

# Principles of Modern CDMA/MIMO/OFDM Wireless Communications

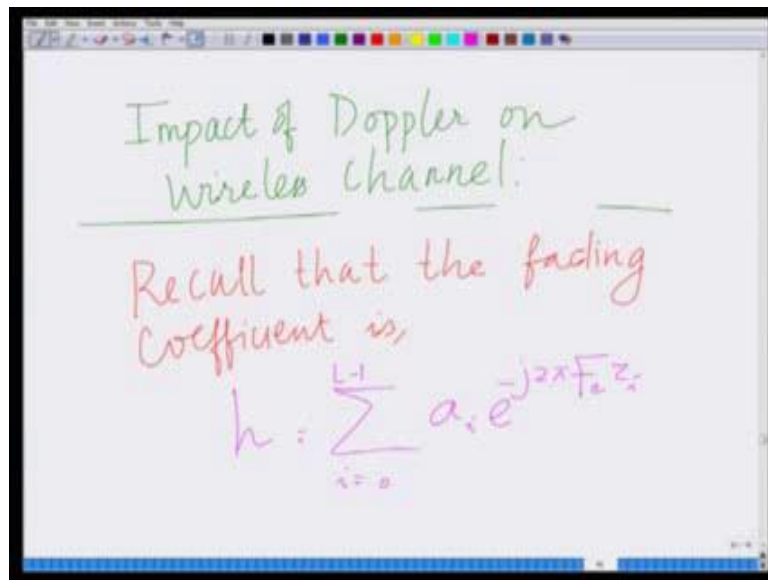
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## Lecture – 25

### Impact of Doppler Effect on Wireless Channel

Hello. Welcome to another module in this Massive Open Online Course on the Principles of CDMA, MIMO, and OFDM Wireless Communications Systems. In the previous module we have seen the Doppler Effect why the Doppler Effect arises and how to calculate the Doppler Effect. Now let us examine what is the impact of the Doppler Effect on wireless communication or the wireless channel.

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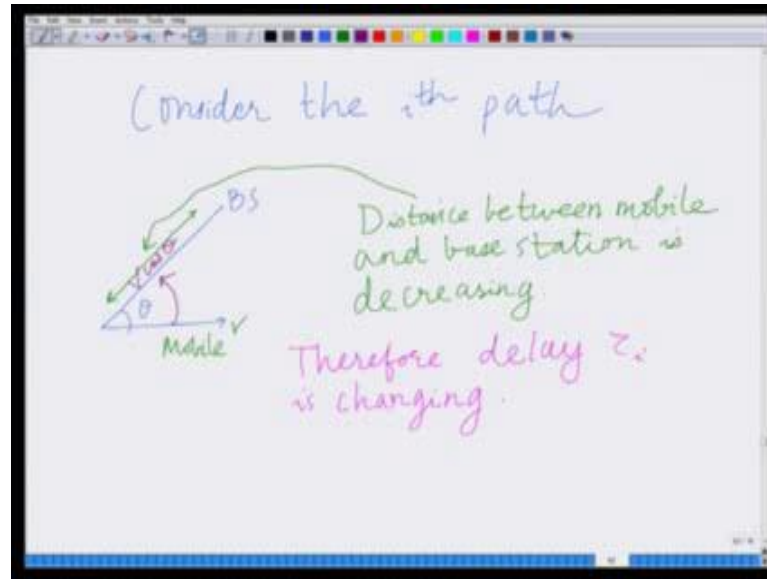


So, let us look at the impact of the Doppler on the wireless channel and for that recall that the wireless channel coefficient is defined as fading coefficient

$$h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$$

So, this is the definition of the fading channel coefficient  $h$ .

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Now considered  $i$ -th path; when I look at  $i$ -th path and when I look at the mobile. So, let us say this is my base station and I have my mobile which is moving at an angle of  $\theta$ , and the  $i$ -th path which as. So, this is my mobile which is moving with the velocity of  $v$  at an angle  $\theta$ .

Now if I look at that the distance between the base station and the mobile this is decreasing, the distance between the mobile and the base station is decreasing, as the mobile is moving at an angle of  $\theta$ , the distance between the base station and the mobile is decreasing. While, if the angle  $\theta$  is between  $0$  and  $\frac{\pi}{2}$  the distance is decreasing and when  $\frac{\pi}{2} \leq \theta \leq \pi$ , the distance is increasing.

