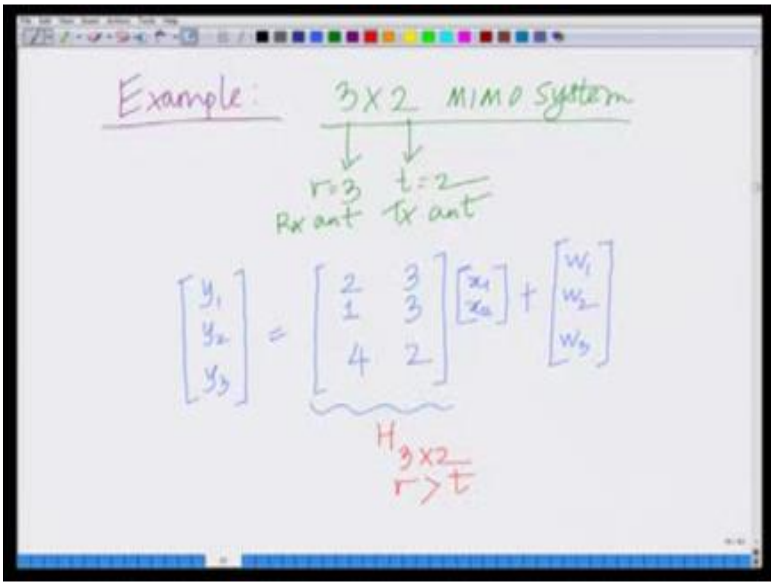
Principles of Modern CDMA/MIMO/OFDM Wireless Communications
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 37
BER Performance of ZF Receiver

Hello, welcome to another module in these massive open online courses in the Principles of CDMA MIMO OFDM wireless communication system. In the previous module we have seen the MIMO Zero Forcing receiver.

(Refer Slide Time: 00:29)



In this module let us look at simple examples to understand this concept of MIMO Zero Forcing Receiver. So let us look at simple example consider 3 x 2 MIMO system that is a MIMO system with r = 3 receive antennas, t = 2 transmit antennas. So we have r = 3 receive antennas, t = 2 transmit antennas, let us say this system of model is given as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

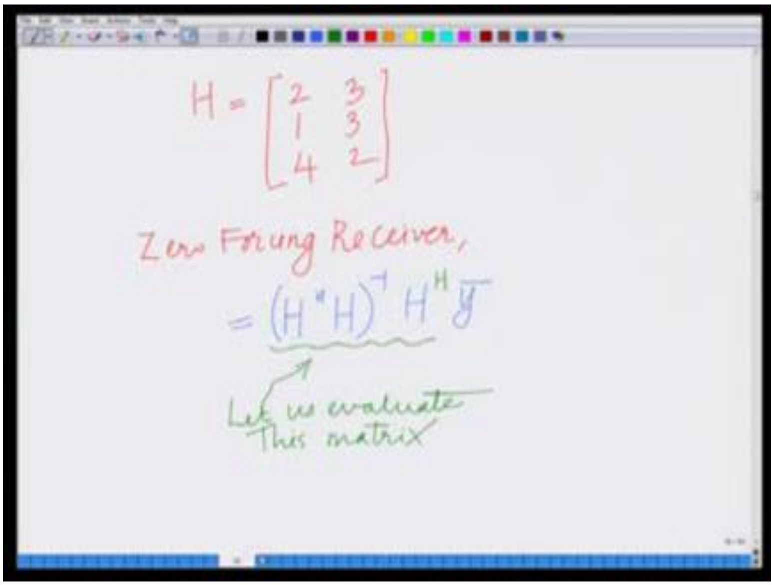
Since I have 3 receive antennas and 2 transmit antennas, so we can see that r is strictly greater than t.

