Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture – 38
Transmit Beam forming in MISO Systems

Hello, welcome to another module in this massive open online course on the principles of CDMA, MIMO, OFDM wireless communication systems. So, currently we are looking at the principles with the various techniques and the theory behind MIMO wireless communication system. Today let us look at a new aspect; let us look at specifics especially case on MIMO wireless communication system that is MISO wireless communication system.

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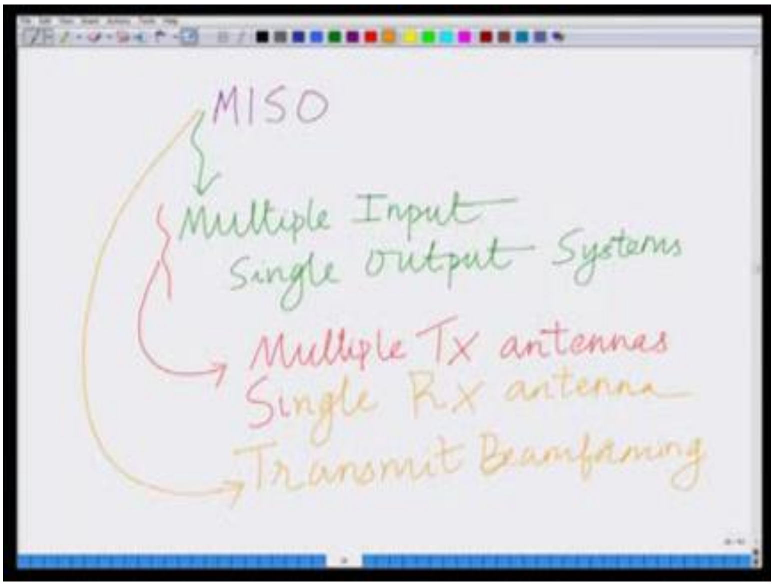

Today's module let us start looking at MISO wireless communication systems that is multiple input, single output system and we have already seen this before multiples input single output systems means we have multiple Transmit antennas and a single output that is a single receive antenna. So, by definition, these systems have multiple inputs that is multiple transmit or multiple T x antennas and a single receive antenna or a single output antenna. In particular what we are going to be interested in these systems is what is known as a new technique that is known as Transmit Beam Forming.

We have previously seen maximum ratio combining at the receiver for a single input multiple output system. We have Maximal Ratio Combining which is also in a receive beam forming. Now if you have multiple inputs and a single output we can do something known as Transmit Beam Forming and this is what we are going to explore in this module.

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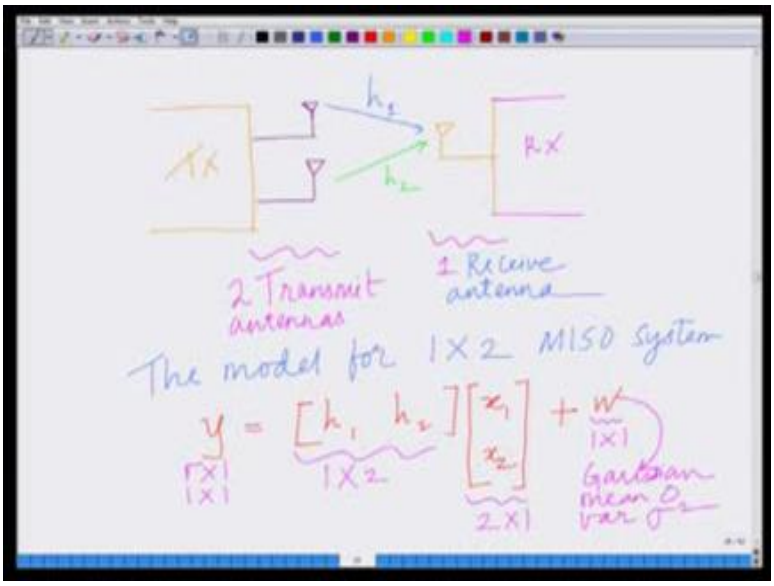

So, let us start by looking at a simple example, in which have a Transmit or we two Transmit antennas and a receiver. And I have a receiver with a single receive antenna. Therefore, I have two transmit antennas and single receive antenna, and therefore, let h_1 denotes the channel coefficient between transmit antenna 1 and the receive antenna and simultaneously h_2 denote the channel coefficient between transmit antenna 2 and the receive antenna therefore, the model for this MISO system. In fact, this is a 1 x 2 MISO system. And the model for this is the receive symbol y equals

$$Y = [h_1 \quad h_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w$$

So, what do we have over here, we have an r x 1 which is basically 1 x 1 received symbol this is my 1 x 2 the MISO channel vector. This is my T x 1 or 2 x 1 transmit vector and the same R x 1 or 1 x 1 noise, and similar to before no surprises here we will assume Gaussian noise with mean 0 and variance σ^2 so W is Gaussian noise with mean

