Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture – 32
Multipath Diversity in CDMA Systems

Hello, welcome to another module in this massive open online course in the Principles of CDMA, MIMO, OFDM Wireless Communication Systems, what we are going to look at in this module, how to extract multipath diversity in a CDMA system, that is we are concerned with new form of diversity that is, termed as multipath diversity in the CDMA system and this is possible through what is known as the Rake Receiver.

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This is the Rake Receiver, for our CDMA or code division multiple access system. So, to illustrate this concept of multipath diversity, let us consider frequency selective channel we are not consider, we are only consider flat fading channel. So, far let us now consider a frequency selective channel, let us see how a frequency selective channel can be modeled in a wireless communication system.

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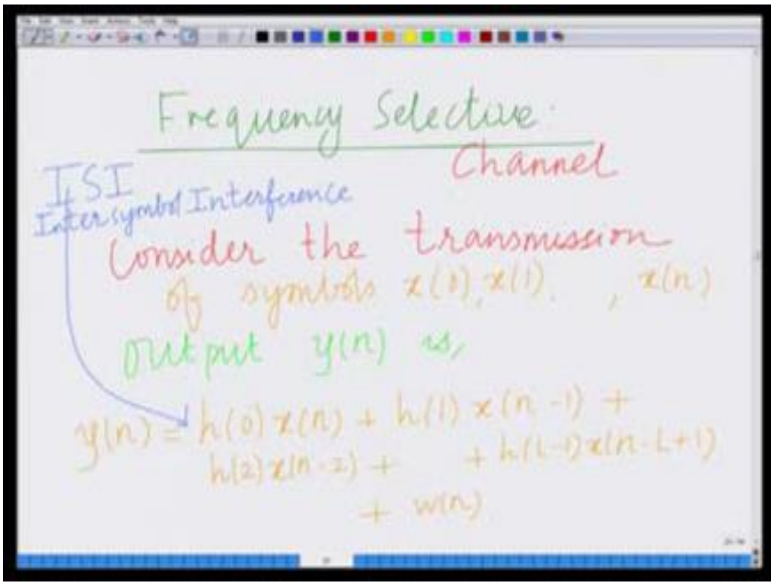

So, let us look at a frequency selective, we have our frequency selective channel. So, for this frequency selective channel consider the transmission of symbols $x(0)$, $x(1)$ up to $x(n)$ then the output $y(n)$ is given as,

$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + \ldots + h(L-1)\,x(n-L+1) + w(n)$$

So, we are seeing something very interesting, we are seeing that the output symbol $y(n)$ depends not only on the current input symbol $x(n)$, but it also depends on the past input symbol namely $x(n-1)$, $x(n-2)$ so on until $x(n-L+1)$. Therefore, this is precisely inter symbol interferes, this system model you can clearly see, in this system model we have ISI or Inter Symbol Interference.

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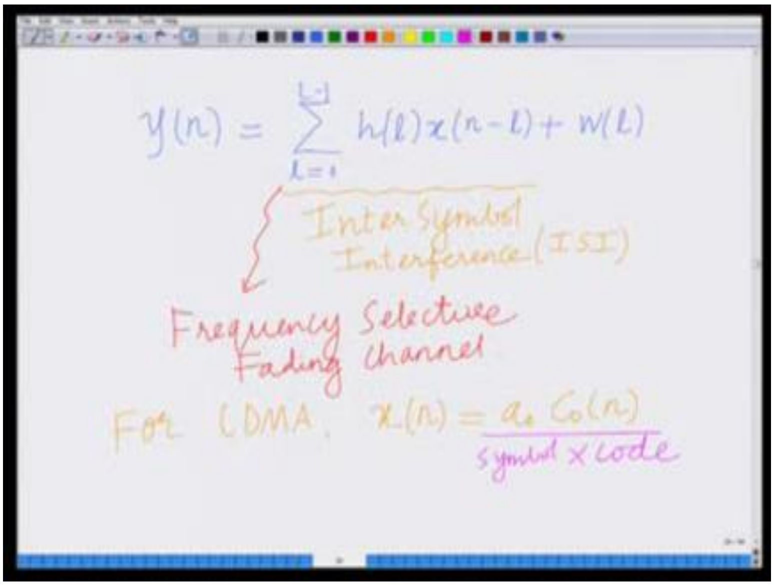

This can be subsequently represented in the summation form as

$$y(n) = \sum_{l=0}^{L-1} h(l)\, x(n-l) + w(l)$$

This represents Inter Symbol Interference or basically, ISI right. So, this represents Inter Symbol Interference correct, because the output $y(n)$ depends not only on $x(n)$, but also on the past symbols. So, the past symbols $x(n-1), x(n-2)$ so on, are interfering with the current symbol that is $x(n)$.