So, therefore, H^{-1} that is there is nothing known as this a matrix which is number of rows strictly greater than the number of columns, therefore there is no H in this; matrix is not invertible, that is H inverse does not exist. However, we know how to construct the zero forcing receiver, the zero forcing receiver is given as, so for this system the Zero Forcing, we have H - the channel matrix H is equal to

$$H = \begin{bmatrix} 2 & 3 \\ 1 & 3 \\ 4 & 2 \end{bmatrix}$$

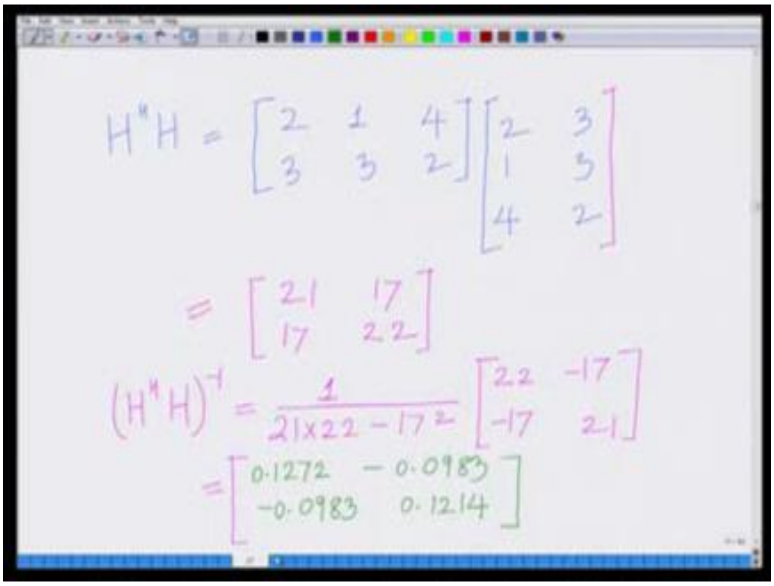
Therefore, our Zero Forcing receiver is equal to $(H^{H}H)^{-1}H^{H}\bar{y}$.

(Refer Slide Time: 02:25)



Let us now evaluate this matrix $(H^{H}H)^{-1}H^{H}$. So, let us now evaluate this matrix.

(Refer Slide Time: 03:37)

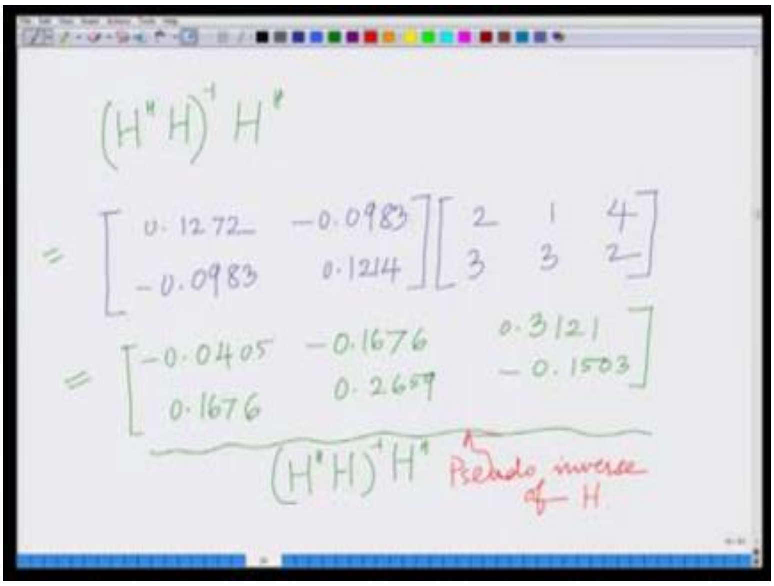


Now, let us first start with $\mathrm{H}^H H$ is basically transpose and complex conjugate of H that is

$$\mathrm{H}^H H = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 21 & 17 \\ 17 & 22 \end{bmatrix}$$

$$(\mathrm{H}^H H)^{-1} = \begin{bmatrix} 0.1272 & -0.0983 \\ -0.0983 & 0.1214 \end{bmatrix}$$

(Refer Slide Time: 05:20)



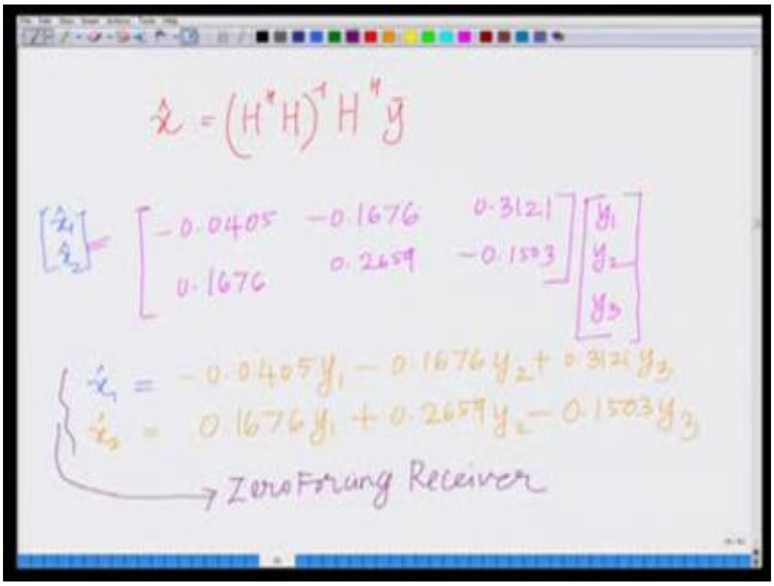
Therefore, now all I have to do is now, I have

