Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture – 42
SVD in MIMO

Welcome, another module in this massive open online course on the Principles of CDMA, MIMO, OFDM Wireless Communication Systems. So, in the previous module we have seen the SVD the Singular Value Decomposition, and we have said that the singular value decomposition can be used to gain insides and analyse the properties of MIMO wireless communication, let us now look at the applications of the SVD that is the Singular Value Decomposition in MIMO wireless communication systems.

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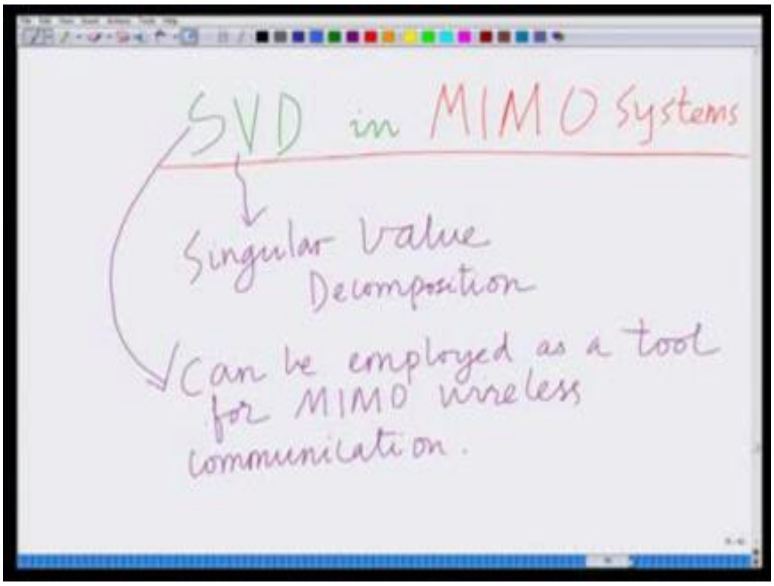

So, what we are going to look at today is the application of SVD that is Singular Value Decomposition in MIMO transmission or in MIMO systems basically, and SVD as we have already mention stands for Singular Value Decomposition, and SVD can be employed as a valuable tool for MIMO wireless. So, SVD can be employed as a valuable tool for MIMO wireless communication. Let us look at an example, lets us look at how the SVD is employed in MIMO wireless system.

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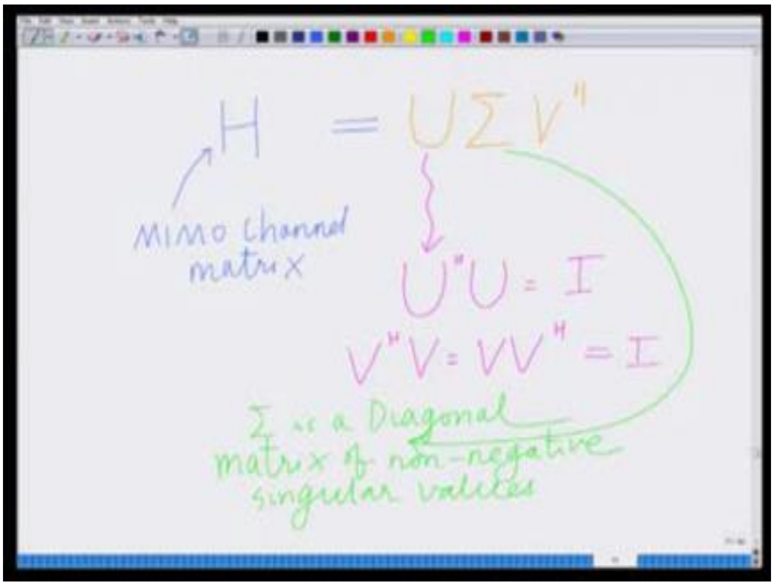

So, we have the channel matrix H, this is the MIMO channel matrix and we said that the MIMO channel matrix can be decomposed using the singular value decomposition as the product of three matrices

$$H = U \Sigma V^{H}$$

where we have the following property is

$$U^{H} U = I_{txt}$$

$$V^{H} V = V V^{H} = I_{txt}$$

Σ is a diagonal matrix containing the non negative singular values,

$$\sigma_1, \sigma_2, \ldots, \sigma_t > 0$$

In fact, these non negative singular values are arranged in the decreasing order or descending order that is what we said is singular value decomposition.

