Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture - 40
BER of Alamouti Coded System

Hello, welcome to another module in this massive open online course in the principles of CDMA, MIMO, OFDM wireless communications systems. In the previous module what we have seen is Alamouti transmission scheme for a MIMO wireless or more specifically for a 1 x 2 MISO wireless communication system. What we are going to start with in this module is the Bit Error Rate performance of an Alamouti Coded MISO Wireless System.

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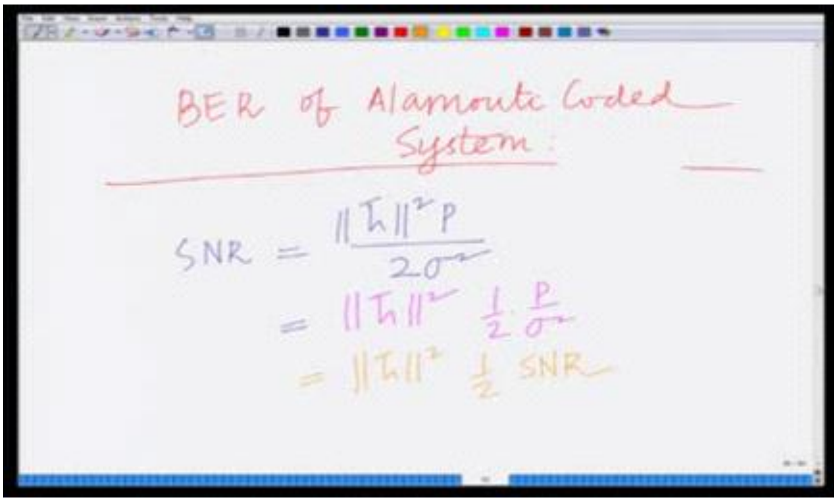

So, let us look at the Bit Error Rate performances of this Alamouti Coded System. As we already seen Alamouti is an orthogonal space time block course. So, the Bit Error Rate of this Alamouti Coded System.

$$\text{SNR} = \frac{1}{2}\frac{||\bar{h}||^2 P}{\sigma^2} = ||\bar{h}||^2 \frac{1}{2}\frac{P}{\sigma^2} = ||\bar{h}||^2 \frac{1}{2}\, SNR$$

Basically it has M R C is identical to M R C with half the SNR half the transmit SNR, $\frac{P}{\sigma^2}$ if there is this factor of half therefore, the Bit Error Rate is similar to that of M R C with two antennas remember the number of antennas in this MISO system Alamouti Coded MISO System is 1 x 2 therefore, the number of antennas is 2 or L = 2.

So, therefore, it is identical took the performance often M R C system with L = 2 antennas, but half the transmit SNR.

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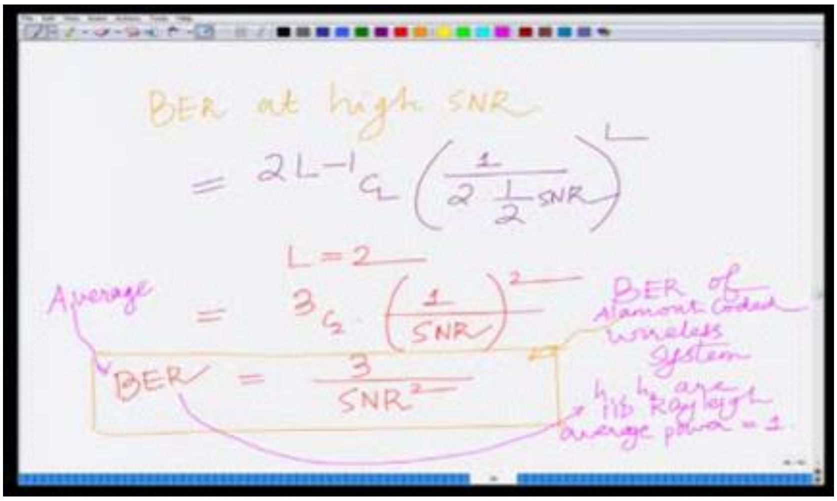

