Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture - 30
BER of CDMA Systems

Hello. Welcome to another module in this Massive Open Online Course on the Principles of CDMA, MIMO, and OFDM Wireless Communication Systems. In the previous module we have seen the properties of the PN sequences employed in a CDMA communication system. Let us now look at the Bit Error Rate performance of a CDMA system. So, today in this module we are going to look at the Bit Error Rate of CDMA system.

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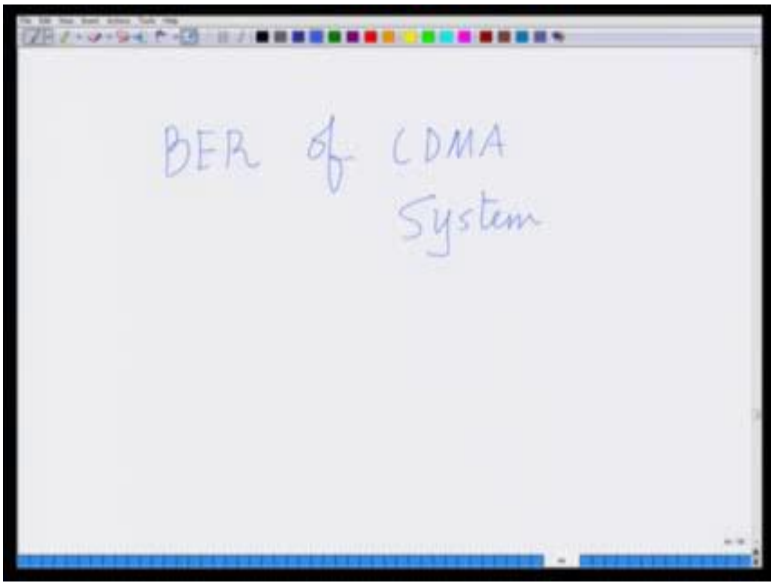

So, what we want look at the Bit Error Rate of a CDMA system.

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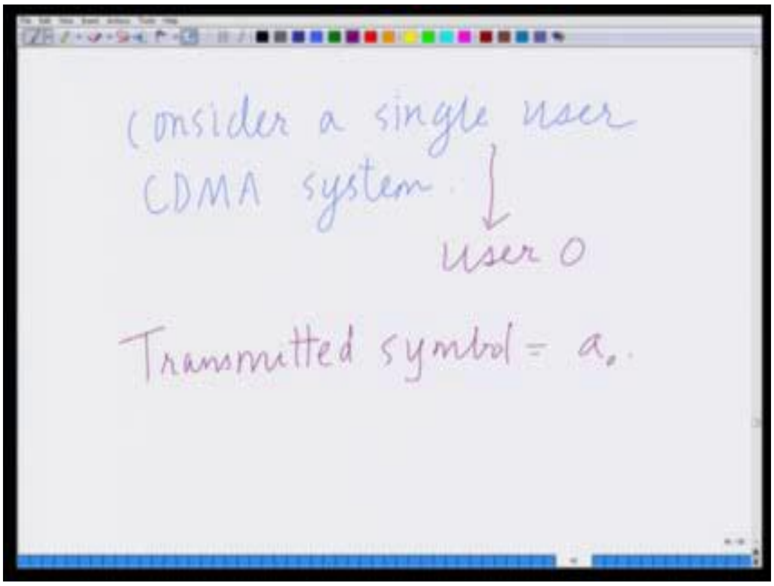

For this purpose consider a single user CDMA system. Let the Transmitted symbol may not the single user by user 0. So, we are calling this user as user 0, let the Transmitted symbol of this user the equal to a_0.