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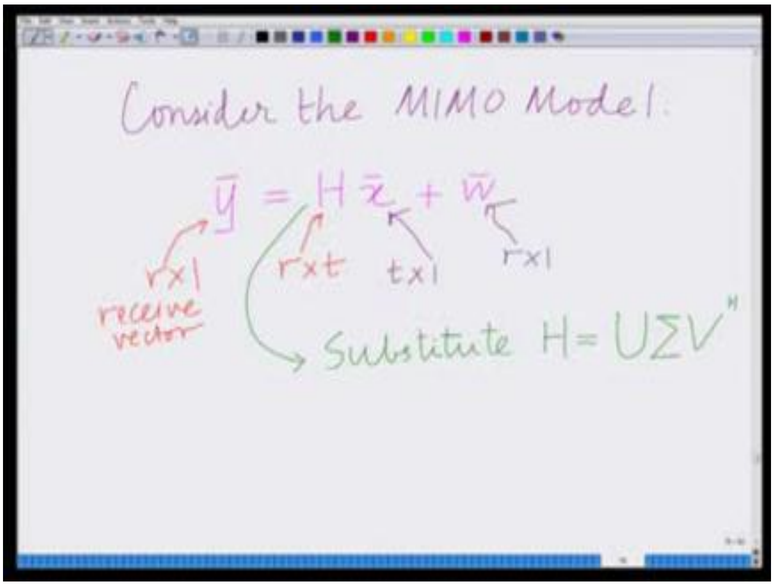

Now, let us go back to our MIMO system model, consider the MIMO communication model, we have

$$\bar{y} = H\bar{x} + \bar{w}$$

this is our MIMO communication system model, which $\bar{y}$ this is our r x1 receive vector, this H is the r x t channel matrix, x is the t x 1 transmit vector and w is basically your r x 1 noise vector, so these are the different components. Now we employee the SVD, now for H in this equation, for H substitute the SVD that is $H = U\Sigma V^H$. So, in this equation in this MIMO system model, what we are going to do is we are going to replace H by it singular value decomposition that is $H = U\Sigma V^H$.

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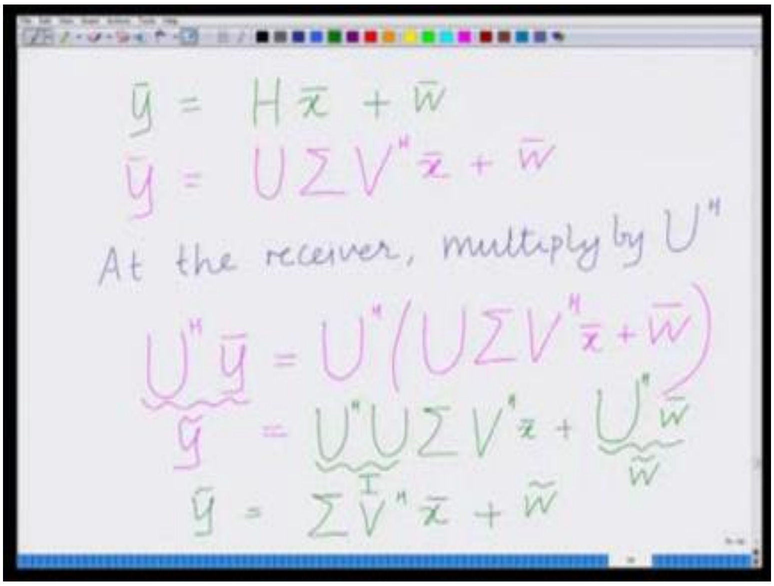