Therefore, this represents Inter Symbol Interference and previously you might remember that is a Inter Symbol Interference is basically, arises from a frequency selective channel, we have said that; inter symbol interference in the time domain is the same thing as frequency selective fading, when looked at in the frequency domain. So, this Inter Symbol Interference channel represents nothing, but a frequency selective fading channel. So, therefore, this ISI channel is basically the model for a frequency selective fading channel since, frequency selective fading in the frequency domain right, where the bandwidth of the signal remember is greater than the coherence bandwidth of the channel this leads to frequency selective fading in the frequency domain which basically, implies inter symbol interference in the time domain. So, therefore, this is a model for frequency selective fading.

Now, let us look at a CDMA transmission or the corresponding symbols that are transmitted in a CDMA system, we know that in a CDMA system, the transmitted symbol x(n) is given as the symbol a_0 multiplied by the code $c_0(n)$. So, for a CDMA system we have

x(n) = $a_0\, c_0(n)$

that is, the product of the symbol times the codes sequence.

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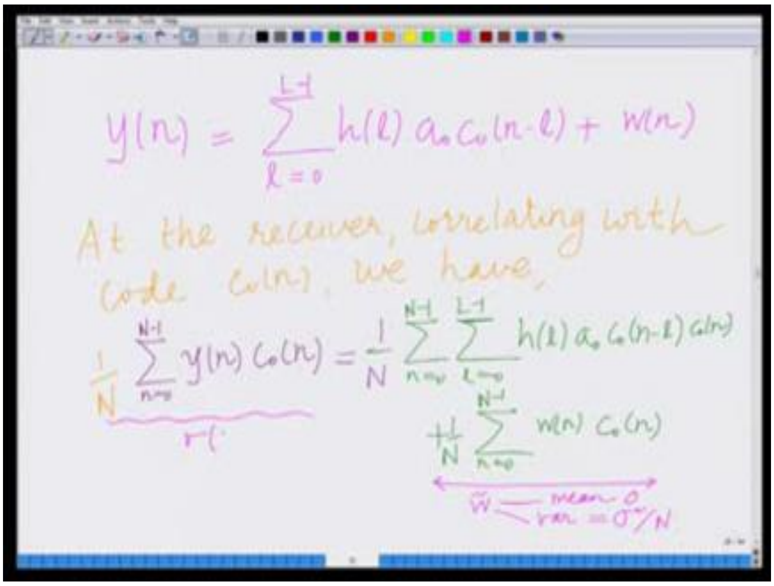

Now let us, substitute this x(n) in the frequency selective system model above and we will receive

y(n) = $\sum_{l=0}^{L-1}$ h(l) $a_0\, c_0(n-l)$ + w(n)

Now, at the receiver we correlate y(n). So, this is the y(n) that is received in this CDMA system across the frequency selective channel at the receiver, we will correlate this y(n) with the code sequence $c_0(n)$ because, at the receiver in CDMA system with we correlate with the spreading sequence of the chip sequence of the particular user therefore, at the receiver we perform the correlation operation, at the receiver correlating with code $c_0(n)$, we have

$\frac{1}{N}\sum_{n=0}^{N-1} y(n)c_0(n) = \frac{1}{N}\sum_{n=0}^{N-1} \sum_{l=0}^{L-1} h(l)\, a_0\, c_0(n-l)\, c_0(n) \quad +$

$$\frac{1}{N}\sum_{n=0}^{N-1} w(n)c_0(n)$$

And we have already seen this,

$$\frac{1}{N}\sum_{n=0}^{N-1} w(n)c_0(n) = \tilde{w}$$

which is Gaussian with mean 0 and variance equal to $\frac{\sigma^2}{N}$.

Now, let us rewrite the signal part. So, that we gain some insides in to this, the signal part can be written as, so now, if I denote this quantity by $r(0)$.

