**Principles of Modern CDMA/MIMO/OFDM Wireless Communications**
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**Lecture - 29**
**CDMA Codes- Properties of PN Sequences**

Hello welcome to another module in this massive open online course on the principles of MIMI, CDMA, OFDM wireless communication systems. So, in the previous lecture we were looking at CDMA and how to generate the code sequences for CDMA which stands for code division for multiple access and we have said that the CDMA code sequences are generated as PN sequences.

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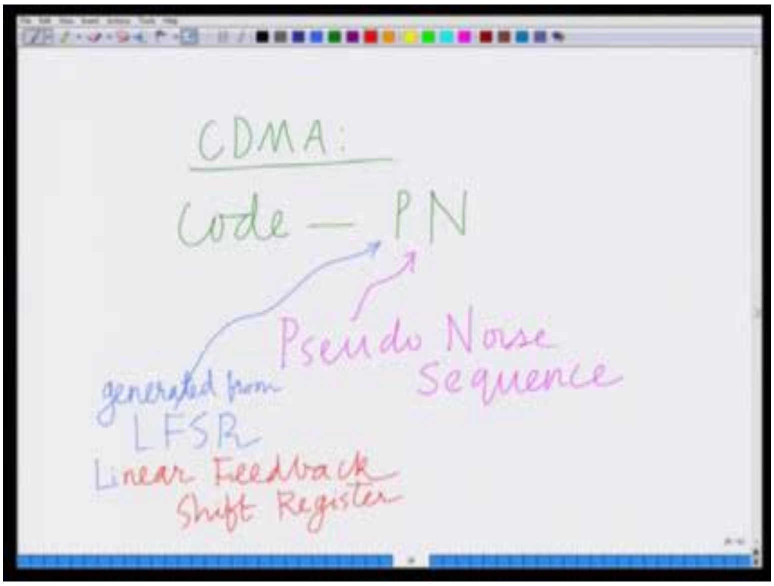


So, for CDMA when we look at CDMA the code sequence of CDMA is generated as a PN sequence which stands for a Pseudo Noise. The sequence it looks like a random series of 1's and +1 and - 1's, this is generated as a PN sequence or a Pseudo Noise sequence. PN sequence means a Pseudo Noise sequence and this is generated using an LFSR structure which stands for a linear feedback shift register structure.

So, the PN sequences these are generated using the LFSR. Which is also known as the linear feedback shift register structure. We had seen an example of an LFSR structure for D = 4 registers, we said that this is a maximal length linear feedback shift register and goes through all the possible states except the all 0 states therefore, it goes through $2^D - 1$ or $2^4 - 1$ that is 16 - 1 or 15 states.

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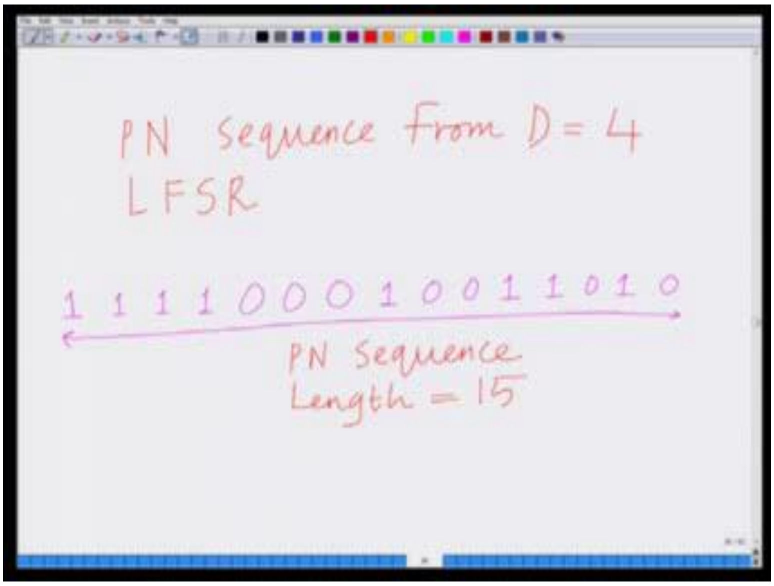


And the PN sequence that is generated by this LFSR that is given as, we had also derived that in the last lecture the PN sequence from D = 4 linear feedback shift register that is given as well 1 1 1 1 0 0 0 1 0 0 1 1 0 1 0. So, this is the PN sequence that is generated by the LFSR and the length of this PN sequence is equal to 15. So, this is the PN sequence that is generated by the linear feedback shift register and the length of this PN sequence that is equal to 15, we have 15 symbols in this PN sequence.