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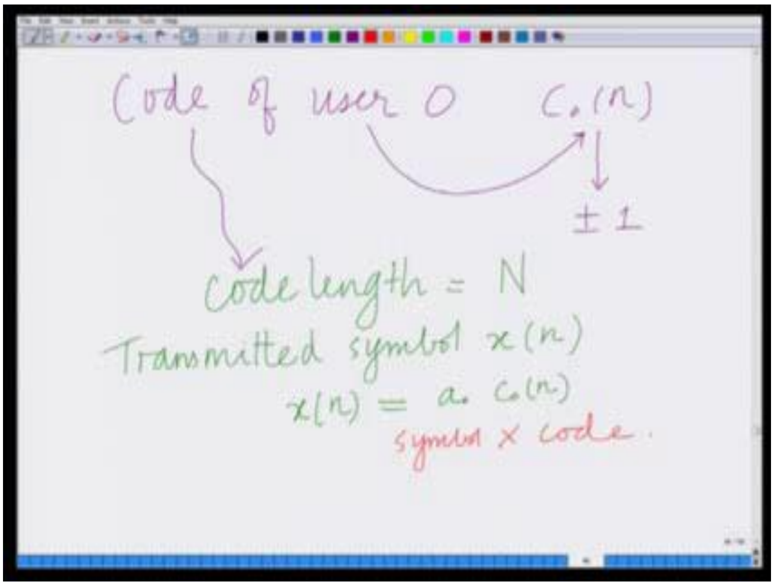

So, a_0 is the Transmitted symbol and the code CDMA code the denoted by $c_0(n)$. So, the code of user 0; $c_0(n)$ and each $c_0(n)$as we have seen previously is either +1 or -1 and we are considering a code length; we are considering a code length of N.

So, we have considering a single user CDMA scenario in which the Transmitted symbol of user is zero is a_0 the code of user 0 is $c_0(n)$ and each code symbol or each chip is plus or minus 1 and the code length is N. Now let us look at the Bit Error Rate performance of this system; we know that the Transmitted symbol x(n) is given as

$$x(n) = a_0 \, c_0(n)$$

that is this is the symbol multiplied with code in the CDMA system we said that the Transmitted symbol is the symbol of the user multiplied by the code. So, we are generating the Transmitted symbol x(n) as the symbol a_0 times the chip or times the code that is $c_0(n)$.

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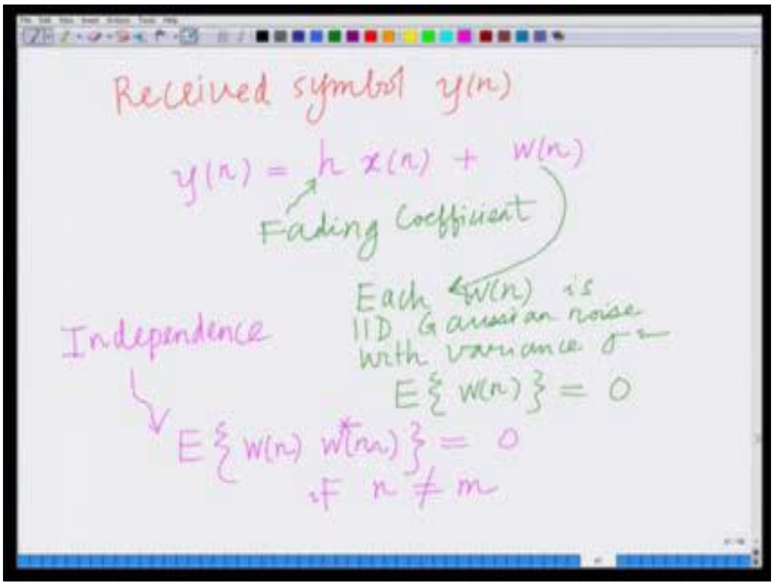

And now the received the symbol therefore, our Received symbol y(n) Received symbol y(n) is given as

$$y(n) = h \, . \, x(n) + w(n)$$

where h is the everyone remembers that h is the Fading Coefficient in particular we are considering Rayleigh Fading Channel. So, this is the Rayleigh Fading Channel Coefficient and each w(n) we are assuming each w(n) is IID Gaussian noise such that with variance sigma w square with. So, we are saying $y(n) = h \, . \, x(n) + w(n)$ where h is the Rayleigh Fading Channel Coefficient, x(n) is the Transmitted symbol and w(n) is the IID; Independent Identically Distributed Gaussian noise sample with zero mean and variance σ^2; that is $E\{ w^2(n)\}$ or the power of noise is equal to σ^2 and also the noise is zero mean. Therefore, $E\{w(n)\} = 0$ further the noise is independent which also means uncorrelated.

$$E\{ w(n) w^*(m)\} = 0 \quad \text{if } n \neq m$$

This follows from the independence of the noise samples; noise samples are independent which means the noise samples are uncorrelated therefore, if we look at two distinct noise samples w(n) and w(m); $E\{ w(n) w^*(m)\} = 0$.