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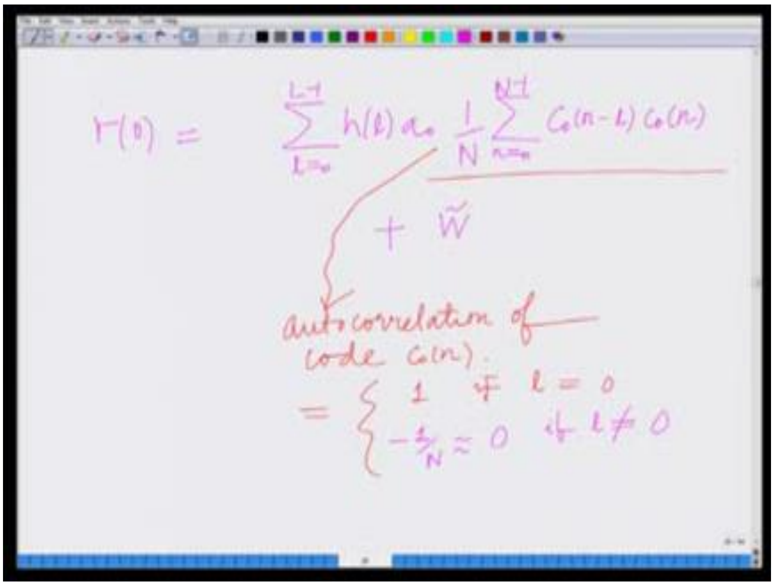

Now, if you look at this correlation,

$$r(0) = \sum_{l=0}^{L-1} h(l)\, a_0\, \frac{1}{N} \sum_{n=0}^{N-1} c_0(n-l)\, c_0(n) + \tilde{w}$$

Now, this is nothing, but the correlation the shifted correlation this is nothing, but the auto correlation of code, this is the auto correlation of the code $c_0(n)$ and we have already seen the auto correlation properties of the p n sequences this is equal to this auto correlation is equal to 1 if l = 0 and this is equal to $-\frac{1}{N}$ which is approximately equal to 0 if, l is not equal to 0. So, I can ignore this correlation, if l is not equal to 0 which means, I can only retain the term corresponding to l is equal to 0.

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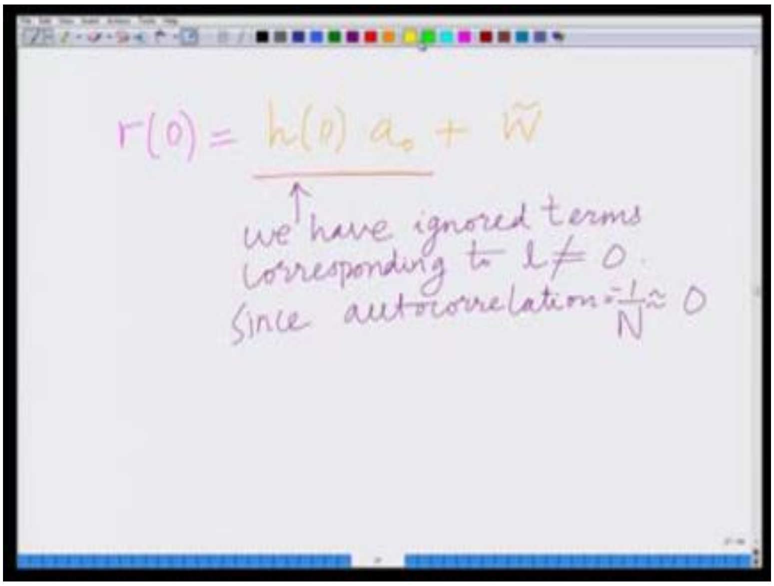

In this summation to basically derives, the expression for $r(0)$ as,

$$r(0) = h(0)\, a_0 + \tilde{w}$$

where we have ignore in this signal component, we have ignored terms corresponding to l not equal to 0 since, auto correlation equals $-\frac{1}{N}$ which is approximately equal to 0. Since, we ignored this terms which correspond to l not equal to 0 therefore, $r(0)$ that is correlation with $c_0(n)$ can be expressed as $h(0)\, a_0 + \tilde{w}$ where $\tilde{w}$ again the Gaussian noise with means 0 variance $\frac{\sigma^2}{N}$.

Now, let us look at something slightly different, let us look at correlation with the delayed version chip sequence let us look at correlation of $y(n)c_0(n-l)$.

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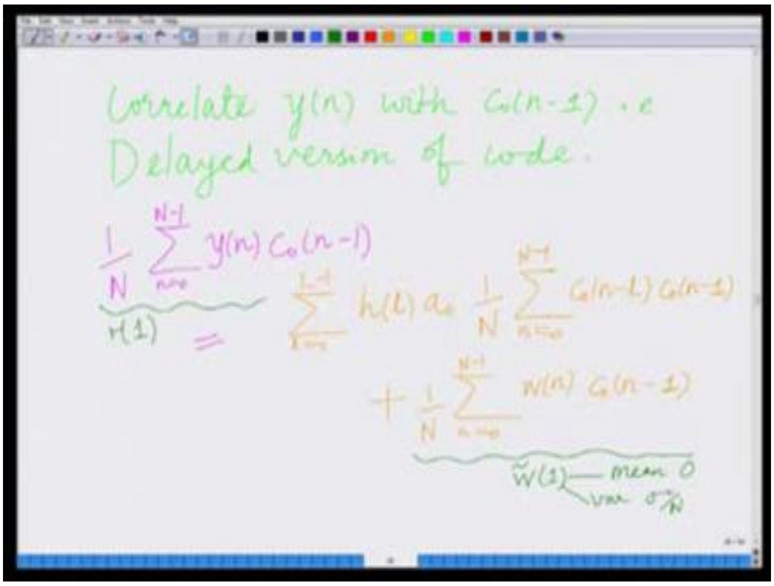