$$(\mathrm{H}^H H)^{-1}\mathrm{H}^H = \begin{bmatrix} 0.1272 & -0.0983 \\ -0.0983 & 0.1214 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 3 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0405 & -0.1676 & 0.3121 \\ 0.1676 & 0.2659 & -0.1503 \end{bmatrix}$$

Therefore, this is basically my matrix $(\mathrm{H}^H H)^{-1}\mathrm{H}^H$ or this is also the pseudo inverse of H. So what we have calculated now is basically the matrix $(\mathrm{H}^H H)^{-1}\mathrm{H}^H$ which is also the pseudo inverse of this matrix H.

(Refer Slide Time: 07:04)



Now, therefore, now the Zero Forcing receiver is simple, the Zero Forcing receiver is

$$\hat{\mathrm{x}} = (\mathrm{H}^H H)^{-1}\mathrm{H}^H \bar{\mathrm{y}}$$

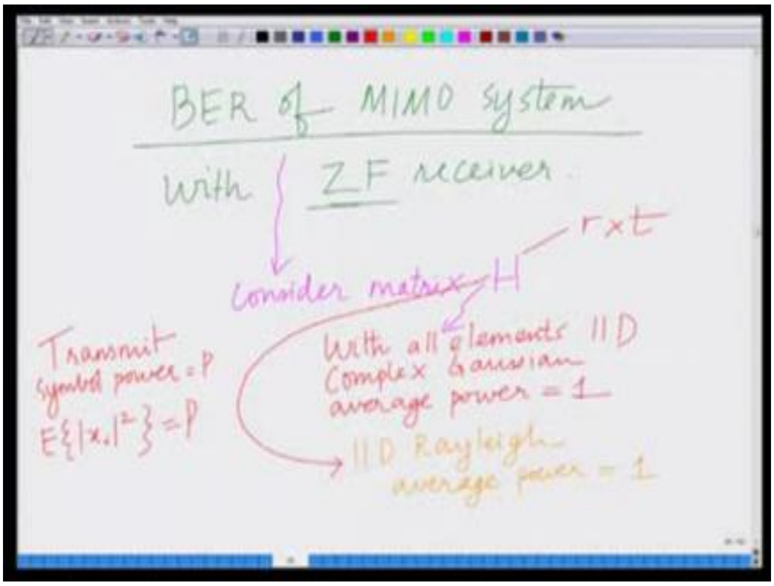
$$\begin{bmatrix} \hat{\mathrm{x}}1 \\ \hat{\mathrm{x}}2 \end{bmatrix} = \begin{bmatrix} -0.0405 & -0.1676 & 0.3121 \\ 0.1676 & 0.2659 & -0.1503 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

This is the Zero Forcing Receiver for this 3 cross 2 MIMO system, which we have illustrated by this simple example.

So, what have we done in this case we have consider a simple channel matrix H for a 3 x 2 system which means number of receive antennas is 3, number of transmit antennas is 2. So, H inverse does not exist. So, we have constructed the pseudo inverse that is $(\mathrm{H}^H H)^{-1}\mathrm{H}^H$ and from the pseudo inverse we have constructed the Zero Forcing Receiver for this MIMO system. So, this is simple examples for a MIMO Zero Forcing receiver.

Let us now look at another aspect, let us look at the bit error rate of this MIMO system, with the Zero Forcing Receiver because the bit error rate in the MIMO system depends upon the type of the receiver, right now we are considering the Zero Forcing receivers. So, let us look at the bit error rate of this MIMO system with Zero Forcing reception.