Therefore the Bit Error Rate at high SNR equals

$$\text{BER} = {}^{2L-1}C_L \left(\frac{1}{2\,\frac{1}{2}\,\text{SNR}}\right)^{L}$$

we have to substitute L = 2. Therefore, it is

$$\text{BER} = {}^{3}C_2 \left(\frac{1}{\text{SNR}}\right)^{2} = \frac{3}{SNR^{2}}$$

The Bit Error Rate of this Alamouti Coded Wireless System is interestingly we are able to derive an expression for the Bit Error Rate of this Alamouti Coded Wireless System and that is $\frac{3}{SNR^{2}}$.

So, this is the Bit Error Rate of Alamouti Coded Wireless System is $\frac{3}{SNR^{2}}$. This is the average Bit Error Rate mind average over the distribution on the fading channel coefficients. We are assuming the fading channel coefficients h_1, h_2, to be IID Rayleigh with average power 1 that is the standard assumption when we derived the Bit Error Rate of this system with max well ratio combined. So, remember this is average Bit Error Rate under the assumption h_1, h_2, are IID Rayleigh average power equal to with average power equals 1.

This is the assumption that we are using here. Now let us look at some examples of this Alamouti Coded Wireless Communication system to understand this better let us look at some simples examples.

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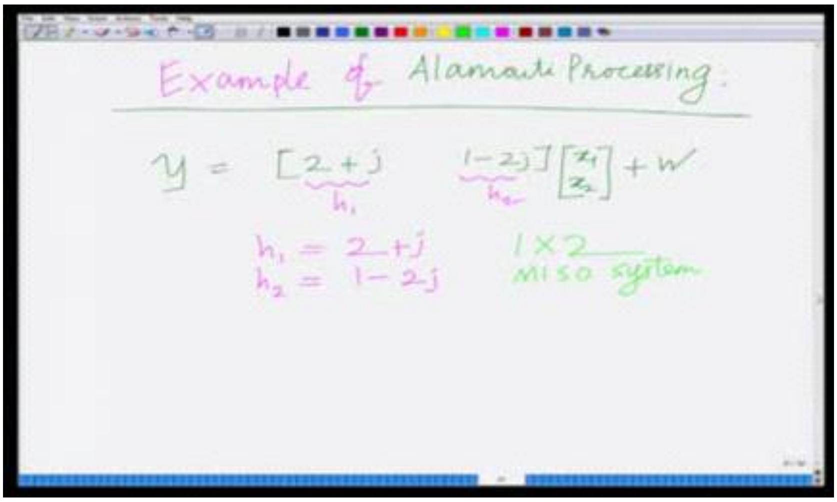

Let us start with example of Alamouti Processing. Consider a simple system

$Y = [2 + j \quad 1 - 2j] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w$

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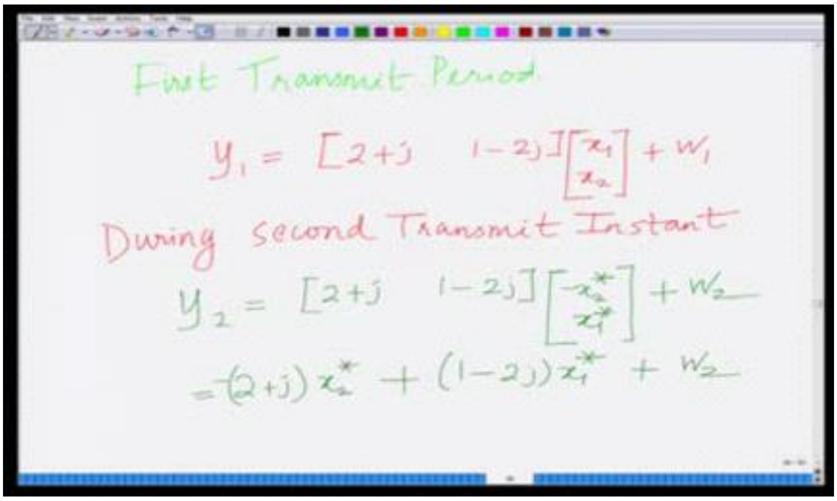