So, the expected value or the correlation between these two different noise samples is basically equal to 0. So, the noise samples are Gaussian, they are Independent Identically Distributed, each has power or variance σ^2 each is zero mean and since there independent and 0 mean it follows that the correlation between two distinct noise samples is equal to 0.

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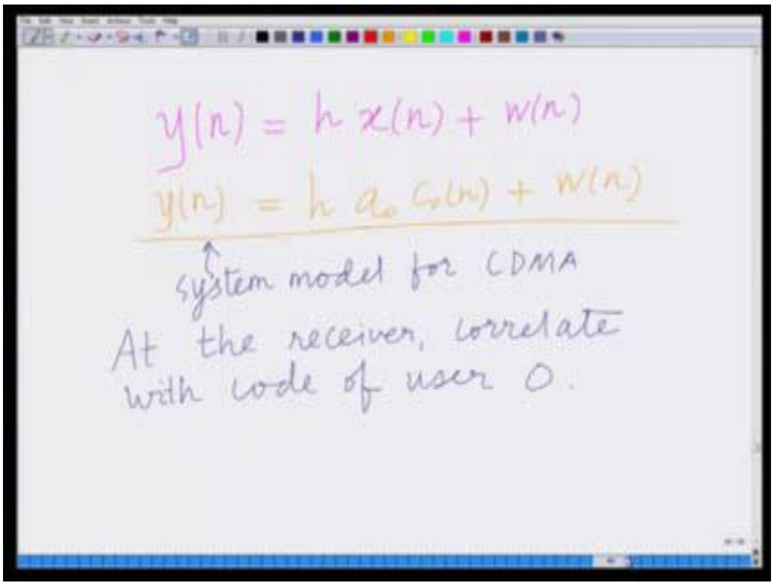

So, this is the system model that we have this is the standard system model; now let us substitute our CDMA symbol we have

$y(n) = h \cdot x(n) + w(n)$

but $x(n) = a_0\, c_0(n)$

we already seen that that is the product of the symbol and the code and as a result I have

$y(n) = h \cdot a_0\, c_0(n) + w(n)$

Therefore, this is the system model for our CDMA wireless communication system. Now at the receiver we have to correlate as we have seen for CDMA have to correlate with the code of user 0.

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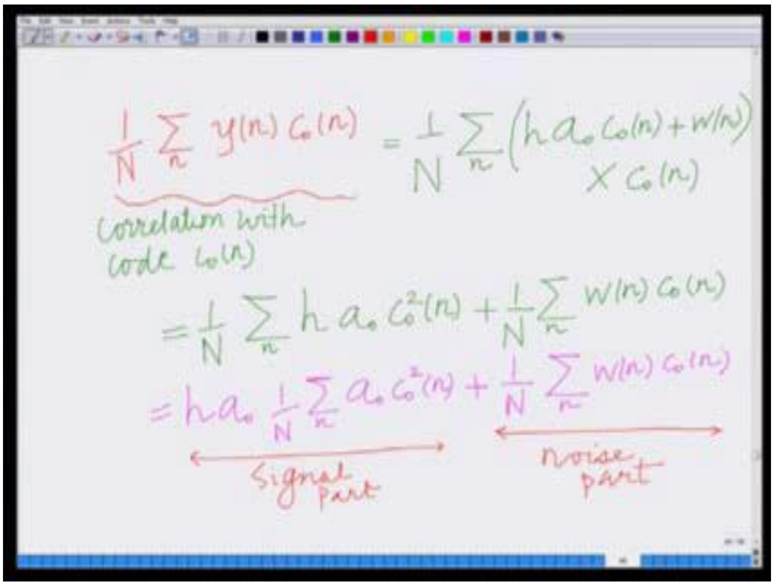

At the receiver we process correlate with the code of user 0 which means I have to perform

$= \frac{1}{N} \sum_n y(n) c_0(n)$

$= \frac{1}{N} \sum_n (h \cdot a_0\, c_0(n) + w(n)) c_0(n)$

$= \frac{1}{N} \sum_n h \cdot a_0\, c_0^2(n) + \frac{1}{N} \sum_n w(n) c_0(n)$

$= \text{h} \, . \, a_0 \, \frac{1}{\text{N}} \sum_n \; c_0^2(n) \; + \frac{1}{\text{N}} \sum_n \; \text{w(n)} c_0(n)$

So, we have substituted for y(n) equals $\text{h} \, . \, a_0 \; c_0(n) \; + \; \text{w(n)}$ in this expression for the correlation and we have simplified this resulting expression to derive the expressions for the signal part and the noise part. Now if you look at the single part we have $\frac{1}{\text{N}} \sum_n \; c_0^2(n)$.

