Principles of Modern CDMA/MIMO/OFDM Wireless Communications
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 39
Alamouti Code and Space-Time Block Codes

Hello. Welcome to another module in this massive open Online Course in the Principles of CDMA MIMO OFDM Wireless Communication System. So, in the previous module we had seen transmit beam forming in the MISO system that is Multiple Input Single Output wireless communication system. But we said one of the challenges in transmit beam forming, is that we need to know the channel state information at the transmitter, which is the challenge in wireless communication system.

So, from particle perspective we also want to look at schemes when the channel state information or the channel coefficients the knowledge of the coefficients is not available at the transmitter and for that purpose we are going to look at new scheme, which is termed as Space Time Coding which is a very popular MIMO wireless transmission scheme. In particular we are going to look at a special case of this space time code which is immensely popular also known as Alamouti code.

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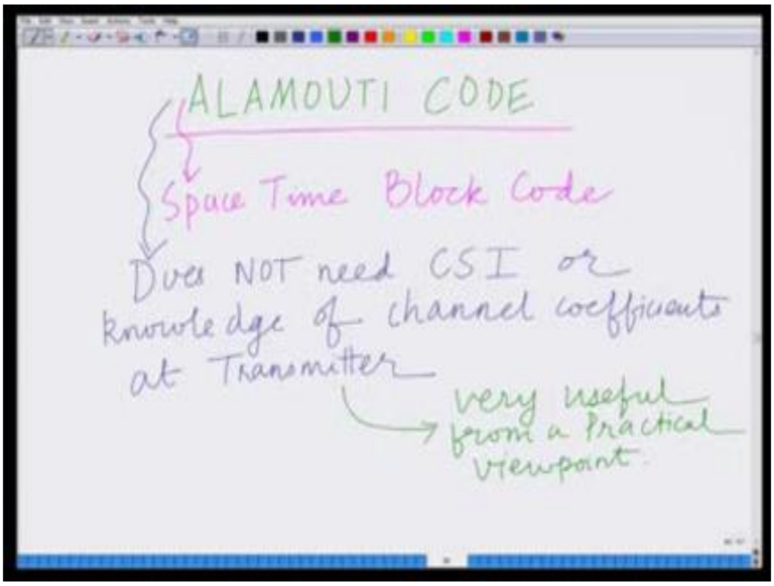

So, in this module we were going to focus on what is known as the Alamouti? We are going to focus on in the Alamouti Code, which is the Space Time Block Code for and this does not need channel state information or knowledge of the channel coefficients, that is unlike transmit beam forming does not need knowledge of the channel coefficients at the transmitter, hence this is very useful from a practical perspective or from a practical view point this is.

So, what we are saying is this is Alamouti, this is space time block code which is the transmission scheme for a MISO system. So, specifically for a 1 x 2 MISO systems and this does not need the channel state information at the transmitter, hence it is very practical scheme; it is useful from practical perspective, for practical implementation in a wireless communication system.

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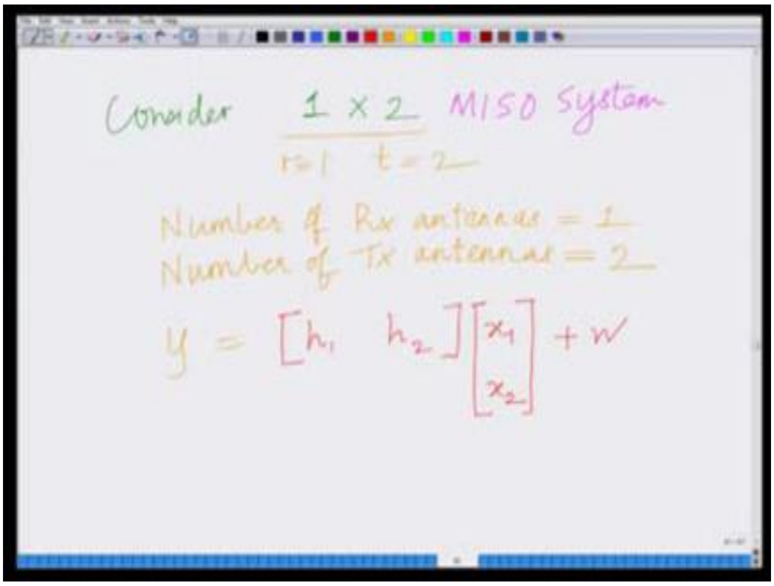

So, let us start by considering to explore this Alamouti scheme, consider of 1 cross 2 MISO system, 1 x 2 as we have already seen which means r = 1, t = 2, that is number of receive antennas equals 1, number of transmit antennas is equal to 2 therefore, we have the system model y is equal to

$$Y = [h_1 \quad h_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w$$

This is the system model that we have, we are considering 1 x 2 MISO system, 1 received antenna, 2 transmit antennas and this is the system model that we had. What we are going to do now is, we are going to consider 2 transmit symbols, x_1, x_2. So, now, what we are going to do is consider 2 symbols x_1, x_2.