So, let us look at the correlation

$$\frac{1}{\mathrm{N}}\sum_{n=0}^{N-1} y(n)c_0(n-1) = \sum_{l=0}^{L-1} \mathrm{h(l)}\, a_0\, \frac{1}{\mathrm{N}}\sum_{n=0}^{N-1} c_0(n-l)\, c_0(n-1) \;+ \frac{1}{\mathrm{N}}\sum_{n=0}^{N-1} w(n)c_0(n-1)$$

Now, let me call this as $r(1)$, this correlation with $c_0(n-1)$ as $r(1)$, now again this noise which is $\frac{1}{\mathrm{N}}\sum_{n=0}^{N-1} w(n)c_0(n-1)$ I can denote this by $\tilde{\mathrm{W}}(1)$ which will be again Gaussian noise with mean 0 variance $\frac{\sigma^2}{N}$. Now if you look at this inner summation that is, this is the auto correlation of the shifted sequence $c_0(n-1)$ right. Therefore, this will be equal to 1 if the shift between these two sequences is 0 that is l is equal to 1 and this is equal to $-\frac{1}{\mathrm{N}}$ if, l is not equal to 1 or approximately 0, if l is not equal to n therefore, if we examine this inner auto correlation.

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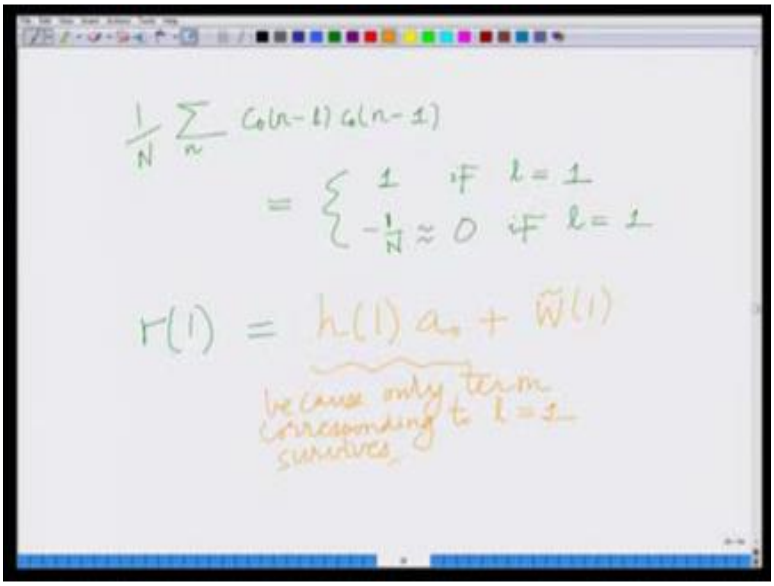

The inner auto correlation is

$$\frac{1}{N}\sum_n c_0(n-l)c_0(n-1) = 1 \qquad \text{if } l=1$$

$$= -\frac{1}{N} \sim 0 \qquad \text{if } l \neq 1$$

therefore, when you look at this inner summation only the term corresponding to l equal to 1 will survive and therefore, again I have $r(1)$ is interestingly that is

$$r(1) = h(l)\, a_0 + \tilde{w}(1)$$

because, only term corresponding to l equal to 1 only the term corresponding to l equal to 1 will survive after this correlation with the shifted sequence $c_0(n-1)$. Similarly, 1 can correlate with $c_0(n-2)$ that is shifted by 2 chips, so on and so for $c_0(n-l+1)$.