Now, therefore, now what we are going to do we have to convert this into a sequence of +1 and - 1's.

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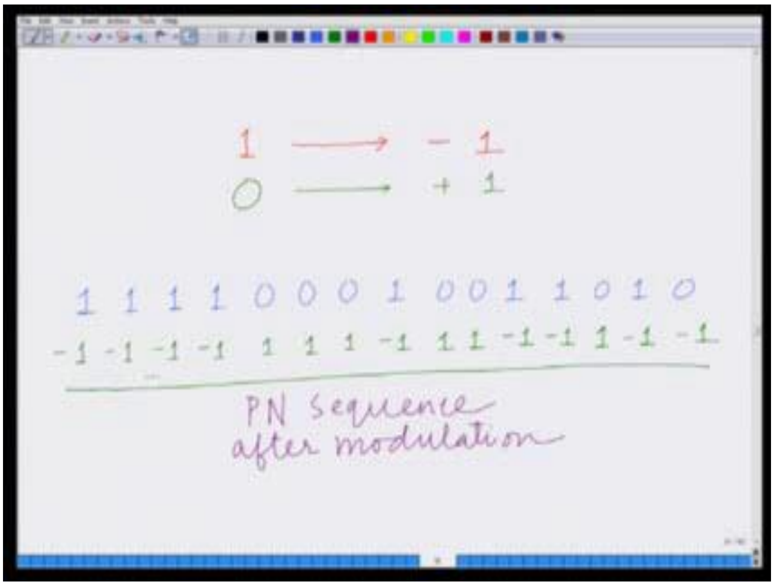


So, what we are going to do we are going to digitally modulate this PN sequence by mapping by modulating, we are going to use BPSK modulation, we are going to map the one information symbol 1 goes to a - 1 this is the standard modulation. And information symbol 0 goes to + 1. So, we are going to map 1 the information symbol 1 to a - 1 to a voltage level this is the typical convention. The information symbol 1 is mapped to the voltage level - 1, the information symbol 0 is map to the voltage level + 1 therefore, now my PN sequence becomes. So, my earlier PN sequence let me write that or let me write it over here, let me write that again my PN sequence information sequence is 1 1 1 1 0 0 0 1 0 0 1 1 0 1 0.

Now, after modulation this becomes remember 1 is mapped to a - 1, so this becomes - 1, - 1, - 1, - 1, 1, 1, 1, - 1, 1, 1- 1, - 1, 1, - 1, - 1. So, after modulation this PN sequence becomes this is the PN sequence after modulation and this sequence is given as - 1, - 1, - 1, - 1, 1, 1, 1, - 1, 1, 1- 1, - 1, 1, - 1, - 1 and the sequence is again of length 15 which remains unchanged. So, this the modulated PN sequence. So, this is the PN sequence, this is the Pseudo Noise sequence which looks like a random sequence or noise like sequence of +1 and - 1's which is used as a code in the CDMA system. The code by which the user symbol is multiplied.

Now, to understand properties of CDMA better let us examine the properties of this PN sequence. So, let us start examining the properties.

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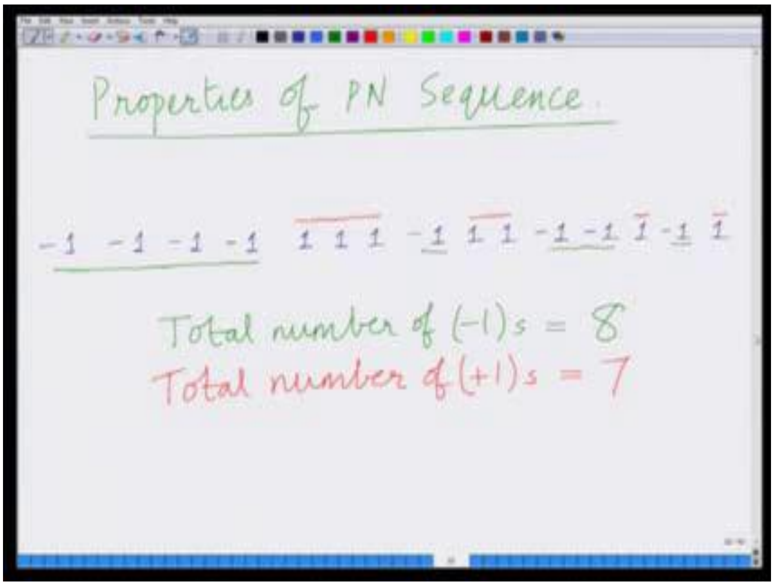


Let us examine the properties of this PN sequence. Now, let me write this PN sequence this PN sequence is as - 1, - 1, - 1, - 1, 1, 1, 1, - 1, 1, 1,- 1, - 1, 1, - 1, 1, this is the sequence as we all ready seen this is the sequence of length 15.