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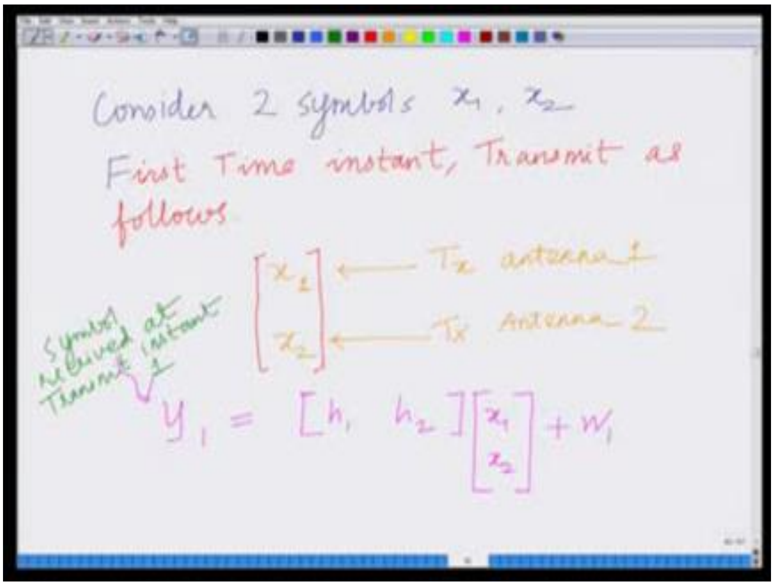

Now, in the first time instant, what we are going to do is transmit as follows, what we are going to do is x_1 is transmitted from transmit antenna 1, and x_2 is transmitted from transmit antenna 2.

I have

$$y_1 = [h_1 \quad h_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_1$$

So, what is this? This is the symbol received at transmit instant 1 of the first time period. So, y_1 is the symbol that is received at the output of the first transmission period in this Alamouti coded MISO system.

Now, what are we going in to do in the second transmit period is something interesting, in the second transmit period we are going to transmit the same symbols x_1, x_2, but from the first transmit antenna we are going to transmit $-x_2{}^*$ and from the second transmit antenna we are going to transmit $x_1{}^*$.

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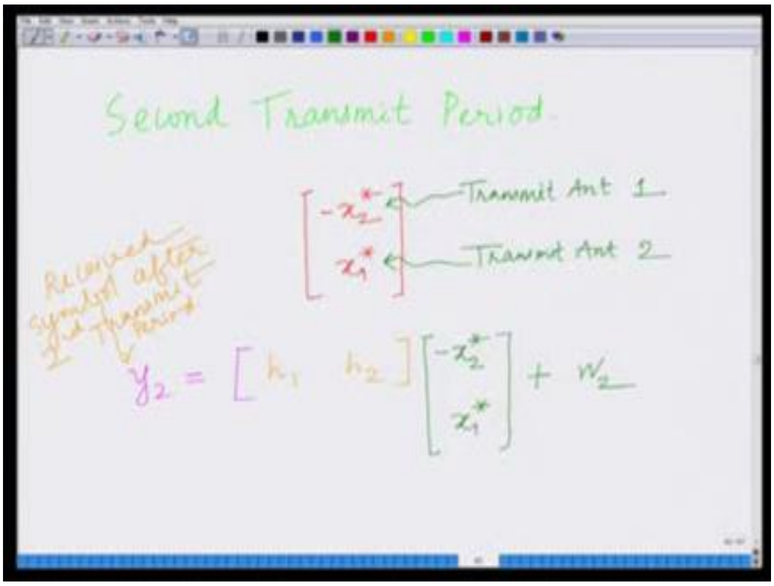

So, in the second transmit period, what we are going to do is we are going to transmit $-x_2{}^*$ from the first transmit antenna. So, this is from transmit antenna 1 and $x_1{}^*$ is transmitted from transmit antenna 2, therefore at the receiver

$$y_2 = [h_1 \quad h_2] \begin{bmatrix} -x_2{}^* \\ x_1{}^* \end{bmatrix} + w_2$$

y_2 is the received symbol after the second transmits period.