So, now substituting what I am going to have is, I have

$$\bar{y} = H\bar{x} + \bar{w}$$

$$\bar{y} = U\Sigma V^H\bar{x} + \bar{w}$$

Now, we employee receive processing, at the receiver we perform multiply by U^H that is what we are going to have is,

$$U^H\bar{y} = U^HU\Sigma V^H\bar{x} + U^H\bar{w}$$

$$\tilde{y} = \Sigma V^H\bar{x} + \tilde{W}$$

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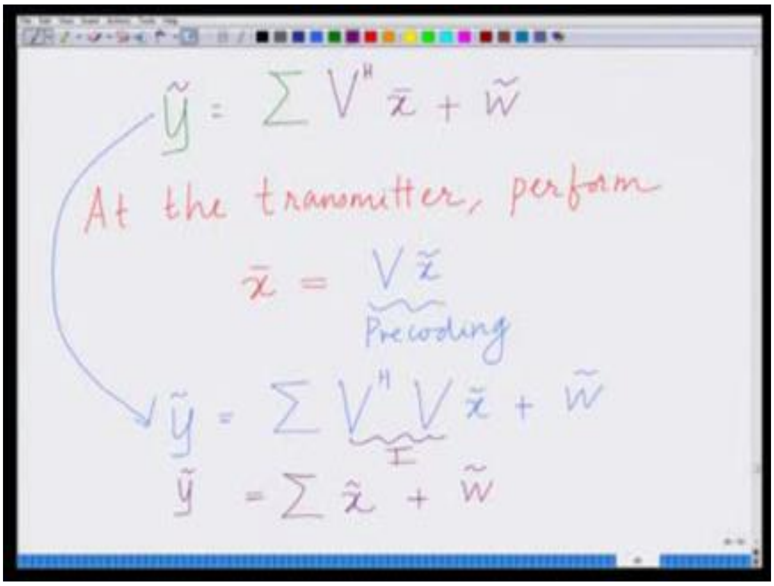

I have at the receiver that is

$$\tilde{y} = \Sigma V^H \bar{x} + \tilde{W}$$

Now, what I am going to do at the transmitter, we perform

$$\bar{x} = V \tilde{x}$$

This is termed as Pre coding, what we are doing is we are pre processing the transmit vector, that is we are transmitting $\bar{x}$. So, we are pre processing the transmit symbol vector that is pre processing $\tilde{x}$ by multiplying by matrix V. So we have $\bar{x} = V \tilde{x}$.

Now, let us substitute this

$$\tilde{y} = \Sigma V^H V \tilde{x} + \tilde{W}$$

$$\tilde{y} = \Sigma \tilde{x} + \tilde{W}$$

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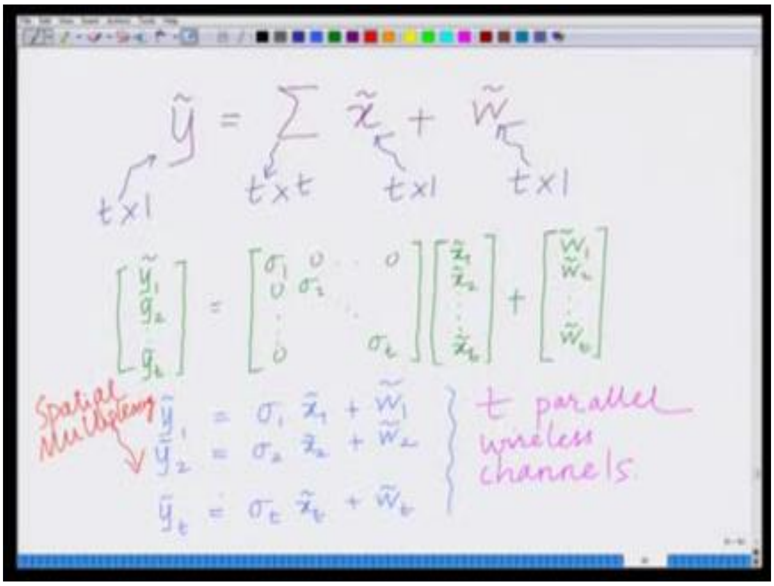

This is something very interesting, let me rewrite this again over here, I have

$$\tilde{y} = \Sigma\, \tilde{x} + \tilde{W}$$

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_t \end{bmatrix} = \begin{pmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_t \end{pmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_t \end{bmatrix} + \begin{bmatrix} \tilde{W}_1 \\ \tilde{W}_2 \\ \tilde{W}_t \end{bmatrix}$$

And therefore what you have, is you have the net result is that you have this matrix y tilde, is equal to well the diagonal matrix sigma times the symbol vector x tilde, plus the noise vector w tilde, and now if u look at this because, sigma is a diagonal matrix you have a very interesting structure what you have is basically, you have

$$\tilde{y}_i = \sigma_i \tilde{x}_i + \tilde{W}_i$$

So, what you have is you have a combination of t parallel wireless channels.