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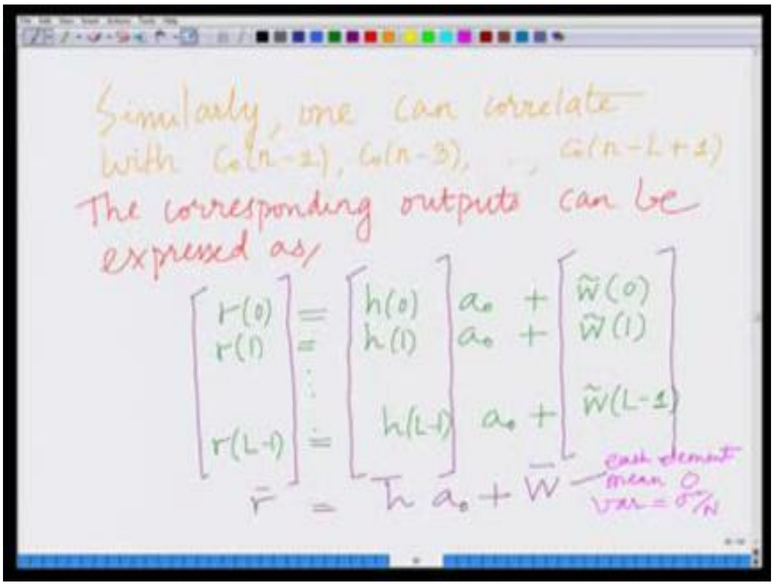

Similarly, 1 can correlate with $c_0(n-2)$ so on, $c_0(n-l+1)$. the corresponding outputs can be expressed as, after correlation with these various shifted versions of the code sequence c_0 the corresponding outputs can be expressed as well, we have

$$\begin{bmatrix} r(0) \\ r(1) \\ r(L-1) \end{bmatrix} = \begin{bmatrix} h(0)a_0 \\ h(1)a_0 \\ h(L-1)a_0 \end{bmatrix} \mathrm{x} + \begin{bmatrix} \tilde{\mathrm{W}}(0) \\ \tilde{\mathrm{W}}(1) \\ \tilde{\mathrm{W}}(\mathrm{L}-1) \end{bmatrix}$$

$$\bar{\mathrm{r}} = \bar{\mathrm{h}} a_0 + \bar{\mathrm{w}}$$

where each element of $\bar{\mathrm{w}}$ is Gaussian noise with mean 0 and variance $\frac{\sigma^2}{N}$.

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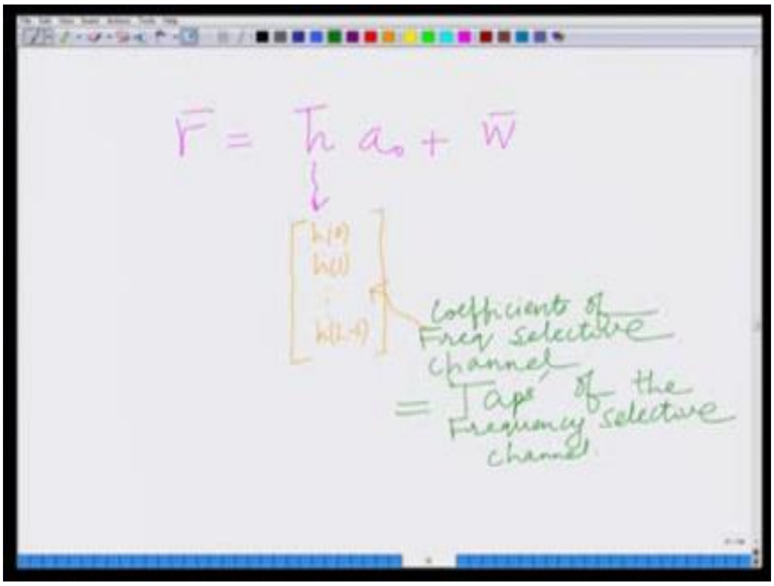

Therefore, I have my model is equal to $\bar{r} = \bar{h} a_0 + \bar{w}$

, and if you look at this model, this model is very similar. In fact, exactly identical to the model of the multiple antenna wireless communication system where the different channel coefficients across the antenna are $h(0),$ $h(1)$ up to $h(L-1)$. We had an exactly similar model that is $\bar{r} = \bar{h} a_0 + \bar{w}$ here, we have $\bar{r}$ that is these different statistics at the output of the correlate, what are the statistics. These statistics correspond to correlation with different shifted versions of code sequence $c_0(n)$.

So, $r(0)$ corresponds to correlation with $c_0(n)$ so on $r(L-1)$ corresponds to correlation with $c_0(n-L+1)$, which is equal to $\bar{h}$ which is the vector of channel coefficient, $\bar{h}$ the vector contains the coefficients $h(0),$ $h(1)$ up to $h(L-1)$. These are the coefficients of the frequency selective channel these are also known as taps of the frequency selective channel and the noise vector w bar is at each element is Gaussian with means 0 and variance $\frac{\sigma^2}{N}$. And now, therefore, the optimal processing is once again similar to that of the multiple antenna wireless communication system the optimal processing is through the maximal ratio combiner.