So basically the distance is changing, as the distance is changing the corresponding delay of the wireless signal that is  $\tau_i$ , because remember  $\tau_i$  is the propagation delay. As the distance is changing, the corresponding propagation delay is also changing. Now how is a propagation delay  $\tau_i$  changing for that, let us look at this expression therefore, the corresponding delay  $\tau_i$  is.

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The whiteboard shows the following derivation:

$$\tau_i(t) = \tau_i - \frac{v \cos \theta t}{c}$$

Therefore,

$$h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \left( \tau_i - \frac{v \cos \theta t}{c} \right)}$$

$$= \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} e^{j2\pi f_c \frac{v \cos \theta}{c} t}$$

A red arrow labeled  $F_d$  points to the term  $\frac{v \cos \theta}{c}$  in the second equation.

Now, how fast is this changing, so if I write  $\tau_i(t)$ . Remember  $\tau_i$  is a function of the distance. So, it is equal to

$$\tau_i(t) = \tau_i - \frac{V \cos \theta t}{c}$$

therefore, the time or the delay is decreasing by  $\frac{V \cos \theta t}{c}$ . If you look at this figure, you will realize that the component of velocity in the direction of the base station is  $V \cos \theta$ . Therefore for a given time  $t$ , the distance in the direction of the base station is decreasing as  $V \cos \theta t$  and there is a result the delay is decreasing as  $\frac{V \cos \theta t}{c}$ .

Therefore, what we have is at the net delay  $\tau_i(t)$  is decreasing as,  $\tau_i$  - the initial delay minus  $\frac{V \cos \theta t}{c}$ . Now if I substitute this in my expression for  $h$ , I have

$$h = \sum_{i=0}^{L-1} a_i e^{-2\pi f_c \left( \tau_i - \frac{V \cos \theta t}{c} \right)}$$

$$h = \sum_{i=0}^{L-1} a_i e^{-2\pi f_c \tau_i} e^{-2\pi f_c \frac{V \cos \theta t}{c}}$$

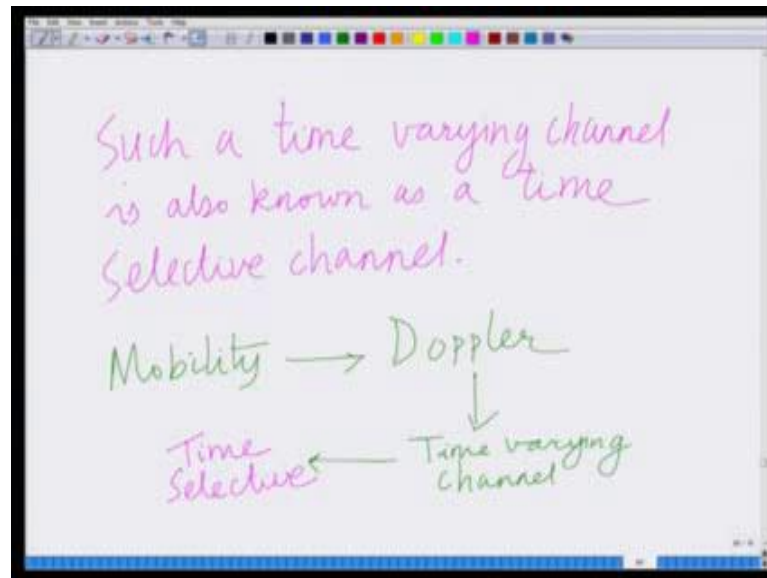
$$h = \sum_{i=0}^{L-1} a_i e^{-2\pi f_c \tau_i} e^{-2\pi f_d t}$$

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The image shows a handwritten derivation on a whiteboard. At the top, the equation  $h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} e^{j2\pi f_d t}$  is written. A bracket groups the two exponential terms, with a label  $f_d = \text{Doppler Freq}$  pointing to it. Below this, a note says "Changing with respect to time" with an arrow pointing to the time-varying term. The final conclusion is written in purple: "Therefore, the channel coefficient  $h$  is time varying."