Now, let us calculate the number of +1 and - 1's in this sequence, so we have. So, if we want to calculate the number of plus - 1's, 1st we have 4 - 1's here, 1 - 1 here, 2 - 1's here, 1 - 1 here. So, the total number of - 1's in the system is equal to 8.

Now, let us calculate the number of +1 symbol in this CDMA in this PN sequence. I want to calculate what is the total number of plus 1's, so total number of +1's says we have 3 +1's over here, 2 +1's over here, another 1 over here. So total number of +1 is equal to 7. Now, if you see we observe something very interesting the total number of +1 out of these 15 symbols, there is 15 +1's and - 1's we have number of - 1's is equal to 8 and number of +1's is equal to 7 therefore, what we have is a roughly the number of +1's and number of - 1's is equal.

In fact, the precise relation is that the number of - 1's is exactly 1 more than the number of +1's.

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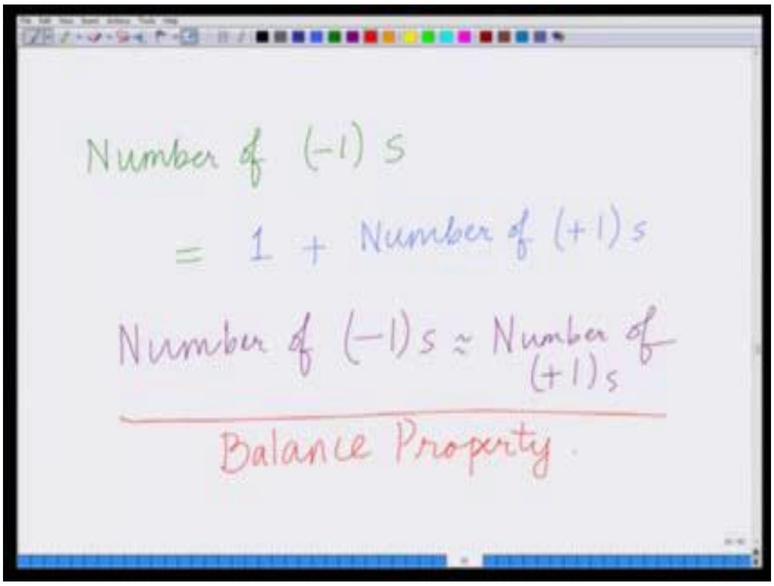


So, what we observe is that

number of - 1's = 1 + the number of +1's

the number of minus 1 is 8, which is 1 plus the number of +1's that is 1 plus 7.

So, what you can see also is that the number of - 1's and number of +1's are approximately equal, that is out of a length of 15 total length of 15 we have 8 - 1's and 7 +1's. So, what we observe is that the number of - 1's and number of +1's is roughly equal and this is known as the balance property. This property is known as the balance property. So, what we have is number of - 1's is approximately equal to number of +1's and this is known as the balance property. That is approximately if I take a length n PN sequence, approximately half of the symbols in the PN sequence are - 1 and approximately half of the symbols in the PN sequence are +1. So, approximately half and half are +1 and - 1 each this is known as the balance property. And more precisely speaking the number of - 1 symbol is exactly 1 more than the number of +1 symbol in the PN sequence. This is known as the balance property of the PN sequence.

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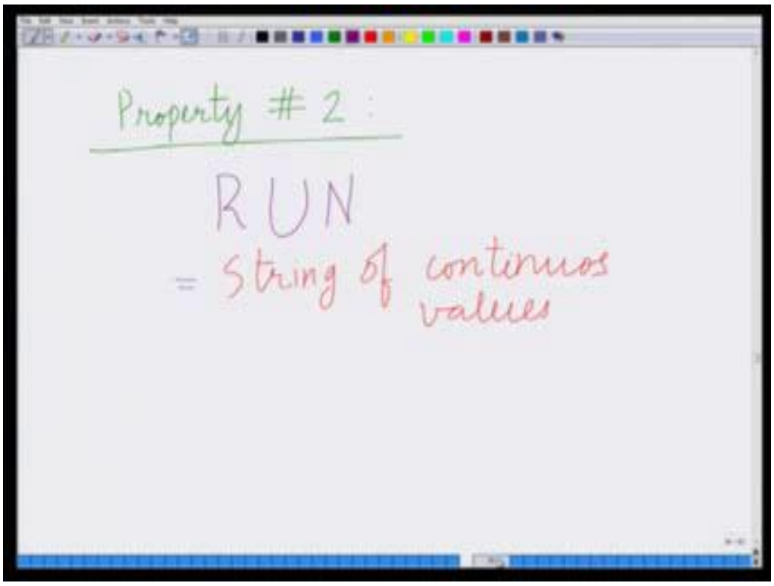


So, this is property number 1, let us now look at property another interesting property let us now look at property number 2. For this PN sequence and for this let us look at the runs what we call as the run, what is the run? A run in a PN sequence is a string of a run in a PN sequence is a run is the string of continuous values.