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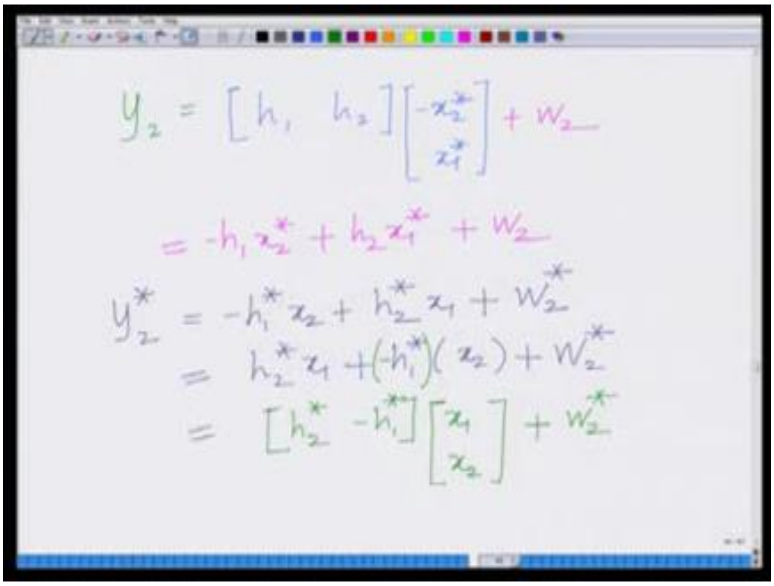

Now, let us look at this, let us expand this slightly I have

$$y_2 = -h_1 x_2{}^* + h_2 x_1{}^* + w_2$$

Now I can take the conjugate of y_2 at the receiver,

$$y_2{}^* = -x_2 h_1{}^* + h_2{}^* x_1 + w_2{}^*$$

$$= \begin{bmatrix} h_2{}^* & -h_1{}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_2{}^*$$

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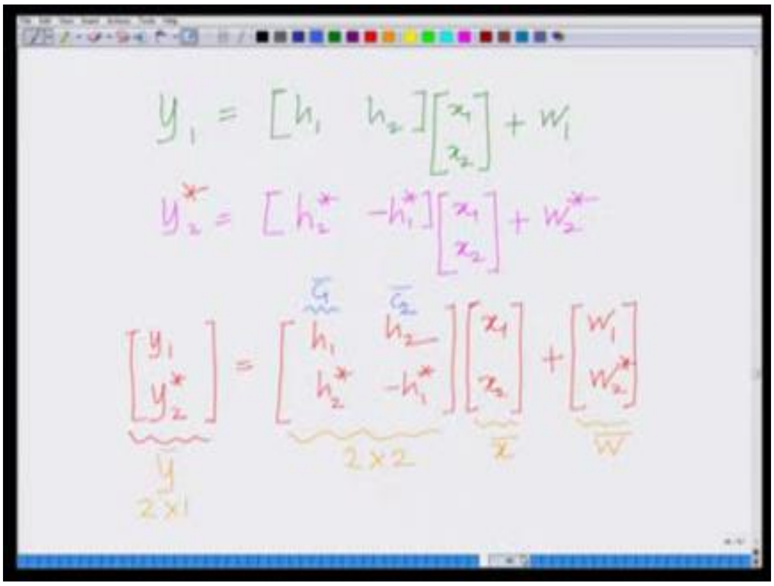

So, know I have 2 equations the from the first transmit period I have

$$y_1 = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_1$$

From the second time period I have

$$y_2{}^* = \begin{bmatrix} h_2{}^* & -h_1{}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_2{}^*$$

$$\begin{bmatrix} y_1 \\ y_2{}^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2{}^* & -h_1{}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2{}^* \end{bmatrix}$$

So, now, what I have done is basically I have stacked y_1 and ${y_2}^*$ and what I have received an effective system model for this Alamouti coded system, in which I can represent these various quantities. Now as this quantity here this is y bar which is 2 x 1 vector, this quantity here this is the 2 x 2 matrix, this quantity here is my transmit vector x bar and this quantity here is the noise vector w, this quantity here is the noise vector w bar, I can denote this column of this matrix by $\bar{c}_1$ the first column, the second column of this matrix by $\bar{c}_2$.