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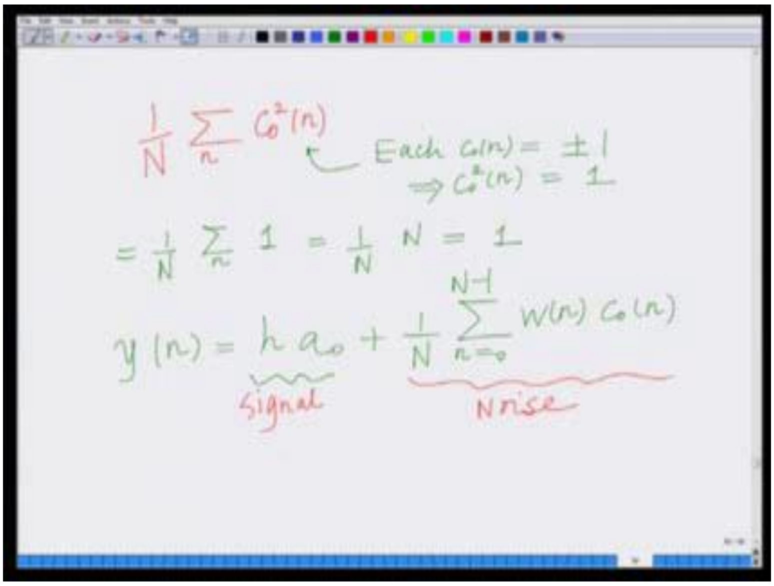

Now, if you look at this; this part is equal to $\frac{1}{\text{N}} \sum_n \; c_0^2(n)$

Now let us look at this $c_0^2(n)$, but each $c_0(n)$ is equal to is either +1 or -1 which means $c_0^2(n)$ which implies $c_0^2(n)$ is equal to 1. Therefore,

$= \frac{1}{\text{N}} \sum_n \; 1 \; = N . \frac{1}{\text{N}} = 1$

Therefore, now we substitute this expression here and I get my y(n) is equal to

$\text{y(n)} = \text{h} \, . \, a_0 \, + \frac{1}{\text{N}} \sum_{i=0}^{N-1} \; \text{w(n)} c_0(n)$

This is basically the signal part.

So, the signal is $h \cdot a_0$ and this is the same as before; this is the noise at the receiver. We have again simplified this after the correlation in terms of the signal part and the noise part and we have seen that the signal part is simply $h \cdot a_0$ that is the signal part.

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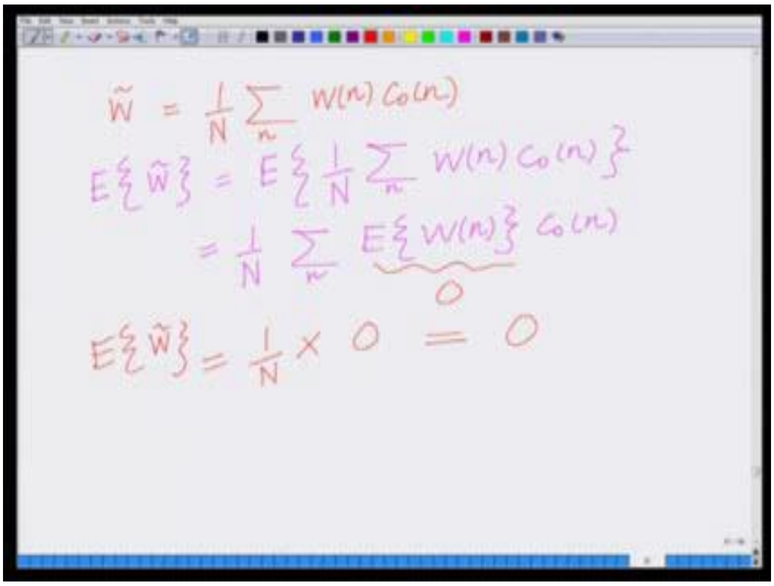

Now, let us look at the noise part; let us try to simplify the noise part and see what intuition we can get. Let us denote this by $\tilde{W}$; $\tilde{W}$ is

$$\tilde{W} = \frac{1}{N} \sum_n w(n) c_0(n)$$

$$E\{\tilde{W}\} = E\{\frac{1}{N} \sum_n w(n) c_0(n)\}$$

$$= \frac{1}{N} \sum_n E\{w(n)\} c_0(n)$$

$$= \frac{1}{N} \times 0 = 0$$

since w(n) is zero mean noise.