0 variance σ^2 received symbol Y is a single symbol 1 x 1 the channel vector the channel vector h_1, h_2 is the 1 x 2, that is R x T, MISO channel vector, and x_1, x_2 is the T x 1 or 2 x 1 transmit symbol vector in this MISO system. Now, let us look at what is the Appropriate Transmission Scheme for this MISO System? For this MISO system one can ask the question what is the Appropriate Transmission Scheme?

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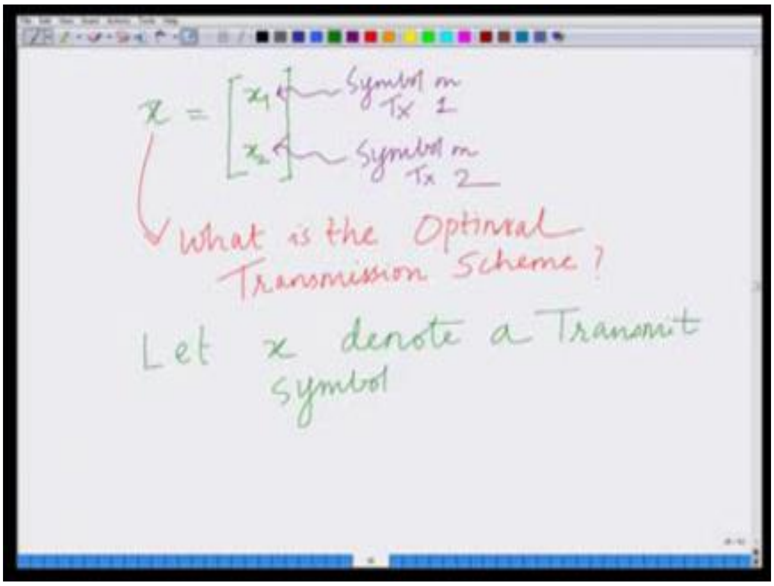

So, we have $\bar{x}$ is the transmit vector $\bar{x}$ out of which I have x_1 is the symbol on transmit antenna and x_2 is the symbol on transmit antenna 2 and one can ask the question what is Optimal Transmission Scheme for this MISO system.

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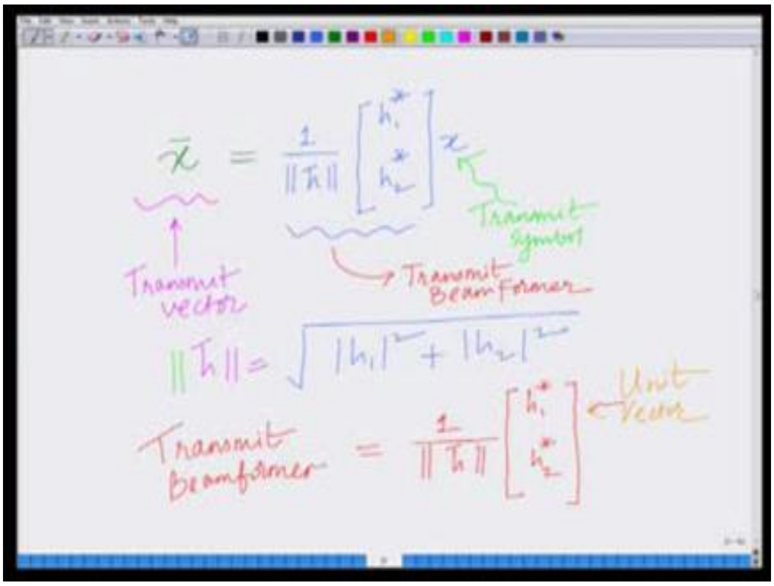

So, let us start with the Transmit symbol x, let x denote a Transmit Symbol, then the vector $\bar{x}$ can be formed as the transmit vector x bar is given as

$$\bar{x} = \frac{1}{||\bar{h}||}\begin{bmatrix} h_1{}^* \\ h_2{}^* \end{bmatrix} x$$

So, what do I have over here $\bar{x}$ is the Transmit vector, my x is the Transmit Symbol and this $||\bar{h}||$ is obviously is equal to

$$||\bar{h}|| = \sqrt{|h_1|^2 + |h_2|^2}$$

$$\text{Transmit Beam Former} = \frac{1}{||\bar{h}||}\begin{bmatrix} h_1{}^* \\ h_2{}^* \end{bmatrix}$$

So, this is a unit vector, the norm of this vector is unity this is known as the Transmit Beam Former.