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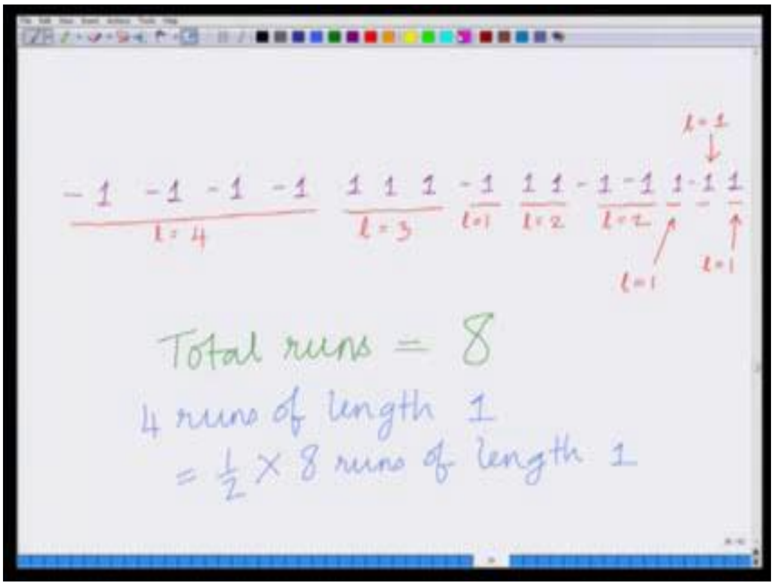


So, let us look again at our PN sequence our PN sequence what is our PN sequence? Our PN sequence is well - 1, - 1, - 1, - 1, 1, 1, 1, - 1, 1, 1,- 1, - 1, 1, - 1, 1. So, this is my length 15 PN sequence.

Now, let us look at the runs, that is then a strings continuous strings of continuous values I have 1 string of 4 - 1's here, so this is my 1st run of length equal to 4; I have another string of length 3 1's, so this is of n is equal to 3; I have another string of length 1. So, I have n is equal to 1 I have another string of 1's of length is equal to 2, I have another string of - 1's of length is equal to 2 then the rest of the string I have 3 strings of 1 - 1 1 each of these is of length L is equal to 1. So, I have the 1st string which is of 4 - 1's. So, the length is equal to 4, I have the 2nd string of length equal to 3 that is 3 1's, 3 consecutive plus 1 symbols, I have 1 string of length is equal to 1 which is a - 1, 1 string of a length equal to 2 of plus 1 symbols 1 string of length equal to 2 of - 1 symbols again 3 strings of length 1 each 1 - 1 and 1.

So, the total number of strings, if you look at the total number of runs or strings, total number of runs that is the number of strings is equal to 1 2 plus 1 2 3 4 5 6 7 8. So, the total number of runs in this PN sequence that is total number of strings of continuous values in this PN sequence is equal to 8. Now, out of these total number of runs, I have 4 runs are of length 1, out of these total number of runs 8, I have 4 runs of length 1. So, the length of the run is basically the length of the string is the length of the run and we have 4 strings of length 1 which means we have 4 runs of length 1 and the total number of runs is 8 and now look at this 4 is basically half of 8. So, basically which means that half of the 8 runs are of length, 1.

Now, let us look at the number of runs of length 2, we have 2 runs of length 2 which means one-fourth of 8,  8 is the total number of runs one-fourth of 8 is 2.

