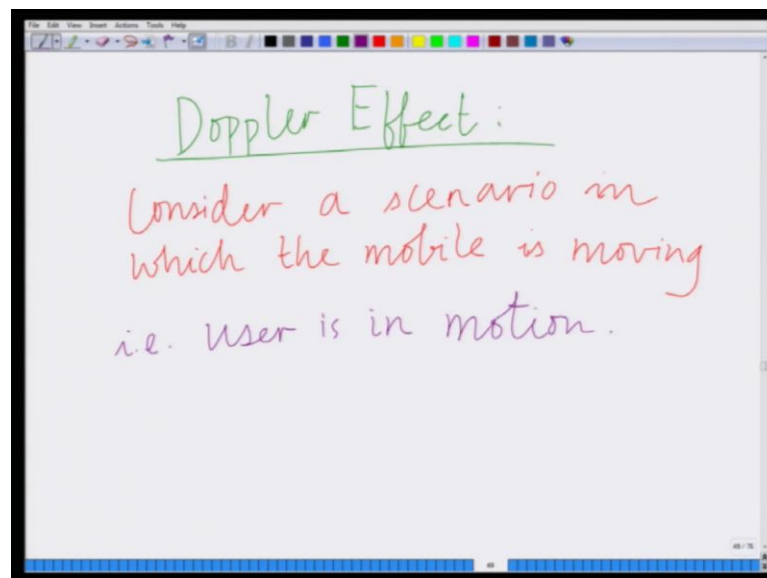


Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture – 24
Mobility and Doppler Effect in Wireless Channels

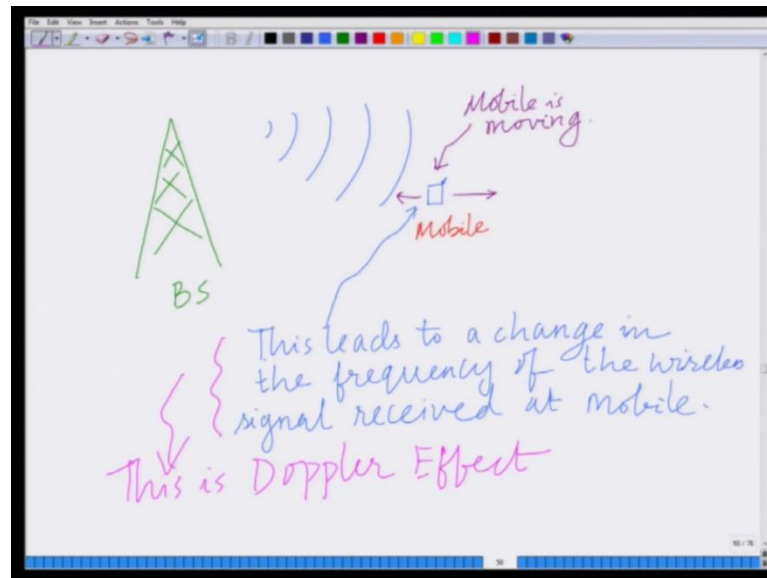
Hello, welcome to another module in this massive open online course on the Principles of CDMA, MIMO, OFDM Wireless Communication Systems. So, we are looking at a characterization of the wireless channel.

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And today, we are going to start talking about a new aspect that is the Doppler Effect. So, we are going to start focusing on the Doppler effect in a wireless channel, and now, for the Doppler effect basically we have to consider a scenario, in which the mobile is moving or the user is in motion, that is basically to say the wireless user is in motion there is a relative velocity is relative motion between the transmitter and the receiver or there is a relative motion between the base station and the mobile because the user is moving correct.