What this means is you are forming a beam in the direction given by this unit vector, so x is the symbol that is being transmitted in the direction of this Transmit Beam Former. Therefore, this scheme is also known as the Transmit Beam Forming Scheme. So, what is my Transmit Beam Former? Now, let us look at what is the output, why going to be once we employ this Transmit Beam Forming Scheme?

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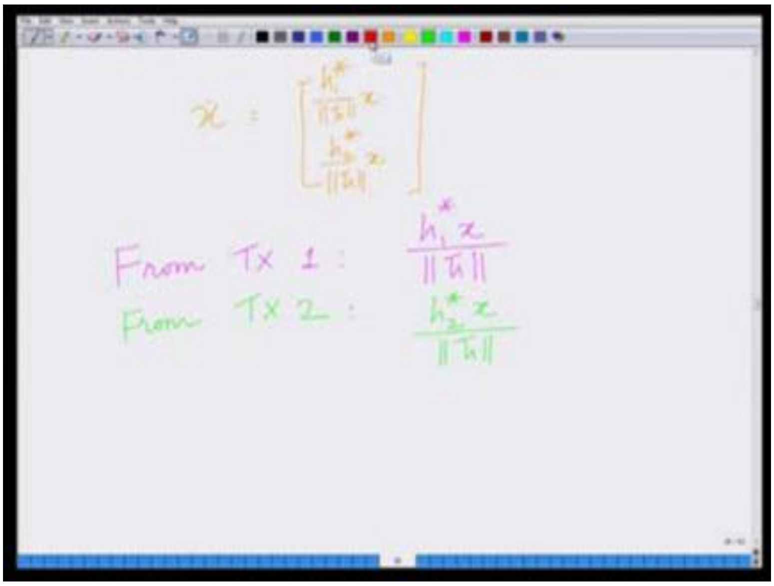

So when we employ this Transmit Beam Forming Scheme, therefore, what we are saying is

$$\bar{x} = \begin{bmatrix} \frac{h_1^*}{||\bar{h}||} x \\ \frac{h_2^*}{||\bar{h}||} x \end{bmatrix}$$

Therefore, what we are saying is

symbol from TX antenna 1 = $\frac{h_1^*}{||\bar{h}||} x$,

symbol from TX antenna 2 = $\frac{h_2^*}{||\bar{h}||} x$,

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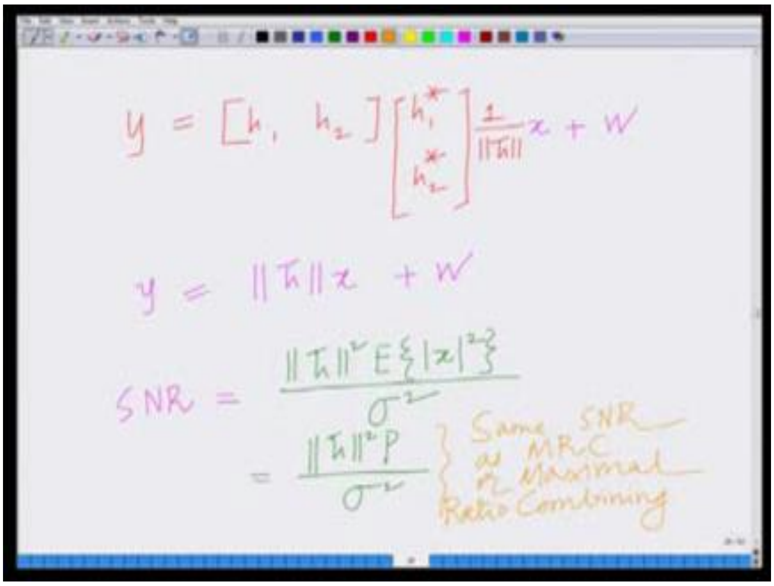

So, this is going to be equal to

$$Y = [h_1 \quad h_2]\begin{bmatrix} h_1^{\ *} \\ h_2^{\ *} \end{bmatrix}\frac{1}{||\bar{h}||}x + w$$

$$Y = ||\bar{h}||\, x + w$$

and very interestingly what you see here is now if I compute the SNR of this scheme is

$$SNR = \frac{||\bar{h}||^2 P}{\sigma^2}$$

This exactly the same SNR as MRC or Maximal Ratio Combining.