Therefore, $\tilde{W}$ also has mean equal to 0; since each w(n) has 0 mean $\tilde{W}$ also has mean equal to 0.

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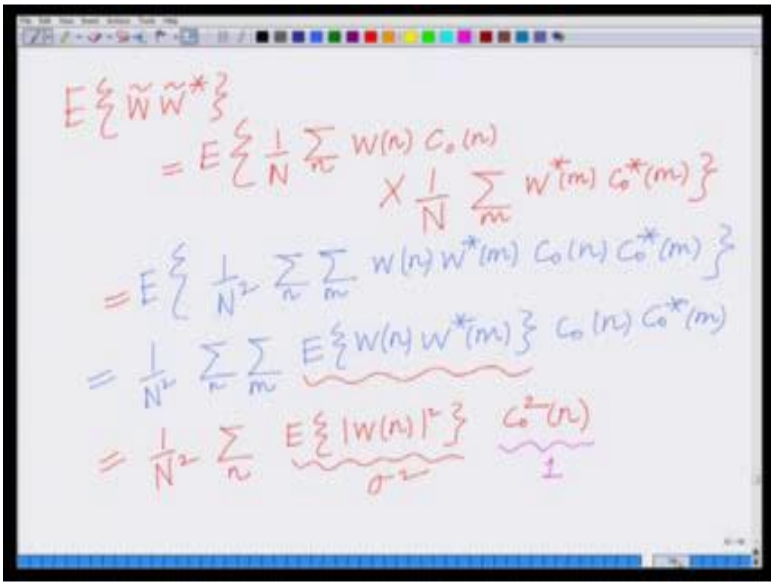

Now, let us look at the variance of this noise; variance is

$$E\{\tilde{W}\,\tilde{W}^*\} = E\left\{\frac{1}{N}\sum_n w(n)c_0(n)\ X\ \frac{1}{N}\sum_m w^*(m)\,c_0^*(m)\right\}$$

$$= E\left\{\frac{1}{N^2}\sum_n\sum_m w(n)w^*(m)\,c_0(n)\,c_0^*(m)\right\}$$

$$= \frac{1}{N^2}\sum_n\sum_m E\{\,w(n)w^*(m)\,\}\,c_0(n)\,c_0^*(m)$$

Now, we know that expected the noise samples w(n) and w(m) are independent. Therefore, we have already said that expected value of w(n) into w(m) conjugate is equal to 0; if n is not equal to m therefore, only those terms in the summation will survive therefore, we can simplify this as

$$= \frac{1}{N^2}\sum_n E\{\,|w(n)|^2\,\}\,c_0^2(n)$$

Since $c_0^2(n)$ is a real quantity, but that $E\{\,|w(n)|^2\,\}$ is the variance of the noise σ^2 expected.

$c_0^2(n)$ is equal to 1 since $c_0(n)$ is ± 1.

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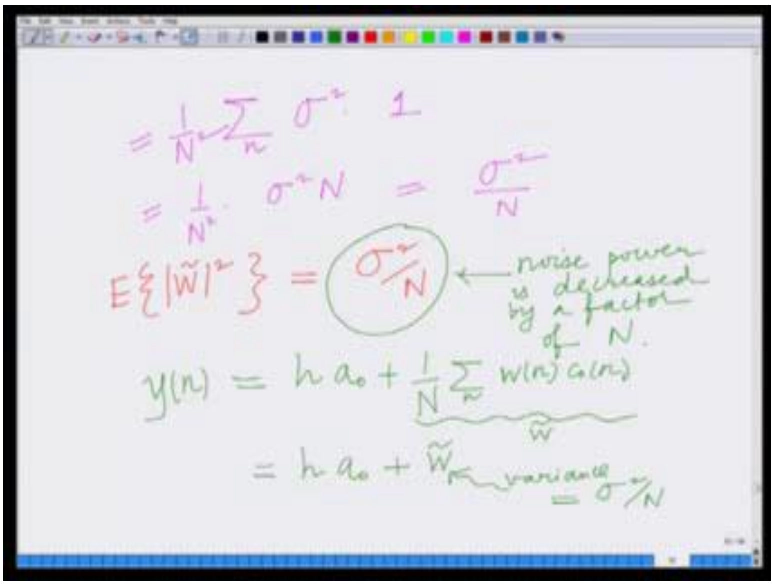