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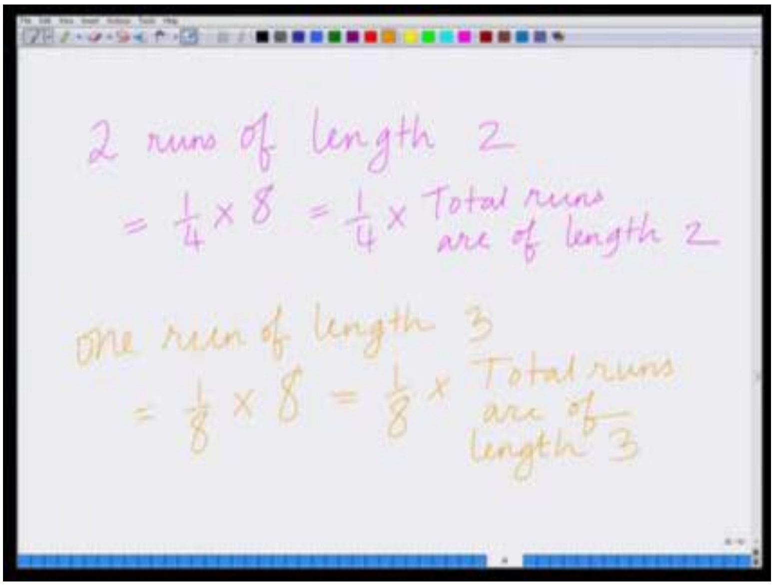


So, one-fourth of 8 which is basically one-fourth of total runs are of length 2. So, what we are observing is half of the runs are of length 1, one-fourth of the runs are of length 2 and interestingly if you look at this you will further realize that you have 1 run of length 3. So, I have 1 run of length 3, which is one-eighth of the total number of runs. So, one-eighth of 8 is 1.

So, one-eighth of the total runs are of length 3. So, what we are observing is something very interesting, we are observing that half of the runs are of length 1, one-fourth of the runs are of length 2, one-eighth of the runs are of length 3 and, if you have a longer PN sequence we will have one-sixteenth of the runs of length 4, one more 32 of the runs of length 5. In general $\frac{1}{2^n}$ of the total runs, of the total numbers of fraction 1 over 2 to the power of n of the total number of runs is of length n.

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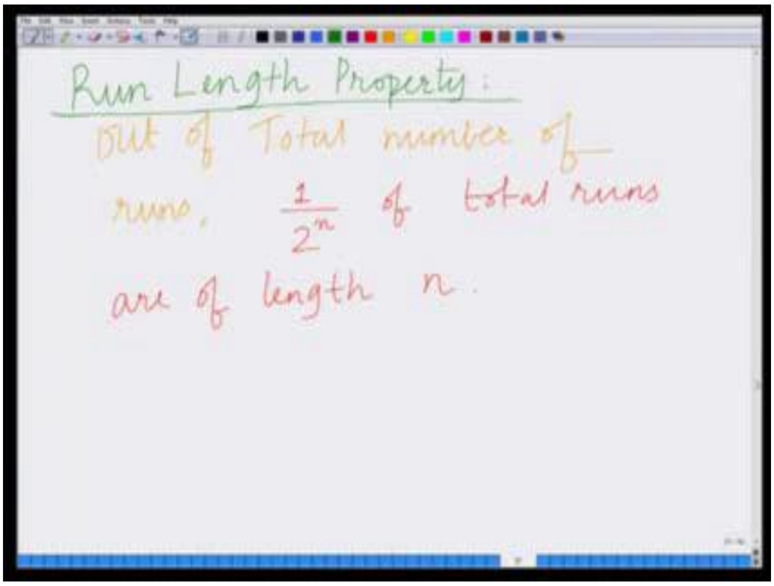


So, what we are observing is the 2nd interesting property, this is the run length property out of total number of runs. $\frac{1}{2^n}$ of total runs are of length n. So, what we are observing is and this is known as the run length property of the CDMA sequences which says that $\frac{1}{2^n}$ fraction of runs are of length n, which means half that is $\frac{1}{2^1}$ or half of the total runs are of length 1, one-fourth of the runs are of length 2, one-eighth of the runs are of length 3 and so on, this is 1 as the run length property of the PN CDMA, PN sequence. This is known as the run length property and this another important property of a CDMA sequence, which basically is a fundamental property it must to obey if it indeed has to look at like a noise like sequence of plus and minus 1, that is the randomly generated sequence of +1's and -1's.

Now we want to look at another property that is property number 3 that is a 3rd property. So, I want to look at another property, property number 3 of this PN sequence and this is the most important property, this is known as the auto correlation property of the PN sequence.