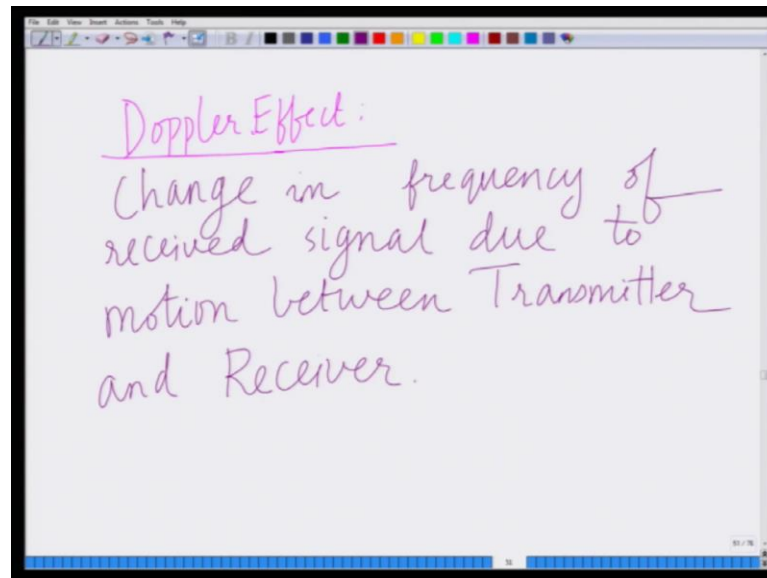
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And let me just schematically depict the scenario here, I have a base station which is transmitting its signal and I have my mobile, this is the mobile. So, the base station is transmitting the signal and the mobile is moving either towards, the base station or it can move away from the base station. So, mobile is in motion, so the mobile is moving and this motion leads to a change in the frequency of the signal received at the mobile this relative motion between the base station mobile leads to a change in the frequency of the signal received at the mobile.

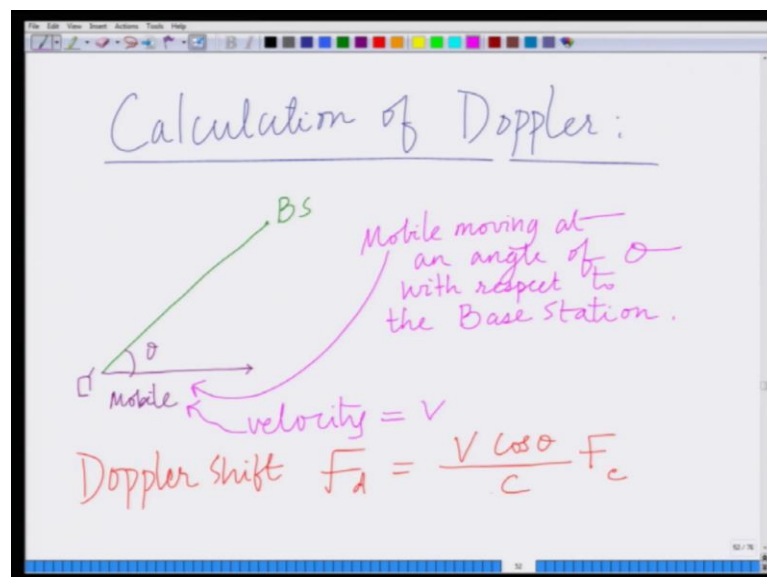
So, this leads to this motion change in the frequency of the wireless signal received at mobile. So, this leads to a change in the frequency of the signal received at the mobile and this phenomenon this is termed as this phenomenon is termed as Doppler effect that is a change in the frequency of the signal received, because of the motion of the mobile because, it is a relative motion between the base station mobile, this is termed as the Doppler effect.

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So, what is the Doppler Effect Doppler effect, which is also sometime simply termed as Doppler in the wireless channel, what is this? This is the change in frequency of received signal due to motion between the transmitters and there is a change in the frequency of the wireless signal due to motion or relative motion between the transmitter and the receiver.

