**Principles of Modern CDMA/MIMO/OFDM Wireless Communications**
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**Lecture – 31**
**Analysis of Multi-user CDMA**

Hello, welcome to another module in this massive open online course on the principles of CDMA, MIMO and OFDM wireless communication systems and previously the previous module we have looked at the Bit Error Rate performance of CDMA system. Today let us look at the performance of a Multi-user CDMA wireless communication systems, so what we are trying to look at today is a Multi-user CDMA scenario.

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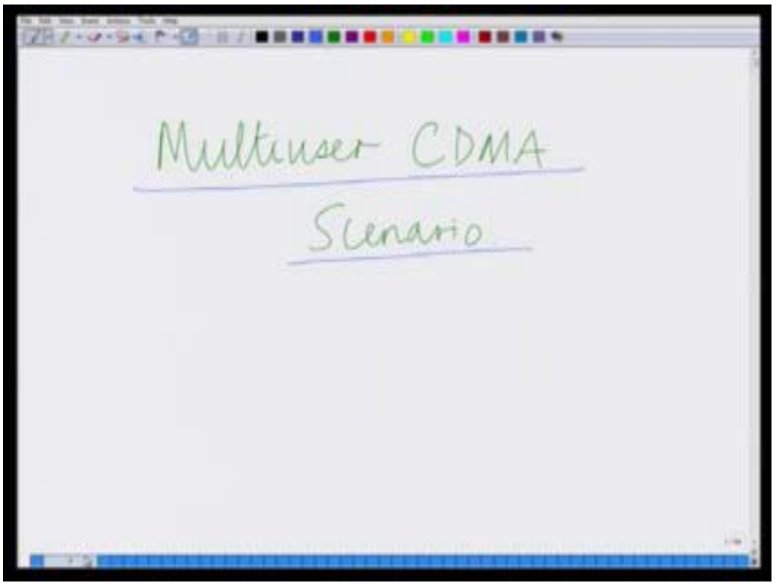


So, let us look at a Multi-user CDMA scenario, so for a simple illustration of this consider two users, two CDMA users. So, we have basically what we have is; we have two users, we are going to denote by user 0 who has symbol $a_0$ and code $c_0(n)$.

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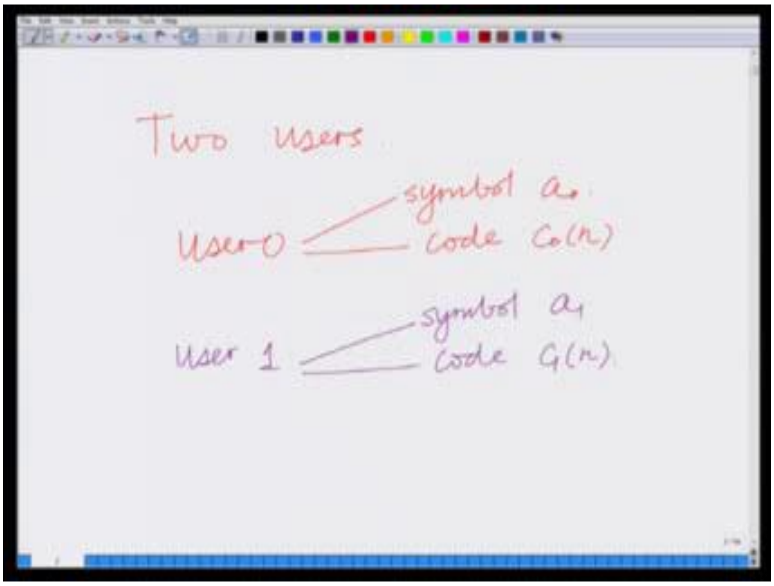


Then we have user 1, who has symbol a 1 and code c 1 of n, so we have two user CDMA scenario, we are starting with the simple two user CDMA scenario. We are considering user 0 who has symbol $a_0$ and code $c_0(n)$ and user 1, who has symbol $a_1$ and code $c_1(n)$.

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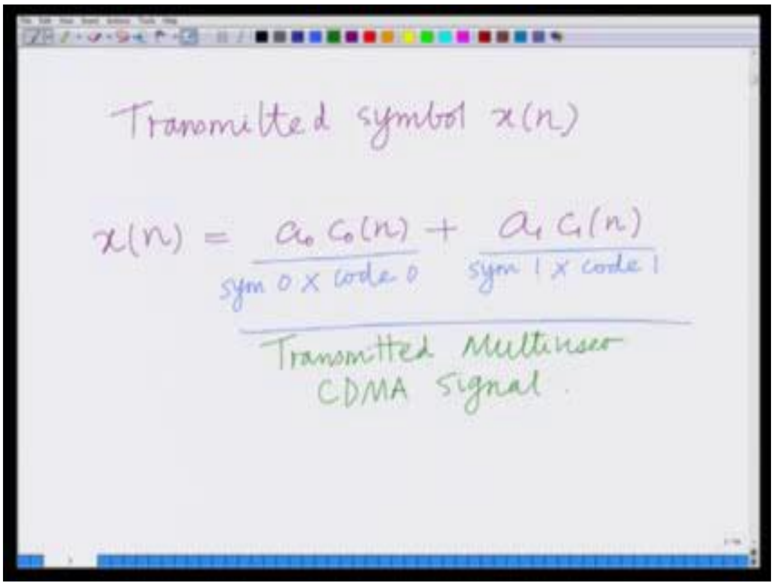


Therefore the Transmitted symbol x(n) in this scenario, the Transmitted symbol x(n) is given as

$x(n) = a_0\ c_0(n) + a_1\ c_1(n)$

The sum of these two components to generate the multiuser signals. So, this is the Transmitted Multi-user CDMA signal.

