Principles of Modern CDMA/ MIMO/ OFDM Wireless Communications
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Lecture – 46
Transmission in Multicarrier Systems

Hello, let us look at another module in this Massive Open Online Course on the Principles of CDMA MIMO OFDM Wireless Communication System. In the previous module we have seen an introduction to a multicarrier modulated system. Now, let us look at the transmission scheme that is a basic principle of operation the transmission and reception in multicarrier modulated system.

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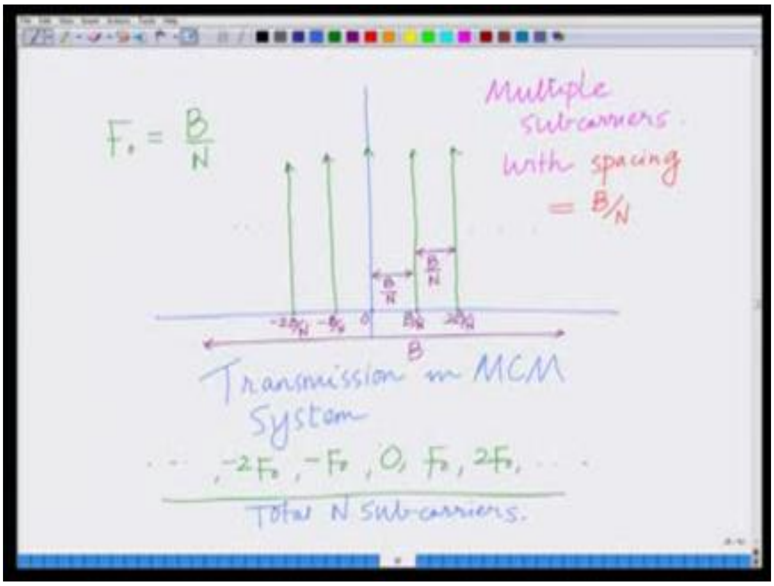

So, what we are going to see is basically MCM system and we are going to see the transmission; that is what I want to see is I want to see what is the transmission the scheme for transmission in an MCM that is a multicarrier modulated system. As we have already said in a multicarrier modulated system I have a large bandwidth that is I have a large bandwidth which is B and I am dividing it in to sub bands each of band $\frac{B}{N}$.

So, this spacing between these sub bands or the subcarrier is B by N at each sub band I am placing a subcarrier. So, I have the first subcarrier at 0, second subcarrier at $\frac{B}{N}$, third another subcarrier at $\frac{2B}{N}$, another subcarrier at $\frac{-B}{N}$, another subcarrier at $\frac{-2B}{N}$, so on and so forth. So, what I have is I have a subcarrier at each. So, we have multiple subcarriers with subcarrier spacing remember with the spacing between the subcarriers equals B by N. So, I have subcarrier for instance at if I call f_0 as a fundamental frequency equals B by N let us denote by

$$f_0 = \frac{B}{N}$$

Therefore equal I have subcarriers at 0, at f_0 ; $2f_0$ so on. As well as $-f_0$, $-2f_0$ and so on. So, basically I have total of N subcarriers I have a total of n subcarrier. So, what we are saying is we have divided the bandwidth B into N sub bands, the bandwidth of each some band is $\frac{B}{N}$ and what I am doing in each sub band I am placing a subcarrier right. So, we have a subcarrier at 0, $\frac{B}{N}$, $\frac{2B}{N}$ on the negative side we have subcarriers at $\frac{-B}{N}, \frac{-2B}{N}$.

So, if we denote this $\frac{B}{N}$, by a quantity f_0 where f_0 can be thought of as a fundamental frequency of this system then I have subcarriers at 0, f_0, $2f_0$, $3f_0$, so on and on the negative side at $-f_0$, $-2f_0$, and so on. So, I have a total of N such subcarriers. So, the k th, so we are denoting the k-th subcarrier.