Therefore, we know that this can be simplified as

$$\frac{1}{N^2}\sum_n \sigma^2 .1 \;\; = \frac{1}{N^2}.\, \sigma^2 \;.\, \mathrm{N} = \frac{\sigma^2}{N}$$

$$\mathrm{E}\,\{\, \tilde{\mathrm{W}}^2 \,\} = \frac{\sigma^2}{N}$$

Therefore we are finding something very interesting. We are finding that this noise w tilde and the output of the CDMA receiver after correlation the noise is zero mean the variance $\frac{\sigma^2}{N}$ that is the noise power is decreased by a factor of N therefore, the noise variance is suppressed or noise power decreased by a factor of N.

Now we have our net system module remember our system model equals

$$\mathrm{y(n)} = \mathrm{h}\,.\, a_0 + \frac{1}{\mathrm{N}}\sum_{i=0}^{N-1} \mathrm{w(n)}c_0(n)$$

$$\mathrm{y(n)} = \mathrm{h}\,.\, a_0 + \tilde{\mathrm{W}}$$

The variance of this $\tilde{\mathrm{W}}$ equals $\frac{\sigma^2}{N}$.

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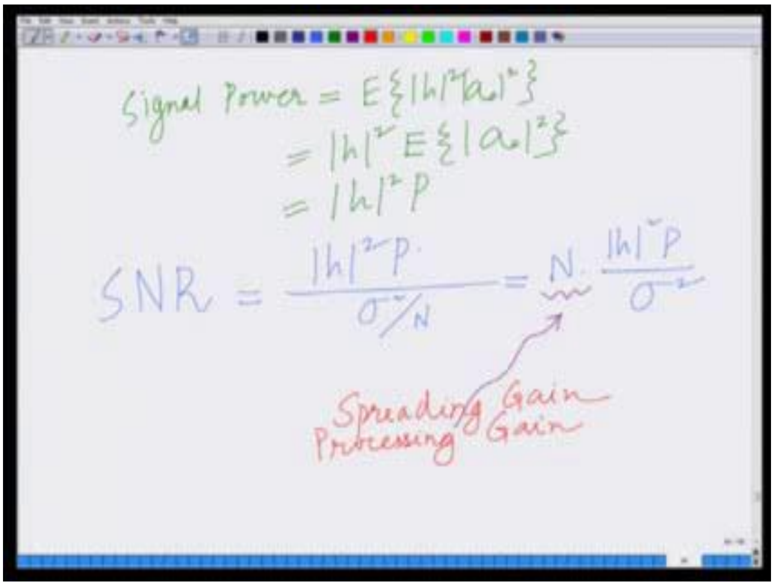

Now, let us look at the signal power; signal power equals

$$E\{ |h|^2\ |a_0|^2\} = |h|^2\ E\{ |a_0|^2\}$$

$$= |h|^2\ P$$

$$SNR = \frac{|h|^2 P}{\sigma^2 \frac{1}{N}} = N.\ \frac{|h|^2 P}{\sigma^2}$$

And therefore, now you can see there is a factor of N that is the Signal to Noise power Ratio is amplified by a factor of N in this CDMA system. This is a very important aspect of CDMA system; in a CDMA system this which is arising due to the spreading code of length N, this processing this gain of N which is arising due to the spreading code of length N this is known as the Spreading Gain or the Processing Gain of the CDMA system.

So, this factor of N increase in the SNR is known as the Spreading Gain or this is known as the Spreading Gain or the Processing Gain of the CDMA system this is a very important aspect of CDMA system this spreading code resulting in a gain which is known as the Spreading Gain or the Processing Gain of the CDMA system. Now let us look at the Bit Error Rate of this CDMA system; let us look at to the Bit Error Rate previously we have seen the Bit Error Rate of a normal wireless communication system.