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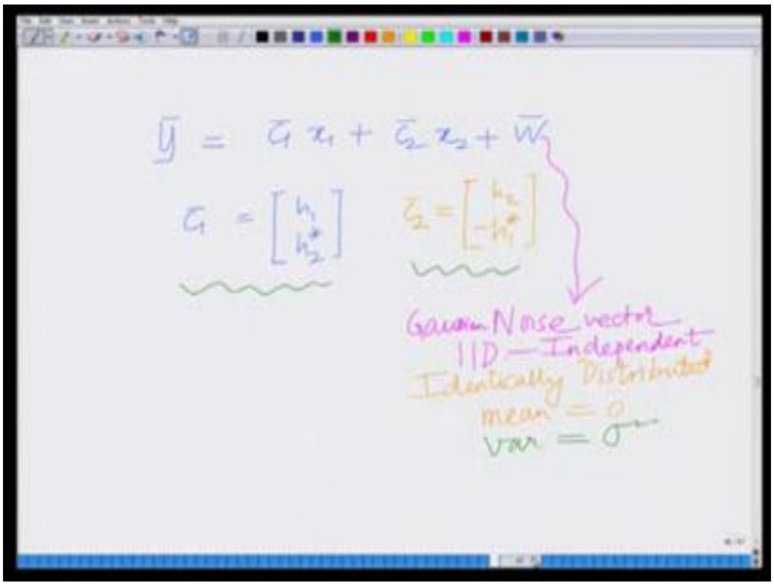

Therefore, what I will have now is

$$\bar{y} = \bar{c}_1 x_1 + \bar{c}_2 x_2 + \ldots + \bar{w}$$

$$\bar{c}_1 = \begin{bmatrix} h_1 \\ {h_2}^* \end{bmatrix}$$

$$\bar{c}_2 = \begin{bmatrix} h_2 \\ -{h_1}^* \end{bmatrix}$$

So, these are the 2 columns of the matrix and also w bar as usual is the noise vector is in fact the Gaussian noise vector, with IID Independent Identically Distributed noise elements, which means the mean is 0 variance of the noise element is same and the correlation between the 2 different noise elements is 0. So, I have noise elements which are IID which means Independent Identically Distributed noise elements, the mean is equal to 0, the variance is equal to σ^2.

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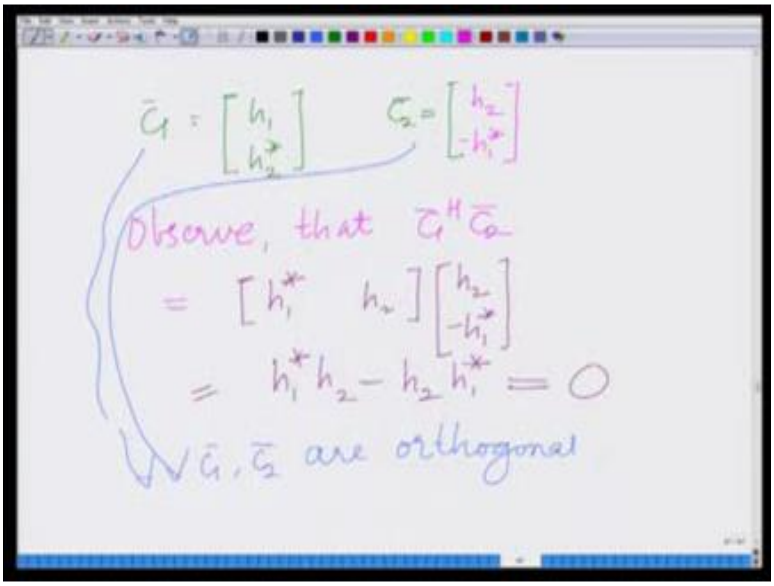

I have

$$\bar{c}_1 = \begin{bmatrix} h_1 \\ h_2{}^* \end{bmatrix}$$

$$\bar{c}_2 = \begin{bmatrix} h_2 \\ -h_1{}^* \end{bmatrix}$$

$$\bar{c}_1{}^H \bar{c}_2 = \begin{bmatrix} h_1{}^* & h_2 \end{bmatrix} \begin{bmatrix} h_2 \\ -h_1{}^* \end{bmatrix} = 0$$

So, $\bar{c}_1{}^H \bar{c}_2$ or the inner product between these 2 columns of the matrix is 0, which means $\bar{c}_1$ and $\bar{c}_2$ are orthogonal vectors because $\bar{c}_1{}^H \bar{c}_2$ equal 0, it means the dot product or the inner product between these 2 vectors is 0 therefore, these 2 vectors are Orthogonal. So, from a geometrical perspective these vectors $\bar{c}_1$, $\bar{c}_2$ these are Orthogonal and this is an interesting property of the Alamouti code.