So, Transmit Beam Forming, when we transmit beam form in the direction of $\begin{bmatrix} h_1^{\ *} \\ h_2^{\ *} \end{bmatrix}\frac{1}{||\bar{h}||}$ the SNR at the receiver is the same as what we get for Maximum Ratio Combining. That is with Transmit antennas at the receiver multiple antennas at the Transmitter and single antenna at the receiver we are able to achieve the same SNR as a scheme with multiple antennas at the receiver and a single antenna at the Transmitter.

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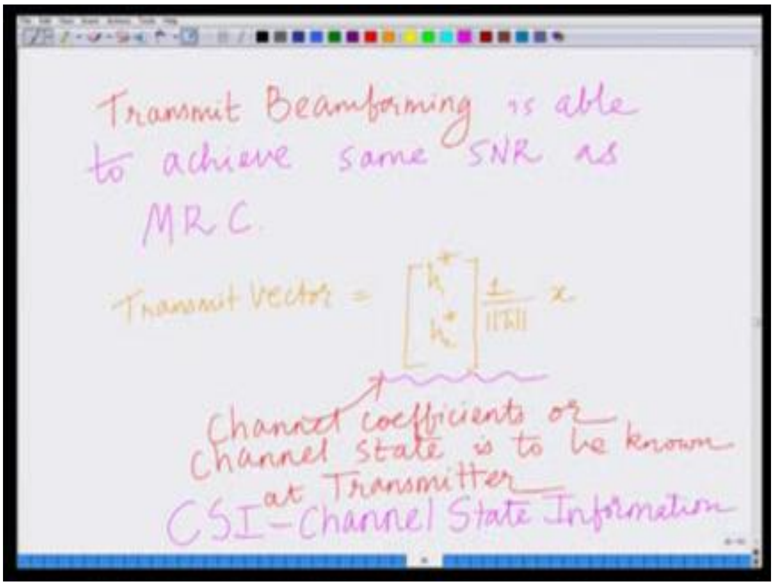

So, Transmit Beam Forming is able to achieve the same SNR as Maximal Ratio Combining.

However, if you notice, we are transmitting the Transmit Vector equals $\begin{bmatrix} h_1{}^* \\ h_2{}^* \end{bmatrix} \frac{1}{||\bar{h}||} x$

So, what we need at the Transmitter? We need knowledge of these fading coefficients h_1, h_2 to form the Transmit Beam Former. So, which means we need knowledge of the channel coefficients or the channel state at the Transmitter this is also known as the Channel State Information and frequently this is difficult to obtain in the Wireless Communication System at the Transmitters. So, this is the challenge in Transmit Beam Forming. The challenge in Transmit Beam Forming is basically the channel coefficients or channel state is to be known at the Transmitter. This is also known as Channel State Information CSI.

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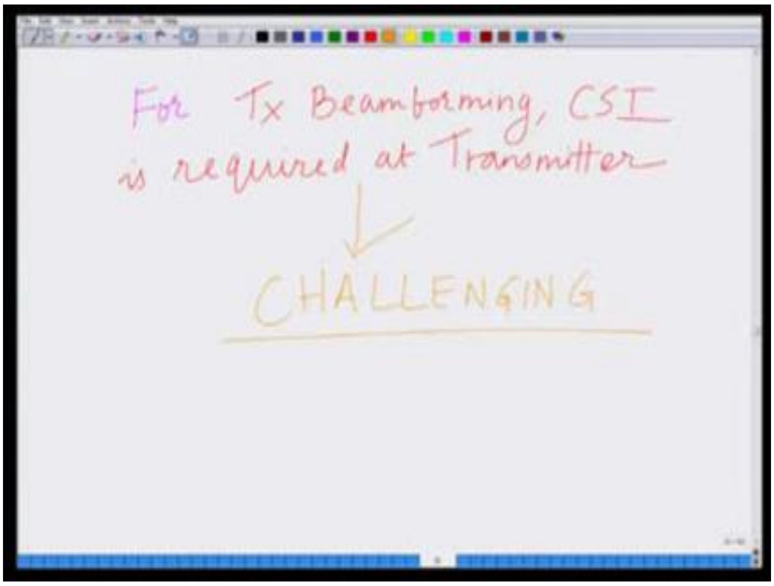

That is what we are saying is for Maximal Ratio Transmission or for Transmit Beam Forming CSI or Channel State Information is required at transmitter and this is CHALLENGING that is for CSI, for the Channel State Information to be known at the Transmitter is Challenging.

So, maximum ratio transmission is or so Transmit Beam Forming is possible only when the challenge state information is available or the knowledge of channel coefficients h_1, h_2 is available at the Transmitter. Now what happens when the Channel Coefficients h_1, h_2 are not known at the Transmitter then we have to explore alternative schemes one such scheme is known as Space Time Coding which is something that we are going to look at in the subsequent modules. So, transmit beam forming optimal in the sense it yields the same SNR as maximal ratio combining.