So, between the transmitter and the receiver you have t parallel channels where you are transmitting $\tilde{x}_1$, $\tilde{x}_2$, $\tilde{x}_t$ and correspondingly you are receiving $\tilde{y}_1$, $\tilde{y}_2$, $\tilde{y}_t$ that is basically on the i-th channel you are transmitting $\tilde{x}_i$ and you are receiving $\tilde{y}_i$ and therefore, you have t such channels or t parallel channels.

Therefore you are able to transmit t parallel data streams or t data streams in parallel; this is termed as spatial multiplexing. So, what you have as net is basically for i is equal to that is one each channel.

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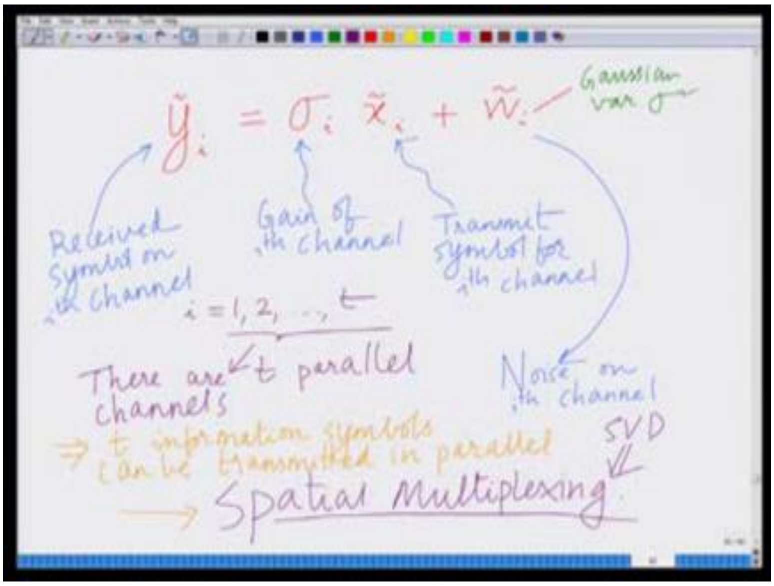

That is you have

$$\tilde{y}_i = \sigma_i \tilde{x}_i + \tilde{W}_i$$

where $\tilde{y}_i$ is received symbol on i th channel, σ_i is the gain of i th channel, $\tilde{x}_i$ is transmitted symbol or transmit symbol for i th channel, and $\tilde{W}_i$ is noise for noise on i th channel, which is Gaussian this noise $\tilde{W}_i$ is Gaussian with variance σ^2. So, what we have as a result is we have, t decoupled channels, we also have we can also call this as decoupled channels because these different channels you can see these have this decoupled inputs $\tilde{x}_1$, $\tilde{x}_2$, $\tilde{x}_t$ and the d coupled outputs that is $\tilde{y}_1$, $\tilde{y}_2$, $\tilde{y}_t$. So, we have these t parallel channels between the transmitter and receiver. So, we are able to transmit t streams of information or t symbols across these parallel channels between the transmitter and receiver, this principle in a MIMO system is precisely what is termed as spatial multiplexing.

So, there are t parallel channels. So, for i equals 1 2 up to t, there are t parallel channels which implies the t streams of information, which implies t information symbols can be transmitted in parallel and this is what is termed as spatial multiplexing as we discussed in one of the first modules on MIMO spatial multiplexing, multiple streams of information using the space dimension. So, what are the multiple streams of information? The multiple streams of the information of the multiple symbols $\tilde{x}_1$, $\tilde{x}_2$, $\tilde{x}_t$.

We are multiplexing these symbols over the same frequency band at the same time, by exploiting the space dimension in this multiple input, multiple output, and wireless communication system. This is termed as spatial multiplexing and therefore, what we have seen is basically we have seen how the SVD or the singular value decomposition now helps us basically decouple this MIMO wireless communication system into t parallel channels, for the transmission of t independent information symbols and this phenomenon is termed as spatial multiplexing.

Therefore, what we have seen is basically this is termed as spatial multiplexing and basically what we have seen is how the SVD can be employed for spatial multiplexing. So, with this we will conclude this module over here and we will continue with other modules on MIMO in the subsequent lectures.

Thank you.