So this quantity  $f_d$  is the Doppler frequency of the system further now you can see, this component  $e^{-2\pi f_d t}$  is a function of time. So, it is varying with time and therefore, as a result the coefficient  $h$  is now a function of time or it is varying with time. Such a channel coefficient  $h$  which is time varying is also known as the Time selective channels.

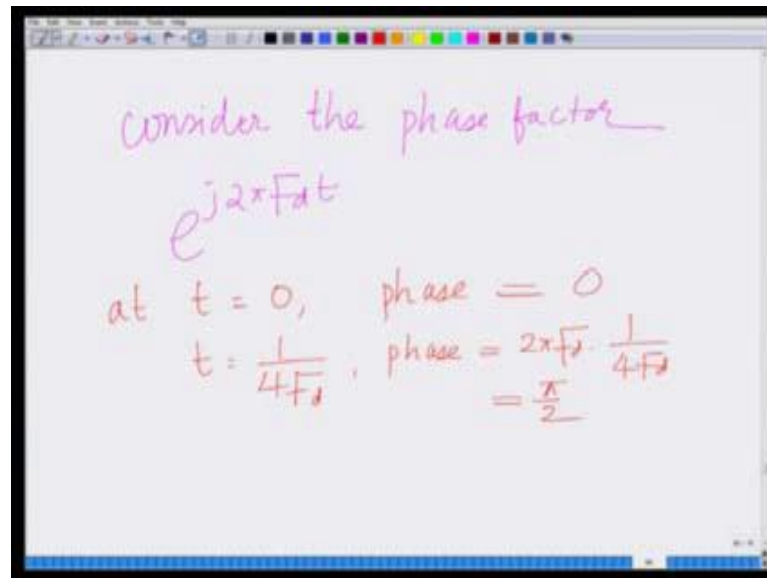
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Since, this coefficient  $h$  is varying with time, the wireless channel is known as a “Time Selective Channel”, therefore what is the reason for this time selective channel? That is because of the Doppler therefore, what is happening in this scenario because of mobility that is because of the mobile leisure because the user is moving, so this mobility is giving **rise** to Doppler. This Doppler is resulting in a time varying channel and therefore this time varying channel this is a time, this is a time selective channel.

So, the mobility results in the Doppler Effect, the Doppler Effect is resulting the time varying phase or the time varying channel and this is therefore a time selective wireless channel. So, this Doppler Effect is leading to the variation of the wireless channel, with respect to time. Now how fast or how slow is this channel changing with respect to time for that, let us look at of the phase factor, and consider the typical consider the phase factor  **$e^{-2\pi f_d t}$** .

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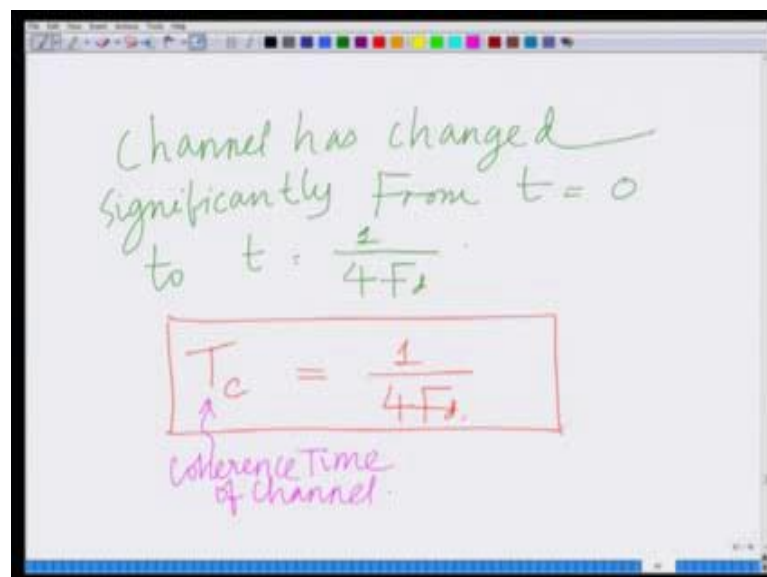


Now, at  $t = 0$ , the phase equals = 0.