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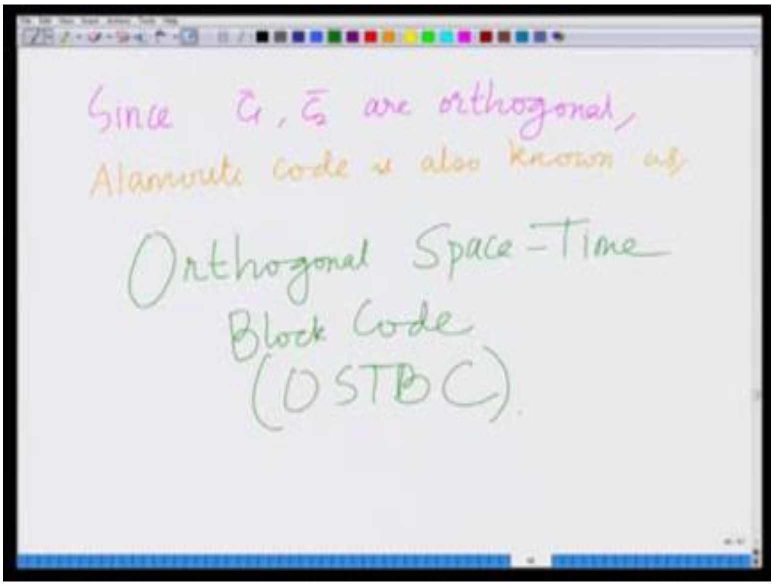

Therefore since $\bar{c}_1$, $\bar{c}_2$ are Orthogonal Alamouti code is also known as an orthogonal it is an "Orthogonal Space Time", Alamouti code is an Orthogonal Space Time Block Code or basically it is an OSTBC, this is a very interesting property of the Alamouti code, where the column vectors $\bar{c}_1$, $\bar{c}_2$ are orthogonal therefore, the Alamouti code is also known as an Orthogonal Space time Block code.

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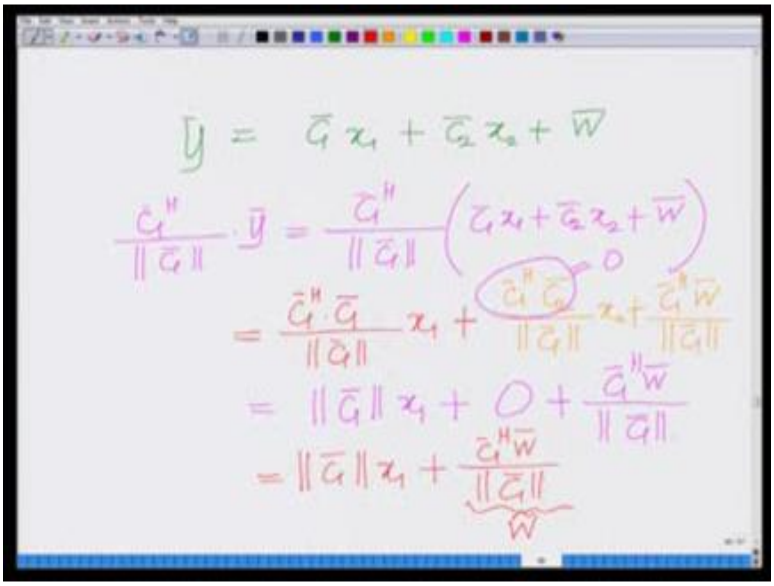