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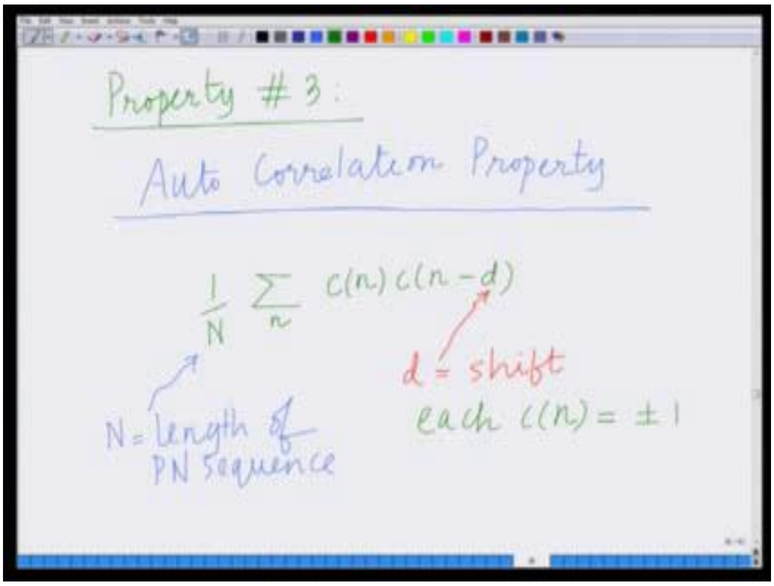


And auto correlation of the property and sequence is defined as

$$= \frac{1}{N}\sum_n c(n)c(n-d)$$

where, this quantity d is the shift, this d is equal to shift, this quantity n is equal to the length of the PN sequence. The capital N is equal to length of and this is the auto correlation, this is the definition of the auto correlation which is basically $\frac{1}{N}\sum_n c(n)c(n-d)$ .

So, what are we doing? We are taking each chip $c(n)$ we are multiplying by the chip corresponding to the shifted PN sequence. What is $c(n-d)$? $c(n-d)$ is basically shifting the PN sequence by the quantity d and therefore, we are shifting the PN sequence by this amount d, we are doing in element wise multiplication of these PN sequence and the shifted PN sequence by d, we are summing this element wise product and then we are dividing by capital N, which is the length of the PN sequence this is defined as the this is known as the auto correlation of the PN sequence, this is the auto correlation of the PN sequence.

Now, let us see what happens? Now, realize also that each $c(n)$, each element c n is either equals to +1 or -1. Now, let us see what happens to this out of the behavior of this auto correlation function of this PN sequences is a function of the shift d.

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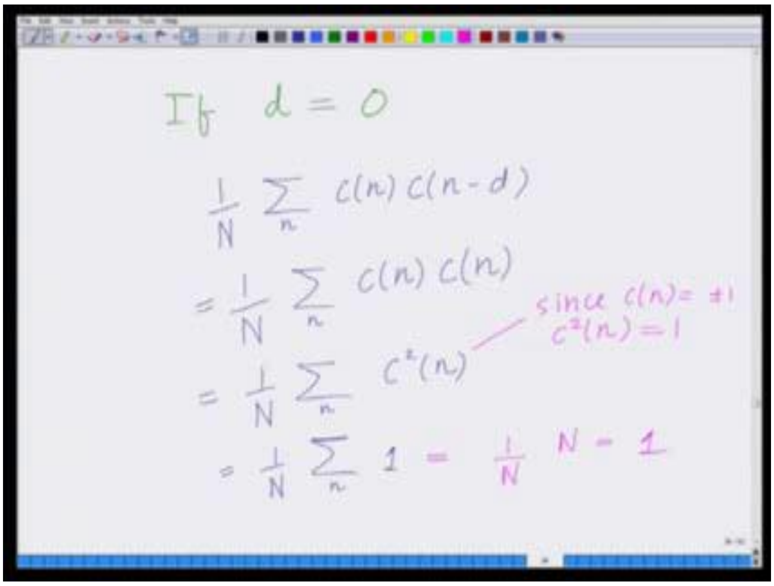


Now, if d = 0, then my auto correlation is

$$= \frac{1}{N}\sum_n c(n)c(n-d)$$

$$= \frac{1}{N}\sum_n c(n)c(n)$$

$$= \frac{1}{N}\sum_n c^2(n)$$

$$= \frac{1}{N}\sum_n 1$$

$$= \frac{1}{N}N = 1$$

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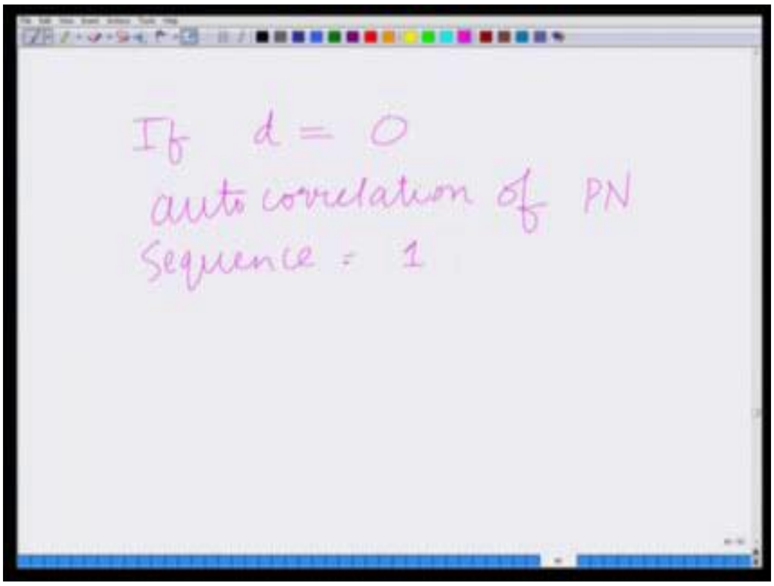