So, at  $t = \frac{1}{4f_d}$ , the phase =  $2\pi f_d \frac{1}{4f_d} = \frac{\pi}{2}$

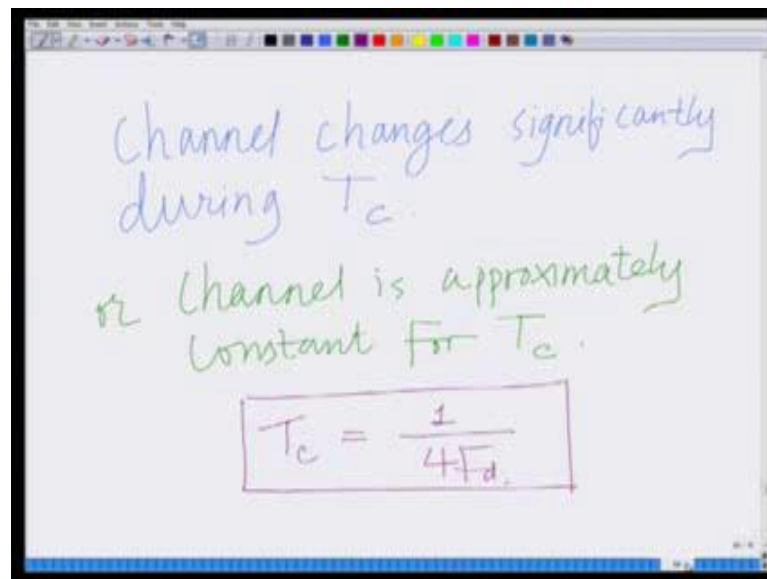
So, therefore the phase has changed significantly from  $t = 0$  to  $t = \frac{1}{4f_d}$  therefore, we can consider that this channel coefficient has change significantly from  $t = 0$  to  $t = \frac{1}{4f_d}$ .

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So, in this interval of duration between  $t = 0$  to  $t = \frac{1}{4f_d}$  or this interval of duration  $\frac{1}{4f_d}$ , the channel has change significantly this can be defined as the coherence time of the channel. So, coherence time  $T_c = \frac{1}{4f_d}$ , this is the definition of the coherence time; this is termed as the coherence time.

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So, the channel is changing significantly during the coherence time interval. So we can say channel changes significantly during the coherence time  $T_c$  or another way to say this is that a channel is approximately. So, we can say that, the channel is changing significantly during one coherence time interval, that is  $T_c = \frac{1}{4f_d}$  or the channel is approximately constant for 1 coherence time interval that is  $T_c = \frac{1}{4f_d}$ .

So, let us remember and this is an important principle to remember that the coherence time  $T_c = \frac{1}{4f_d}$ , this is the coherence time  $T_c$  and to understand this further let us do a simple example to understand the concept of coherence time.

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Example: Consider  $f_d = 143 \text{ Hz}$   
What is  $T_c$ ?

$$T_c = \frac{1}{4f_d}$$
$$= \frac{1}{4 \times 143}$$

Channel is approximately constant for 1.7ms

$$T_c = 1.7 \text{ ms}$$

or channel is changing after 1.7ms.

For instance, let us do a simple example for the coherence time consider as in the previous example of or Doppler shift equals, 143 Hertz, in the previous example what is the coherence time  $T_c$  for this scenario, we have

$$T_c = \frac{1}{4f_d}$$

$$= \frac{1}{4 \times 143}$$

$$= 1.7 \text{ ms}$$

So, coherence time is of the order of milliseconds, the coherence time for this scenario is roughly 1.7 milliseconds, which means that the channel is constant for 1.7 milliseconds approximately and the channel is changing after every duration or a after every slot of 1.7 milliseconds. So, there are the way to interpret this is channel is approximately constant 1.7 milliseconds or channel is changing after 1.7 milliseconds.

So, there are 2 ways to look at it, the coherence time is 1.7 milliseconds, for this scenario with the Doppler of 143 Hertz, which means that the channel is approximately constant for a duration of the coherence time that is 1.7 milliseconds or the channel is changing after the duration of 1.7 milliseconds, therefore the channel is a time vary channel.



So, this concludes our characterization of our wireless channel in terms of the coherence time.

Thank you very much.