Now, let us look at the receiver for this Alamouti code and the receiver has a very simple structure, I have

$$\bar{y} = \bar{c}_1 x_1 + \bar{c}_2 x_2 + \ldots + \bar{w}$$

$$\frac{\bar{c}_1^{\,H}}{||\bar{c}_1||}\bar{y} = \frac{\bar{c}_1^{\,H}}{||\bar{c}_1||}(\bar{c}_1 x_1 + \bar{c}_2 x_2 + \ldots + \bar{w})$$

$$= \frac{\bar{c}_1^{\,H}\bar{c}_1}{||\bar{c}_1||}x_1 + \frac{\bar{c}_1^{\,H}\bar{c}_2}{||\bar{c}_1||}x_2 + \frac{\bar{c}_1^{\,H}\bar{w}}{||\bar{c}_1||}$$

$$= ||\bar{c}_1||x_1 + 0 + \frac{\bar{c}_1^{\,H}\bar{w}}{||\bar{c}_1||}$$

$$= ||\bar{c}_1||x_1 + \tilde{w}$$

Since $\bar{c}_1$, $\bar{c}_2$ are Orthogonal that will remove the interference that is called by symbol x_1 and I can does decode symbol x_2 at the receiver.

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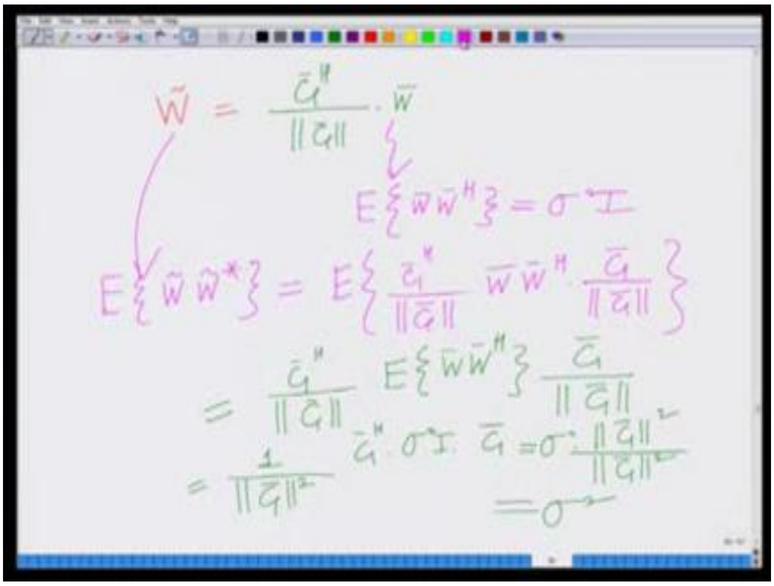

Now, let us look at the property of this noise $\tilde{W}$. So, now, let us look at the property

$$\tilde{W} = \frac{\bar{c}_1^{\,H}\bar{w}}{||\bar{c}_1||}$$

Where $E\{ \bar{w}, \bar{w}^H \} = \sigma^2 I$

$$E\{ \tilde{w}, \tilde{w}^* \} = E\{ \frac{\bar{c}_1^H}{||\bar{c}_1||} \bar{w}, \bar{w}^H \frac{\bar{c}_1}{||\bar{c}_1||} \}$$

$$= \frac{\bar{c}_1^H}{||\bar{c}_1||} E\{ \bar{w}, \bar{w}^H \} \frac{\bar{c}_1}{||\bar{c}_1||}$$

$$= \frac{1}{||\bar{c}_1||} \bar{c}_1^H \sigma^2 I \bar{c}_1 \frac{1}{||\bar{c}_1||} = \sigma^2 \frac{||\bar{c}_1||^2}{||\bar{c}_1||^2} = \sigma^2$$

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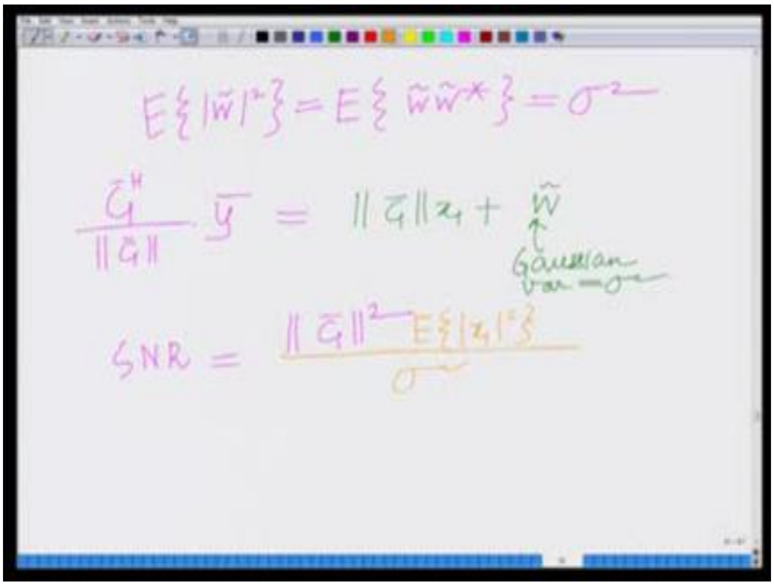