So, if d = 0 the auto correlation of this PN sequence is equal to 1. So, if d = 0 that is the auto correlation of the self correlation of this PN sequence is equal to 1. Now, let us examine what happens if d is not equal to 0 for instance let us take a simple example.

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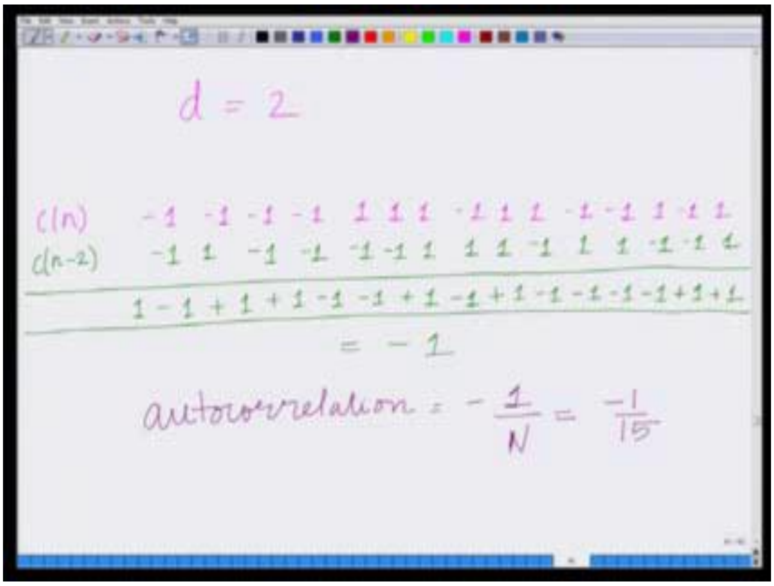


Let us consider d is equal to 2, let us see what happens when d is equal to 2, let us circularly shift this PN sequence by 2 symbols and let us see what the value of the auto correlation is going to be.

So, I am going to write this sequence. So, I have $c(n)$ which is - 1, - 1, - 1, - 1, 1, 1, 1, - 1, 1, 1,- 1, - 1, 1, - 1, 1. Now, I am going to circularly shift this by 2. So, I am taking this

PN sequence and I am circularly shifting this PN sequence by 2 symbols. So, I am considering a shift of d = 2 therefore, this will become is - 1, 1, - 1, - 1, - 1, - 1, 1, 1, 1, - 1, 1, 1,- 1, - 1, 1, and this last symbols will be will come to the front because this is a circular shift. So, this is this, so I have the sequence I have the circularly shifted sequence.

Now, I have to take the element wise multiplication and I have to take the sum which is going to be 1,- 1, 1, 1, - 1, - 1, 1, -1, 1, -1, - 1, -1, -1, 1, 1, and now, dividing by N therefore, the auto correlation now, I am taking the element wise sum and now I have to divide by N. So, therefore, the final auto correlation will be equal to $\frac{-1}{N}$, which in this case N is equal to 15. So, this is $\frac{-1}{15}$. and what we are observing is something very interesting, we are observing that if the auto correlation, if d shift is equal to 0 then the auto correlation is equal to 1 for any shift other than 0 the auto correlation decreases drastically it is $\frac{-1}{N}$.

