If you look at these to subcarriers k and l we said this

$$f_0 \int_0^{\frac{1}{f_0}} e^{j2\pi(k-l)f_0 t}\, dt \quad = 1 \quad \text{if } k = l$$

$$= 0 \quad \text{if } k \neq l$$

which means the subcarriers k and l any two difference subcarriers k and l in this system are orthogonal, and this orthogonality between the subcarriers is being used to resolve the symbols is transmitted on the different subcarriers at the receiver that is the first aspect.

The second aspect of this OFDM system is if you look at this it is frequency division, right we are dividing this broad band width B amongst the N subcarriers with each band having band with $\frac{B}{N}$. So, this is the frequency division based multiplexing which means we are multiplexing multiple symbols over this multiple subcarriers over the band with B as we have all ready seen we are transmitting n symbols in parallel over the N subcarriers.

So, we are multiplexing n symbols over the n difference subcarriers in the system. Therefore, the word multiplexing together this is orthogonal frequency division multiplexing orthogonal for the orthogolity of the subcarriers frequency division, because this is the frequency division scheme in which you are using multiple subcarriers and multiplexing, because your multiplexing several symbols n symbols over the n subcarriers. And as we have all ready seen in today's module the IFFT proposing at the transmitter and the FFT proposing at the receiver is a key step in this OFDM system which helps easy implementation of this OFDM system. So, let us conclude this module here.

Thank you.