However, it is also challenging to implement because the channel state information is required at the transmitters. So, this is the summary of this Transmit Wave Forming.

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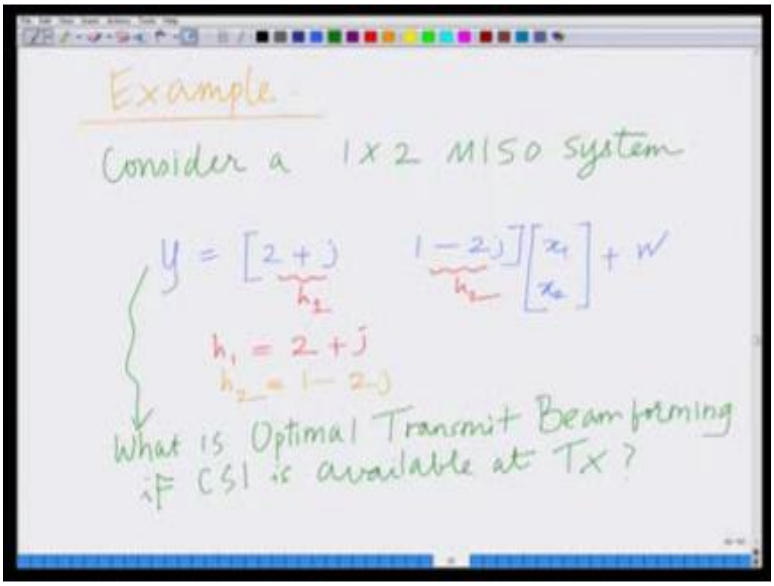

Now, let us try to look at a simple example to understand this Transmitting Beam Forming. So, consider a 1 x 2 MISO system, now in this 1 x 2 MISO system, let or system model be given as

$$Y = [2 + j \quad 1 - 2j] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w$$

We are asking what is the Optimal Transmit Beam Forming Procedure.

What is Optimal Transmit if CSI is available or Channel State Information is available at the Transmitter that is for this system what is the Optimal Transmit Beam Forming Procedure, when the Channel State is Information available at the Transmitter or the Channel Coefficients are known at the Transmitter; obviously, the Optimal Procedure is Transmit Beam Forming.

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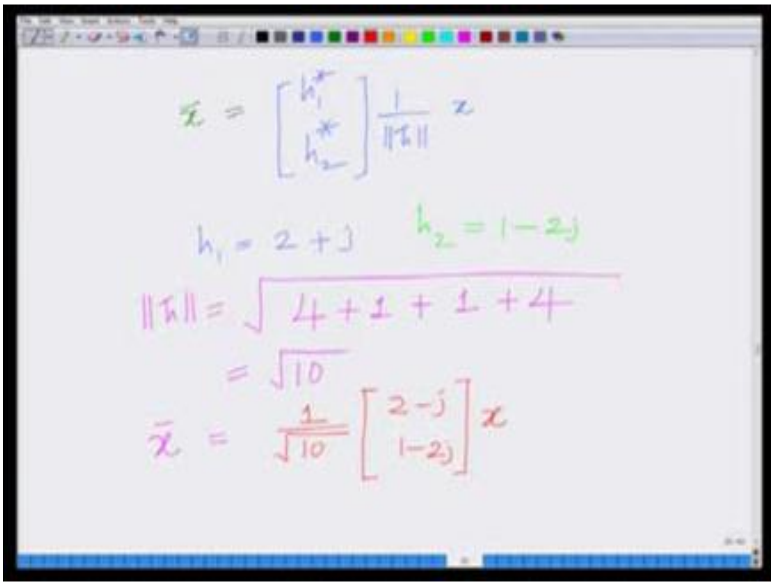

And the Transmit Beam Former is

$$\bar{x} = \begin{bmatrix} h_1{}^* \\ h_2{}^* \end{bmatrix} \frac{1}{||\bar{h}||} x$$

$$||\bar{h}|| = \sqrt{4+1+1+4} = \sqrt{10}$$

$$\bar{x} = \frac{1}{\sqrt{10}} \begin{bmatrix} 2-j \\ 1-2j \end{bmatrix} x$$

So, Optimal Transmit Vector in this scenario is $\bar{x} = \frac{1}{\sqrt{10}} \begin{bmatrix} 2-j \\ 1-2j \end{bmatrix} x$

So, x is the Transmit Symbol in this Scenario. So, now, let us substitute this Transmit a Vector in the System Module to see what is the SNR that we obtain?