(Refer Slide Time: 10:19)

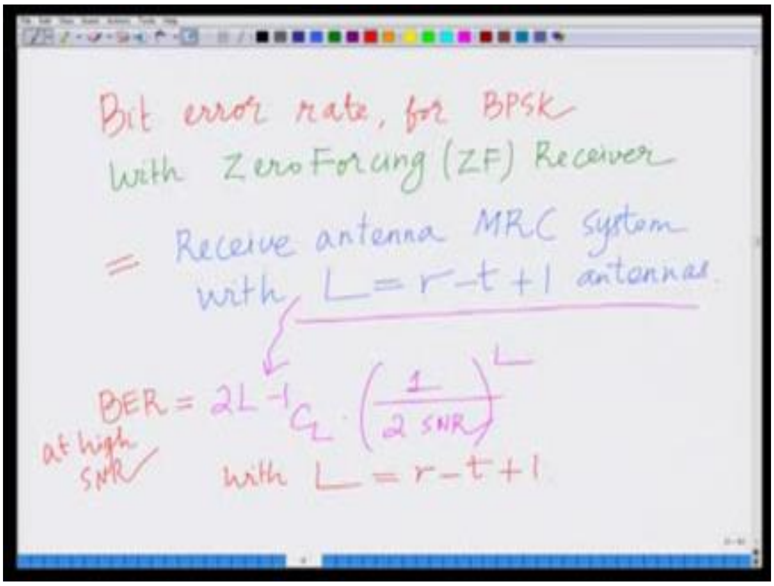


So, what we want to look at next is the Bit Error Rate of MIMO system, with the Zero Forcing receiver. Now if you look at this consider a channel matrix H with all elements IID Complex Gaussian with average power equal to 1.

So, we are considering a channel matrix H, basically in which all the elements that is all the channel co efficient h_{ij} are IID Complex Gaussian that is they have a complex that is a Gaussian real part, Gaussian imaginary parts with average power 1, which mean to say all the channel coefficient are Rayleigh fading with average power 1, these are IID Rayleigh fading coefficient with average power 1. So, we are considering an r x t channel matrix, so there r times t channel co efficient. Further we are considering this channel coefficient to be IID Rayleigh fading average power or average magnitude square of each Rayleigh fading coefficient is average power 1. Let the transmit symbol power P_t.

So, also consider Transmit symbol power equals P that is $E\{ |x_i|^2 \} = P$. That is the transmit power symbol is P alright, the channel coefficients are IID Rayleigh fading with average power, that is independent and identical Rayleigh fading channel coefficients with average power 1, average gain magnitude square is equal to 1 and the transmit power of each symbol in this transmit vector is P, then the bit error rate for BPSK modulated symbols.

(Refer Slide Time: 13:08)



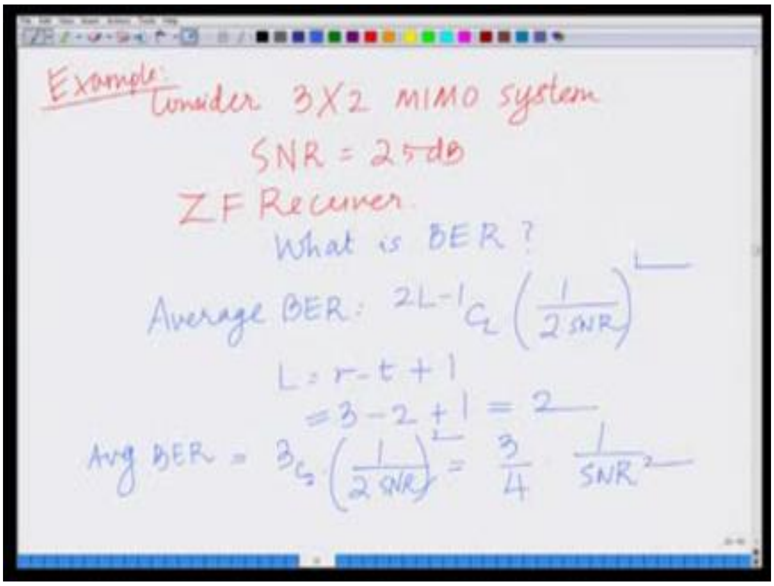
Bit error rate for BPSK with Zero Forcing receptions, that is considering the Zero Forcing or ZF receiver is basically equal to that of a receive antenna system or receive diversity system with maximal ratio combining that is receive antenna MRC system, which we have seen before with $L = r - t + 1$ antennas, and this is something that is very interesting, which one not going to prove rigorously here, but which were going to take on the face of it.