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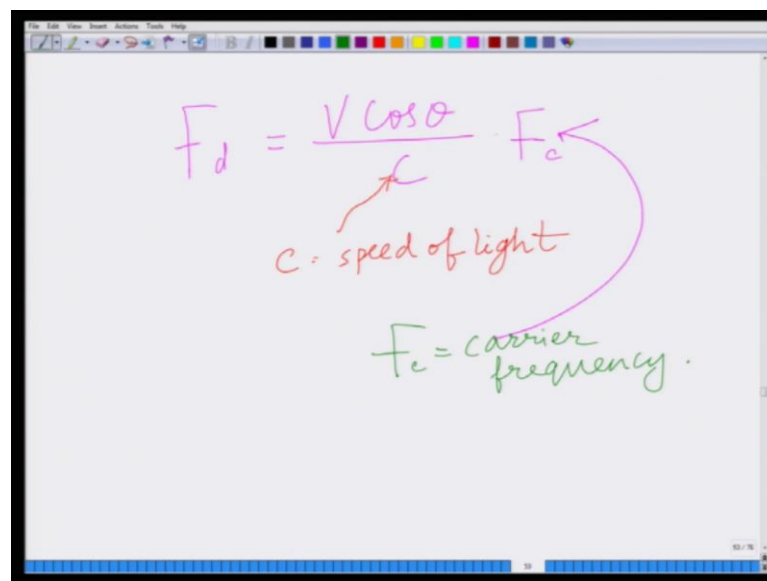
Now, how to calculate this Doppler Effect or the shift in the frequency or the change in the frequency that is arising because, of this Doppler effect for that we have the

following relation or the following principle; let us consider, we would like to calculate the Doppler shift and for that consider a scenario in which I have a base station this is let us say my base station and let us say I have a mobile which is moving at an angle of θ . This is my mobile which is moving at an angle of θ . So, I have the mobile moving at an angle of θ with respect to the base station.

Lets further say that, this mobile the velocity of this mobile equals V and then in this scenario the Doppler shift f_d is equal to

$$f_d = \frac{V \cos \theta}{c} f_c$$

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A screenshot of a whiteboard with a handwritten equation for the Doppler shift. The equation is $f_d = \frac{V \cos \theta}{c} f_c$. A red arrow points from the text 'c = speed of light' to the variable 'c' in the denominator. Another green arrow points from the text 'f_c = carrier frequency' to the variable 'f_c' in the numerator. The whiteboard has a standard toolbar at the top and a blue progress bar at the bottom.

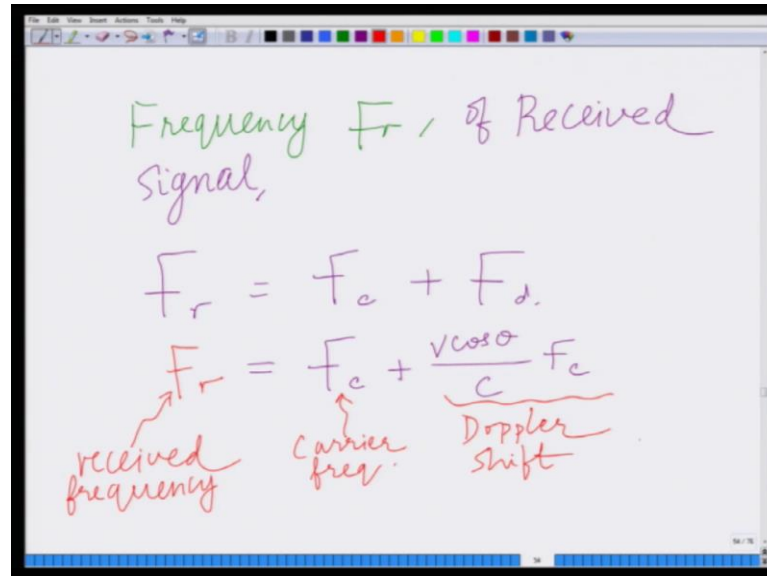
So, let me write this clearly over here, the Doppler shift equals

$$f_d = \frac{V \cos \theta}{c} f_c$$

V is the velocity, θ is the angle, c is the speed of light or basically, the speed of an electromagnetic wave and f_c is the carrier frequency of the signal alright. Where V is the velocity of the of the of the mobile θ is the angle of the mobile with respect to the base

station c is the velocity of the light or the velocity of the electromagnetic wave and f_c is the carrier frequency.

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Frequency F_r of Received Signal,

$$F_r = F_c + F_d$$
$$F_r = F_c + \frac{V \cos \theta}{c} F_c$$

received frequency carrier freq. Doppler Shift

And now you can also observe from this expression that if 0 and therefore, the received frequency f_r of the received signal this is given as,

$$f_r = f_c + f_d$$

$$f_r = f_c + \frac{V \cos \theta}{c} f_c$$

So, the received frequency f_r this is the received frequency of the received signal equals f_c this is the carrier frequency plus f_d this is your Doppler shift.