So, what I am finding interestingly that

$$E\{ |\tilde{w}|^2 \} = E\{ \tilde{w}, \tilde{w}^* \} = \sigma^2$$

$$\frac{\bar{c}_1^H}{||\bar{c}_1||} \bar{y} = ||\bar{c}_1|| x_1 + \tilde{w}$$

$$\text{SNR} = \frac{||\bar{c}_1||^2 E\{ |x_1|^2 \}}{\sigma^2}$$

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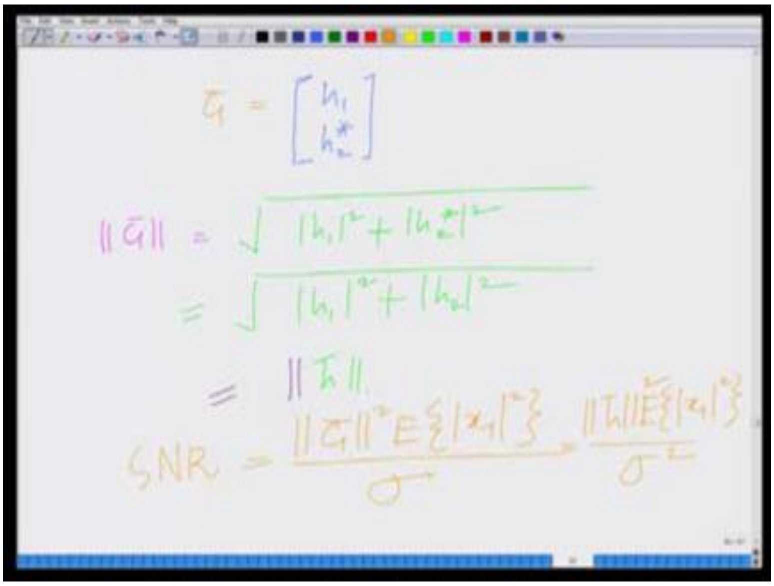

Now let us look at this quantity

$$\bar{c}_1 = \begin{bmatrix} h_1 \\ h_2{}^* \end{bmatrix}$$

$$||\bar{c}_1|| = \sqrt{|h_1|^2 + |h_2{}^*|^2} = ||\bar{h}||$$

$$\text{SNR} = \frac{||\bar{c}_1||^2 \mathrm{E}\{|x_1|^2\}}{\sigma^2} = \frac{||\bar{h}||^2 \mathrm{E}\{|x_1|^2\}}{\sigma^2}$$

Now what we have to do is we have to find out this quantity, $\mathrm{E}\{|x_1|^2\}$ or the power in the transmitted symbol x_1 and this can be found as follows.

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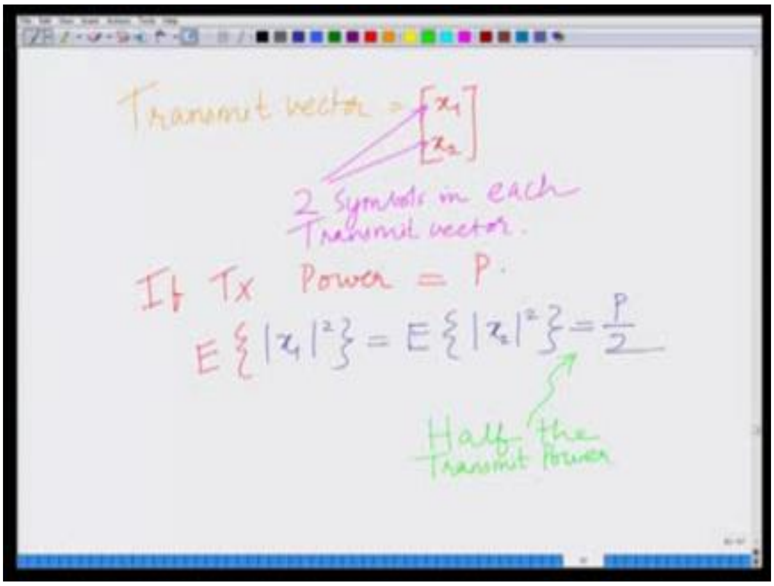

Let us look at the transmit vector, the transmit vector here is x_1, x_2. So, therefore, what we have is we have 2 symbols, 2 symbols in each transmit vector.