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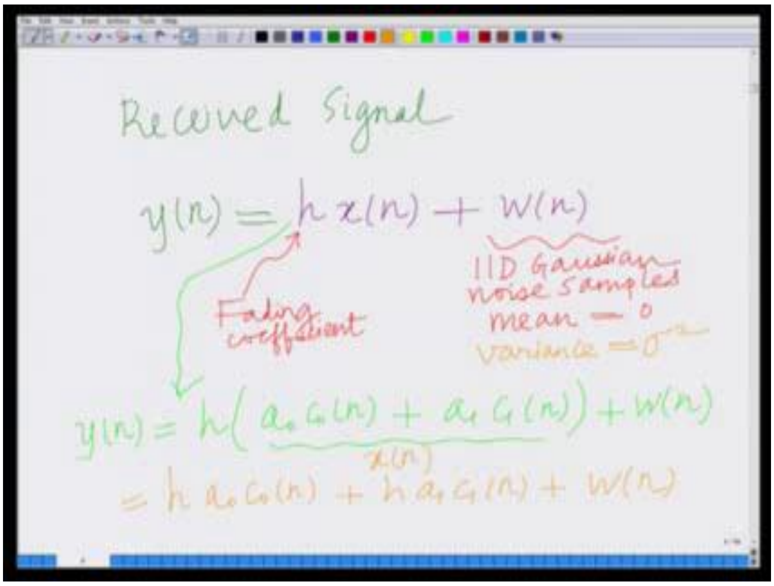


The Received Signal as we already know, the Received Signal

$y(n) = h\ .\ x(n) + w(n)$

where h is the fading coefficient and w (n) is IID Gaussian noise samples with mean equal to 0 and variance equal to $\sigma^2$.

So, we are saying the received system model $y(n) = h\ .\ x(n) + w(n)$ where h is the fading channel coefficient the Rayleigh fading channel coefficient and w (n) are IID Gaussian noise samples with 0 mean and variance $\sigma^2$, this is the same system module that we have seen before in the previous module.

But we are now extending it to a multiuser scenario and now I am going to transmit or going to incorporate the multiuser transmitted signal and I have

$y(n) = h\ .\{\ a_0\ c_0(n) + a_1\ c_1(n)\} + w(n)$

$y(n) = h.\{ a_0\, c_0(n) \} + h.\{ a_1\, c_1(n)\} + w(n)$

Now, what we are going to do is we are going to correlate with the code $c_0(n)$ of user 0 to extract the symbol $a_0$ of user 0; this is similar to what we have been doing in along in a CDMA system.

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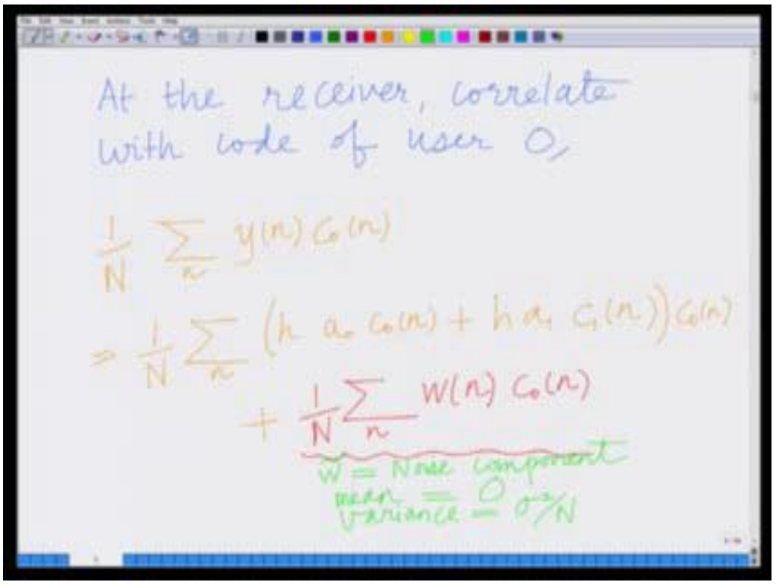


We said that it is CDMA receiver, we correlate; we perform the correlation operation with the code of the particular user to extract the symbol belonging to that particular user therefore, we are going to perform the correlation with the code 0.

So at the receiver, we correlate with code of user 0 and that gives us

$$\frac{1}{N}\sum_n y(n)c_0(n) = \frac{1}{N}\sum_n\{ h.a_0\, c_0(n) + h.a_1\, c_1(n)\}\, c_0(n) + \frac{1}{N}\sum_n w(n)c_0(n)$$

which is the noise component, we have already seen this in the previous module; we defined this as $\tilde{W}$; $\tilde{W}$ is Gaussian noise with 0 mean and variance $\frac{\sigma^2}{N}$, this we have seen in the previous module.

So, $\tilde{W}$ so we have our this is $\tilde{W}$ which is the noise component, which is Gaussian since it is a linear combination of Gaussian noise samples the mean of this noise is equal to 0 and the variance, the interesting aspect of this is the variance is equal to $\frac{\sigma^2}{N}$ that is the variance of the noise is suppressed by a factor of N.