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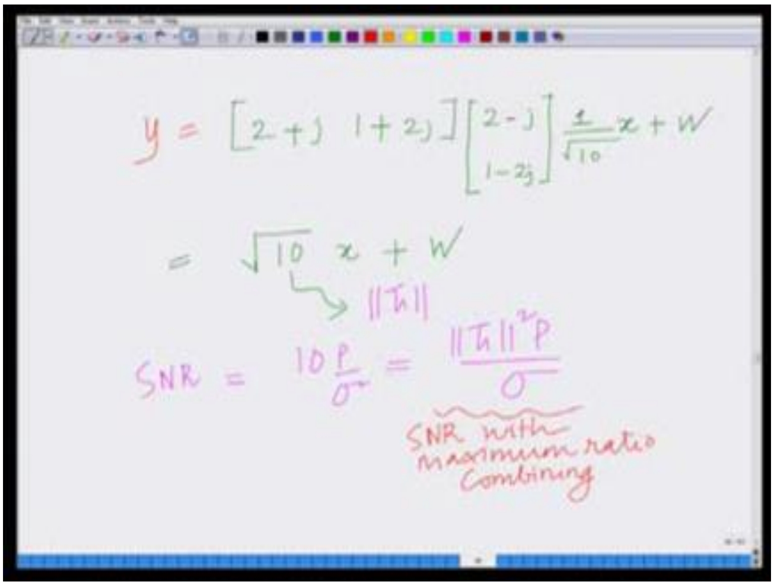

So, once I substitute this Transmit Vector I obtain

$$Y = [2 + j \quad 1 - 2j] \begin{bmatrix} 2 - j \\ 1 - 2j \end{bmatrix} \frac{1}{\sqrt{10}} x + w$$

$$= \sqrt{10}x + w$$

So, the SNR equals

$$\text{SNR} = \frac{||\bar{h}||^2 P}{\sigma^2} = 10\ \frac{P}{\sigma^2}$$

this is the same as SNR with Maximum Ratio Combining.

So, what we are saying is that the SNR at the receiver is $10\ \frac{P}{\sigma^2}$, which is the same as $\frac{||\bar{h}||^2 P}{\sigma^2}$, which is the same as the SNR with Maximum Ratio Combining. So, Transmit Beam Forming is achieving the same SNR as Maximum Ratio Combining. And now it is also straight forward to compute to Bit Error Rate of the system. So, let us look at the Bit Error Rate of this system of the Transmit Beam Forming.

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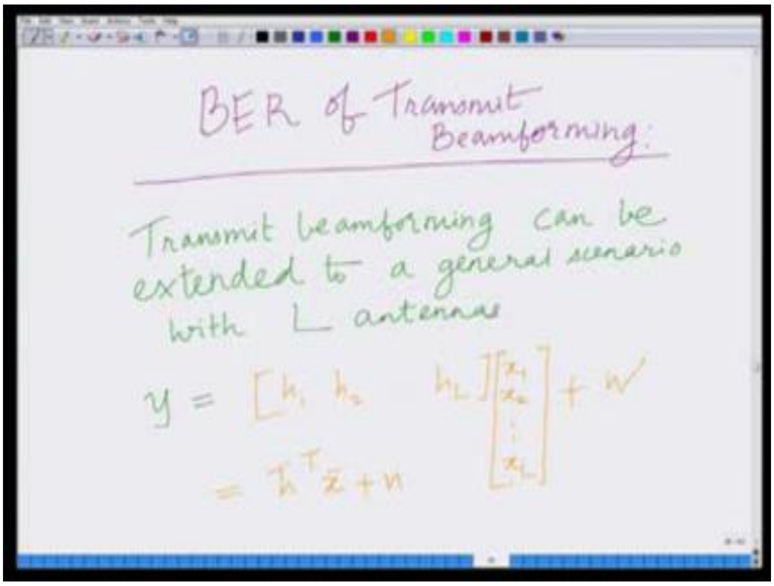