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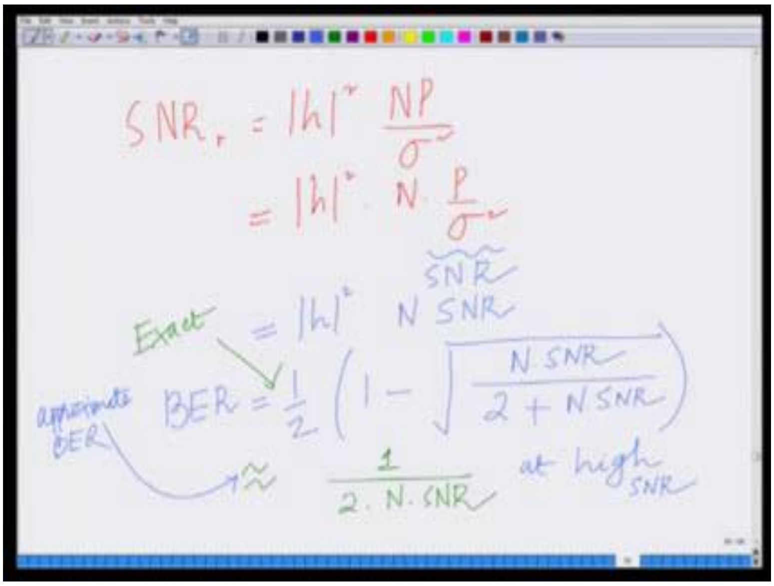

So, now let us look at the Bit Error Rate of a CDMA communication system we observe that SNR at the receiver is

$$SNR_r = \frac{|h|^2 P}{\sigma^2 \frac{1}{N}} = |h|^2 . \frac{NP}{\sigma^2}$$

$$= |h|^2 . N . \frac{P}{\sigma^2}$$

$$= |h|^2 . N . SNR$$

So, this is the same expression before that we have seen for the Rayleigh Fading Channel except instead of SNR we have N times SNR. Therefore, the Bit Error Rate using the same expression as before for the Bit Error Rate of a BPSK system across Rayleigh Fading Channel we can write the Bit Error Rate of this system as

$$BER = \frac{1}{2}\left(1 - \sqrt{\frac{N.\ SNR}{2 + N.\ SNR}} \right)$$

$$\approx \frac{1}{2.N.SNR} \text{ at high SNR}$$

This is the approximate Bit Error Rate expression.

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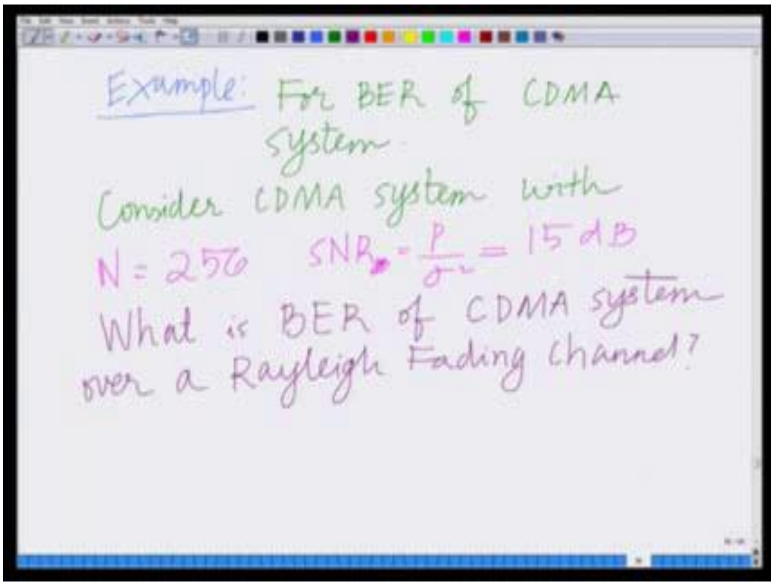

Let us now look at an example to understand this better. So, let us look at a simple example to understand. So, this simple example is for the Bit Error Rate this is for the Bit Error Rate of the CDMA system now consider CDMA system with $N = 256$ that is code length $N = 256$ and my SNR that is the transmit SNR equals $\frac{P}{\sigma^2}$ equals 15 dB; this is the SNR in dB. What is the Bit Error Rate of this CDMA system over a Rayleigh Fading Channel? So, this is the question that we are asking that is to calculate the Bit Error Rate of the CDMA system in which the dB SNR is 15 dB and the code length of the spreading code is $N = 256$.