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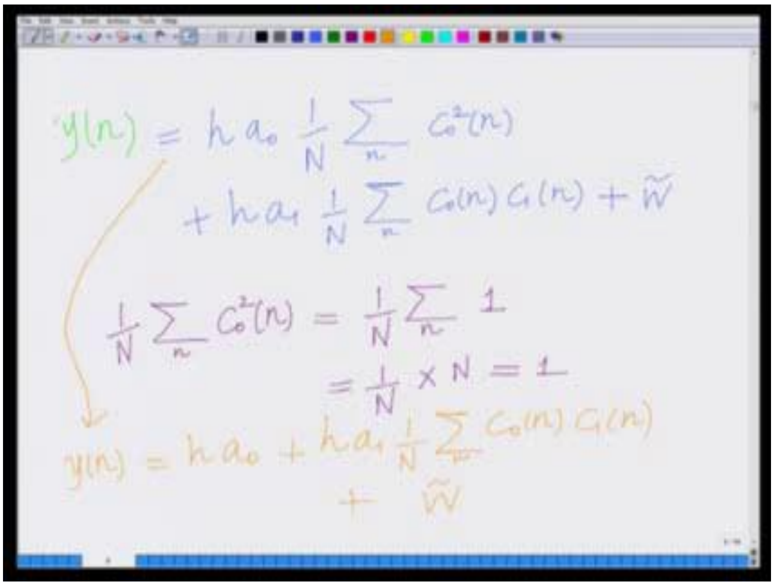

Therefore now simplifying this further, I have

$$y(n) = h \, . \, a_0 \frac{1}{N} \sum_{i=0}^{N-1} c_0^2(n) \; + h \, . \, a_1 \frac{1}{N} \sum_{i=0}^{N-1} c_0(n) \, c_1(n) \; + \tilde{W}$$

$$\frac{1}{N} \sum_{i=0}^{N-1} c_0^2(n) \; = \frac{1}{N} \sum_n 1 \; = N . \frac{1}{N} \; = 1$$

Therefore, this expression can be further simplified as

$$y(n) = h \, . \, a_0 + h \, . \, a_1 \frac{1}{N} \sum_{i=0}^{N-1} c_0(n) \, c_1(n) \; + \tilde{W}$$

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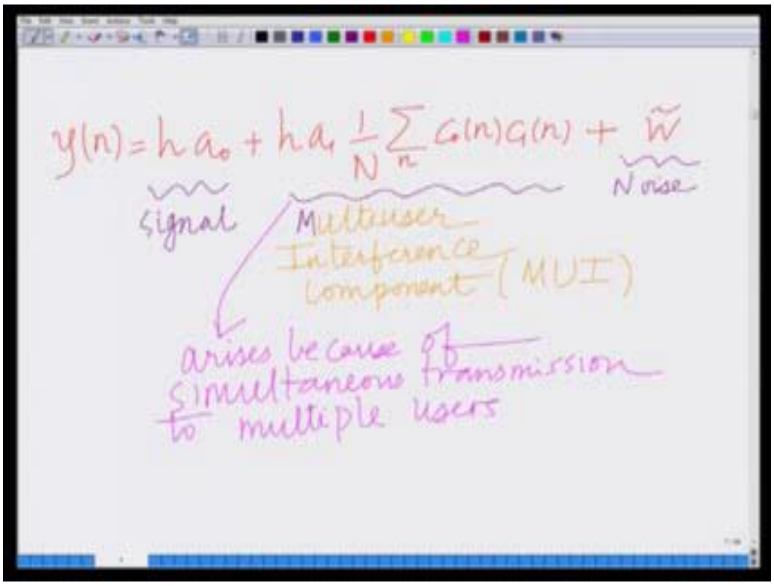


So we have the final expression we write this again, we have

$$y(n) = h \cdot a_0 + h \cdot a_1 \frac{1}{N} \sum_n c_0(n)\, c_1(n) + \tilde{W}$$

Out of these three components we have seen two already this $h.a_0$ is the signal component, the last one $\tilde{W}$ is the noise component and this component in between is the Multiuser Interference component, this is the new component which arises in a CDMA system because of the simultaneous transmission to multiple users, this is a Multiuser Interference Component. So, this is the new component in a CDMA system, this is the Multiuser Interference Component and this arises we are going to denote this by MUI, which is for multi user interference and this arises in a CDMA system arises because of simultaneous transmission.

The Multiuser Interference component in the CDMA system arises because of the simultaneous transmission to multiple users. Now we would like to characterize the impact of this Multiuser Interference in terms of the power or in terms of the noise power or the interference power of this Multiuser Interference component and let us look at that now. Now for this, let us first look at this correlation component that is $\frac{1}{N} \sum_{i=0}^{N-1} c_0(n)\, c_1(n)$.

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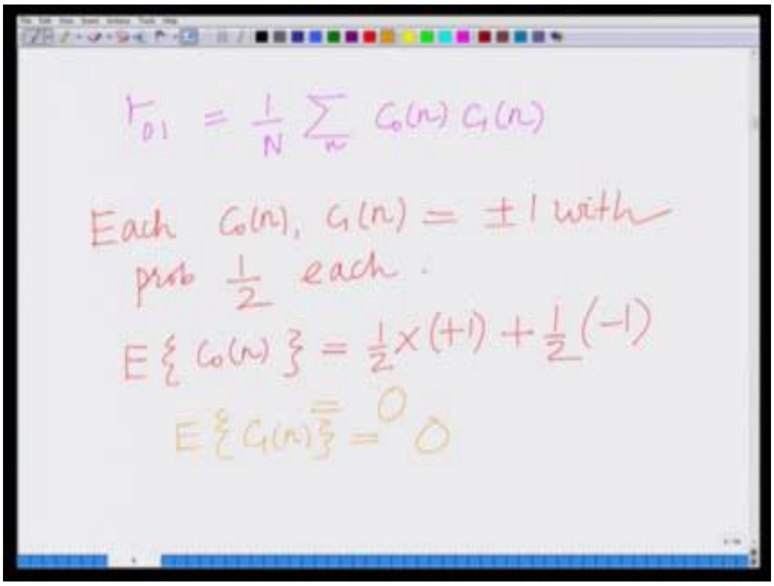