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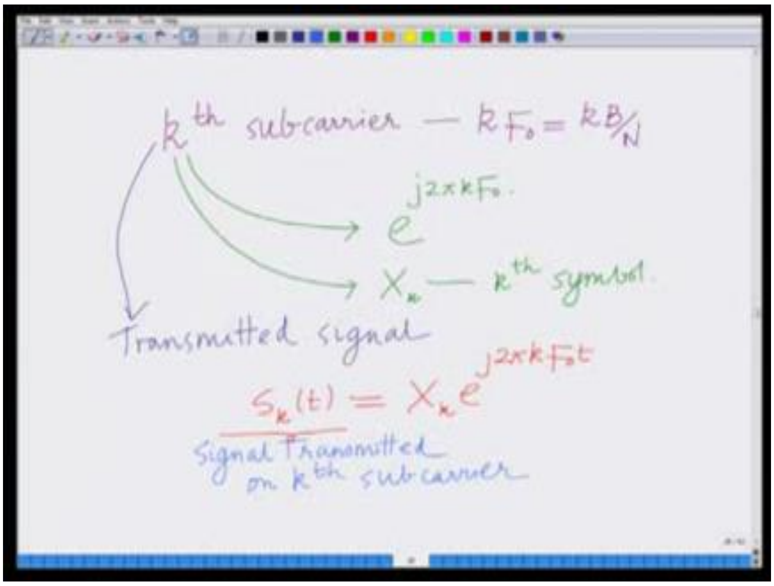

So, the k-th subcarrier is centered at $k\ f_0$ that is equal to $\frac{k\ B}{N}$. So, this is k-th sub carrier which is that $k\ f_0$, this subcarrier is given as $e^{j2\pi k f_0}$.

So, we have the k-th subcarrier in this system which is given by $e^{j2\pi k f_0}$. and let the k-th symbol be given by X_k . Then the transmitted signal, the transmitted signal is X_k which is equal to the transmitted signal is $s_k(t)$ which is equal to

$$s_k(t) = X_k e^{j2\pi k f_0 t}$$

So, what is this? This is the signal transmitted on the k-th subcarrier. So, the signal transmitted on the k-th sub carrier is $X_k e^{j2\pi k f_0 t}$. Now, you have to sum across all this N sub carrier frequency. So, the net transmitted signal or the net transmitted MCM signal multicarrier modulated signal.

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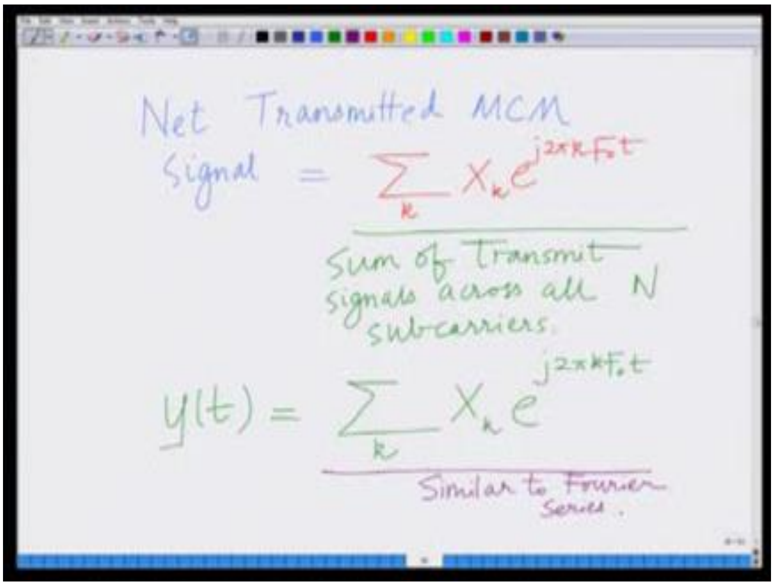

The net transmitted MCM or multicarrier modulated signal is basically, given by

$$= \sum_k X_k e^{j2\pi k f_0 t}$$

that is the sum of the transmitter signals across all transmit signals across all N sum of transmit signal across all the N subcarrier.