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If $0 \leq \theta \leq \pi/2$, then $\cos \theta \geq 0$
Therefore,
$$F_r = F_c + \frac{V \cos \theta}{c} F_c$$
$$\geq F_c$$

Received Freq $\geq$ Carrier Freq.
 $\Rightarrow$ Mobile is moving towards Base station

And now you can see from this expression that if

$$0 \leq \theta \leq \frac{\pi}{2}$$

$$\text{then, } \cos \theta \geq 0$$

$$\text{therefore, } f_r = f_c + \frac{V \cos \theta}{c} f_c$$

$$\geq f_c$$

since, cosine theta is positive this is greater than equal to f_c . Therefore, received frequency is what does this mean this means that, received frequency is greater than or equal to the carrier frequency and when does this occurs, this occurs when $0 \leq \theta \leq \frac{\pi}{2}$;

that means, the mobile is moving towards the base station. And this implies, occurs when

$$0 \leq \theta \leq \frac{\pi}{2}$$

this implies the mobile is moving towards. So, if the mobile is moving towards the base station then, $0 \leq \theta \leq \frac{\pi}{2}$, θ is positive therefore, the receive frequency f_r is greater than the carrier frequency f_c .

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If $0 \leq \theta \leq \frac{\pi}{2}$, then $\cos \theta < 0$

$$f_r = f_c + \frac{V \cos \theta}{c} f_c$$

$< f_c$

$\Rightarrow$ Received Freq. $<$ Carrier Freq.

$\Rightarrow$ Mobile is moving away from Base station.

On the other hand,

if $\frac{\pi}{2} < \theta \leq \pi$, then $\cos \theta < 0$

therefore, $f_r = f_c + \frac{V \cos \theta}{c} f_c$

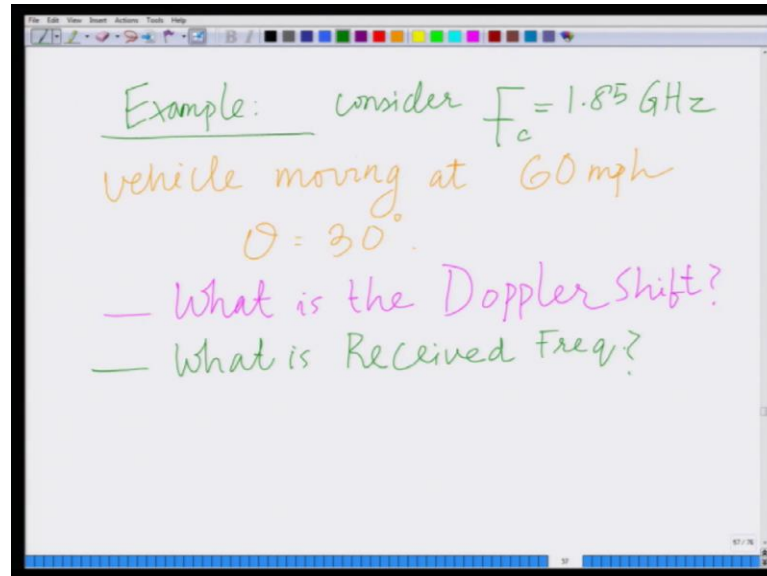
$< f_c$

which implies that basically my received frequency is less than the carrier frequency and this occurs when $\frac{\pi}{2} < \theta \leq \pi$, which implies basically, occurs in the mobile is moving away from the base station.