Let us define that as $r_{01}$,

$$r_{01} = \frac{1}{N} \sum_n c_0(n)\, c_1(n)$$

Now we know that each $c_0(n),\ c_1(n)$ equals $\pm 1$ with because this is a random sequence, this is a pseudo noise sequence of $\pm 1$, each $c_0(n),\ c_1(n)$ equals plus or minus 1 with probability half each because these are P N sequences each $c_0(n)\ and\ c_1(n)$, each of the chips is either equal to $\pm 1$ with probability half each. Therefore, now we want to compute the Expected value of these chips,

$$E\{\ c_0(n)\ \} = \frac{1}{2} x(1) + \frac{1}{2} x(-1) = 0$$

$$E\{\ c_1(n)\ \} = 0$$

further these two different chip sequences of user 0 and user 1 can be assumed to be independent.

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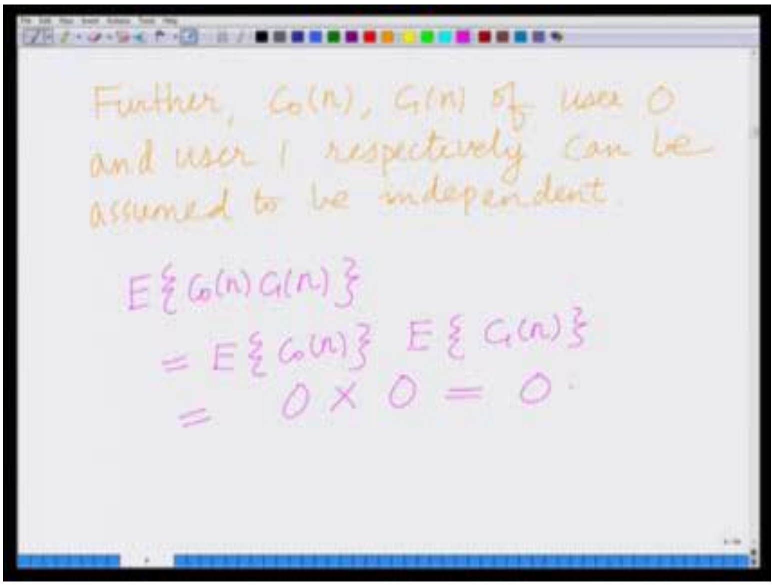


Further $c_0(n)$ and $c_1(n)$ of user 0 and user 1 respectively can be assumed to be independent. Therefore, what we have is we have since they are independent we have

$$E\{ c_0(n)\, c_1(n) \} = E\{ c_0(n)\}\, E\{ c_1(n) \} = 0 \times 0 = 0$$

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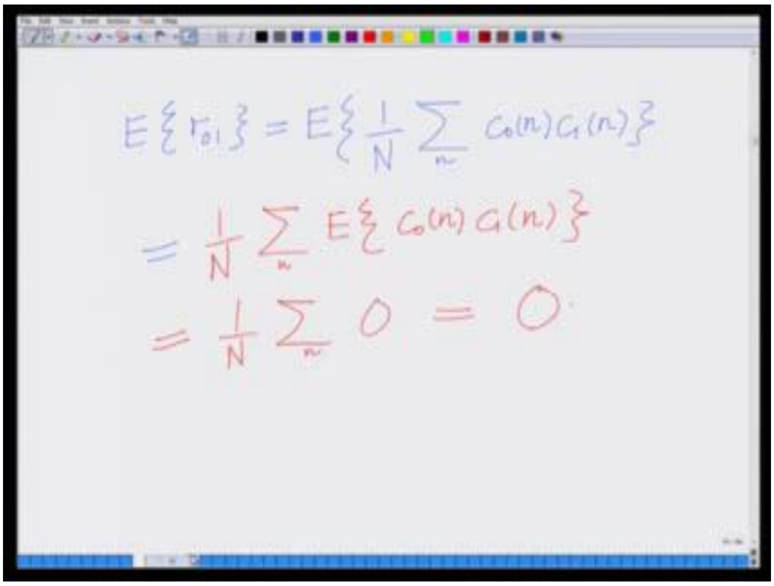

Therefore we have the Expected value of the product is equal to 0 since the chip sequences $c_0(n)$ and $c_1(n)$ belonging to user 0 and user 1 respectively can be assumed to be independent. Therefore, we have

$$E\{ r_{01} \} = E\{ \frac{1}{N}\sum_n c_0(n)\, c_1(n) \} = 0$$

So, what we have seen is that $E\{ r_{01} \}$ which can be thought of as the cross correlation between the chip sequence $c_0(n)$ and $c_1(n)$ belonging to user 0 and 1 respectively, the cross correlation the Expected value of this cross correlation is 0.