And now let us say that the received signal. So, this is the transmitted signal. So, let us say I have the received signal which is given as

$$y(t) = \sum_k X_k e^{j2\pi k f_0 t}$$

so obviously, here am ignoring the noise at the receiver what I am saying is am taking the signal across each subcarrier I am adding up these transmit signals across the different N subcarrier and therefore, I have the sum signal or the composite signal which I am transmitting over the channel and I am receiving the corresponding signal $y(t)$. In the absence of noise I am saying that the received signal is equal to the transmit signal, I am ignoring the presence of the noise component for the moment and now we are going to look at how to reconstruct that is how to recover these different symbols because our ultimate interest is in recovering these difference symbols X_k that have been transmitted on the various of subcarrier.

Now how to recover these difference symbols? Now if you look at this expression the multicarrier transmitted signal, you will see this is similar to the Fourier series - notice that this is similar to a Fourier series expansion. So, therefore, X_k is the k-th Fourier series coefficient.

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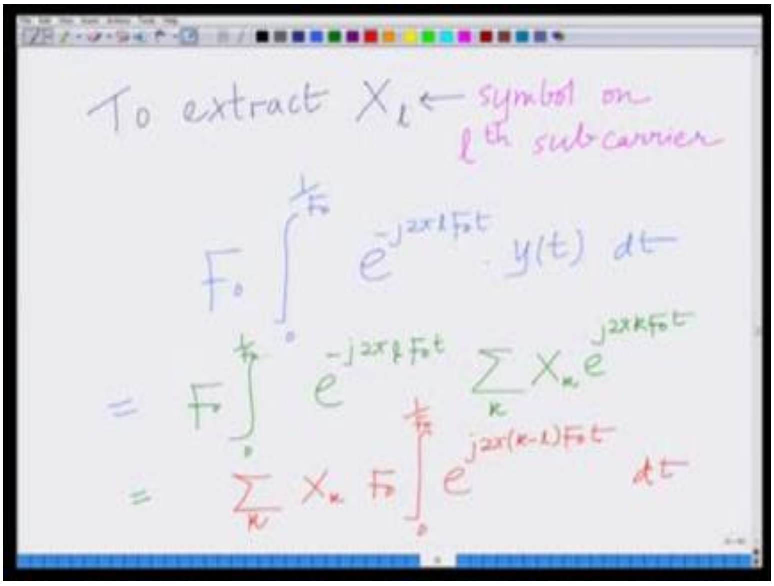

And therefore, the way to extract l-th symbol to extract X_l which is basically symbol on l-th subcarrier, symbol on l-th sub carrier we have

$$= f_0 \int_0^{\frac{1}{f_0}} e^{-j2\pi l f_0 t}\, y(t)\, dt$$

$$= f_0 \int_0^{\frac{1}{f_0}} e^{-j2\pi l f_0 t} \sum_k X_k e^{j2\pi k f_0 t}\, dt$$

$$= \sum_k X_k\, f_0 \int_0^{\frac{1}{f_0}} e^{j2\pi (k-l) f_0 t}\, dt$$

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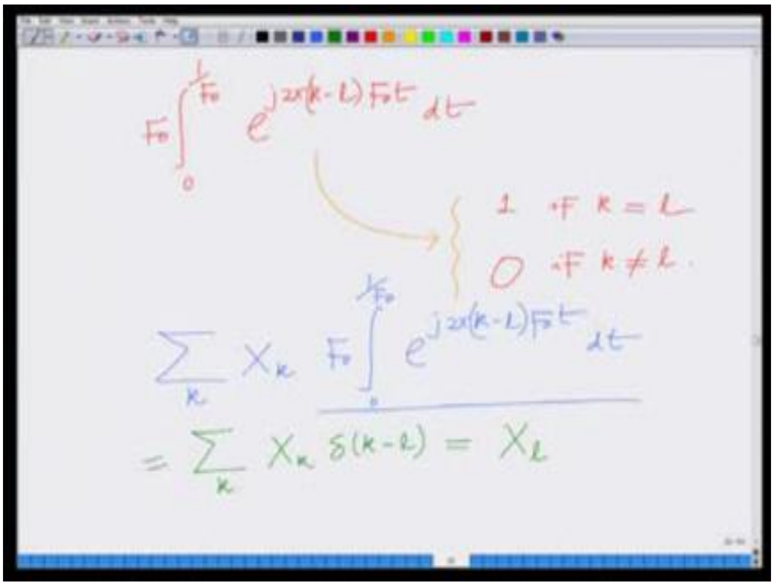