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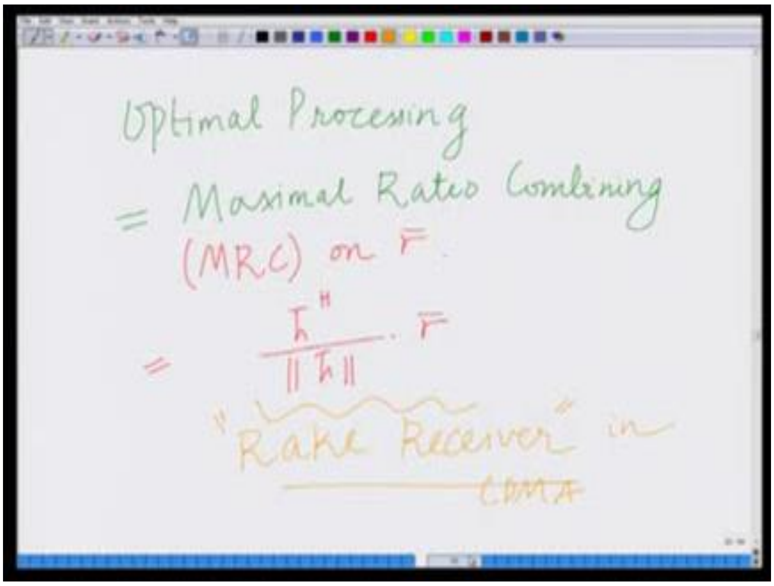

So, the optimal processing, now, is to use maximal ratio combiner on $\bar{r}$ which is basically,

$= \frac{\bar{h}^H}{||\bar{h}||} \cdot \bar{r}$ in the context of CDMA this is precisely what is known as the rake receiver. That is performing, this maximal ratio combining, operation or coherently combining this statistics $r(0)$, $r(1)$ up to $r(L-1)$. This process is known as Rake Reception or this is known as Rake Receiver in CDMA system.

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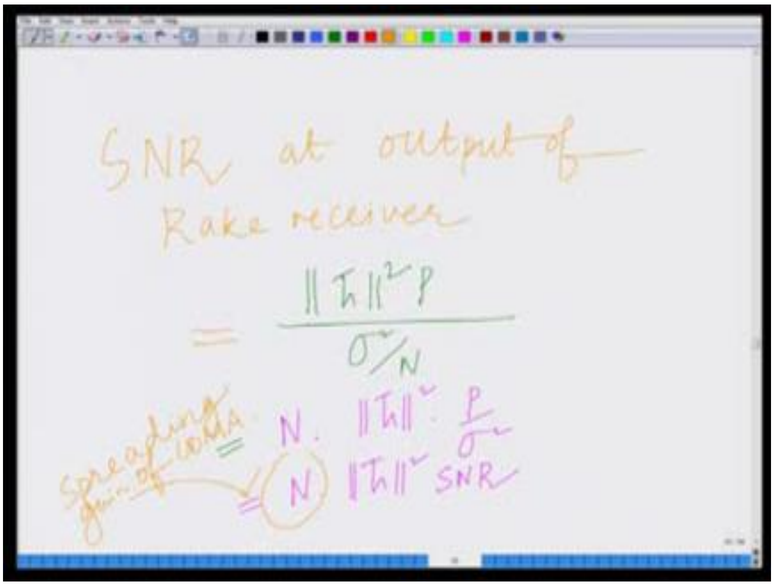

And therefore, the SNR at the output of the Rake Receiver, we all ready seen the SNR at the output maximal ratio combiner SNR at output of Rake Receiver equals well, this is equal to

$$= \frac{\|\bar{h}\|^2 P}{\frac{\sigma^2}{N}}.$$

$$= N. \|\bar{h}\|^2 \frac{P}{\sigma^2}$$

$$= N. \|\bar{h}\|^2 \text{ SNR}$$

Therefore the output SNR at the output of the Rake Receiver, $N. \|\bar{h}\|^2 \text{ SNR}$ which is the transmitters that we consider, and this N as we know is the spreading gain that is the associated with the CDMA system, this is something that we all ready seen this factor of N this is the spreading gain of the CDMA system.