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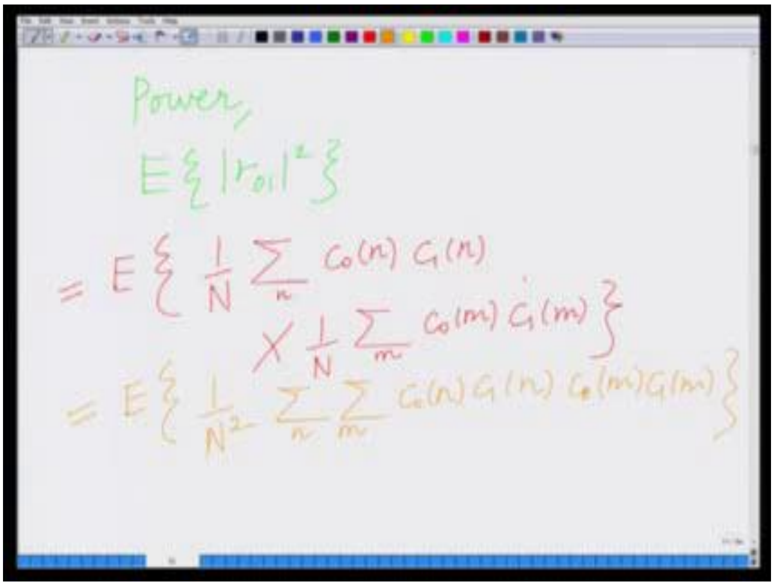


Now let us look at the power of this cross correlation, let us look at

$$E\{ |r_{01}|^2 \} = E\{ \frac{1}{N}\sum_n c_0(n)\, c_1(n) \frac{1}{N}\sum_n c_0(m)\, c_1(m) \}$$

$$= E\{ \frac{1}{N^2}\sum_n \sum_m c_0(n)\, c_1(n)\, c_0(m)\, c_1(m) \}$$

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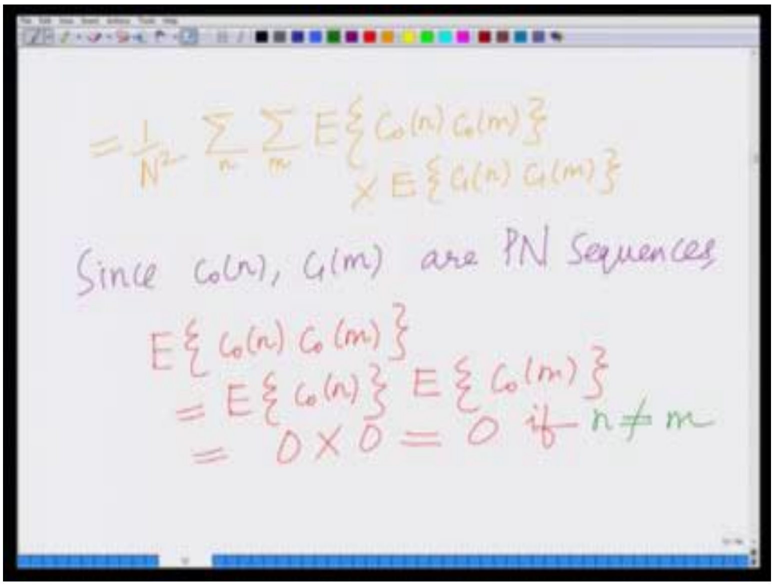


$$= \frac{1}{N^2} \sum_n \sum_m \ \mathrm{E}\{\ c_0(n)\ c_1(n)\}\ \mathrm{E}\{\ c_0(m)\ c_1(m)\ \}$$

$$\mathrm{E}\{\ c_0(n)\ c_0(m)\} = \mathrm{E}\{\ c_0(n)\}\mathrm{E}\{\ c_0(m)\} = 0 \text{ x } 0 = 0$$

but this occurs if $n \neq m$.

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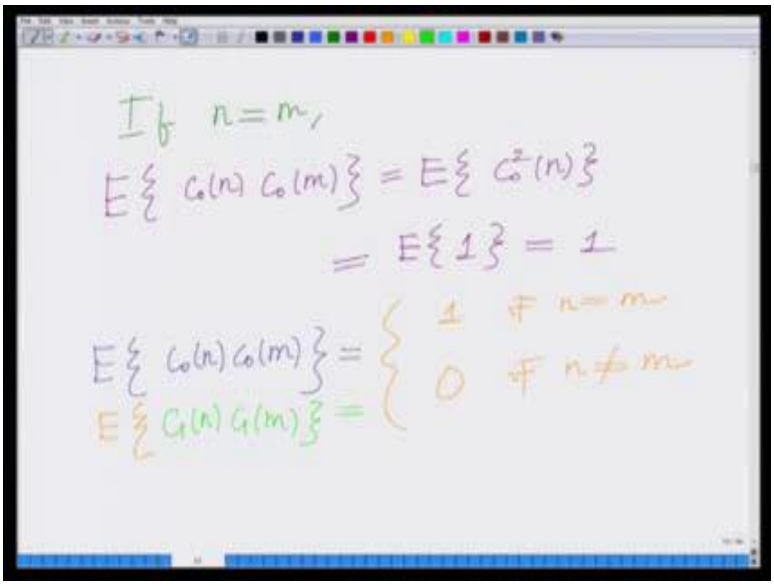

However if $n = m$, we have

$$\mathrm{E}\{\ c_0(n)\ c_0(m)\} = \mathrm{E}\{\ c_0^2(n)\}\ = \mathrm{E}\{\ 1\ \} = 1$$

if n = m , it is 1 and if n ≠ m, it is 0.