In the 1st transmit period we are transmitting x_1 from the 1st transmit antenna and x_2 from the 2nd transmit antenna.

$$y_1 = [2+j \quad 1-2j] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_1$$

During the second transmit instant what do we have we are transmitting

$$y_2 = [2+j \quad 1-2j] \begin{bmatrix} -x_2^{\ *} \\ x_1^{\ *} \end{bmatrix} + w_2$$

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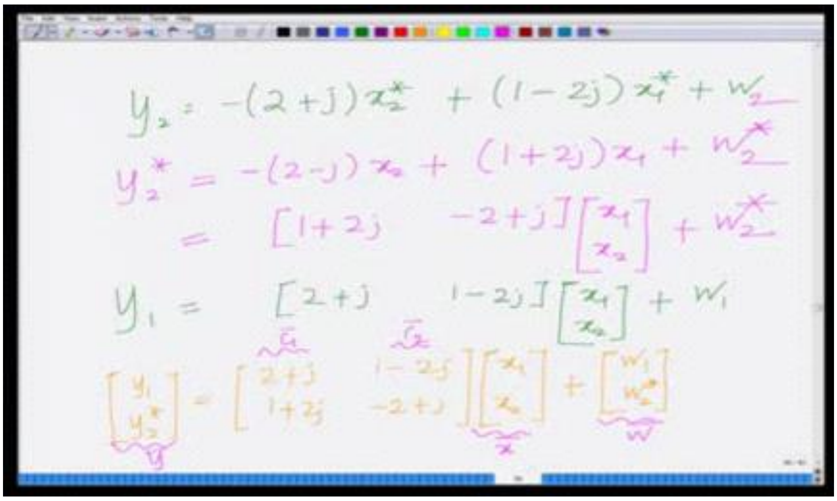

I have

$$y_2 = -(2+j)\, x_2^{\ *} + (1-2j)\, x_1^{\ *} + w_2$$

$$y_2^{\ *} = -(2-j)\, x_2 + (1+2j)\, x_1 + w_2^{\ *}$$

$$y_2^{\ *} = [1+2j \quad -2+j] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_2^{\ *}$$

$$y_1 = [2+j \quad 1-2j] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_1$$

$$\begin{bmatrix} y_1 \\ y_2^{\ *} \end{bmatrix} = \begin{bmatrix} 2+j & 1-2j \\ 1+2j & -2+j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2^{\ *} \end{bmatrix}$$

This is my vector $\bar{y}$ this is my 2 x 2 matrix this is my vector X bar this is my additive Gaussian noise vector $\bar{w}$ and these are my columns $\bar{c}_1$, $\bar{c}_2$. Now let us look at $\bar{c}_1$ and $\bar{c}_2$.

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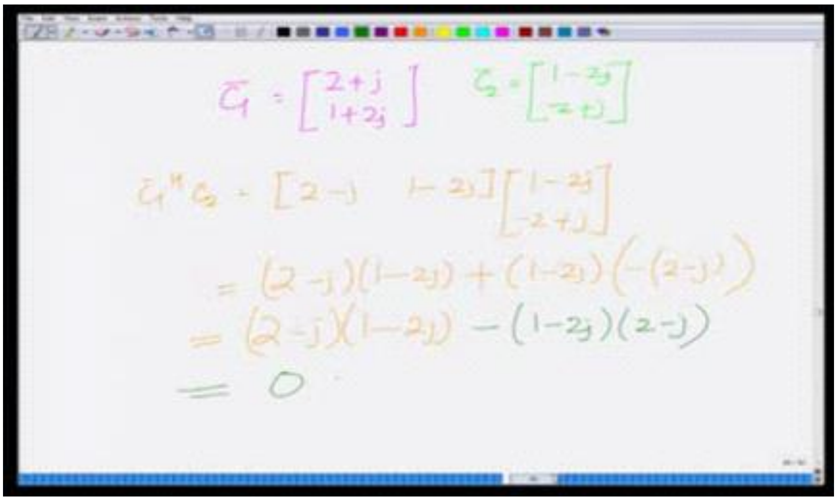