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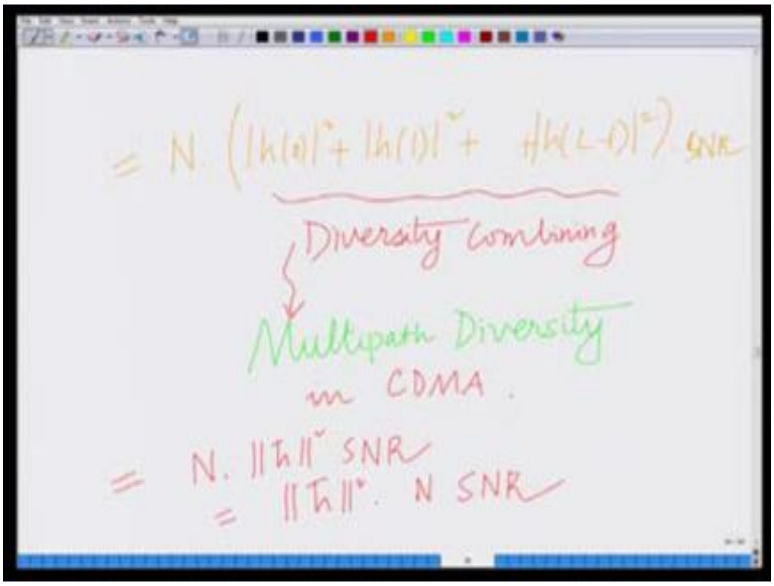

This can also be this can also look at this norm, this is also equal to

= $N. \left(|h_1|^2 + |h_2|^2 + \cdots + |h_L|^2 \right) SNR$

and you can also see this is basically, the diversity combining across the L branches and this is what leads to your multipath, this is diversity combining and this is a new form of diversity in a CDMA this is known as multipath diversity, in the CDMA system, that is $||\bar{h}||^2$ which is $|h_1|^2 + |h_2|^2 + \cdots + |h_L|^2$ which is similar to that of multiple antenna system is what leads to the diversity in CDMA code division for multiple access system and this is also termed as multipath diversity in the context of a CDMA system.

And now, you can also see that this expression is basically $N. ||\bar{h}||^2 \ SNR$, which is N times which if you can write the these,

we can also write. $||\bar{h}||^2 \ N \ . \ SNR$ which is basically replacing the SNR in a multiple antenna wireless system by N times SNR therefore, the bit error rate performance can be derived by substituting N times SNR in the expression.

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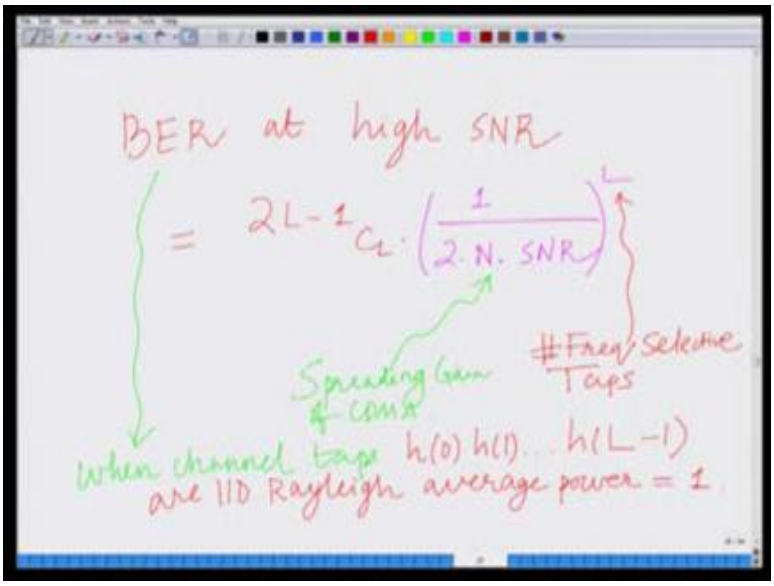

We have for the multiple antenna wireless system therefore, the bit error rate remember at high SNR of at L antenna wireless communication system, bit error rate remember at high SNR of n L antenna wireless communication system, bit error rate at high SNR is equal to

$${}^{2L-1}C_L \left(\frac{1}{2.\text{N}.\ \text{SNR}}\right)^L$$

where L is the number of frequency selective taps of the channel, and this N this arises from the spreading gain this arises from the spreading gain of the CDMA system this is the expression for the bit error rate and of course, for this expression to hold we must be need that the channel taps $h(0)$, $h(1)$ up to $h(L-1)$ are IID that is, independent identically distributed Rayleigh with average power 1.

So, for this is a bit error rate at high SNR when my channel taps, when the channel taps $h(0)$, $h(1)$ up to $h(L-1)$ with these are IID Rayleigh with average power, is equal to 1. So, this is the expression for bit error rate that is ${}^{2L-1}C_L \left(\frac{1}{2.\text{N}.\ \text{SNR}}\right)^L$, this is a expression for the bit error rate of a CDMA system with recombining and with channel taps distributed as IID Rayleigh random variables IID Rayleigh channel coefficients of average power of unity or average power 1, and you can see therefore, is bit error rate is a proportional to $\left(\frac{1}{2.N.\ SNR}\right)^{L}$ the diversity order of this system is L and this system therefore, is able to extract multipath diversity therefore, this implies that this also implies that diversity order is equal to l.