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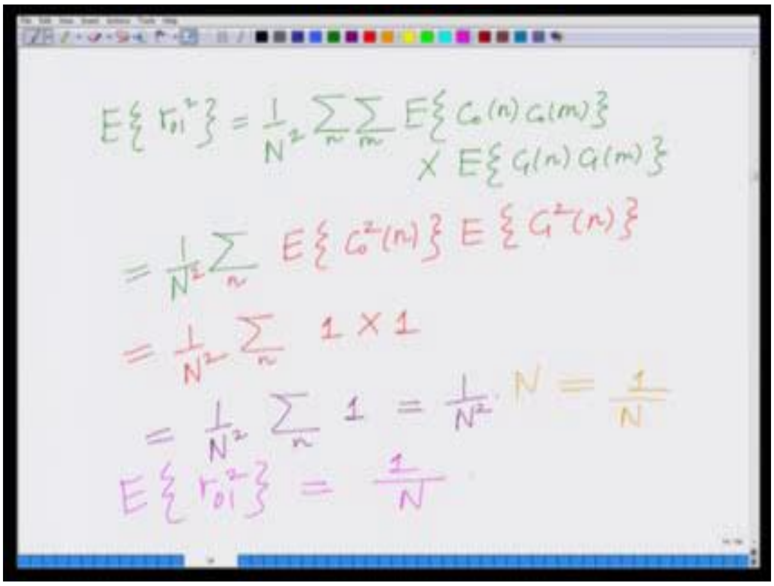

Therefore the power of $r_{01}$, that is

$$E\{|r_{01}|^2\} = \frac{1}{N^2}\sum_n \sum_m E\{c_0(n)\,c_1(n)\}\,E\{c_0(m)\,c_1(m)\}$$

$$= \frac{1}{N^2}\sum_n E\{c_0^2(n)\}\;E\{c_1^2(n)\}$$

$$= \frac{1}{N^2}\sum_n 1$$

$$= \frac{1}{N^2}\,.\,N = \frac{1}{N}$$

Therefore in summary what we have is something interesting, we have

$E\{|r_{01}|^2\} = \frac{1}{N}$ , that is the variance of $r_{01}$, the cross correlation between the code sequence of user 0 and user 1 $E\{|r_{01}|^2\} = \frac{1}{N}$ which means it is inversely proportional to the code length N.

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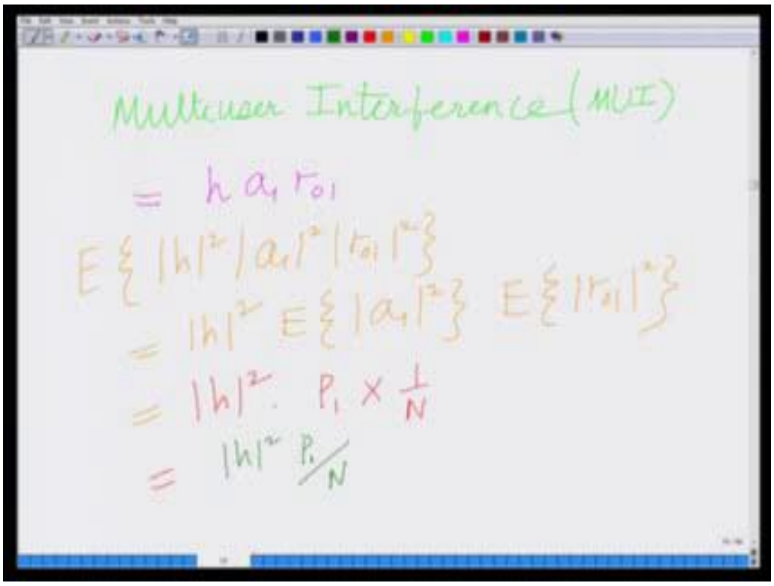


Therefore now if we go back to our Multiuser Interference, now realize our Multiuser Interference our MUI, our Multiuser Interference is equal to $h\,.\,a_1 r_{01}$

$$E\{\,|h|^2 |a_1|^2 |r_{01}|^2\} = |h|^2\, E\{\,|a_1|^2\}\, E\{\,|r_{01}|^2\}$$

$$= |h|^2 P_1 \times \frac{1}{N}$$

Therefore now going back to the signal model, if we look at this signal model; in a CDMA system we have the signal interference and noise components.

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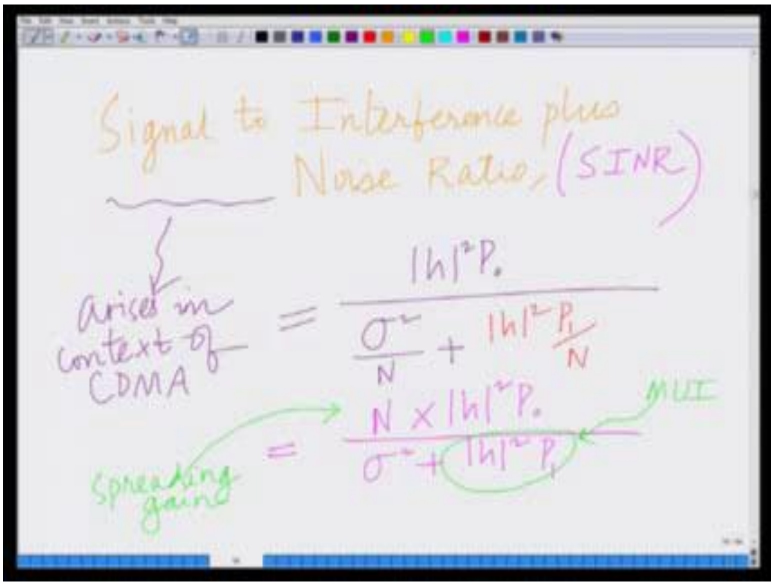


Therefore the interference and the noise are the undesirable components therefore; we want to compute the Signal to Interference plus Noise Ratio that is SINR. Normally we look at the Signal to Noise power Ratio, but in a CDMA system since we have signal Multiuser Interference and the noise, we look at the signal to the interference plus noise ratio. So, this is a new term for a CDMA system, so arises in the context of this arises in the context of because of the Multiuser Interference in CDMA because of the simultaneous transmission to multiple users that is equal to the signal power.