Now, before that we will extend this to L Antenna, Transmit Beam Forming can naturally be extended to L antennas, to a general scenario with L antennas. So, let say I have 1 x L MISO system Y equals

$$y = \begin{bmatrix} h_1 & h_2 & h_t \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_t \end{bmatrix} + w$$

$$= \bar{h}^T \bar{x} + w$$

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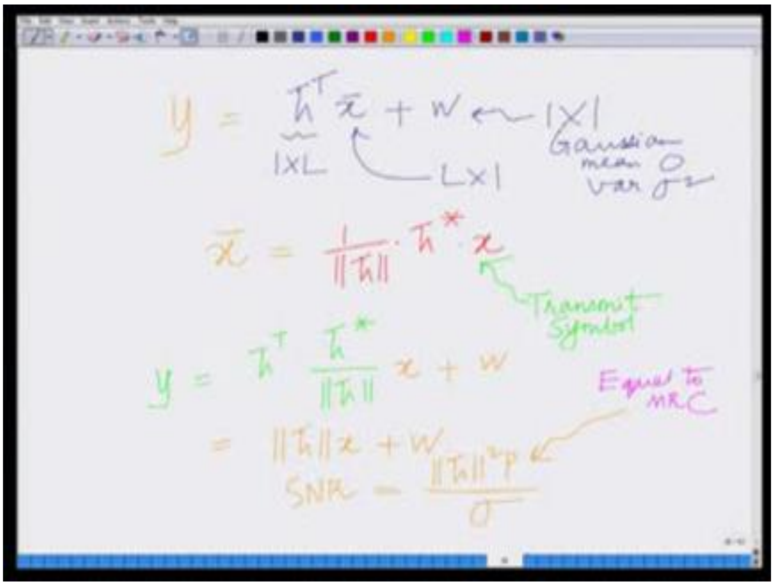

So, what I have is

$$y = \bar{h}^T \bar{x} + w$$

$\bar{h}^T$ is my 1 x L channel vector, $\bar{x}$ is my L x 1 transfer vector this is my 1 x 1 Gaussian noise mean 0 variance σ^2 as usual and then what I am going to Transmit with my Transmit Beam Former is

$$\bar{x} = \begin{bmatrix} h_1{}^* \\ h_2{}^* \end{bmatrix} \frac{1}{||\bar{h}||} x$$

is my Transmit Symbols. So, x is my Transmit Symbol.

So, my system model is Y equals

$$y = \bar{h}^T \frac{\bar{h}^*}{||\bar{h}||} x + w$$

$$= ||\bar{h}|| \, x + w$$

$$\text{SNR} = \frac{||\bar{h}||^2 P}{\sigma^2}$$

Therefore, the Bit Error Rate performance of Transmit Beam Forming is identical to that of MRC, this is the SNR is identical to that of MRC. The Bit Error Rate for Transmit Beam Forming is identical to that of Maximal Ratio Combining.

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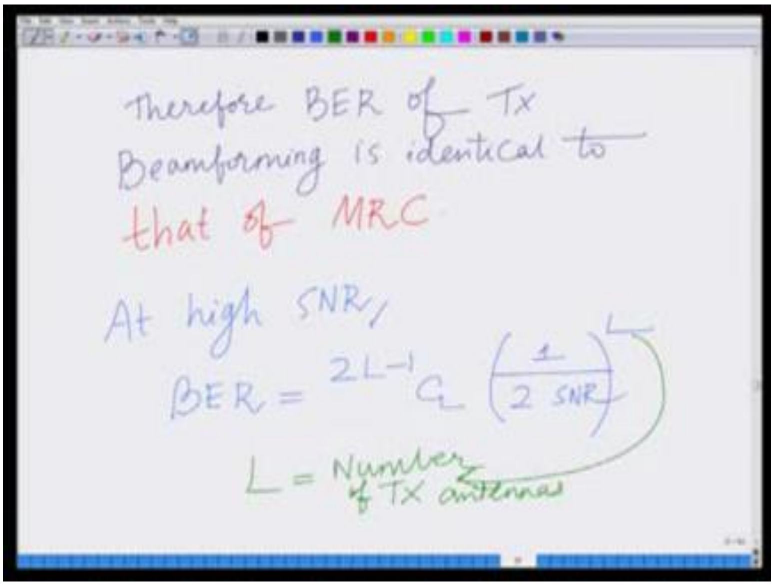

So, therefore, Bit Error Rate of Transmit Beam Forming is identical to that of MRC which means at high SNR the Bit Error Rate equals

$$\text{BER} = {}^{2L-1}C_L \left(\frac{1}{2\ \text{SNR}}\right)^L$$

where L is the number of Transmit antennas. That is similar to the Bit Error Rate of Maximal Ratio of Combining at high SNR.

So, basically this the SNR is identical to that MRC, the Bit Error Rate performance is also identical to that of MRC, except the procedure is different there we perform Maximal Ratio Combining at the receiver here we have to perform Transmit Beam Forming at the Transmit. So, the procedure is different, the challenge is different because a Transmit Beam Forming you needs knowledge of the channel coefficients of the Transmitter.

However, if you are able to do that ensuing performance and the Bit Error Rate is identical to that of MRC because at the end of the day the SNR is basically identical in both these.