What we are saying is something very interesting, that is in the channel coefficients are IID for this MIMO channel matrix, that is $r \times t$ MIMO channel matrix and we are using the Zero Forcing receiver at there is ZF receiver, then the bit error rate when the symbols are BPSK modulated, is basically as if we have multiple receive antennas. How many receive antennas? That is $L = r - t + 1$ receives antennas and we are doing maximal ratio combining. We know that is the approximate bit error rate expression for approximate bit error rate, maximum ratio combining is

$$\text{BER} = {}^{2L-1}C_L \left(\frac{1}{2\ \text{SNR}}\right)^L$$

So, this is my expression for bit error rate at high SNR, with $L = r - t + 1$. So, what this is saying a something very interesting all we have to do is to look at the maximum ratio of combining system, set $L = r - t + 1$ in that formula, and basically that gives the bit error rate of this MIMO wireless communication system with Zero Forcing. To understand this better so let us look at example, let us look at example to understand this better.

(Refer Slide Time: 15:53)



So, what we want to do is consider a 3×2 MIMO system, with SNR = 25 dB and then we want to consider the Zero Forcing Receiver and question is what is the bit error rate of this system? We said average bit error rate equals

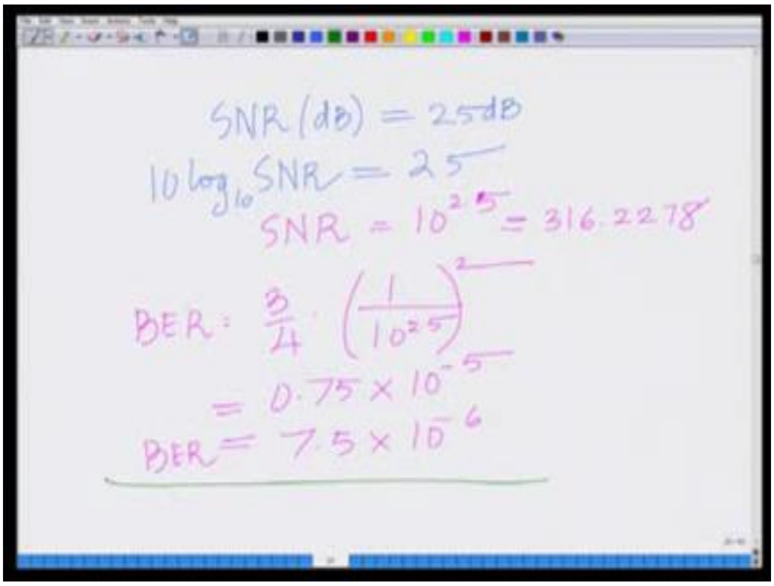
$$\text{BER} = {}^{2L-1}C_L \left(\frac{1}{2\ \text{SNR}}\right)^L$$

, where $L = r - t + 1$, in this case $r = 3, t = 2$. So, $L = 3 - 2 + 1 = 2$.

Therefore, the average bit error rate equals

$$BER = {}^{3}C_{2} \left(\frac{1}{2\,SNR}\right)^{2} = \frac{3}{4}.\ \frac{1}{SNR^{2}}$$

(Refer Slide Time: 17:23)



Now, what is SNR?

$$SNR_{dB} = 10 \log_{10} SNR = 25$$

$$SNR = 316.2278$$

$$BER = \frac{3}{4}.\ \frac{1}{316.2278^{2}} = 0.75 \times 10^{-5} = 7.5 \times 10^{-6}$$

Therefore this is the bit error rate of a MIMO wireless communication system; with that is a 3×2 MIMO wireless communication system, that is channel matrix is 3×2 SNR is 25 dB and basically what we have computing is the bit error rate with BPSK modulation and Zero Forcing receiver. Remember, the bit error rate depends of the kind of the receiver, for MIMO system there is many different kinds of receiver that are possible.

So, here we are calculating the bit error rate with the Zero Forcing receiver. So in this concept we have comprehensively looked an example of a Zero Forcing receiver in a MIMO system and also an example to how to calculate the bit error rate of MIMO system with Zero Forcing receiver, and a simple example to illustrate what is the bit error rate of this of a 3 x 2 MIMO system with Zero Forcing receiver. So, we will conclude this module here.

Thank you very much.