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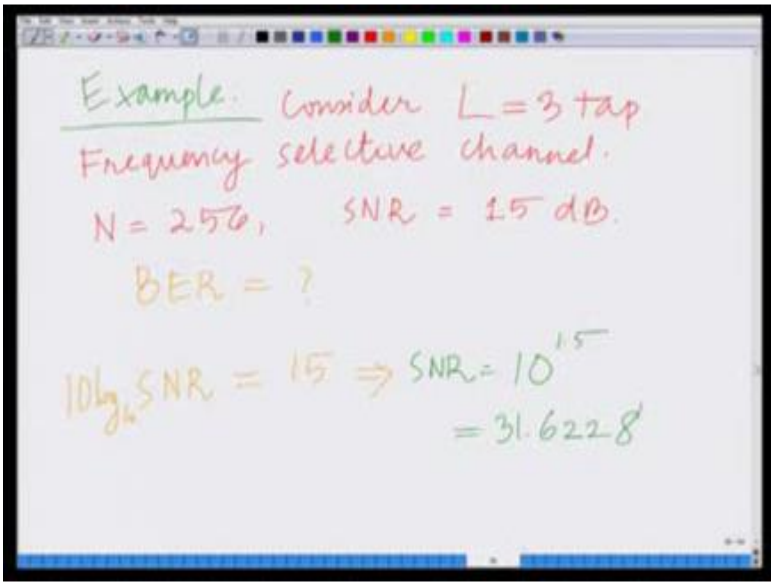

So, to understand this better, let us do a simple example to understand this better let us do simple example in this context of the CDMA consider an $L = 3$ tap frequency selective channel in CDMA system, the code length $N = 256$ and the $SNR = 15$ dB and the question we want ask is, what is the bit error rate of the CDMA system that is we considering and $L = 3$ tap frequency selective channel code length $N = 256$, $SNR = 15$ dB. What is the bit error rate, first let us look at the SNR,

$$SNR_{dB} = 10 \log_{10} SNR = 15$$

$$SNR = 10^{1.5} = 31.6228$$

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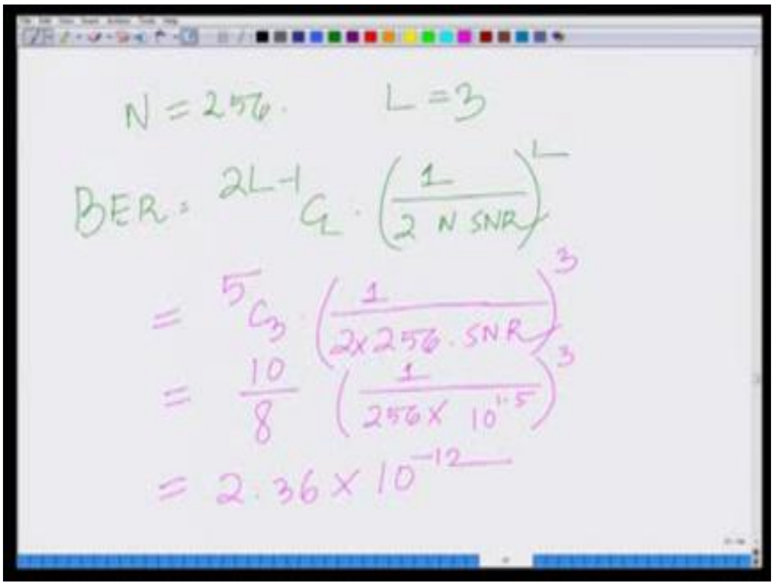

Further given the code length N = 256 also we have

$$\text{BER} = {}^5C_3 \left(\frac{1}{2.\ 256.\ SNR}\right)^3 = 2.36 \times 10^{-12}$$

So, this is a bit error rate of the CDMA system in L equal to multipath channel with code length N = 256 which is 2.36×10^{-12}.

So, what we seen in this module is a new concept of multipath diversity in a CDMA system, you seen how to model frequency selective channel how do we correlate with various shifts of the code sequences at the receiver how do we coherently combining them using the maximal ratio combiner or basically what is also known as Rake Receiver, in CDMA system how this is able extract multipath diversity in the CDMA system and also the associated bit error rate performance of the CDMA system. With this Rake Reception we were able to derive the expression and do simple example to illustrate we will end this module here.

Thank you very much.