$$\bar{c}_1 = \begin{bmatrix} 2+j \\ 1+2j \end{bmatrix}$$

$$\bar{c}_2 = \begin{bmatrix} 1-2j \\ -2+j \end{bmatrix}$$

$$\bar{c}_1^H \bar{c}_2 = (2-j)(1-2j) + (1-2j)(-2+j) = 0$$

Now in this example we are seeing that we have constructed this Alamouti matrix, we extracted this 2 columns $\bar{c}_1$, $\bar{c}_2$ and what we are seeing is that $\bar{c}_1^H \bar{c}_2$ is that is why we said Alamouti is an orthogonal space time block ward. This orthogonality of the columns at the receiver in this Alamouti matrix makes the decoding operation very easy notably it is a simple correlation operation and not a matrix inversion.

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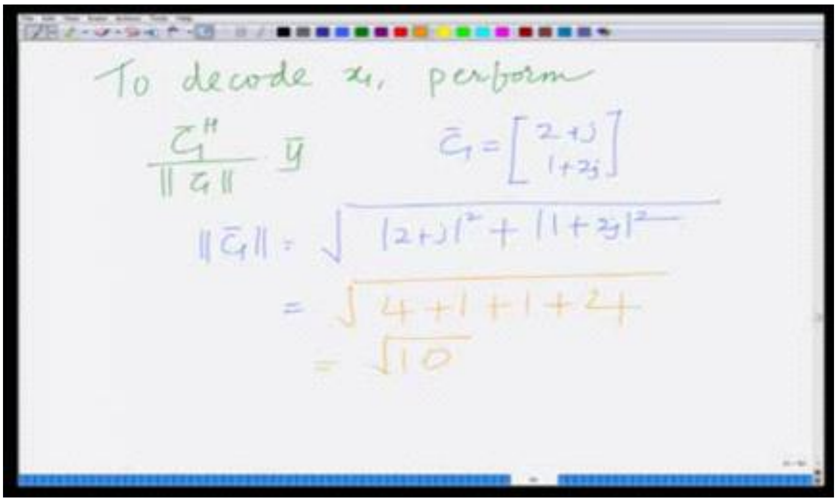

It is an orthogonal spaced and block code therefore, to decode x_1 we perform,

$$\frac{\bar{c}_1^{H}}{||\bar{c}_1||}\bar{y} \qquad \bar{c}_1 = \begin{bmatrix} 2+j \\ 1+2j \end{bmatrix}$$

$$||\bar{c}_1|| = \sqrt{|2+j|^2 + |1+2j|^2} = \sqrt{4+1+1+4} = \sqrt{10}$$

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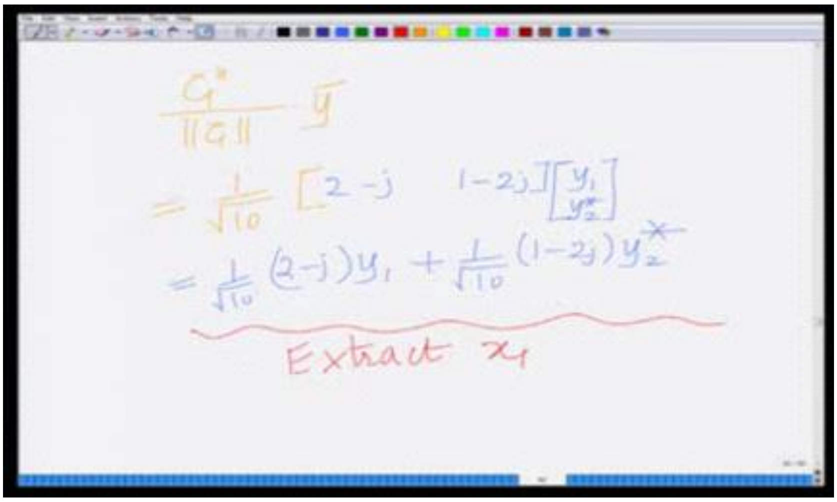