Now, look at this quantity if I look at this quantity

$$f_0 \int_0^{\frac{1}{f_0}} e^{j2\pi(k-l)f_0 t}\, dt \quad = 1 \quad \text{if } k = l$$

$$= 0 \quad \text{if } k \neq l$$

So, what are you doing you are taking a sinusoid with the fundamental period $\frac{1}{f_0}$ and you are integrating it over $\frac{1}{f_0}$ that is a you are integrating it over a period.

Therefore, this is going to be non zero if it is a sinusoid that is a case not equal to l it as it is a sinusoid at a multiple of the fundamental frequency f_0 its integral over the fundamental period one over f_0 is zero and the only scenario in which the integral is non zero if k is equal to l which means the sinusoid reduces to a dc signal and for a dc signal the integral over a fundamental period is equal to 1.

Therefore, in this summation only the term corresponding to k equal to l will survive and therefore, I can recover X_l.

$$X_l = \sum_k X_k \delta(k-l)$$

Similarly for all the N sub carrier to recover all then symbols transmitted on the various sub carriers this system as coherent demodulation. So, at the receiver we employ coherent demodulation.

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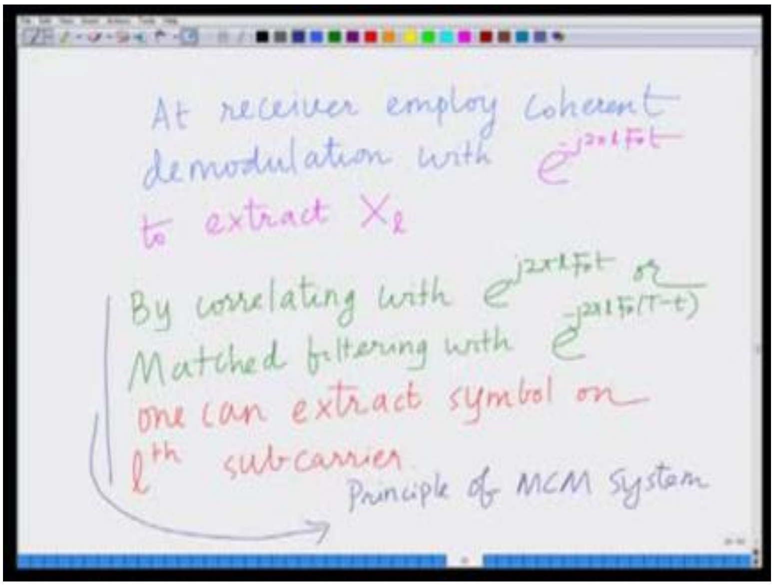

We employ coherent demodulation with $e^{-j2\pi l f_0 t}$ to extract to extract X_l thus by correlating with $e^{j2\pi l f_0 t}$ or matched filtering $e^{-j2\pi l f_0 (T-t)}$ the match filtering h conjugate $T - t$ one can extract symbol on the l-th one can extract the symbol on the l-th sub carrier this is the principle and this is the transmission and reception scheme.

So, this is the principle of MCM this is the principle of a multicarrier modulated system. So, basically to summaries what we have is earlier.