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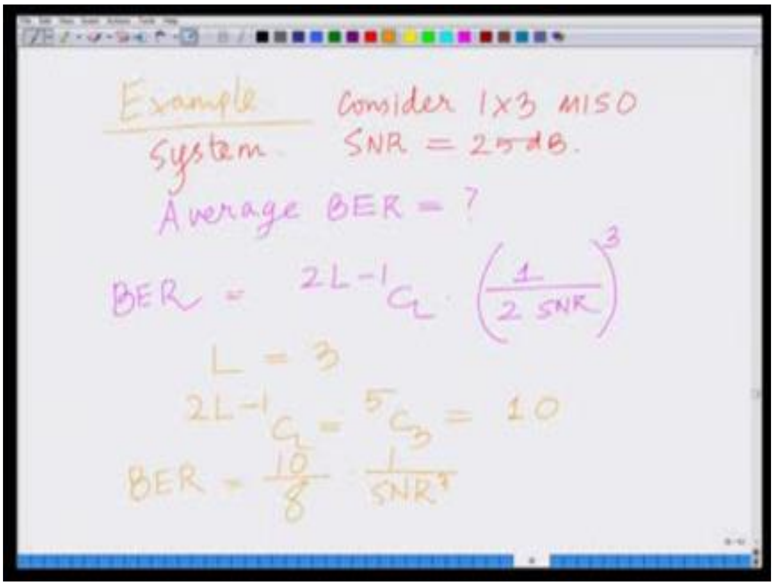

Let us do a simple example again to understand this thing it is very simple example consider a 1 x 3 MISO system and SNR = 25dB. Question you want to ask is what is what is the average Bit Error Rate of this system? the Bit Error Rate equals

$$\text{BER} = {}^{2L-1}C_L \left(\frac{1}{2\ \text{SNR}}\right)^L$$

$$= {}^{5}C_3 \left(\frac{1}{10 \wedge 2.5}\right)^3 = 6.3 \times 10^{-8}$$

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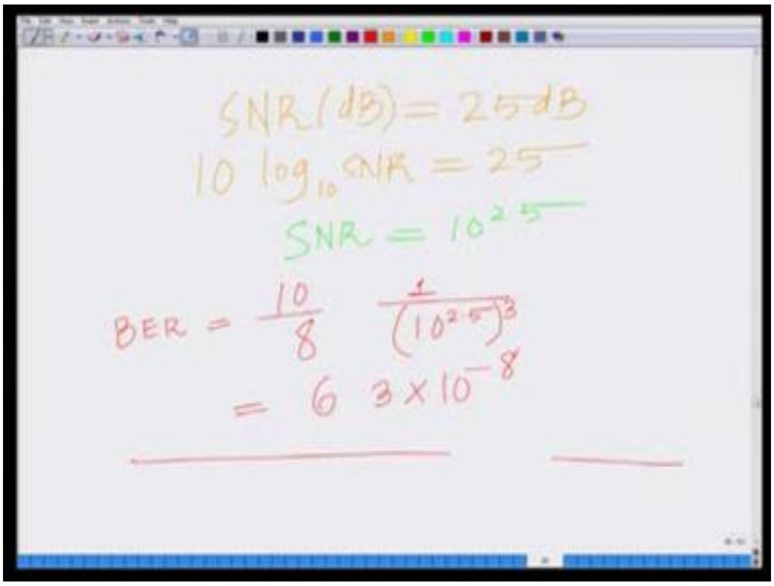

So, that is the Bit Error Rate of this system with Transmit.

So, what we have seen in this module is, we have seen an interesting case what happens with multiple Transmit antennas? What we have seen so far in the previous modules? In fact, in the very early part of this course when you have Multiple Receive antennas at the receiver and a single Transmit antenna now we have seen the other side of a that is when we have the analog SK when we have Multiple Transmit antennas, at the Transmit and single receive antenna that is a MISO system multiple input single output system.

In this system the Optimal Transmission Scheme is Transmit Beam Forming and Transmit Beam Forming Achieves a performance that is identical to Maximal Ratio Combining in terms of the SNR.

However, it is challenging to implement because this requires channel state information at the Transmitter. But if that is available we can implement Transmit Beam Forming you computed the SNR Transmit we demonstrated the Transmit Beam Former computed the SNR for Transmit Beam Forming derived the Bit Error Rate expression with Transmit Beam Forming and is also seen (Refer Time: 34:13) simple examples.

Now, in the subsequent modules, in the next modules, we are going look at what can we do when we do not have channel state information at the transmitter and then we have to use space time codes or space time block codes and this we are going to look at in the subsequent modules. So, we will conclude this module here.

Thank you very much.