Therefore we perform

$$\frac{\bar{c}_1^{H}}{||\bar{c}_1||}\bar{y} = \frac{1}{\sqrt{10}}[2-j \quad 1-2j]\begin{bmatrix} y_1 \\ y_2^{*} \end{bmatrix}$$

$$= \frac{1}{\sqrt{10}}(2-j)\, y_1 + \frac{1}{\sqrt{10}}(1-2j)\, y_2{}^*$$

which is the vector in the Alamouti Coded System and that helps us extract the symbol x_1.

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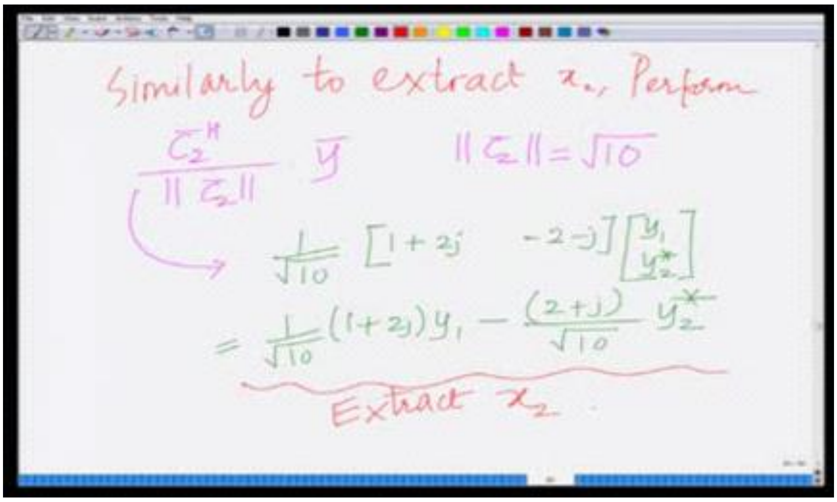

Similarly, to extract the symbol x_2 we perform

$$\frac{\bar{c}_2{}^H}{||\bar{c}_2||}\bar{y} \qquad ||\bar{c}_1|| = \sqrt{10}$$

$$\frac{\bar{c}_2{}^H}{||\bar{c}_2||}\bar{y} = \frac{1}{\sqrt{10}}\begin{bmatrix} 1+2j & -2-j \end{bmatrix}\begin{bmatrix} y_1 \\ y_2{}^* \end{bmatrix}$$

$$= \frac{1}{\sqrt{10}}(1+2j)\, y_1 - \frac{1}{\sqrt{10}}(2+j)\, y_2{}^*$$

that helps us extract the corresponding symbol x_2.

Now, let us look at a simple example to evaluate the Bit Error Rate performance of this Alamouti Systems. So, let us evaluate Bit Error Rate.