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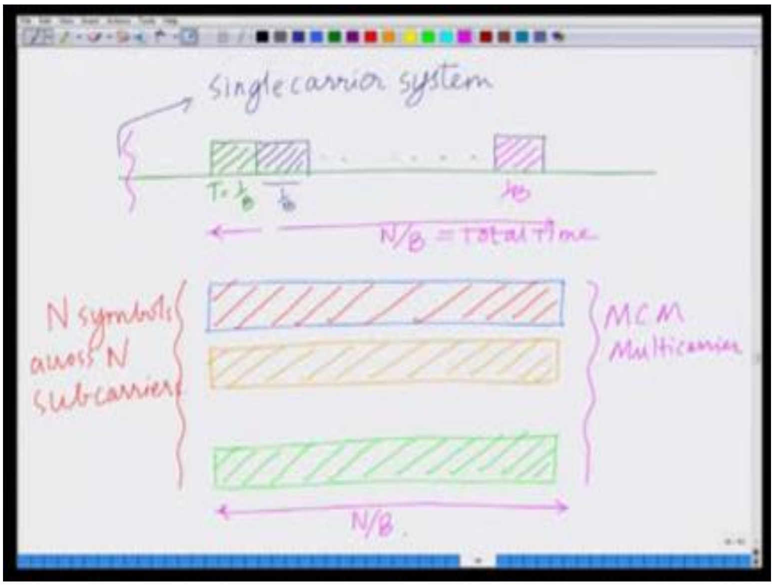

If you remember we had a single carrier system in a single carrier system I have a bandwidth of B and I have a symbol time $T = \frac{1}{B}$. So, I have a symbol time $T = \frac{1}{B}$ and therefore, in $\frac{N}{B}$ I am transmitting therefore, I am transmitting N symbols in symbol time $\frac{N}{B}$ equals the total time and this is for a single carrier system.

However, in our multiple carrier system what we are doing is each symbol on each sub carrier for instance if I look at each symbol on each sub carrier this has a time of. So, if in a multicarrier system if I look at this, each symbol on each subcarrier has a time of $\frac{N}{B}$. However, I have n subcarriers; therefore I have transmitting N symbols across these n subcarriers in this multicarrier system. So, what I have if you look at this net, I am transmitting N symbols across n subcarriers. And the total symbol time remains fixed, the total symbol time is $\frac{N}{B}$.

So, here we have a comparison between a single carrier and this is my MCM system or multicarrier. So, the (Refer Time: 20:07) there is a pictorial comparison between a single carrier and a multicarrier system in a single carrier system we have a bandwidth of B the symbol time is one over B therefore, each symbol is $\frac{1}{B}$. So, we have symbol at $\frac{1}{B}$. So, in time $\frac{N}{B}$ we are transmitting N symbols.

However, in a multicarrier modulated system we have N subcarriers the symbol trans symbol time is $\frac{N}{B}$. So, where increasing the symbol time that is symbol time has been increase to n times that of a single carrier system symbol time is $\frac{N}{B}$, but we are transmitting n symbols in parallel across the N subcarrier. So, each symbol has a time $\frac{N}{B}$ that is a difference between single carrier and multicarrier modulated system. So, multicarrier modulated system can be thought of as a parallel transmission as having several parallel channels between the transmitter and receiver each of bandwidth $\frac{B}{N}$ and each width symbol time $\frac{N}{B}$ and N such channel or N such sub channels or N such sub bands are used with n sub carrier to convey N symbols in parallel between the transmitter and receiver.

This is in short this summarizes the philosophy and principles of multicarrier modulation as we have already seen this helps avoid inter symbol interference because the symbol time in each sub channel is much larger than the delay spread and also what we have seen is we have seen how to modulate the symbols on the subcarriers how to demodulate using coherent demodulation based on our principle of the Fourier series.

So, this is a very intuitive way to modulate and demodulate in this multicarrier modulated system and this is a precursor or this forms a basis for the OFDM system which we are going to start exploring in the subsequent module system.

Thank you very much.