So, we have 2 symbols in each transmit vector therefore, if the total power is P, this transmit power P has to be divided between the 2 symbols, therefore each symbol x_1, x_2 will have half the transmit power or $\frac{P}{2}$, therefore if transmit power is equal to P, if T x power is equal to P, then

$$\mathrm{E}\{\, |x_1|^2\} = \mathrm{E}\{\, |x_1|^2\} = \frac{P}{2},$$

that is each symbol has half, that is if the total power is P, then this power P has to be divided between the 2 symbols equally therefore, the power is symbol is $\frac{P}{2}$.

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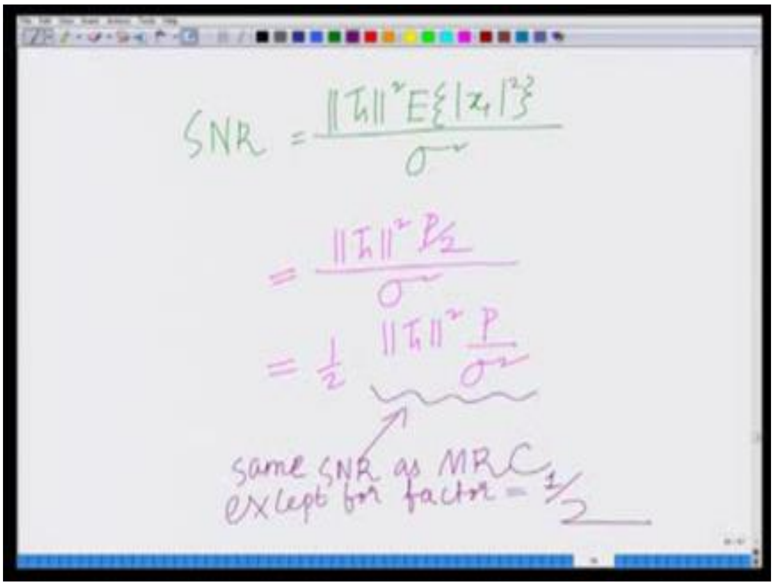

Therefore the final SNR for this Alamouti coded quantity Alamouti scheme,

$$\mathrm{SNR} = \frac{||\bar{\mathrm{h}}||^2 \mathrm{E}\{\,|x_1|^2\}}{\sigma^2} = \frac{||\bar{\mathrm{h}}||^2 \frac{P}{2}}{\sigma^2} = \frac{1}{2}\,\frac{||\bar{\mathrm{h}}||^2 \mathrm{P}}{\sigma^2}$$

So, we get half the SNR, that we get with maximal ratio combine and this is very interesting because remember previously we had seen that in a maximum ratio out that is in a transmit beam forming system, we get the exact identical SNR as a maximal ratio combining system. So, in that sense transmit beam forming yields a performance that is exactly identical to maximal ratio combing.

But transmit beam forming requires channel rate information at the transmitter which is challenging. Now what we are seeing here is that Alamouti code does not need channel state information at the transmitter, but it achieves half the SNR as that of maximal ratio combining. So, the price that we are paying for not having the channel state information at the transmitter is that we are losing a factor of half in terms of SNR. So in basically in terms of the log scale, we are losing 3 dB. So, the SNR is an Alamouti scheme is - 3 dB, 3 dB lower than that of the maximal ratio or the transmit beam forming scheme or the maximal ratio combining scale. So, 3 dB is the loss in the Alamouti coded scheme.

But it is practical, it is effective from a practical perspective because we do not need knowledge of the channel coefficients and what you can see here is we have this factor of norm h square, which means the diversity order is preserved therefore, the diversity order of this scheme is tend to therefore, the despite the loss of 3 dB despite not having the channel state information at the transmitter, it is still yields the same diversity order as that of the Maximal ratio combining base system. So, this is the interesting property of the Alamouti code.

With this we are going to conclude this module and we are going to look at the other aspects notably that bit error rate performance of this Alamouti coded system in the subsequent module.

Thank you very much.