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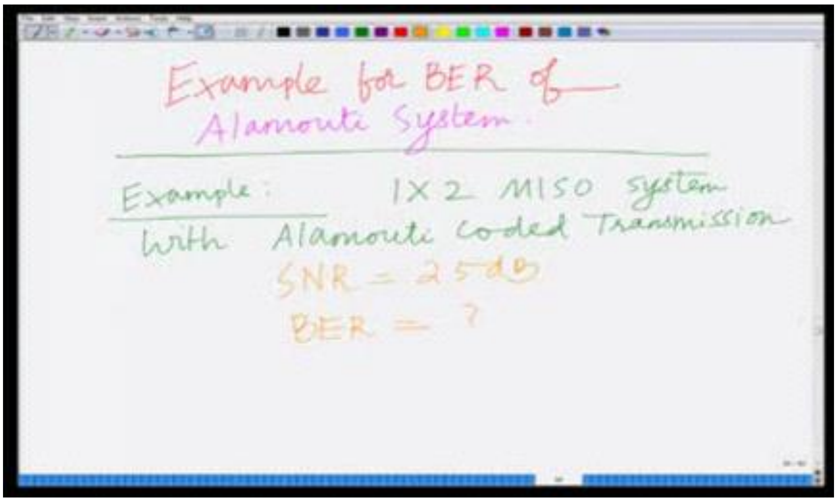

Example for BER, Bit Error Rate of the Alamouti Coded System for instance let us look at these simple example consider a 1 x 2 MISO system with Alamouti Coded Transmission and what we want to do is we want to consider SNR = 25 dB and we want to ask the question what is the Bit Error Rate? So, what we are doing is we are considering a simple example for the Bit Error Rate evaluation in an Alamouti Coded 1 x 2 MISO system we have an SNR = 25 dB we want to ask what is the average Bit Error Rate at this SNR?

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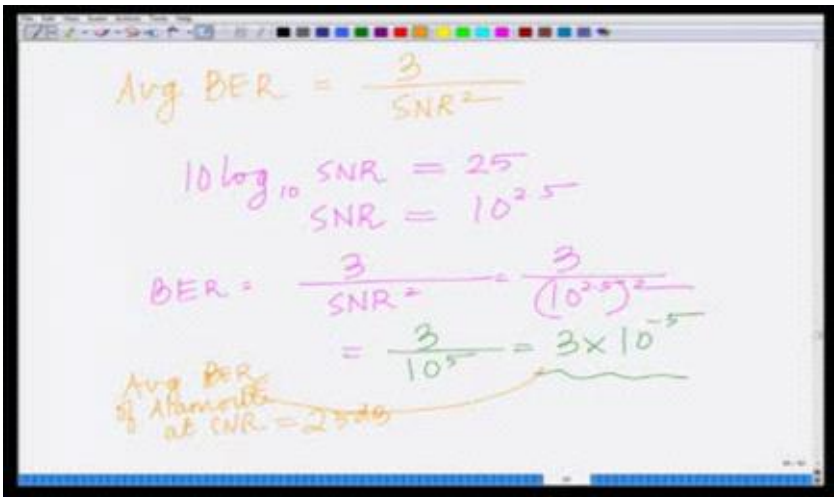

We already seen the average Bit Error Rate of Alamouti, equals

$$\text{BER} = {}^{3}C_2 \left(\frac{1}{\text{SNR}}\right)^2 = \frac{3}{SNR^2}$$

$10 \log_{10} SNR = 25$

$SNR = 10^{2.5}$

$BER = \frac{3}{SNR^2} = \frac{3}{10^5} = 3 \times 10^{-5}$

This examples and theory before that competency in illustrate the analysis, the theory, the scheme, the transmission, the decoding and also the Bit Error Rate performance of this Alamouti Coded System and Alamouti is a very important transmission scheme or considered key mile step in the key milestone in the progress or in the development of wireless communication system. Because of its elegance because at which simplicity because of its convenience remember we said Alamouti Coded System it is an orthogonal space time block code it has 3 aspects. One - it does not need the channel state information at the transmitter, two - the decoding at the receiver is very simple because of the orthogonal structure of Alamouti and therefore, these 2 techniques combined make Alamouti Scheme which very convenient for implementation in a practical wireless communication system. Hence it is a very included in 3G and 4G wireless standards and what we have seen is we seen the theory behind the Alamouti Scheme and we also seen is performance in terms of in the Bit Error Rate.

So, we will conclude this module here and explore other aspects of MIMO and subsequent modules.

Thank you very much.