This is what we have already seen in the previous module that is

$$\frac{|h|^2 P_0}{\sigma^2\frac{1}{N}+|h|^2\frac{P_1}{N}} = \frac{N.\ |h|^2 P_0}{\sigma^2+|h|^2 P_1}$$

where as we have seen previously this N is the Spreading gain and this $P_1$ in the denominator is arising because of the Multiuser Interference.

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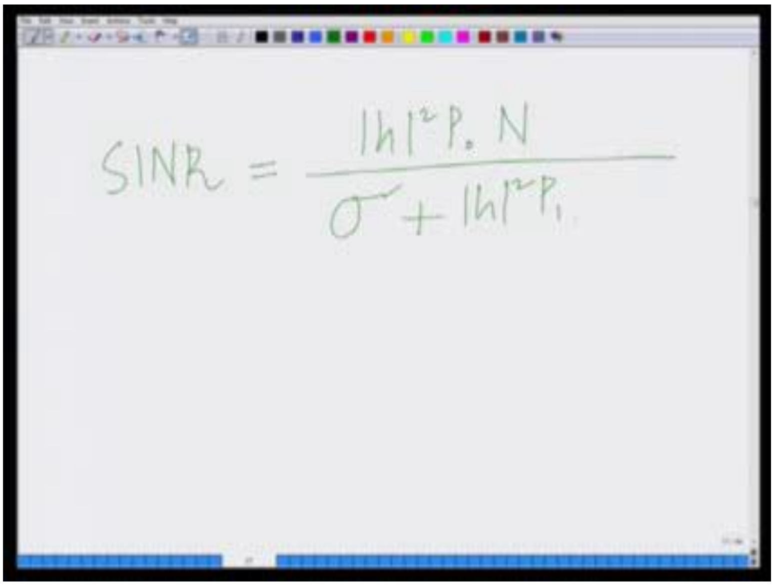


Therefore let me write again the expression for this SINR, the SINR in this CDMA system the SINR equals

$$\text{SINR} = \frac{N.\ |h|^2 P_0}{\sigma^2 + |h|^2 P_1}$$

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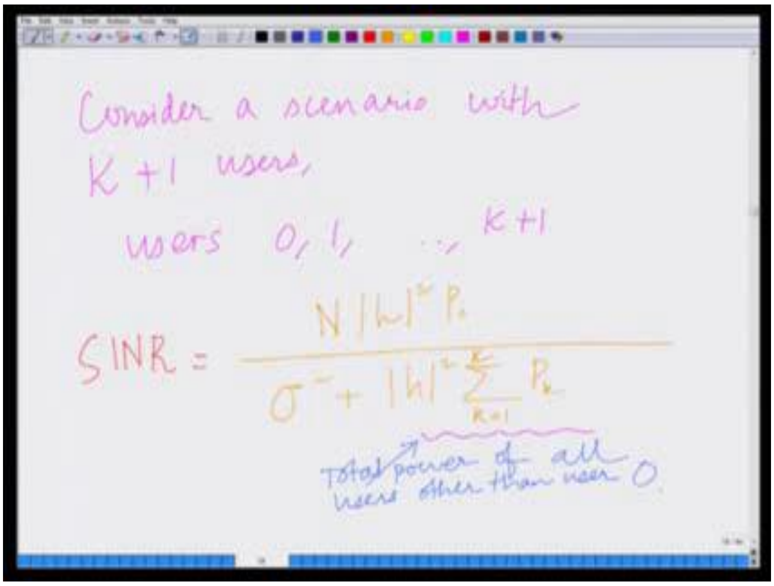


This can now be easily extended to a scenario with K + 1 user, so now consider a scenario with K + 1 user; where we have 0 that is where we have users 0, 1 up to K + 1. In this scenario the SINR of this scenario will be given as

$$\text{SINR} = \frac{N.\ |h|^2 P_0}{\sigma^2 + |h|^2 \sum_{k=1}^{k}\ P_k}$$

$P_k$ is total power of all users other than user 0. So, this is in the denominator we have interference from total power of all users other than user 0. So, the SINR is $\frac{N.\ |h|^2 P_0}{\sigma^2 + |h|^2 \sum_{k=1}^{k}\ P_k}$ that is signal power divided by sigma square that is the noise power plus magnitude h square times summation k equal to 1 to K; $P_k$ that is the Multiuser Interference from all the users other than user 0, so this is the SINR of a Multi-user CDMA system. Now let us look at a simple example to understand this.