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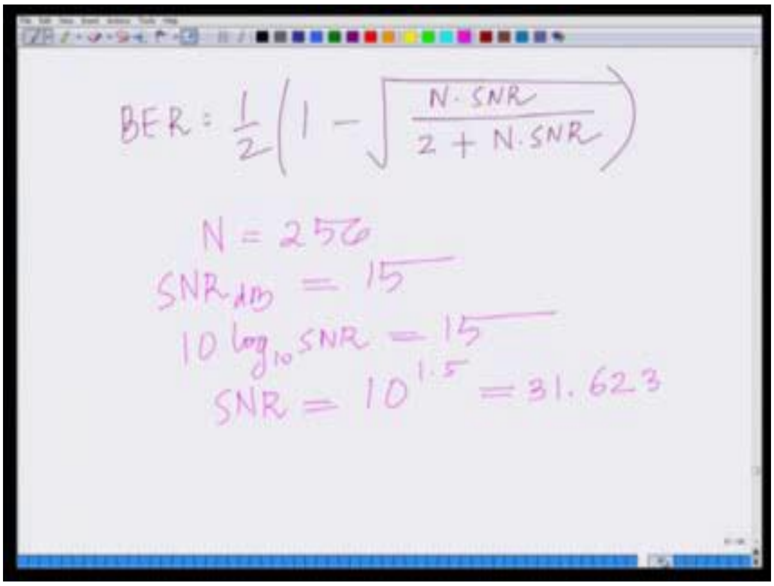

Now, we can calculate the Bit Error Rate as follows; we have the expression for the Bit Error Rate as we already seen is given as

$$\text{BER} = \frac{1}{2}\left(1 - \sqrt{\frac{N.\,SNR}{2+N.\,SNR}}\right)$$

We have N = 256, my

$$SNR_{dB} = 10 \log_{10} SNR = 15$$

$$SNR = 10^{1.5} = 31.623$$

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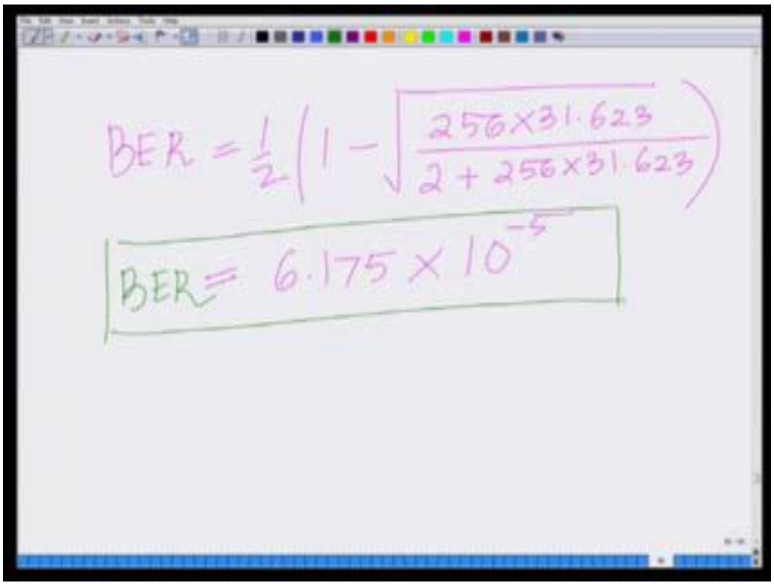

And therefore, the Bit Error Rate equals 6.175 into 10^{-5}. Therefore, my Bit Error Rate equals 6.175 into 10^{-5}. So, what we have done is we have calculated the Bit Error Rate of the CDMA system with spreading code length of N is equal to 256 and the dB SNR is 15 Db. We are saying that the Bit Error Rate of this system is 6.175 into 10^{-5}; that is a Bit Error Rate of this system.

So, what we seen in this module is we have seen how to compute the Bit Error Rate that is what is a system model for a CDMA that is the input output system model for a CDMA wireless system and how to compute we have seen another important aspect that is the Processing Gain or the Spreading Gain at the receiver arising from the correlation with the spreading code in this CDMA system and finally, we have seen what is the expression of the Bit Error Rate for this CDMA system in a Rayleigh Fading Channel and we have done a simple example to illustrate this principle or how to get the procedure to calculate the Bit Error Rate of a CDMA wireless system.

So, we will conclude this module here and we will look at other aspects in the subsequent modules.

Thank you very much.