Therefore, we have seen 2 cases one is when $0 \leq \theta \leq \frac{\pi}{2}$ that is when mobile is moving towards the base station and then we have the received frequency Doppler shift is positive therefore, the received frequency is greater than the carrier frequency on the

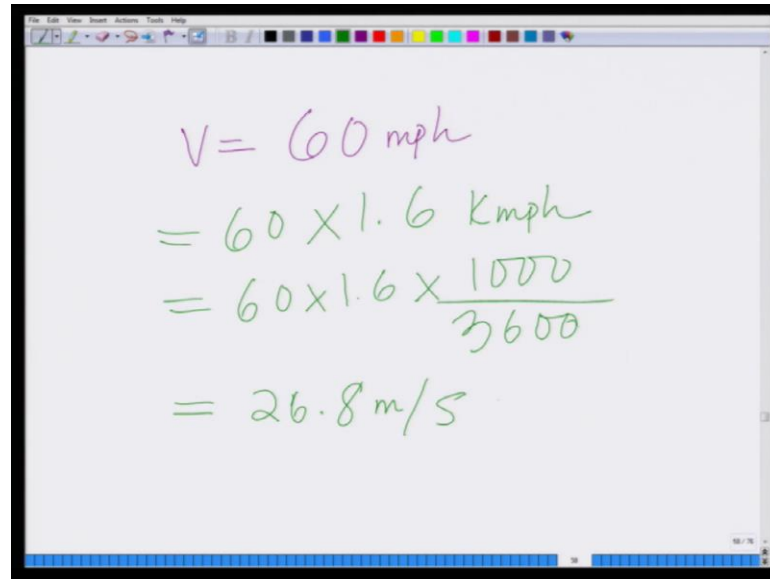
other hand, when $\frac{\pi}{2} \leq \theta \leq \pi$ therefore, $f_r < f_c$ the carrier frequency and this occurs when the mobile is moving away from the base station alright.

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And now let us look at a simple example to understand the Doppler Effect better let us consider simple example. So, let us consider $f_c = 1.85$ gigahertz and we have vehicle moving at 60 miles per hour at $\theta = 30^\circ$ that is, the angle of the mobile with respect to the base station, we would like to ask, what is the Doppler shift and what is the received frequency.

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A digital whiteboard interface with a toolbar at the top. The whiteboard contains the following handwritten text in green ink:

$$\begin{aligned} V &= 60 \text{ mph} \\ &= 60 \times 1.6 \text{ kmph} \\ &= 60 \times 1.6 \times \frac{1000}{3600} \\ &= 26.8 \text{ m/s} \end{aligned}$$

For this purpose, now look at the following I have the velocity equals 60 miles per hour which is equal to

$$V = 60 \text{ mph}$$

$$= 60 \times 1.6 \text{ kmph}$$

$$= 60 \times 1.6 \times \frac{1000}{3600}$$

$$= 26.8 \text{ m/s}$$

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Handwritten calculation of Doppler shift frequency f_d on a digital whiteboard. The formula $f_d = \frac{V \cos \theta}{c} \times f_c$ is written in green. Below it, the values are substituted: $f_d = \frac{26.8 \times \cos 30^\circ}{3 \times 10^8} \times 1.85 \times 10^9$. This is then simplified to $f_d = \frac{26.8 \times \sqrt{3}/2}{3 \times 10^8} \times 1.85 \times 10^9$. The final result, $f_d = 143 \text{ Hz}$, is boxed in green.

Now, let us look at the Doppler shift equals

$$f_d = \frac{V \cos \theta}{c} f_c = \frac{26.8 \times \cos 30^\circ}{3 \times 10^8} \times 1.85 \times 10^9$$

And this is equal to basically, this is approximately equal to 143 hertz. So for this scenario the Doppler shift is given as 143 hertz that is for a vehicle moving at 60 miles per hour at triangle $\theta = 30^\circ$ and carrier frequency 1.85 gigahertz.

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Handwritten calculation of Received Frequency f_r on a digital whiteboard. The formula $f_r = f_c + f_d$ is written in green. Below it, the values are substituted: $f_r = 1.85 \times 10^9 + 143$. The text "Received Freq." is written in blue next to f_r .

We have calculated the Doppler shift as 143 hertz and the received frequency that is, f_r
 $= f_c + f_d$ which is equal to

$$f_r = 1.85 \text{ GHz} + 143 \text{ Hz}$$

where f_r is the received frequency. So, this what we have seen is basically this module we have defined the Doppler shift we have illustrated how to compute the Doppler shift for a given velocity and angle θ at a given Doppler frequency and also we have seen a simple example of the Doppler shift.

So, we will end this module here, and look at other aspect in the subsequent modules.

Thank you very much.