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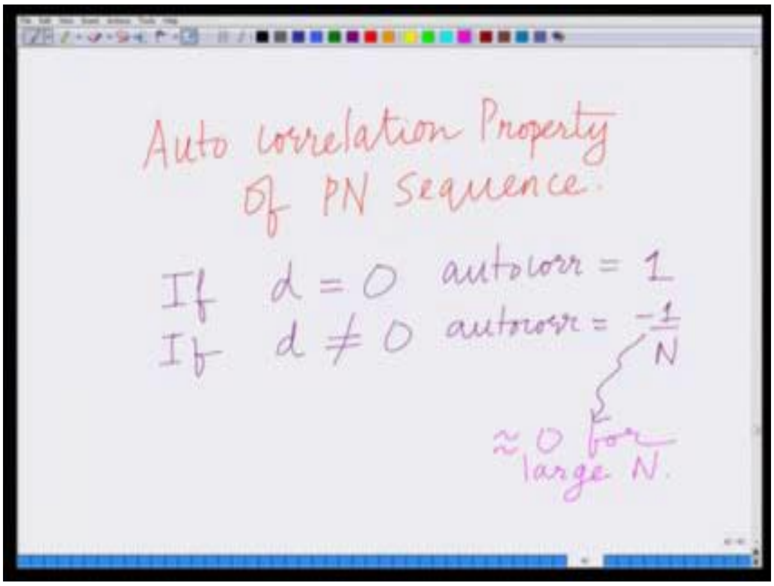


So, the auto correlation property can be summarized as follows the auto correlation property of the PN sequence.

If d = 0, autocorrelation = 1

If d ≠ 0, autocorrelation = $\frac{-1}{N}$

For a non-zero shift auto correlation is $\frac{-1}{N}$. We had seen an example of this by considering a shift of d = 2. Therefore, what you can see something very interesting as soon as the shift moves away from 0 the auto correlation decreases drastically, it becomes $\frac{-1}{N}$ which means as N increases. Remember N is the length of the spreading sequence of the PN sequence, if N is large then $\frac{-1}{N} \sim 0$ which means $\frac{-1}{N}$ becomes progressively is approximately equal to 0 for larger, which means the auto correlation of this PN sequence for any shift other than the 0 shift is approximately equal to 0 for a PN sequence which a large length N which is a very desirable property as we are going to see shortly or in subsequent modules.

So, what happens is this N, this approximately equal to 0 for large N or the PN sequence decreases as $\frac{-1}{N}$ with large N.

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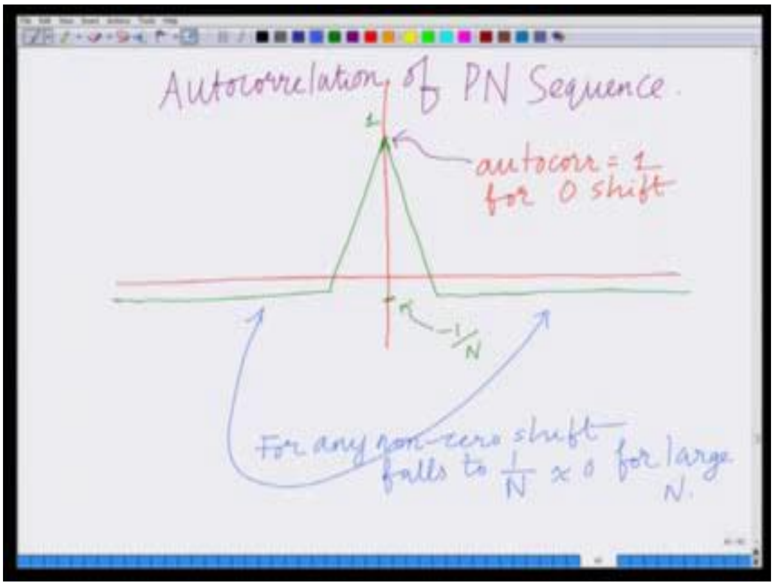

Therefore I can plot this auto correlation, The auto correlation function of the PN sequence it looks like this that is at 0 this is equal to 1, but away from 0 what happens is this auto correlation it was sharply and this is equal to $\frac{-1}{N}$. So, the moment I have any other shift other than the 0 shift, this auto correlation fall sharply. So, this is the auto correlation property of the PN sequence this is the plot of the auto correlation. This is the plot of the auto correlation of the PN sequence.

So, for a 0 shift the auto correlation, so this auto correlation equal to 1 for a 0 shift and for any non-zero shift, for any shift other than 0, it falls to $\frac{-1}{N} \sim 0$ for a large N. So, the auto correlation property of the PN sequence relates that the correlation is equal to 1 or the self correlation is equal to 1 for the 0 shift, but for any non-zero shift the auto correlation is $\frac{-1}{N}$ which is approximately equal to 0 for large N and these are the salient properties of the PN sequences which are used in a CDMA system. One is the balance property, two is the run length property and third the most important property is the auto correlation property, which we are going to see is will help us in extracting the multipath components in the wireless channel in a CDMA system and that gives raise the multipath diversity and this is a very important property of the PN sequences in a CDMA system.

So, this module which deals with the properties of the PN sequences we will stop here and continue with other aspects of the CDMA systems in subsequent modules.

Thank you very much.