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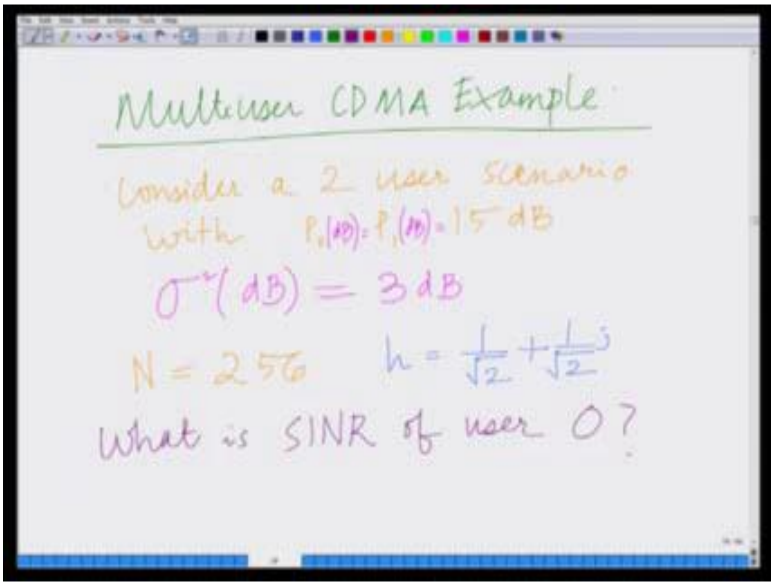


So, let us look at a simple example for this Multi-user CDMA scenario, let us consider a CDMA scenario. So, let us look at an example; let us look at a Multi-user CDMA example, let us consider a scenario in which consider a two user scenario with $P_0 = P_1$; equal to 15 dB, that is the power in dB; $P_0(dB) = P_1(dB) = 15$ dB. Similarly that the noise power in dB is $\sigma^2$ dB = 3 dB, further we have the code length N = 256; we have the channel coefficient h is $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j$. Now, what we want to ask is what is SINR of user 0; for this scenario we want to find out what is the SINR of user 0 and the SINR of user 0 is given as.

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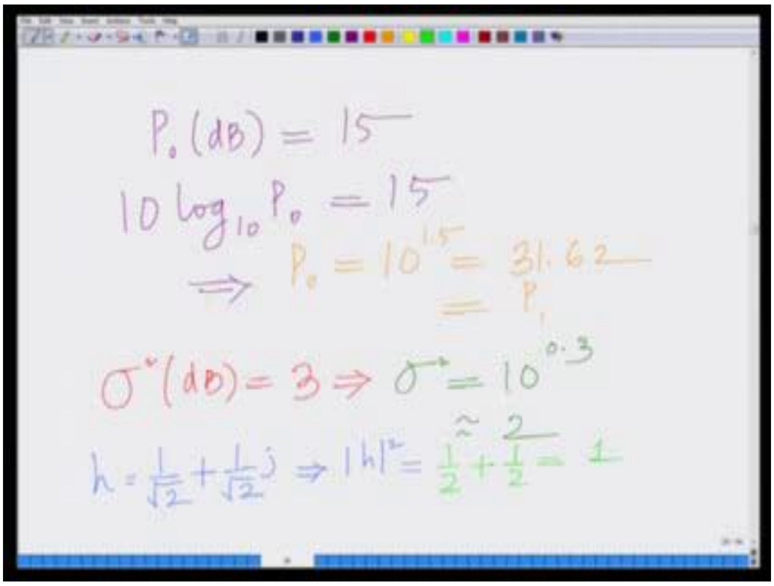


Now, let us convert first all these figures we have

$P_0(dB) = 15$ dB

$10\log_{10}P_0$ 15dB

$P_0 = 10^{1.5} = 31.62 = P$

$\sigma^2$ dB = 3 dB $\Rightarrow \sigma^2 = 10^{0.3} = 2$

$h = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \Rightarrow |h|^2 = \frac{1}{2} + \frac{1}{2} = 1$

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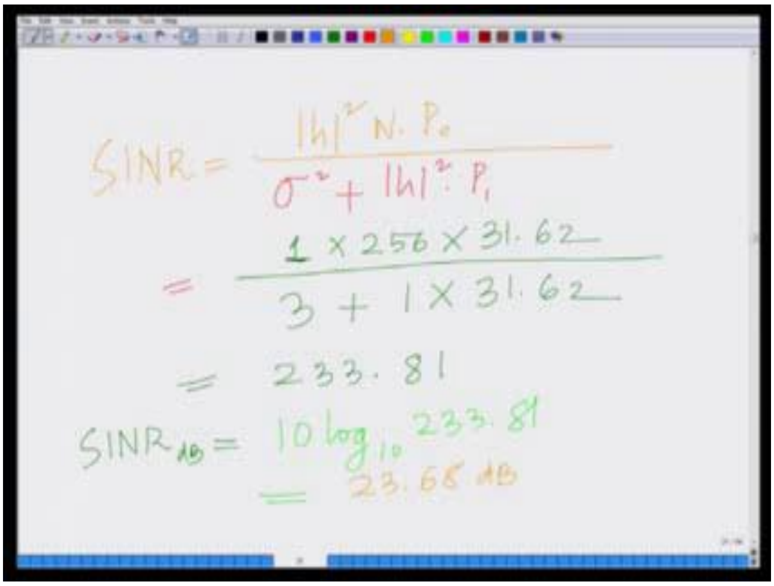


Therefore our SINR which is equal to

$$\text{SINR} = \frac{\text{N. } |\text{h}|^2 P_0}{\sigma^2 + |\text{h}|^2 P_1}$$

$= 233.81$

Therefore, the SINR in dB equals $10\log_{10} 233.81 = 23.68$ dB.

So, we can compute the SINR for this simple two user CDMA example and we have seen that this SINR is 23.68 dB. So, this basically; this module basically summarizes the idea of the SINR, idea of the most importantly the idea of Multiuser Interference in a CDMA system, how to characterize the SINR that is a Signal to Interference plus Noise Ratio in a Multi-user CDMA scenario and we have looked at a simple example to understand that this multiuser idea of SINR in a Multi-user CDMA scenario.

So, we are going to stop this module here; we are going to look at other aspects in the subsequent modules.

Thank you very much.